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BEAM-BLOWUP STUDY FOR A WEAK-STRONG CASE*

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The formula suggested in[1] presumably describes the weak beam blowup when it interacts with a fixed counter-rotating strong beam in an electron storage ring. This gives us an opportunity to make a comparison of experimental results obtained on two SLAC storage rings PEP and SPEAR with the theoretical calculations as well as to study the dependence of the phenomenon on different machine parameters.

In the present paper we present such a comparison with reasonably good agreement between the experiment and the theory. The important conclusion from our study is that any valid theory of the beam-beam phenomenon should take into account the asymmetries of the machine parameters arising in any storage ring from all kinds of machine imperfections.

Theoretical Description The theoretical formula is obtained[2] by solving the Fokker-Planck equation with the help of a perturbation method. In our case the beam-beam force $F(x,y)$ plays the role of the perturbation. For head-on collisions, considered here, this force contains the linear part:

$$F_{x\ lin} \sim x \quad (1)$$

$$F_{y\ lin} \sim y \quad (2)$$

For particles of the weak beam, which happen to be in the vicinity of the strong bunch center, the linear part of the force changes the tunes ν, τ and the β functions β_x, β_y by maximal values consistent with the magnitude of the force. For particles in the tails of distribution the force has the reverse slope. For such a particle the unperturbed machine parameters probably better represent the particle motion. The distribution of the particles on the tune shift axis for a flat beam is found in Ref. 3. The maximum of the distribution appears to be at $\simeq 0.6 \div 0.7$ of the maximum incoherent tune shift That suggests that the correct solution should be somewhere between the two following solutions

Approach A. Consider whole beam-beam force as a perturbation. Then the beam blowup, i.e., ratio of the vertical rms size Σ_y of the weak beam perturbed by the interaction to the unperturbed value σ_y of the same parameter, is described by the formula

$$\frac{\Sigma_y}{\sigma_y} = \sqrt{\frac{E_y}{\epsilon_y}} \quad (3)$$

where the ratio E_y/ϵ_y is found in Ref. 2. It is expressed in terms of the unperturbed machine parameters (tunes, β functions, space charge parameters ξ).

Approach B. Let us define

$$F = F_{lin} + \Delta F \quad (4)$$

$$\Delta F = F - F_{lin} \quad (5)$$

Consider now only ΔF as perturbation including at the same time F_{lin} into the machine lattice. In this case

$$\frac{\Sigma_y}{\sigma_y} = \sqrt{\frac{E_y\beta_y}{\epsilon_y\beta_{y0}}} \quad (6)$$

where β_y, as well as β_x, ν, τ are the values of the β functions and tunes perturbed by the linear part of the beam-beam force (dynamic β and tune). Formula for E_y/ϵ_y in this case can be found in Ref. 1.

The essential difference of this case from case A is that case B takes into account the dynamic change of the tunes and the amplitude functions of the machine.

Results for the beam blowup to the first order in ξ (or the strong beam current), are the same in both cases, of course. But they are quite different in the second order in ξ.

Experimental Method and Results

PEP Vertical beam height measurements in PEP were obtained by means of an X-ray monitor.[4] In this device synchrotron radiation was detected by a fluorescent screen viewed by a vertically scanning TV camera. All the video scans in a single frame are averaged to give an analogue signal representing the vertical beam profile. This profile signal was processed to obtain a signal proportional to the full width at half maximum, which was then digitized and displayed at the control console. Absolute calibration was not available. Relative measurement errors were estimated subjectively at about 20%.

Operating conditions for both PEP and SPEAR measurements are listed in Table 1. Data for PEP on beam height versus current are given in Table 2 and compared to theory in Figs. 1, 2 and 3. Curves 1, 2, and 3 on Fig. 1 present the results of calculations using the case B formula assuming the coupling factor (ϵ_y/ϵ_x) before collision to be 0.015, 0.020, and 0.025, respectively. Curves 4, 5 and 6 present the results of calculations for the same machine parameters but using the case A formula.

In both cases the asymmetries in the amplitude functions at IP $\Delta\beta_x$, $\Delta\beta_y$ and in the tunes per superperiod $\Delta\nu$, $\Delta\tau$ are defined by random numbers uniformly distributed in the interval $\pm a$, where a is 0.1 for $\Delta\beta_x/\beta_x$, $\Delta\beta_y/\beta_y$ and a is 0.01 for $\Delta\nu$, $\Delta\tau$.

Table 1. Nominal Machine Parameters

Parameter	PEP	SPEAR
Particle energy (GeV)	14.5	1.885
Strong beam current (mA)	30.0	8.0
Horizontal β function at IP (m)	3.0	1.2
Vertical β function at IP (m)	0.11	0.10
Coupling factor (ϵ_y/ϵ_x)	0.02	0.01
Number of interaction points (IP)	6	2
Horizontal tune per superperiod ν	3.545	2.640
Vertical tune per superperiod τ	3.032	2.590

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Table 2. Experimental Data

SPEAR				PEP			
No.	I (mA)		Σ_y/σ_y	No.	I (mA)		Σ_y/σ_y
	e^-	e^+			e^-	e^+	
1	2.0	2.0	1.25 ± 0.15	1	9.70	3.00	1.057 ± 0.2
2	3.0	2.0	1.45 ± 0.20	2	14.74	2.95	1.18 ± 0.2
3	4.0	1.8	1.82 ± 0.25	3	19.55	2.88	1.30 ± 0.2
4	6.0	1.8	1.91 ± 0.31	4	22.48	2.82	1.50 ± 0.3
5	7.0	1.7	1.91 ± 0.29	5	25.19	2.70	1.95 ± 0.4
6	8.0	1.7	1.78 ± 0.23	6	30.53	2.59	[illegible]03 ± 0.4
7	9.3	1.0	3.72 ± 1.52	7	31.72	2.44	2.39 ± 0.5
8	9.6	2.1	4.73 ± 0.76				

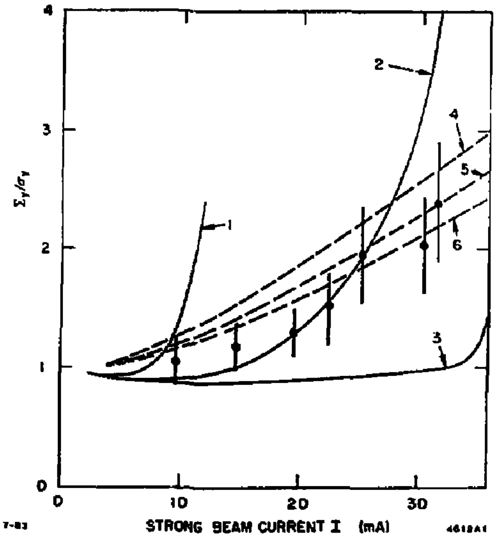

Fig. 1. Theoretical (curves) and recent experimental (points) results for the weak beam blowup versus the strong beam current for PEP (see text).

In the small current region the solid curves are below the line $\Sigma_y/\sigma_y = 1.0$, while the dashed curves are above it. This occurs due to the focusing produced by the strong current – the effect taken into account in case B by using perturbed (dynamic) β functions.

The results of calculations in case A (dashed curves) seem to be less sensitive to the machine asymmetries. This is the consequence of the assumed independence of the (unperturbed) tunes on the strong beam current in case A.

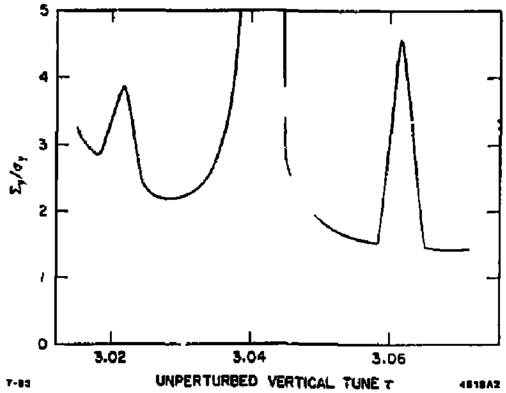

Fig. 2. Beam blowup versus unperturbed vertical tune (per one superperiod). The calculations are done using the case A formula for strong beam current 30 mA. The same machine parameters are assumed as for the curve 2 on Fig. 1.

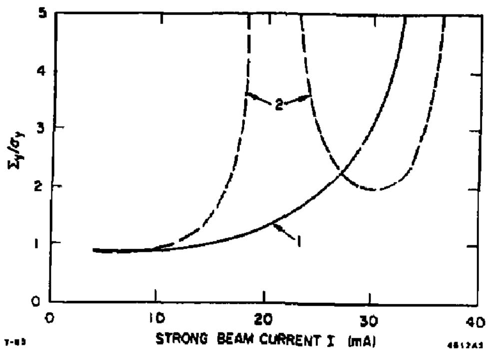

Fig. 3. Strong dependence on the assumed asymmetry values is illustrated in this figure. Curve 1 is the same as curve 2 on Fig. 1. Curve 2 is calculated with $\Delta\nu$ and $\Delta\tau$ twice as big (the rest of the parameters are the same).

SPEAR The beam profile measurements in SPEAR employed optically imaged synchrotron radiation which was scanned across a photodiode by a rotating mirror. The profile signals were recorded by an x-y plotter.

Data[5] are given in Table 2 and compared to theory in Figs. 4 and 5. Curves 1, 2, and 3 on Fig. 4 present the results of calculations using the case B formula for coupling factors 0.0025, 0.010, and 0.020 respectively. The asymmetries in the machine functions are assumed to be in the interval ± 0.05 for $\Delta\beta_x/\beta_x$, $\Delta\beta_y/\beta_y$, and ± 0.02 for $\Delta\nu$, Δr.

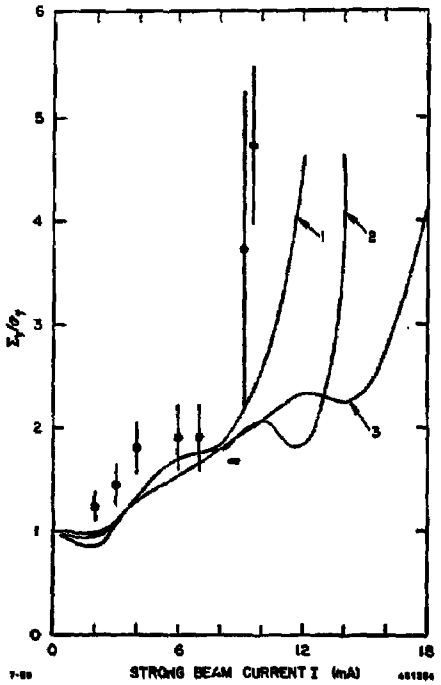

Fig. 4. Theoretical (curves) and experimental (points) results for the weak beam blowup versus the strong beam current for SPEAR (see text).

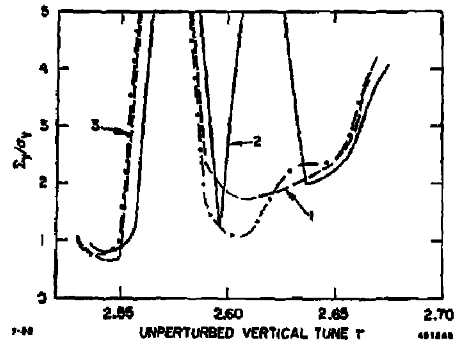

Fig. 5. Beam blowup versus unperturbed vertical tune (per one superperiod). The calculations are done using the case B formula for strong beam current 9 mA. The same machine parameters are assumed as for the curve 1 on Fig. 4 but the unperturbed horizontal tune. Curves 1, 2, and 3 correspond to values ν = 2.82, 2.84 (nominal value for SPEAR), and 2.88. The nominal vertical tune for SPEAR is 2.59.

Acknowledgements

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