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ENGINEERING DESIGN OF SUPERCONDUCTING MAGNETS FOR A TORSATRON EXPERIMENT

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Summary

The design of superconducting magnets for a torsatron plasma confinement experiment are presented. A torsatron is a variation of a stellarator which requires half the helical windings and no toroidal field coil. Both the toroidal and poloidal fields are produced by a single set of helical winding.

Due to the particular distribution of magnetic forces in the helical windings, the design of a Torsatron configuration requires special consideration. In this paper, we present of the force distribution on the helical coils. For the special case of uniformly distributed winding, magnetic forces and energy are calculated directly from formula for the inductance or toroids and dipoles. Significant reduction in the standard requirement can be achieved if the winding angle can be set at a particular value.

Introduction

In the present paper, we report on several aspects of the design of magnets for a superconducting torsatron experiment. A torsatron is a variation of the stellarator concept for magnetic confinement of a plasma, which requires half the helical windings and no toroidal coils. Both the toroidal and poloidal fields are produced by a single set of helical coils. However, it is necessary to cancel the net vertical field on the magnetic axis. This is accomplished by one of two means: either establishing large vertical field coils separate from the helical windings or varying the helical coil winding law to compensate for this effect. The latter winding law has been called an "ultimate" torsatron configuration since it minimizes the coil requirements.

Since torsatrons are inherently steady state devices from the standpoint of the plasma physics, it is desirable for full steady state operation to have superconducting magnetic field coils. The following paper presents certain aspects of the coil requirements for two superconducting torsatron experiment designs. These designs have evolved out of a combined effort by the Wisconsin Stellarator Group, The Wisconsin Applied Superconductivity Group, Chicago Bridge and Iron Corporation and Gruman Aerospace Corporation[1]. The emphasis of the paper is on the magnetic field and force calculations and how they affect the design of the helical magnets for toroidal plasma confinement.

Magnet Designs

The design parameters for the two torsatron magnet systems are listed in Table 1. The devices are inherently similar in that they are both steady-state and superconducting with similar toroidal magnetic fields. Since the energy stored is substantial, the magnets are designed for full cryostatic stabilization with current densities across the winding of $2900\ A/cm^2$.

Besides these similarities, there are substantial differences on the design of these two devices. Torsatron A is larger, has a constant pitch winding law, $\phi = m\theta$, and is an $\ell=3$ devices. The ℓ-value refers to the number of coils cutting a plane perpendicular to the magnetic axis. Since Torsatron A has a conventional winding law, large vertical field coils are necessary to cancel the vertical field on the magnetic axis. An artist sketch of Torsatron A is shown in Fig. 1. A further point of distinction between the two devices is that the location of the plasma vacuum chamber. For Torsatron A, this component is inside the helical windings, while for Torsatron B the vacuum chamber is outside the coils.

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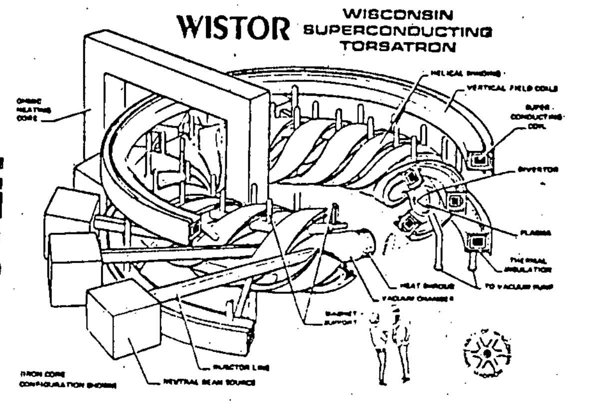

Fig. 1. Artist sketch of superconducting torsatron experiment.

Torsatron B is a more compact device and eliminates the need for large vertical field coils by using the "ultimate" torsatron winding law,

$$\theta = 1/m(\phi - \alpha \sin \phi) \qquad (1)$$

α is the modulation coefficient for Torsatron B, where m=6, and has been empirically determined to be 3.78. Figure 2 shows the two winding laws in the top view. The "ultimate" winding law has the coils pitched more steeply on the outside than on the inside of the torus. For this device, the vertical field coils are reduced to the point of providing field trimming capability only. For Torsatron B, the vacuum chamber is placed outside the coils to allow better access between the coils and to take advantage of the natural divertor action of the helical windings.

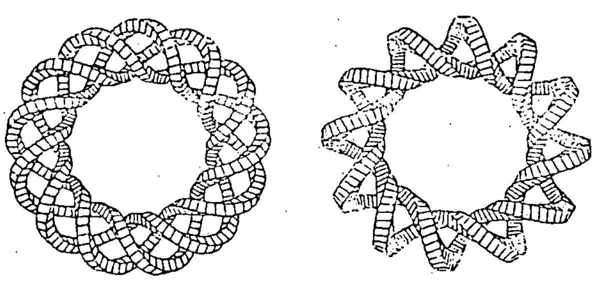

Fig. 2. Schematic of two winding laws. Torsatron A is constant pitch, Torsatron B is ultimate winding law.

Magnetics

Inductance and Energy Calculations

In the following section the method of calculating the energy stored and forces for torsatron geometries is discussed. The magnetic energy stored in a system of helical windings consisting of uniformly distributed currents over a toroidal surface can be calcu-

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lated by addition of the energy stored in the poloidal and toroidal fields.

$$W_h = W_T + W_p \quad (2)$$

This addition is possible because the two field components of the helical windings do not inductively interact. For a given helical winding pitch m, defined by $\phi = m\theta$, W_T and W_p are

$$W_T = 1/2\ L_T (mI_p)^2, \text{ and} \quad (3)$$

$$W_p = 1/2\ L_p\ I_p^{\ 2} \quad (4)$$

where L_T and L_p are the inductances of the toroidal and poloidal components of the winding respectively, I_p is the current per turn. The number of turns in the toroidal windings are m times the number of turns in the poloidal windings because each turn crosses the midplane of the helical windings m times. Summing the energies as in eq. (2), the net inductance L_H of the helical windings is

$$L_H = L_p + m^2\ L_T \quad (5)$$

To calculate L_p and L_T, we consider a helical winding of finite thickness as shown in Fig. 3. The toroidal volume between $r=a-\Delta$ and $r=a$ is completely covered by helical windings with a constant pitch winding law. The inductance of the toroidal component L_T is

$$L_T = \mu_o N^2 R\{(1-\sqrt{1-\beta^2(1-\delta)^2}) + \frac{1}{2\pi\delta^2\beta^2}\int_{\beta(1-\delta)}^{\beta}\int_0^{2\pi}\frac{x(\beta-x)^2}{1-x\cos\phi}dx\,d\phi\} \quad (6)$$

where β and δ are aspect ratios $\beta=a/R$, $\delta=\Delta/a$. The first term in the brackets results from the toroidal geometry and the second term is due to the finite thickness of the winding. For $\beta<0.5$, the inductance of the poloidal component L_p has been shown to be approximately[2]

$$L_p \simeq \mu_o R\{1-\beta^2/4(1-\delta^2)^2\} \times\{[\ln\frac{8}{\beta} - \frac{(1-\delta)^4}{\delta^2(2-\delta)^2}\ln(1-\delta) - \frac{1}{4}\,\frac{3\delta^2-6\delta^2-6\delta+2}{\delta(2-\delta)}]-2\} \quad (7)$$

Using the Neumann formula for mutual inductance between two elements[3], the values for L_p and L_T can be calculated directly. Listed in Tables II and III are these values as a function of aspect ratios β and δ. The agreement between eq. (6) and Table II are exact, while eq. (7), is approximately in agreement with Table III.

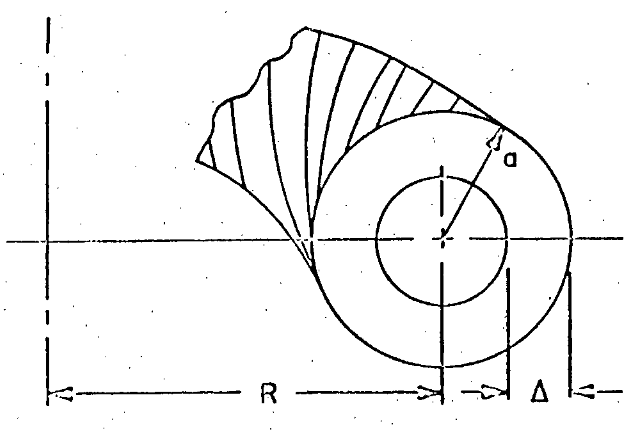

Fig. 3. Helical winding of finite thickness.

The actual windings are not uniformly distributed over the torus as shown in Fig. 3, but are discrete. This leads to somewhat higher inductance, because the current does not uniformly cover the surface. For Torsatron A, the difference is notable with L_H=11.2μh for uniform current and L_H=18.2μh for the discrete current when calculated by the Neumann formula.[3]

The total energy stored in the torsatron windings is due to the helical windings and the vertical field coils. We can write the energy as a sum of three terms,

$$E = 1/2\ L_H I_H^{\ 2} + 2(1/2\ L_V I_V^{\ 2}) - 2M_{HV} I_V I_H \quad (8)$$

The first two terms are the self energy of the helical windings and vertical field windings. The third term is the energy of the mutual coupling between the two coils.

The inductances calculated for the actual windings are L_H=18.8μh, L_V=21.4μh and M_{HV}=4.18μh for Torsatron A. Based on these values and the currents of $I_H=3(3.5\times10^6$ amperes) and $I_V=4.5\times10^6$ amperes, the total energy stored in the magnet system is 1004 MJ.

Forces

The forces on the torsatron helical windings are generated by the currents in the windings and by the currents in the two external vertical field coils. The dominant force on the helical windings is radially outward and results from the toroidal currents. In general, the azimuthal force is smaller and changes direction around the circumference. Figure 4 is a plot of the radial force F_ρ and azimuthal for F_ϕ versus the angle around the minor circumference, ϕ, for a constant pitch winding law. The force pattern is periodic with azimuthal angle.

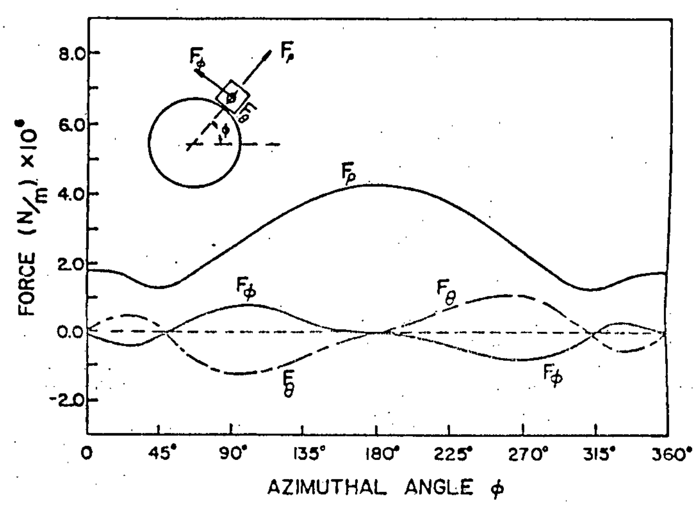

Fig. 4. Forces on helical windings of constant pitch.

In the case of helically wound magnets such as the torsatron system, electromagnetic forces can be reduced significantly, about one order of magnitude, if the winding law or the winding angle can be selected close to an optimum value. To examine the behavior of the electromagnetic forces versus the winding law, we considered a simple model of uniformly distributed surface currents flowing over a toroidal surface with the constant winding law.

Figure 5 shows two forces $\sum_i f_i$, F_R vs. the winding pitch m. $\sum_i f_i$ is the scaler sum of the total force per unit area at different points on the surface of the helical winding F_R is the net radial force and is related to the total energy stored W_H by

$$F_R' = \frac{dW_H}{dR} \tag{9}$$

It is shown that the scaler sum of the forces has a minimum around m=3 while the net force F_R has a minimum at m=6.5. The use of compensating coils can change the sum of forces shown in Fig. 4, and hence the amount of structure required.

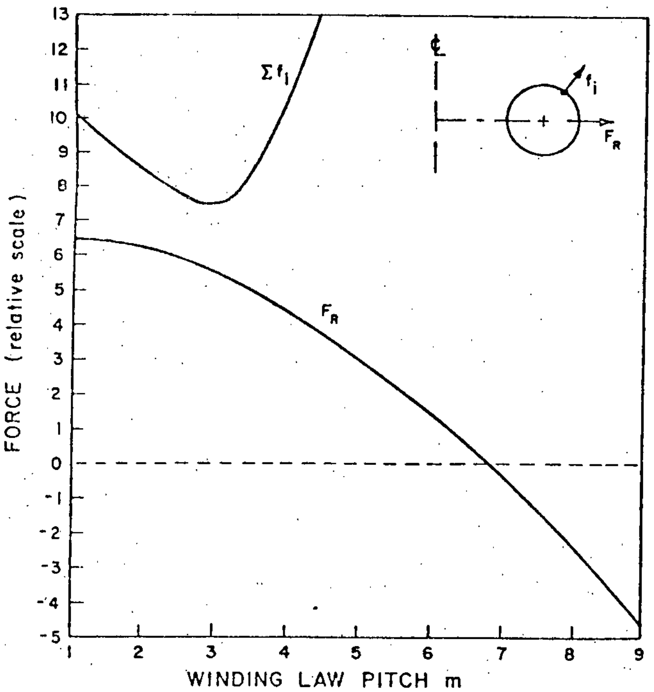

Fig. 5. Scaler sum, $\sum f_i$, and total radial force, F_R, on helical windings versus pitch, m.

Acknowledgements

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Table 1

Magnet Design Parameters For Two Torsatron Experiments

Major radius	3.63 m	2.5 m
Minor radius	1.07 m	0.56 m
Helical windings	conventional (ℓ=)	"ultimate (ℓ=2)
Number of periods	16	12
Toroidal field	6T at coils,3T on axis	6T at coils,3T on axis
Helical ampere-turns	3.5 MA	3.0 MA
Vertical field ampere turns	4.5 MA	0.14 MA
Overall current density	2900 A/cm^2	2900 A/cm^2
Energy stored	1004 MJ	141 MJ

Table 2

Inductance of a Hollow Dipole, $L_p/\mu_o Rn^2$ as a Function of the Aspect Ratios β and δ

	δ=0	.2	.4	.6	.8	.1
β = 0.1	2.39	2.46	2.52	2.55	2.62	2.63
β = 0.2	1.73	1.79	1.85	1.90	1.94	1.96
β = 0.3	1.36	1.41	1.47	1.52	1.56	1.57
β = 0.4	1.12	1.16	1.21	1.26	1.30	1.31
β = 0.5	0.95	0.99	1.03	1.07	1.10	1.12

R is the major radius and
n is the number of turns

Table 3

Inductance of a Hollow Torus, $L_T/\mu_o Rn^2 \times 10^{-3}$ as a Function of the Aspect Ratio β and δ

	δ=0	.2	.4	.6	.8	.1
β = 0.1	5.01	3.77	2.73	1.90	1.26	0.83
β = 0.2	20.2	15.2	10.9	7.63	5.08	3.34
β = 0.3	46.0	34.5	24.9	17.3	11.5	7.57
β = 0.4	83.4	62.2	44.8	30.9	20.6	13.5
β = 0.5	134.	99.1	71.0	49.0	32.5	21.4

R is the major radius and
n is the number of turns