BNL- 43103
Informal Report

ION RELATED PROBLEMS FOR THE XLS RING

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July 11, 1989

NATIONAL SYNCHROTRON LIGHT SOURCE

BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC.

Under Contract No DE-AC02-76CH00016 with the

UNITED STATES DEPARTMENT OF ENERGY

BNL--43103

DE90 000532

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INTRODUCTION

The electron beam in the XLS will collide with the residual gas in the vacuum chamber. The positive ions will be trapped in the potential well of the electron beam [1,2]. They will perform stable or unstable oscillations around the beam under the repetitive Coulomb force of the bunches [1,3]. If not cleared, the captured ions will lead to partial or total neutralization of the beam, causing both, a decrease of life-time and a change in the vertical tunes as well as an increase in the tune-spread. They can also cause coherent transverse instabilities [4].

The degree of neutralization η that one can tolerate, is primarily determined by the allowable tune shift, which for the XLS is between 1 and 5 10^{-3}. Electrostatic clearing electrodes will be used to keep the neutralization below the desired limit. In order to determine their location and the necessary clearing-rate and voltage, we examine the (i) ion production rate, (ii) longitudinal velocity of ions in field-free regions and in the dipoles to see what distance the ions can travel without clearing before the neutralization of the beam reaches the prescribed limit, (iii) beam potential to see the locations of the potential wells, (iv) voltage requirements for ion clearing, (v) critical mass for ion capture in the bunched beam, (vi) tune shift caused by neutralization of the beam, (vii) pressure rise due to trapped ions and (vii) power dissipation due to beam image current.

ION PRODUCTION and BEAM NEUTRALIZATION

For relativistic electrons ($\beta \approx 1$) the rate of production of ions is [1,5]:

$$R_p = \frac{N}{\tau} = N \sum_i \frac{1}{\tau_i} \qquad \text{and} \qquad R_i = \frac{1}{\tau_i} = d_i \sigma_i c = \sigma_i c \frac{P_i}{R_g T}$$

where N = total number of electons in the beam, σ_i = ionization cross section in m^2, d_i = molecular density in m^{-3}, P_i = partial pressure in Torr, T = temperature in ^oK, c = speed of light, R_g = ideal gas constant (1.0356 10^{-25}) and the summation is for all species of ions present. σ_i can be calculated as

$$\sigma_i = 4\pi \left[\frac{h}{mc}\right]^2 [c_1(\beta^2 \ln\frac{\beta^2}{1-\beta^2} - 1) + c_2\beta^2]$$

$$\beta = v/c, \qquad \gamma = \sqrt{1-\beta^2} \qquad \text{and} \qquad 4\pi \left[\frac{h}{mc}\right]^2 = 1.874\ 10^{-24}\ m^2.$$

The neutralization factor is defined as the ratio of the number of ions to the number of electrons in the ring:

$$\eta = \frac{d_{ions}}{d_e}$$

The clearing rate can be calculated from the equilibrium condition

$$R_p d_e = R_{clear} d_{ion} \quad as \quad R_{clear} = \frac{R_p}{\eta}$$

The time to reach η neutralization is $t = \eta\tau$ sec. With thermal ion velocities [1] of

$$V_{th} = \sqrt{\frac{3kT}{m_p}}\frac{1}{\sqrt{A}} \qquad [m/sec],$$

the ions (of mass Am_p) have to be removed after travelling a distance:

$$l = V_{th} t \qquad [m]$$

Above, k = 1.3807 e^{-23} joul/^{o}K = Boltzman constant.

The calculations were carried out for different ionic species under Phase-1 and 2 conditions (γ = 391 and 1362, respectively). Table-1 contains the parameters used in the calculation; the $c_{1,2}$ constants as given in [1], the P_m partial pressures from [6] and the d_m molecular density at T = 300^{o}K. Table-2 show the calculated ionization cross sections, ionization times and rates.

Table-1
Atomic number, partial pressure, ion density at T=300 ^{o}K
and the $c_{1,2}$ coefficients for calculating σ_i

molecule	A	c_1	c_2	P_i 10^{-9} Torr	d_i 10^{13} m^{-3}
H_2	2	0.50	8.1	1.3	4.18
CO	28	3.70	35.1	0.7	2.25
CO_2	44	5.75	55.9	0.1	0.32

Table-2
Ionization cross section, ionization time and rate for different ions at T=300 ^{o}K with $\gamma = 391$ and 1362.

molecule	σ_i $10^{-22}\ m^2$	τ_i sec	R_i ion/sec	σ_i $10^{-22}\ m^2$	τ_i sec	R_i ion/sec
	Phase-1	($\gamma = 391$)		Phase-2	($\gamma = 1362$)	
H_2	0.254	3.13	0.32	0.278	2.87	0.35
CO	1.41	1.04	0.96	1.59	0.93	1.07
CO_2	2.23	4.65	0.21	2.50	4.15	0.24
total		0.67			0.61	

With I = 0.5 Amp current in the ring, in order to keep η under 10^{-3}, one needs a clearing rate of

$$R_{clear} \geq \begin{cases} 1.32\ 10^{14} \\ 1.47\ 10^{14} \end{cases} [ion/sec] \quad \text{for} \quad \begin{matrix} Phase-1 \\ Phase-2 \end{matrix}$$

The V_{th} thermal ion velocities and the l_{th} distances are given in Table-3 for different ionic species under Phase-1 and 2 conditions and assuming I = 0.5 Amp current, T = 300 ^{o}K temperature and $\eta = 10^{-3}$ neutralization.

One can see, that the heavier ions can travel only ≈0.25 m without clearing before the neutralization will reach 10^{-3}. The higher neutralization we can tolerate; the longer this "clear-free-path" will be; for example, $l_{CO_2} \geq 1.25$ m for η=5 10^{-3}.

Also, since the revolution time of the electrons is 2.83 10^{-8} sec, even with only one bunch in the ring the fastest ion will travel only 0.05 mm between bunches and can not "get out" of the way of the bunches. That is, all types of ions are safely trapped (see also later).

In the bending magnets, the trapped ions perform a cycloidal motion [1,3,7] due to the combined action of the longitudinal electric field of the beam and the vertical magnetic field of the dipole. The resulting longitudinal velocity, the "drift"-velocity is

$$V_D = \frac{\vec{E}_x \ x \ \vec{B}_z}{B_z^2} \frac{\omega_L^2}{\omega_L^2 + \omega_x^2} = \frac{E_x}{B_z} \frac{\omega_L^2}{\omega_L^2 + \omega_x^2} \tag{1}$$

where

$$\omega_L = \frac{eB_z}{Am_p} = k_L \frac{B_z}{A} \tag{2}$$

is the Larmour frequency and ω_x^2 is the cyclotron frequency. It will be shown later (see eqs. (6) and (7) under "Critical mass"), that

$$E_x = \frac{\omega_x^2 x}{e/(Am_p)} \quad (3a) \quad \text{and} \quad \omega_x^2 = \frac{eI}{2\pi\varepsilon_o c}\frac{1}{Am_p}B\frac{1}{\sigma_x(\sigma_x+\sigma_y)} \qquad (3b)$$

Substituting eqs. (2-3) into eq. (1), one can see the dependence of V_D on the beam current I, atomic number A, bunching factor B and the beam size (σ_x and $\Sigma = \sigma_x(\sigma_x+\sigma_y)$):

$$V_D = \frac{k_L B_z}{A + \frac{k_L^2}{k_x}\frac{B_z^2 \Sigma}{BI}}\sigma_x$$

The B bunching factor is defined as the ratio of the ring circumference to the total bunch-length:

$$B = \frac{C}{n_B l_B}$$

The drift velocities and the corresponding l_D distances are given in Table-3 for different molecules under Phase-1 and 2 conditions (B_z=1.1 and 3.85 Tesla) and assuming I = 0.5 Amp current and $\eta = 10^{-3}$ neutralization.

Table-3

Thermal and drift velocities and the corresponding travel-distances for different molecules under Phase-1 and 2 conditions (γ=391 and 1362, B_z=1.1 and 3.85 Tesla) assuming I = 0.5 Amp current, T = 300 ^{o}K temperature and $\eta = 10^{-3}$ neutralization.

molecules	V_{th} 10^3 m/sec	V_D 10^3 m/sec	l_{th} m	l_D m	V_D 10^3 m/sec	l_{th} m	l_D m
		Phase-1			Phase-2		
H_2	1.93	28.9	1.29	19.4	30.2	1.16	18.1
CO	0.52	02.2	0.35	1.5	06.6	0.31	4.0
CO_2	0.41	01.4	0.37	0.9	04.5	0.25	2.7

BEAM POTENTIAL

The electric field due to ρ space charge density in an elliptic beam with horizontal and vertical dimensions of

$$2a = 2\sqrt{2}\sigma_x = 2\sqrt{\varepsilon_x\beta_x + (\sigma_E\,\eta_x)^2} \qquad \text{and} \qquad 2b = 2\sqrt{2}\sigma_y = 2\sqrt{\varepsilon_y\,\beta_y}$$

was calculated [3,8] as:

$$E_{x,y} = \frac{\rho}{\varepsilon_o}\frac{1}{1+\sigma_{x,y}/\sigma_{y,x}}[x,y] \qquad (4)$$

For a round and unbunched beam ($a=b=r_o$, $\rho=\frac{\lambda}{\pi r_o^2}$), this yields:

$$E_r = \frac{\lambda}{2\pi\varepsilon_o} \begin{cases} r/r_o^2 & \text{if } r \le r_o \\ 1/r & \text{if } r \ge r_o \end{cases}$$

where λ is the line charge density. The corresponding $V = -\int E\,ds$ potential in a round vacuum chamber of radius R_o with the boundary condition $V = 0$ at $r = R_o$ is:

$$V = \frac{\lambda}{2\pi\varepsilon_o} \begin{cases} \frac{r^2}{2r_o^2} - \frac{1}{2} - \ln\frac{R_o}{r_o} & \text{if } r \le r_o \\ \ln\frac{r}{r_o} & \text{if } r \ge r_o \end{cases} \tag{5}$$

The more general expression for an elliptic beam displaced by Δx in a round vacuum chamber, V is given in [9].

The potential of an elliptic beam in in an elliptic (rectangular) vacuum chamber of 2w and 2h width and height, was calculated in [10]. For an (x,y) point inside or on the surface of the beam:

$$V(x,y) = \sum_{n=1}^{\infty} \left[1 - \frac{ch\ \zeta_n(h-b)}{ch\ \zeta_n h}\ ch\ \zeta_n y\right] C_n \sin \zeta_n x$$

$$\zeta_n = \frac{\pi n}{2w}, \qquad C_n = \frac{U_o}{4bw}\frac{g_n}{\zeta_n^2}, \qquad U_o = \frac{\lambda}{\varepsilon_o} = \frac{I}{\beta c\,\varepsilon_o}$$

$$g_n = \frac{4a}{\lambda}\int_0^{2w} \rho(x) \sin \eta_n x\ dx = \frac{\cos[\zeta_n(w-a+\Delta x)] - \cos[\zeta_n(w+a+\Delta x)]}{\zeta_n a G_n}$$

$$G_n = \begin{cases} 1 & \text{for } uniform\ \rho(x)\ charge\ distribution \\ 1 - \zeta_n \frac{a^2}{\pi^2} & \text{for } cosine-square\ charge\ distribution \end{cases}$$

For Gaussian charge distribution the potential is calculated in [11].

In case of uniform, unbunched beam, the ions are trapped if their transverse energy is less than E = eV [joule]. In a field-free section of the ring the ions will drift towards the deepest potential well resulting from changing vacuum chamber or beam size.

Figs 1 and 2 show the (w,h) half width and height of the vacuum chamber, the $\sigma_{x,y}$ half beam size and the calculated beam potential on the beam surface for 1/4 of the XLS ring. The calculations were performed assuming I=0.5 Amp beam current. U_o, $U_{\uparrow}$, $U_{\downarrow}$, $U_{\rightarrow}$ and $U_{\leftarrow}$ denotes the potential in the middle of the beam and at the upper, lower, right and left surface of the beam. In case of Fig.2, the vacuum chamber dimensions are modified by the presence of striplines in the straight sections and clearing electrodes in the dipole.

One can see that in the unobstructed chamber, the deepest potential well are before entering the dipoles. There are also shallow wells at the middle of the dipoles and at the middle of the straight sections. The presence of the striplines introduces a potential well of depth equal to that of the one before the dipole, but it is less bothersome since the striplines can also be used as clearing electrodes if necessary.

ION CLEARING

The electric field E of the beam increases linearly with the distance from the beam axis inside the beam and it is the highest on the beam surface. Outside the beam, the field decreases with increasing distance. The minimum E value for clearing of the ions is calculated from eq. (4) for uniform unbunched beam of intensity I Amp:

$$E_{\rm min} = E_x \ (x=\sqrt{2}\sigma_x) = E_y \ (y=\sqrt{2}\sigma_y) = \frac{I}{\sqrt{2}\pi c\,\varepsilon_0 (\sigma_x+\sigma_y)}.$$

The required voltage on a clearing electrode which is separated from the beam by d distance is

$$V \ [Volt] = E_{\rm min} \ [Volt/m] \ d[m]$$

Assuming I=0.5 Amp, $E_{\rm min}$ varies from ≈ 20 V/mm at the middle of the straight to ≈ 35 V/mm at the middle of the dipole. Taking into account the different chamber size (see Figs.1 and 2), the necessary clearing voltage is V ≥ 800 Volt. Bunched beams will tend to increase this value.

CRITICAL MASS

The following is a simplified model of ion trapping by a bunched electron beam [1,3]. The longitudinal velocity of ions is negligible compared with that of the electron beam. The ions are "kicked" by the electron bunches and they drift freely between the kicks. The change in velocity of the ions is proportional to the number of electrons in the bunch and inversely proportional with its mass. At sufficiently high beam current or small ion mass the motion of the ion will be unstable (kicked away far enough from the beam potential, that it becomes "untrapped").

This effect of kick and drift can be described in the usual matrix formalism. The

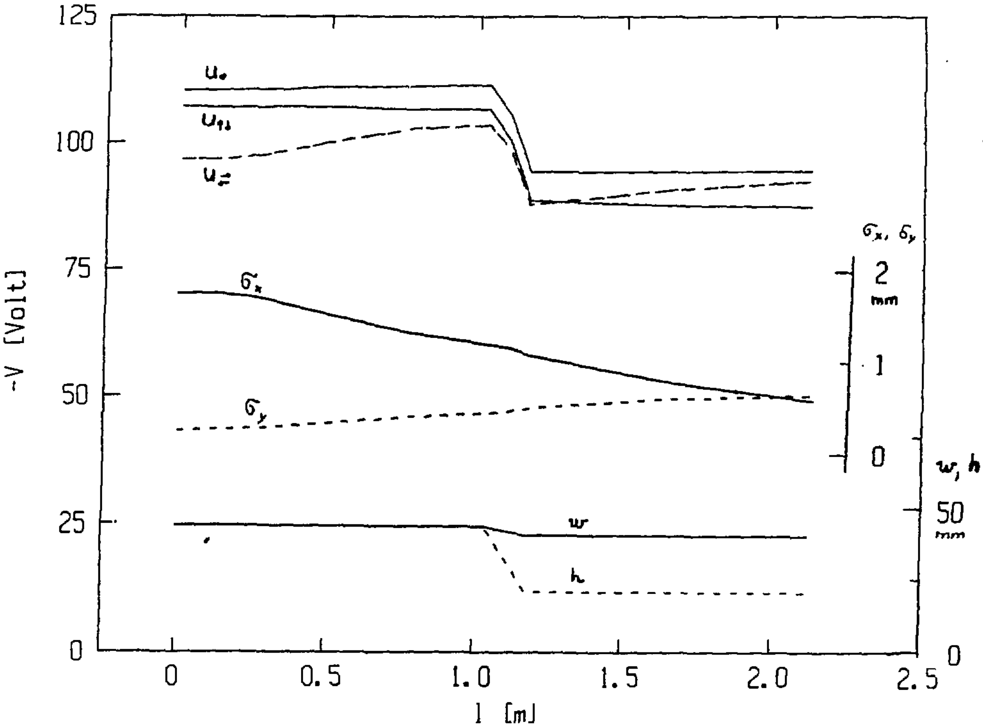

125
100
75
50
25
0
-V [Volt]
0
0.5
1.0
1.5
2.0
2.5
l [m]
2
1
0
50
mm
0
w, h

Fig. 1

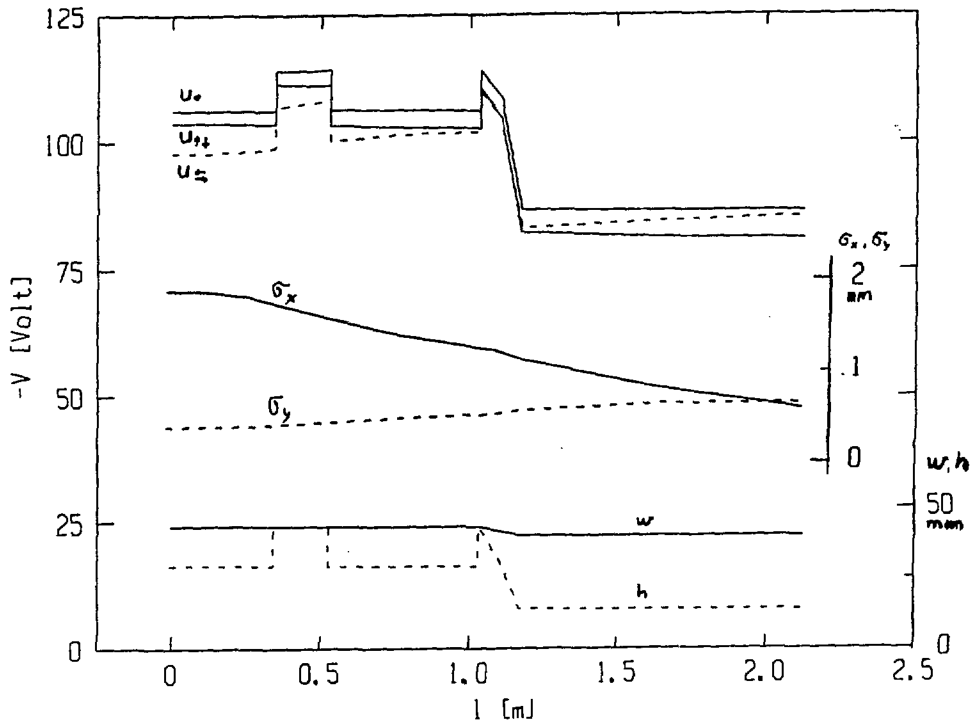

125
100
75
50
25
0
-V [Volt]
0
0.5
1.0
1.5
2.0
2.5
l [m]
2
1
0
50
mm
0
w, h
w
h

Fig. 2

equivalent quadrupole strength of the bunches on the ions is

$$\alpha_x = \frac{e}{Am_p}\frac{\partial E_x}{\partial x}\tau = \omega_x^2\tau \quad \text{where} \quad \tau = \frac{l_B}{c} \tag{6}$$

Using eq. (4) with $\rho = \frac{N}{n_B}\frac{1}{c}\frac{1}{2\sigma_x(\sigma_x+\sigma_y)}$ corresponding to the charge density from one bunch, one obtains:

$$\alpha_{x,y} = \frac{2cr_p}{A}\frac{N}{n_B}\frac{1}{\sigma_{x,y}(\sigma_x+\sigma_y)} \tag{7}$$

From the stability condition imposed on the Trace of the matrix (since $\sigma_y < \sigma_x$, α_y determines the stability condition), one can obtain the critical ion mass, above which the ion motion is stable:

$$A_c = \frac{C^2 r_p I}{2ecn_B^2}\frac{1}{\sigma_y\ (\sigma_x+\sigma_y)}$$

where r_p is the classical proton radius and I is the beam current. With I=0.5 Amp in 6, 3 or 1 bunches, the critical mass was calculated to be $A_c \le .026$, 0.11 and 0.95, respectively. That is, all ions are trapped under usual conditions in the XLS ring. The critical current, under which an ion of atomic number A is trapped was calculated to be $I_c \ge 1.1$ and 14.7 Amp for A=2 and A=28 ions, respectively (the above values correspond to 1 bunch of length $a=\sqrt{2}\sigma_x$ in the ring).

TUNE SHIFT

The tune shift, caused by the ions is:

$$\Delta\nu_{x,y} = \frac{1}{4\pi}\int\beta_{x,y}(s)k_{x,y}(s)ds$$

$$where \quad k_{x,y} = \frac{e}{\gamma m_e c^2}\frac{\partial E}{\partial[x,y]}$$

and E is calculated from eq. (4) using the

$$\rho = \rho_{ion} = \frac{eN\eta}{C}\frac{1}{2\pi\sigma_x\sigma_y}$$

ion charge density. Averaging instead of integration and substituting $<\beta> = R/\nu_{x,y}$, one gets:

$$\Delta\nu_{x,y} = r_e\frac{1}{\gamma}\frac{R}{\nu_{x,y}}\frac{\eta N}{2\pi\sigma_{x,y}(\sigma_x+\sigma_y)}$$

Assuming I=0.5 Amp and $\eta=10^{-3}$, the upper limit for tune change is

$$\Delta \nu_x \le \begin{cases} .0005 \\ .0002 \end{cases}, \qquad \Delta \nu_y \le \begin{cases} .0022 \\ .0007 \end{cases} \qquad \text{for } \begin{matrix} Phase\text{-}1 \\ Phase\text{-}2 \end{matrix}$$

PRESSURE RISE due to IONS

Trapped ions lower beam lifetime in the same way as residual mulecules, ie. by Bremsstrahlung and Coulomb scattering. It is therefore important to determine the pressure or density increase due to trapped ions, which is given by [1]:

$$\Delta P_i = \frac{\lambda}{e} \frac{1}{\pi r_o^2} \frac{1}{3.3\ 10^{22}} \eta$$

For I=0.5 Amp, r_o=1 mm and $\eta = 5\ 10^{-3}$, one obtains $\Delta P_i = 5\ 10^{-10}$ Torr.

This represents more then 10 % of the total pressure and would lower the beam lifetime by the same amount. However, since heavier ions stay longer in the beam, the effect on the lifetime would be worse.

In addition to the pressure bump ΔP_i, the negative voltage applied to the clearing electrodes will accelerate electrons, produced both by the beam and by the photons, away from the clearing electrodes and towards the wall of the vacuum chamber, causing desorption of neutral molecules by electron stimulated desorption. This increase in pressure will be most significant at the beginning of commissioning due to contaminated beam tube and will decrease with beam conditioning.

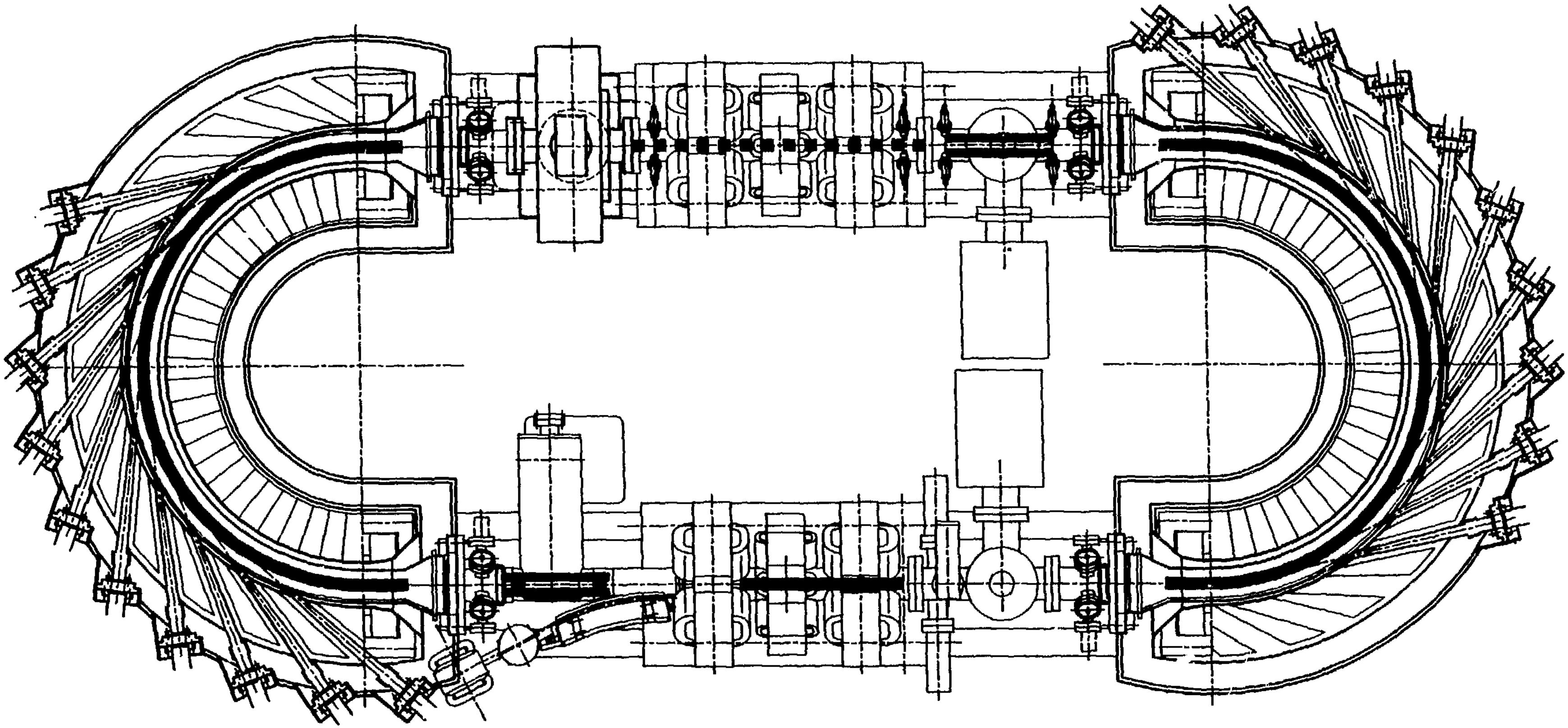
CLEARING ELECTRODES

The ions will be removed by clearing electrodes based on the ISR, SRS and Aladdin experience. While in the ISR there are short electrodes in potential wells only, in the SRS and Aladdin long electrodes cover as much of the machine circumference as possible. The difference lies in the different masses and velocities of electrons (ISR) and ions (SRS, Aladdin).

Similarly, placing clearing electrodes in potential wells only (Fig. 1) would be insufficient in the XLS (Table-3). To obtain efficient clearing, both dedicated clearing electrodes and existing striplines will be used for ion clearing as shown in Fig. 3.

Wherever possible, the electrodes will be capacitively terminated with their characteristic impedance Z_o, in order to minimize machine impedance effects [12]. All dedicated clearing electrodes and their feedthrues will be rated for 5kV, well above the required $\approx$ 1kV (see under "Ion Clearing").

Finally, the heat induced in the electrodes by the circulated beam (see next section) must be removed by providing a thermal path to room temperature. In a cold bore



St.S. Strip: w = 1.5 cm, t = 0.22 cm, Zo = 50Ω
1 cm diameter rod Zo = 50Ω
Existing Strip Lines Zo = 50Ω

Fig. 3 XLS Clearing System

machine the dissipated heat (as given in Table-4) would have to be removed at liquid He temperature, which for the XLS represents a substantial heat load.

POWER DISSIPATION

The surface heat dissipation in the conductors (vacuum chamber wall, clearing electrodes, strip-lines, etc) caused by the induced image current of the circulating beam, can be characterized by the average power per unit length:

$$<P> = \frac{1}{2}\ Re(Z_s\ i_s\ i_s^*)$$

where i_s is the surface current density and Z_s is the impedance per unit length. If $\lambda(s)$ is the line charge distribution in the beam, then the current density can be written as

$$i(s) = \sum_{n=-\infty}^{+\infty} c_n\ e^{i\,n\frac{2\pi}{L}(s-ct)}$$

where L is the separation between bunches and

$$c_n = \frac{1}{L}\int_{-L}^{+L} \lambda(s)\ e^{-in\frac{2\pi}{L}s}\ ds$$

The Z_s impedance for the n-th Fourier component is

$$Z_s^n = \frac{R_s}{2\pi r}\sqrt{n}$$

where r is the separation of the beam from the surface and R_s is the surface resistance

$$R_s = \frac{\rho}{\delta} = \sqrt{\rho\pi f_o \mu_o n_B}$$

Here ρ is the resistivity in [Ohm m], δ is the skin depth in m, f_o is the revolution frequency of the electrons and $\mu_o = 4\pi\ 10^{-7}$ is the permeability of vacuum. With these units, <P> is obtained in Watt/m.

Assuming Gaussian, cosine-square or uniform charge distribution in the bunch, the average power is [13]:

$$<P> = I^2\ \frac{R_s}{2\pi r}\ \sum_{n=1}^{\infty} F_n$$

where

$$F_n = \begin{cases} \sqrt{n}\, e^{-n^2\left[\frac{2\pi\sigma_B n_B}{C}\right]^2} \\ \left[\frac{B}{\pi}\right]^2 n^{-3/2} \left[\frac{\sin(\frac{n\pi}{B})}{1-(\frac{j}{B})^2}\right]^2 \\ \left[\frac{B}{\pi}\right]^2 n^{-3/2} \sin^2(\frac{n\pi}{B}) \end{cases}$$

Calculations were carried out with $\rho = 9\ 10^{-7}$ [Ohm m], I=0.5 Amp beam current, 6 and 3 bunches in the ring and using $l_B = 2(2\sigma_B)$ and $2(3\sigma_B)$ as bunch length. Table-4 shows the results for Phase-1 and 2 (σ_B = 1.6 and 3.8 cm, respectively)

Table-4

n_B	charge distribution	$l_B = 2(2\sigma_B)$	$l_B = 2(3\sigma_B)$
		<P> [Watts/m]	
6	Gauss	1.17	
	cos-square	1.78	0.97
	uniform	1.07	0.59
3	Gauss	2.35	
	cos-square	3.58	1.94
	uniform	2.05	1.15

CONCLUSIONS

As could be expected, the rather small circumference and bunch spacing in the XLS result in complete trapping of all residual gas ions; all ionic species are trapped since under normal working conditions the critical mass is $A_c < 1$.

The worst neutralization pocket occurs before entering the dipole, therefore one must have clearing in that region. However, depending on the upper limit of η that we can tolerate, clearing is needed by intervalls not much longer than the "clear free path" for the CO_2 ions. Clearing is also needed in the straight sections, where the ions travel with thermal velocity.

Cooling of the clearing electrodes will be provided.

A clearing rate of $\simeq 10^{14}$ ion/sec is required to maintain $\eta \leq 10^{-3}$.

The horizontal and vertical tune shifts are tolerable even for $\eta = 5\ 10^{-3}$. The higher is the vertical value, which is $\nu_y \leq .01$ and 0.002 for Phase-1 and 2, respectively.

Research has been carried out under the auspices of the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

APPENDIX

otation and constants used (values are given in MKSA units):

c = 2.9979 10^8 [m/sec]	speed of light
k = 1.3807 10^{-23} [joule/°K]	Boltzmann constant
e = 1.60206 10^{-19} [Coul]	unit charge
ε_o = 0.88542 10^{-11} [farad/m]	permittivity of vacuum
$\mu_o = 4\pi 10^{-7}$ [henry/m]	permeability of vacuum
	$\varepsilon_o \mu_o c^2 = 1$
$Z_o = \frac{1}{c\,\varepsilon_o} = 120\,\pi$ [Ohm]	impedance of vacuum
A	atomic number
A_c	critical mass (ions with A $\le A_c$ are trapped in the beam)
m_p = 1.6726 10^{-27} [Kg]	proton mass
m_e = 0.91095 10^{-30} [Kg]	electron mass
$r_m = \frac{e^2}{4\pi\varepsilon_o mc^2}$	classical radius of particle having mass m
r_p = 1.53 10^{-18} [m]	classical proton radius
r_e = 2.8179 10^{-15} [m]	classical electron radius
$I_A = \frac{ec}{r_e}$ =17.045 [Amp]	Alfven current
N	number of particles in the beam
n_B	number of bunches
$B = \frac{C}{n_B l_B}$	bunching factor
$\lambda = \frac{Ne}{C}B = \frac{Ne}{n_B l_B}$ [Coul/m]	line charge density
$\lambda_B = \frac{Ne}{n_B}$	charge per bunch
$\rho = \frac{\lambda}{2\pi\sigma_x\sigma_y}$ $[Coul/m^3]$	local charge density in beam
$\rho_{ion} = \frac{\lambda^{(B=1)}\eta}{2\pi\sigma_x\sigma_y} = \frac{Ne\eta}{C}$ $\frac{1}{2\pi\sigma_x\sigma_y}$	local (singly ionized) ion charge density (B=1 since ions are not bunched)
$\eta = \rho_{ion} / \rho$	neutralization factor
$d = \rho / e$	particle density
$I = V\lambda$ [Amp]	beam current (v=$\beta c \approx c$ [m/sec] velocity of the beam particles)
$\sigma_{x,y}$ [m]	RMS beamsize

σ_B [m]	RMS bunch length
l_B [m]	full bunch length
$f_o\ [Hz] = f n_B = \frac{c}{C}$	revolution frequency of beam particles
f [Hz]	revolution frequency of the bunches
$\omega_o = 2\pi f_o$	angular frequency
V [Volt]	potential
E [Volt/m]	electric field
R [Ohm]	resistance
ρ [Ohm m]	resistivity

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