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HAMILTONIAN FLOWS, SU(∞), SO(∞), USp(∞), AND STRINGS

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Abstract: Based on the *infinite-dimensional* algebras we have introduced, SU(∞) is identified with general hamiltonian flows in 2-d phase-space, SO(∞) with flows generated by x-p-odd hamiltonians, and USp(∞) with those of hamiltonians of special symmetry. Gauge theories for SU(∞), SO(∞), and USp(∞) are thus formulated in terms of surface (sheet) coordinates for toroidal phase-space. Spacetime-independent configurations of their gauge fields directly yield the quadratic Schild-Eguchi string action.

This is an eclectic summary of recent observations made with David Fairlie and Paul Fletcher, with whom I introduced new infinite-dimensional algebras involving trigonometric functions in their structure constants[1]. The generators of the algebras we have introduced are indexed by 2-vectors $\mathbf{m} = (m_1, m_2)$. The components of these vectors do not need to be integers to satisfy the Jacobi identities, but we take them to be integral for the sake of interpreting them as Fourier modes:

$$[K_{\mathbf{m}}, K_{\mathbf{n}}] = r \sin(k\, \mathbf{m}\times\mathbf{n})\ K_{\mathbf{m}+\mathbf{n}} + \mathbf{a}\cdot\mathbf{m}\ \delta_{\mathbf{m}+\mathbf{n},0}\ . \tag{1}$$

Here, $\mathbf{m}\times\mathbf{n} = m_1 n_2 - m_2 n_1$, r and k are arbitrary (complex) constants, and $\mathbf{a}$ is an arbitrary 2-vector . The Casimir invariants are

$$\sum_{\mathbf{m}} K_{\mathbf{m}} K_{-\mathbf{m}}\ ,$$

$$\sum_{\mathbf{m},\mathbf{n}} e^{ik\mathbf{m}\times\mathbf{n}}\ K_{\mathbf{m}} K_{\mathbf{n}} K_{-\mathbf{m}-\mathbf{n}}\ , \qquad \dots , \tag{2}$$

$$\sum e^{ik(\mathbf{m}\times\mathbf{n}+\mathbf{m}\times\mathbf{p}+\dots+\mathbf{m}\times\mathbf{r}+\mathbf{n}\times\mathbf{p}+\dots+\mathbf{n}\times\mathbf{r}+\dots+\mathbf{p}\times\mathbf{r})}\ K_{\mathbf{m}} K_{\mathbf{n}} K_{\mathbf{p}} \dots K_{\mathbf{r}} K_{-\mathbf{m}-\mathbf{n}-\mathbf{p}\dots-\mathbf{r}}\ .$$

These algebras include as a special case that of $\mathrm{SDiff}_0(T^2)$, the infinitesimal area-preserving diffeomorphisms of the torus[2,3]: $r = 1/k$ in the limit $k \to 0$ yields the algebra

$$[L_{\mathbf{m}}, L_{\mathbf{n}}] = (\mathbf{m}\times\mathbf{n}) L_{\mathbf{m}+\mathbf{n}} + \mathbf{a}\cdot\mathbf{m}\ \delta_{\mathbf{m}+\mathbf{n},0}\ . \tag{3}$$

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You may find the supersymmetric extension of our algebra (1) and the observations to follow in Ref.[1]. The representation and character theory of these algebras is an open problem.

The algebra (3) is known to be, in a particular basis optimal for the torus, that of the generic area-preserving (symplectic) reparameterizations of a 2-surface. Taking x and p to be local (commuting) coordinates for the surface, and f and g to be differentiable functions of them, a basis-independent realization for the generators of the centerless algebra is[2]:

$$L_f = \frac{\partial f}{\partial x}\frac{\partial}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial}{\partial x} \qquad \Longrightarrow \tag{4}$$

$$[L_f, L_g] = L_{\{f,g\}} , \quad [L_f, g] = \{f,g\} , \tag{5}$$

where

$$\{f,g\} \equiv \frac{\partial f}{\partial x}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial x} , \tag{6}$$

the Poisson bracket of classical phase-space. The generator L_f transforms (x,p) to $(x - \partial f/\partial p \ , \ p + \partial f/\partial x)$. Infinitesimally, this is a canonical transformation[4] generated by f, which preserves the phase-space area element $dxdp$. This element is referred to as a *symplectic form* and the class of transformations that leaves it invariant specifies a symplectic geometry. You may regard it as the flow generated by an arbitrary hamiltonian f. For a small patch of 2-surface, you may expand the functions $f(x,p)$ in any coordinate basis you choose. If the surface is a torus, I shall prefer a globally adequate coordinate system, such as $\exp(inx + imp)$; if it is a sphere, spherical harmonics[3]; if it is a plane, powers[5]; and so on. Nevertheless, for the infinitesimal transformations effected by the algebra generators in a patch, *any* coordinate basis will do, and may be transformed to other ones. (When such transformations are singular, however, a number of generators may be lost, leading to a subalgebra, as noted by Pope and Stelle, and Hoppe[6].)

Choosing the torus basis, $f = -e^{i(m_1 x + m_2 p)}$ and $g = -e^{i(n_1 x + n_2 p)}$, $0 \leq x, p \leq 2\pi$, yields

$$L_f = L_{(m_1,m_2)} = -ie^{i(m_1 x + m_2 p)}(m_1 \partial/\partial p - m_2 \partial/\partial x) , \tag{7}$$

which obey the centerless algebra in the basis (3). Conversely, given the basis (3), any function $f(x,p)$ can be reconstituted through

$$f(x,p) = -\sum_{m_1,m_2} F(m_1,m_2) e^{i(m_1 x + m_2 p)} , \tag{8}$$

and thus the linear combinations

$$L_f = \sum_{m_1,m_2} F(m_1,m_2) L_{(m_1,m_2)} \tag{9}$$

are seen to obey the Poisson-bracket algebra (5).

We have found a corresponding realization for the torus-basis algebra (1) generators:

$$\begin{aligned} K_{(m_1,m_2)} &= (ir/2)\exp(im_1 x + km_2 \frac{\partial}{\partial x} + im_2 p - km_1 \frac{\partial}{\partial p}) \\ &= (ir/2)\exp(im_1 x + im_2 p)\ \exp(km_2 \frac{\partial}{\partial x} - km_1 \frac{\partial}{\partial p}) , \end{aligned} \tag{10}$$

somewhat analogous to the one-variable realization found by Hoppe[3]. Note the triviality in this realization of the Casimir operators, as the indices of each of their terms sum to zero.

To Fourier-compose this to a basis-independent realization, we first define, as in (9),

$$K_f \equiv \sum_{m_1,m_2} F(m_1,m_2) K_{(m_1,m_2)} \equiv \frac{r}{2i} f(x+ik\frac{\partial}{\partial p}, p-ik\frac{\partial}{\partial x}) , \tag{11}$$

where the last side of the equation is a formal expression to evoke (8)/(4): the "normal ordering" of its derivatives is specified in its Fourier-series definition, in which they stand to the right of all coordinates, by virtue of eq. (10).

The analog of the Poisson bracket in this case is the *sine*, or *Moyal, bracket* $\{\{f,g\}\}$. This is the extension of the Poisson bracket $\{f,g\}$ to statistical distributions on phase-space, introduced by Weyl[4] and Moyal[7b], and explored by several authors[7] in an alternative formulation of quantum mechanics, regarded as a deformation of the algebra of classical observables. It is a generalized convolution which reduces to the Poisson bracket as $\hbar$, replaced by $2k$ in our context, is taken to zero:

$$\{\{f,g\}\} = \frac{-r}{4\pi^2k^2}\int dp'dp''dx'dx''\ f(x',p')g(x'',p'') \sin\frac{1}{k}\Big(p(x'-x'')+x(p''-p')+p'x''-p''x'\Big) \ . \tag{12}$$

The argument of the sine above is

$$\frac{1}{k}\det\begin{pmatrix} 1 & p & x \\ 1 & p' & x' \\ 1 & p'' & x'' \end{pmatrix} = \frac{1}{k}\int p\cdot dq \ , \tag{13}$$

i.e. $2/k$ times the area of the equilateral phase-space triangle with vertices at (x,p), (x',p'), and (x'',p''). The antisymmetry of f with g is evident in the determinant. The sine brackets satisfy the Jacobi identities[7d], just as their Fourier components (1) (see the next paragraph) do, and thus determine a Lie algebra. These brackets help reformulate quantum mechanics in terms of Wigner's phase-space distribution[7].

The Fourier transform of the sine bracket results from substitution in (12) of the exponential basis used in (7):

$$\{\{f,g\}\} = \frac{-ir}{8\pi^2k^2}\int dp'dp''dx'dx''\ e^{i(m_1x'+n_1x'')+i(m_2p'+n_2p'')} \ \times$$
$$\times\left(e^{\frac{i}{k}(p(x'-x'')+x(p''-p')+p'x''-p''x')} - (k\leftrightarrow -k)\right) = -r\sin(k\mathbf{m}\times\mathbf{n})\ e^{i(m_1+n_1)x+i(m_2+n_2)p} \ . \tag{14}$$

As in (9), it then follows through the linearity of the operators defined in (11), and (1), that these indeed obey the algebra

$$[K_f, K_g] = r\sum_{m_1,m_2,n_1,n_2} F(m_1,m_2)G(n_1,n_2)\sin(k\mathbf{m}\times\mathbf{n})\ K_{\mathbf{m}+\mathbf{n}} = K_{\{\{f,g\}\}} \ . \tag{15}$$

Our algebra is thus identified with that of sine brackets. *Mutatis mutandis*, you might wish to expand it in alternate bases, such as spherical harmonics, so as to specify the corresponding generalizations of $\mathrm{SDiff}_0(S^2)$, powers for the plane[8], and so on.

Focus now on an interesting centerless family of the algebras (1), namely the *cyclotomic* family: the one for which $k = 2\pi/N$, for integer $N > 2$. In this family, there is an additional $\mathbb{Z}\times\mathbb{Z}$ algebra isomorphism

$$K_{(m_1,m_2)} \longmapsto K_{(m_1,m_2)+(Nt,Nq)} \tag{16}$$

for arbitrary integers t and q. Since the structure constants $\sin\frac{2\pi}{N}(m_1n_2 - n_1m_2)$ are only sensitive to the modulo-N values of the indices, the 2-dimensional integer lattice separates into $N \times N$ cells, each of which may be referred to some fundamental cell, e.g. around the coordinate center of the lattice, by proper N-translations. The fundamental $N \times N$ cell contains N^2 index points, but the operator $K_{(0,0)}$, like its lattice translations $K_{N(t,q)}$, factors out of the algebra: it commutes with all K's and cannot result as a commutator of any two such. Thus the fundamental cell involves only $N^2 - 1$ generators, and there are no more structure constants than those occurring in this cell. In consequence, the infinite-dimensional centerless cyclotomic algebras, with the $K_{N(t,q)}$'s factored out, possess the following finite-dimensional invariant subalgebra of "lattice average" operators $\mathcal{K}$:

$$\mathcal{K}_{(m_1,m_2)} \equiv \sum_{s,v} K_{(m_1+Ns,m_2+Nv)}, \qquad [\mathcal{K}_{\mathbf{m}}, \mathcal{K}_{\mathbf{n}}] = r\sin(\frac{2\pi}{N}\mathbf{m}\times\mathbf{n})\,\mathcal{K}_{\mathbf{m}+\mathbf{n}}, \tag{17}$$

where $\mathbf{m}$,$\mathbf{n}$,$\mathbf{m}+\mathbf{n}$ are indices in the fundamental cell, and an infinite normalization has been absorbed in r.

This (N^2-1)-dimensional ideal specifies, in fact, a basis for SU(N) which may be thought of as a generalization of the Pauli matrices[9]. Consider odd N's first. A basis for SU(N) algebras, for odd N, may be built from two unitary unimodular matrices:

$$g \equiv \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & \omega & 0 & \dots & 0 \\ 0 & 0 & \omega^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \omega^{N-1} \end{pmatrix}, \quad h \equiv \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & \dots & 0 \end{pmatrix}, \quad g^N = h^N = \mathbb{1}, \tag{18}$$

where ω is a primitive N'th root of unity, i.e. with period no smaller than N, here taken to be $e^{4\pi i/N}$. They obey the identity

$$hg = \omega\, gh. \tag{19}$$

You also encounter these matrices in the context of representations of quantum SU(2)[10]. The complete set of unitary unimodular $N \times N$ matrices

$$J_{(m_1,m_2)} \equiv \omega^{m_1m_2/2}\, g^{m_1} h^{m_2}, \tag{20}$$

where

$$J^\dagger_{(m_1,m_2)} = J_{(-m_1,-m_2)}; \qquad \mathrm{Tr}J_{(m_1,m_2)} = 0 \quad \text{except for } m_1 = m_2 = 0 \bmod N, \tag{21}$$

suffice to span the algebra of SU(N). Like the Pauli matrices, they close under multiplication to just one such, by virtue of (19):

$$J_{\mathbf{m}}J_{\mathbf{n}} = \omega^{\mathbf{n}\times\mathbf{m}/2} J_{\mathbf{m}+\mathbf{n}}. \tag{22}$$

They therefore satisfy the algebra

$$[J_{\mathbf{m}}, J_{\mathbf{n}}] = -2i\,\sin(\frac{2\pi}{N}\mathbf{m}\times\mathbf{n})\, J_{\mathbf{m}+\mathbf{n}}. \tag{23}$$

Consequently, in this convenient two-index basis with the above simple structure constants, SU(N) describes the algebra (17) of the ideal $\{\mathcal{K}\}$.

For even N, the fundamental matrices in (18) are not unimodular, as their determinant may now be -1 as well. One might choose to modify them to

$$g \equiv \sqrt{\omega} \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & \omega & 0 & \ldots & 0 \\ 0 & 0 & \omega^2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \omega^{N-1} \end{pmatrix} , \quad h \equiv \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -1 & 0 & 0 & \ldots & 0 \end{pmatrix} , \quad g^N = h^N = -\mathbb{1} , \tag{24}$$

with $\omega = e^{2\pi i/N}$, $\sqrt{\omega} = e^{\pi i/N}$. They again obey (19), and again serve to define the unitary basis

$$J_{(m_1,m_2)} \equiv \omega^{m_1 m_2/2} g^{m_1} h^{m_2} , \tag{20'}$$

$$J_{\mathbf{m}} J_{\mathbf{n}} = \omega^{\mathbf{n}\times\mathbf{m}/2} J_{\mathbf{m}+\mathbf{n}} . \tag{22'}$$

The SU(N) algebra is now

$$[J_{\mathbf{m}}, J_{\mathbf{n}}] = -2i \sin(\frac{\pi}{N} \mathbf{m} \times \mathbf{n})\ J_{\mathbf{m}+\mathbf{n}} . \tag{25}$$

It might appear that the fundamental period be $2N$ instead of N. However, note that, by virtue of the symmetry

$$J_{\mathbf{m}+N(t,q)} = (-1)^{(m_1+1)q+(m_2+1)t} J_{\mathbf{m}} , \tag{26}$$

only indices in the fundamental cell $N \times N$ need be considered. Illustrating this for $N = 2$, the Pauli matrices, may be of use to the reader. Naturally, the algebra (25) also holds for N odd, when $\omega = \exp(2\pi i/N)$ is used in (18). Thus, the ideal (17) amounts to SU(N) for N odd and SU($N/2$) for N even.[2] For example, both $N = 3$ and $N = 6$ yield SU(3), $N = 12$ yields SU(6)'s, etc.

In this basis again, the operators $Q_{(m,n)} \equiv J_{(m,n)} - J_{(n,m)}$ close to a subalgebra of SU(N) with $N(N-1)/2$ generators

$$[Q_{(m,n)}, Q_{(m',n')}] = -2i \sin \frac{2\pi}{N}(mn' - m'n)\ Q_{(m+m',n+n')} + 2i \sin \frac{2\pi}{N}(mm' - nn')\ Q_{(m+n',n+m')}, \tag{27}$$

which is shown by reduction to the Cartan-Weyl basis[11] to amount to SO(N). Alternative SO(N)'s may also be found, such as the subset of the above $Q_{(m,n)}$ with $m + n =$ even together with the operators $J_{(m,n)} + J_{(n,m)}$ with $m + n =$ odd; or else, for even $N = 2M$, $J_{(m,n)} - (-)^n J_{(m,-n)}$. Finally, the subalgebra of SU($2M$): $S_{(m,n)} \equiv J_{(m,n)} - (-)^m J_{(m,-n)}$ is seen to be an USp($2M$).

The 2-index SU(N) basis considered here has a particularly simple large N limit. As N increases, the fundamental $N \times N$ cell covers the entire index lattice; the operators $\mathcal{K}$ supplant the K's and, in turn, since $k \to 0$, the operators L of eq.(3).

More directly, you immediately see by inspection that, as $N \to \infty$, the SU(N) algebra (23) goes over to the centerless algebra (3) of SDiff$_0(T^2)$ through the identification:

$$\frac{iN}{4\pi} J_{\mathbf{m}} \to L_{\mathbf{m}} . \tag{28}$$

[2] Actually, in this case[11], the generators describe SU($N/2$)^4, i.e. four mutually commuting SU($N/2$)'s.

An identification of this type was first noted by Hoppe[3] in the context of membrane physics: he connected the infinite N limit of the SU(N) algebra in a special basis to that of SDiff$_0(S^2)$, i.e. the infinitesimal symplectic diffeomorphisms in the sphere basis. A discussion of the group topology of SU(N), or SDiff$_0(T^2)$ versus SDiff$_0(S^2)$, or other 2-dimensional manifolds for that matter[5], exceeds the scope of this type of local analysis; such a discussion has been suggested in Refs.[6], which consider central extensions that are sensitive to global features of the 2-surface.

In view of the SO(N) subalgebras described above, we may also simply identify the SO(∞) subalgebra with the Poisson Bracket subalgebra whose shift potentials f are odd under interchange of x with p — they correspond to hamiltonians which evolve even functions to even ones, and odd to odd ones. Likewise, USp(∞) is generated by shift potentials of the form $\exp(imx)\,\sin(np - m\pi/2)$, i.e. toroidal phase-space hamiltonians odd under $p \longmapsto -p$, $x \longmapsto x + \pi$. (Merely p-odd hamiltonians generate the "sibling" SO(∞).) Saveliev and Vershik[12], and we[11] have initiated a program of systematizing such results in a unified framework common with that of the finite Lie algebras.

Floratos et al.[13] utilized Hoppe's identification to take the limit of SU(N) gauge theory. Their results are immediately reproduced without ambiguity, again by inspection, on the basis of the orthogonality condition dictated by (21) and (22):

$$\mathrm{Tr} J_{\mathbf{m}} J_{\mathbf{n}} = N \delta_{\mathbf{m}+\mathbf{n},0} \quad \rightarrow \quad \mathrm{Tr} L_{\mathbf{m}} L_{\mathbf{n}} = -\frac{N^3}{(4\pi)^2} \delta_{\mathbf{m}+\mathbf{n},0} \; . \tag{29}$$

As a result, for a gauge field A_μ in an SU(N) matrix normalization with trace 1, the analog of eq. (9) is

$$A_\mu \equiv A_\mu^{\mathbf{m}} \frac{J_{\mathbf{m}}}{\sqrt{N}} \quad \rightarrow \quad \frac{4\pi}{iN^{3/2}} A_\mu^{\mathbf{m}} L_{\mathbf{m}} = \tilde{A}_\mu^{\mathbf{m}} L_{\mathbf{m}} \; , \tag{30}$$

where summation over repeated $\mathbf{m}$'s is implied, and I have defined $\tilde{A}_\mu^{\mathbf{m}} \equiv (4\pi/iN^{3/2}) A_\mu^{\mathbf{m}}$. As $N \rightarrow \infty$, the indices $\mathbf{m}$ cover the entire integer lattice, so that I may define

$$a_\mu^{(x,p)} \equiv -\sum_{\mathbf{m}} \tilde{A}_\mu^{\mathbf{m}} e^{i(m_1 x + m_2 p)} \; . \tag{31}$$

By eq. (5),

$$[A_\mu, A_\nu] \quad \rightarrow \quad [L_{a_\mu}, L_{a_\nu}] = L_{\{a_\mu, a_\nu\}} \; . \tag{32}$$

Hence, by virtue of the linearity of L in its arguments,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \quad \rightarrow \quad L_{f_{\mu\nu}}$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu + \{a_\mu, a_\nu\} \; . \tag{33}$$

The group trace defining the Yang-Mills lagrangian density is then

$$\mathrm{Tr} F_{\mu\nu} F_{\mu\nu} \rightarrow -\frac{N^3}{(4\pi)^2} \tilde{F}_{\mu\nu}^{\mathbf{m}} \tilde{F}_{\mu\nu}^{-\mathbf{m}} = \frac{-N^3}{64\pi^4} \int dx dp \sum_{m_1, m_2, n_1, n_2} e^{ix(m_1+m_2)+ip(m_2+n_2)} \tilde{F}_{\mu\nu}^{(m_1,m_2)} \tilde{F}_{\mu\nu}^{(n_1,n_2)}$$

$$= (-N^3/64\pi^4) \int dx dp \; f_{\mu\nu}^{(x,p)} f_{\mu\nu}^{(x,p)} \; . \tag{34}$$

Thus, in the SU(∞) gauge theory, the group indices are surface (torus) coordinates, and the fields are rescaled Fourier transforms of the original SU(N) fields; the group composition rule for them is given by the Poisson bracket, and the trace by surface integration.

Now note an intriguing connection to strings which emerges, for the first time *directly at the level of the action*: for gauge fields independent of x^μ (e.g. vacuum configurations), this lagrangian density reduces to $\{a_\mu, a_\nu\}\{a_\mu, a_\nu\}$, the quadratic Schild-Eguchi action density for strings[14], where the a_μ now serve as string variables, and the surface serves as the world-sheet. This action amounts to the square of the sheet area and it is easily seen that its equations of motion contain those of Nambu's action. Thus, at zero energy, the gauge theory reduces to a string. Whether a superstring follows analogously from the super-Yang-Mills lagrangian is an open question.

The lagrangian (34) with the sine bracket supplanting the Poisson bracket is also a gauge-invariant theory, provided that the gauge transformation also involves the sine instead of the Poisson bracket:

$$\delta a_\mu = \partial_\mu \Lambda - \{\{\Lambda, a_\mu\}\} , \tag{35}$$

and hence, by virtue of the Jacobi identity,

$$\delta f_{\mu\nu} = -\{\{\Lambda, f_{\mu\nu}\}\} . \tag{36}$$

It then follows that

$$\delta \int dxdp \; f_{\mu\nu} f_{\mu\nu} = -2 \int dxdp \; f_{\mu\nu}\{\{\Lambda, f_{\mu\nu}\}\} = 0 . \tag{37}$$

At the moment, however, it is not clear what physical system is described by the corresponding spacetime-independent lagrangian density $\{\{a_\mu, a_\nu\}\}\{\{a_\mu, a_\nu\}\}$. It is further obscure whether a relation exists between the above theories and the Universal Yang-Mills theory[15].

This compact formulation of SU(∞) gauge theory (and that of its subgroups) ought to be of use in large-N model calculations, or various "master-field" efforts; membrane physics[2,3]; and the exploration of connections between gauge theory and strings, as above.

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