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USE OF KALMAN FILTER METHODS IN ANALYSIS OF IN-PILE LMFBR ACCIDENT SIMULATIONS

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Abstract

Kalman filter methodology has been applied to in-pile liquid-metal fast breeder reactor simulation experiments to obtain estimates of the fuel-clad thermal gap conductance. A transient lumped parameter model of the experiment is developed. An optimal estimate of the state vector chosen to characterize the experiment is obtained through the use of the Kalman filter. From this estimate, the fuel-clad thermal gap conductance is calculated as a function of time into the test and axial position along the length of the fuel pin.

Introduction

Adequate characterization of off-normal nuclear reactor operation is of obvious concern to the reactor safety analyst. Conditions arising from inadequate coolant flow, loss of pressurization, spurious reactivity insertion, etc., can be expected to adversely impact normal operation and must therefore be understood. This is particularly true of the so-called fast reactor systems. These differ significantly from the thermal reactor systems currently in use and have specific characteristics unique to their composition. Thermal reactors typically use low enrichment fuel which is placed in a near optimal reactivity configuration. Quite often water is employed as a coolant. While off-normal conditions can have severe consequences in such systems, as evidenced by the accident at Three Mile Island, off-normal conditions in fast reactor systems can be equally devastating, sometimes with extreme rapidity. Fast reactors use high enriched fuel which is not placed in its most reactive configuration. Often liquid sodium is used as a coolant. Since this reactor type is driven by high energy neutrons which have relatively short lifetimes in the reactor core, the fast breeder can respond quickly to changes in conditions. An understanding of the consequences of these varied responses is imperative.

For many years a concerted effort has been undertaken at Argonne National Laboratory and other installations to investigate hypothetical core disruptive accidents in fast breeder systems. Experiments using small scale assemblies in conjunction with computer code simulations have provided much insight into core behavior during accidents. It has been determined that core disruptive behavior is rimarily influenced by thermal effects. While non-thermal perturbations are the most common source of accident initiation their inevitable expression is in terms of off-normal (and sometimes extreme) thermal effects. To understand and predict their magnitude and consequences a basic knowledge of parameters important in heat transfer is necessary. This knowledge is primarily to be obtained via experiment. It is to an analysis of such experiments that this paper is directed.

Lack of certainty in the determination of the conductance of the fuel-clad gap is often the largest single uncertainty in post-test analysis of fuel failure experiments. Unknown impurities in the gap gas, nonuniform swelling with fuel burnup, asymmetric thermal expansion, and pin bowing during transient heating all contribute to difficulties in analytical modeling [1,2]. Lack of an ability to measure fuel and clad conditions directly results in further uncertainties in the magnitude of the gap conductance [3]. The present work utilizes the methodology of the linear Kalman filter, which has the potential of incorporating these uncertainties into a unified analysis scheme.

The Kalman filter [4] represents a class of linear minimum error variance sequential state estimation algorithms. Both discrete time and continuous time versions have been developed. While this methodology has been applied to numerous physical systems, particularly those involving navigation, space vehicle guidance, and orbit determination, its application to nuclear systems (and in particular to experiment analysis) has been limited [5,6].

Methods

The theory of discrete optimal linear recursive filters is well known and will not be repeated here.

In this paper, the system being modeled is that of a single fuel rod surrounded by flowing sodium enclosed in associated structure. Available measurement data consist of inlet and outlet coolant flow and temperature, as well as structure temperature along the axial length of the exterior surface of the test section. To develop a model of the system involved, a linearized, multinodal, lumped parameter system of equations is formulated. All material properties are assumed temperature dependent. Coolant flow is taken to be one-dimensional, incompressible and its magnitude known as a function of time. Reactor power is assumed to be known both spatially and temporally. Cross-flow effects in the coolant are neglected as is internal resistance. With these assumptions a general energy balance can be established for a representative axial node:

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$$(\rho CV)_j \frac{\partial T_j}{\partial t} = -(CVG)_j \frac{\partial T_j}{\partial z}$$

$$+ H_{j-1,j} A_{j-1,j} (T_{j-1} - T_j) \qquad (1)$$

$$-H_{j,j+1} A_{j,j+1} (T_j - T_{j+1}) + Q_j \ .$$

Here, subscript j refers to the particular component (or material) under consideration whether fuel (j = 1), cladding (j = 2), coolant (j = 3), or structure (j = 4). Therefore, Eq. (1) is a general representation for the system of four energy equations describing the node as a whole. The left side of the equation describes the energy storage in material j as a function of time. The first term on the right side of Eq. (1) represents the convective component of energy from the base of the node to its top. This term is nonzero only for the coolant. The next two terms on the right side of the equation describe the flux of energy to and from the j component from or to its neighboring materials. The final term on the right side of Eq. (1) is a volumetric energy production term. It is nonzero only in fuel and blanket materials.

The equation of continuity for the flow is taken to be:

$$\frac{\partial G_j}{\partial z} = 0 \ . \qquad (2)$$

Furthermore, by neglecting frictional effects in the one-dimensional flow, the momentum equation is reduced to

$$G_j = G(t) \ . \qquad (3)$$

Initial and boundary conditions are:

$$T_j(z,0) = T(0) \ ,$$

$$T_j(0,t) = T(t) \ ,$$

and

$$G(z,t) = G(t)$$

These three equations together with their associated initial and boundary conditions, characterize transient behavior of the system.

Five state variables were chosen to describe the process, the four material temperatures and the heat flux from fuel to clad:

$$\underline{X} = \begin{vmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ q''_{1\to 2} \end{vmatrix} \qquad (4)$$

Measurement data was limited to the structure:

$$y = T_4 + r_4$$

Finite differencing Eq. (1) and casting it in the form required by the recursive algorithm provided the transition and deterministic input matrices as well as the deterministic variables; uncertainties in the system model were estimated via a method described by Ulenbeck and Ornstein [7]. To reformulate Eq. (1) into the form required by the Kalman filter, time derivatives and spatial derivatives were approximated as:

$$\frac{\partial T}{\partial t} = \frac{T^K - T^{K-1}}{\Delta t} \ , \qquad (5)$$

where Δt is the time step employed; and

$$\frac{\partial T}{\partial Z} = \frac{T_i^K + T_i^{K-1} - T_{i-1}^K - T_{i-1}^{K-1}}{2\Delta Z} \qquad (6)$$

where the subscripts refer to node boundaries, and ΔZ is the node length. Discretization of Eq. (1) according to Eqs. (5) and (6) produced a form from which the transition and deterministic input matrices could be obtained. The transition matrix elements were various combinations of materials properties, geometric conditions, and time step. The elements of the deterministic input matrix primarily involved coolant flow and fuel energy generation. Uncertainties in the model were estimated by assuming that each element of the state vector was comprised of a deterministic plus a random component. Utilizing Eqs. (1) through (6) or their equivalents the variance of the first four components could be calculated using the methods of Uhlenbeck and Ornstein [7]. In their method the variance in state variables is determined via a double integration procedure. Assuming no cross covariance between the variables and that the time correlation for each is a sharp function, it can be shown that the desired variances may be expressed as functions of the thermal time constants derivable from Eq. (1). Asymptotic values of temperature variances may be obtained from a 4x4 matrix equation whose coefficient matrix depends solely upon these thermal time constants and which is driven by variances in reactor power and coolant flow. The asymptotic values so derived are modified by a time dependent function whose rate of increase is dependent upon each material's thermal time constant. Variance in the fuel-clad heat transfer was determined by a knowledge of variances in the fuel and clad temperatures.

Two types of accident simulation experiments were analyzed -- a hypothetical loss of coolant leading to fuel melting and slumping causing a rapid rise in fuel power (L5 and L7 experiments); and a large and small reactivity induced transient overpower accident (E7 and H5 experiments). Mixed Pu-U oxide fuel pins used in these experiments were prototypic of LMFBR design composition and geometry but were 1/3 the design length. Three test pins in prototypic lattice geometry were used in the L experiments; seven in the E and H experiments. The assembly of test fuel pins with instrumentation was placed in a recirculating sodium loop and irradiated in the TREAT reactor. Reactor operation was pre-programmed to provide a desired fuel power history that was equivalent to the accident conditions being simulated.

Sodium flow in the test loop for all experiments was held constant during the first 3 seconds of steady state reactor operation to establish the desired fuel and coolant temperature distributions. Loss-of-flow conditions were established by computer control of the sodium pump on the loop. Power histories in the test fuel were established by computer control of the TREAT reactor.

Instrumentation included thermocouples at several axial locations on the exterior of the test fuel holder wall, sodium pressure and temperature measurements at the top and bottom of test pin, and the coolant flow rate into and out of the test flow channel. Loop data

and the TREAT power history were recorded on analog tape that was later digitized in 1 ms time intervals for the present analysis.

Lengths of data tracks used to determine the measurement noise covariance were selected to ensure weak stationarity of the data track. It was found that 0.5-s time tracks provided the required weak stationarity. To begin the calculations with the Kalman filter, the gain was set to zero and an initial estimate was chosen for the gap conductance based upon a representative analytical calculation [2].

Within the context of the model and the available measurements, implementation of the Kalman filter provided optimal estimates of the five state variables. From these estimates an estimate of gap conductance axially and as a function of time could be made. Knowledge of the fuel and cladding temperature together with the heat flux between the two provided the effective gap conductance. This, in turn, depended upon thermal resistances in the fuel and cladding as well as the actual conductance between the fuel and the cladding. Once these former effects were corrected for this latter parameter could be determined:

$$\frac{1}{h_{gap}} = \frac{T_2 - T_1}{q''_{1\text{-}2}} - \frac{r_{12}\ln\left(\frac{r_{12}}{r_1}\right)}{k_1} - \frac{r_2\ln\left(\frac{r_2}{r_{12}}\right)}{k_2}, \qquad (7)$$

where

h_{gap} = gap conductance
r_{12} = the interfacial radius between the fuel and cladding
r_1 = the volume-averaged radius of the fuel
r_2 = the volume-averaged radius of the clad
k = thermal conductivity of the material.

Discussion of Results

Consistent with the lumped parameter assumptions of the physical model, averaged values of gap conductance as a function of space and time were obtained for several experiments. Representative of the general results are those for quasi-steady state operation. Figure 1 presents the axial dependence of the gap conductance during such operations. A significant axial variation is noted. This variation is produced by the effects of the axial power generation within the fuel. Axial power generation increased from the base of the fuel rod roughly to the midplane of the fuel and decreased thereafter. A similar behavior is noted for the gap conductance. Since it is to be expected that gap conductance should be invesely proportional to the gap between the fuel and cladding (which is minimum at the axial location of maximum power generation) the observed behavior confirms expectation.

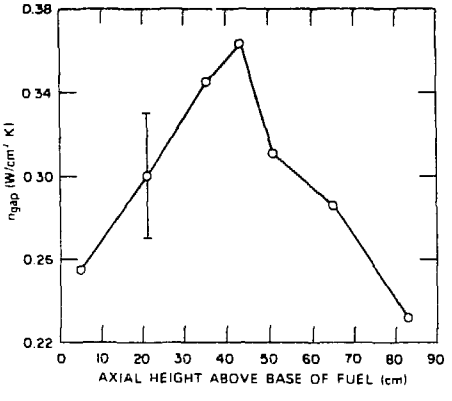

Fig. 1. Axial dependence of the gap conductance at 10 s into the L-7 quasi-steady state test as determined by the Kalman filter methodology.

Figure 2 presents the transient behavior of the gap conductance at the base, the axial midplane, and the top of the fuel rod in the same experiment. As can be seen an initial rapid increase in gap conductivity is followed by a gradual approach to equilibrium. This is due to the approach to steady state thermal conditions in the system as a whole. Decreases in the fuel-cladding gap are significantly greater during the initial heat up phase than later. As a consequence, regardless of axial location a value characteristic of steady state (at the given conditions of power and flow) eventually is achieved.

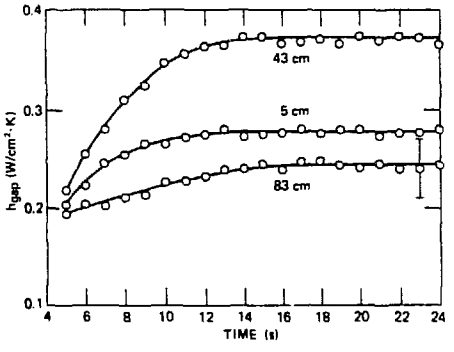

Fig. 2. Time dependence of the gap conductance in the L-7 quasi-steady state test at the top (83 cm), bottom (5 cm), and midheight (43 cm) of the test fuel pin.

In this quasi-equilibrium example the gap between the fuel and clad never closed. As a consequence the gap conductance would be expected to be inversely dependent upon this gap (and it was). Indeed a particularly simple model may be used to explain observed behavior in this realm: thermal gap conductance is an average gap thermal conductivity divided by the sum of an average jump distance plus the fuel-clad gap. the former two parameters, average thermal conductivity and jump distance, are dependent upon a variety of factors associted with conditions in the fuel-clad gap. the helium-fission gas composition of the gas in the gap, the presence and amount of solid fission products in the gap, and even geometry asymmetries all can be important in influencing average thermal conductivity and jump distance. As many of these factors are temperature (and hence time) dependent the value of these quantities must be expected to vary both temporally and spatially. Such is readily inferred from Figs. 1 and 2.

More generally, however, fast reactor accident sequences result in gap closure. Under these circumstances the simple gap conductance model discussed

above becomes inapplicable and instead a more complicated picture results. Once gap closure occurs the bearing of the fuel upon the clad enhances conduction of thermal energy. the interfacial pressure than can be brought to bear in such situations is limited both by the yield characteristics of the clad and by thermal creep in the fuel. These are complicated functions of the synergistics of the system as a whole and cannot easily be quantified. Careful experiment analysis is necessary to aid the understanding of this complex problem.

In the present analysis several experiments were analyzed which had gap closure. Generally, the simple "gap open" behavior proceeded to the more complex "gap closed" behavior in a relatively smooth transition. This can be attributed at least partially to the averaging characteristics of the methodology employed. Abrupt changes in heat transfer between fuel and clad can feedback to result in gap restoration with subsequent reclosure. An alternating sequence of such restorations and reclosures present a fairly smooth transition when averaged via the lumped parameter model employed here. It is also to be anticipated that solid fission products present in the gap will add a further thermal resistance to heat transfer which can serve to moderate any tendency toward abrupt increases in gap thermal conductance. Not to be obscured by the smoothness of the transition, however, is the general tendency for enhanced gap conductance during conditions of gap closure.

In summary, it has been shown that the linear Kalman can provide a useful analysis tool with which to infer fuel-clad gap conductance under conditions of actual LMFBR accident simulation. Knowledge of the magnitude and behavior of this parameter is important to the adequate modeling of off-normal reactor operation. Various models have been proposed to predict gap conductance. Unfortunately, no one model is uniformly effective in prediction. Data derived from careful experiment analysis must be forthcoming to correct these deficiencies.

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