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A STUDY OF CAPITAL REQUIREMENTS FOR SOLAR ENERGY

Final Report: Volume 2
Appendix B: The Hudson-Jorgenson Energy/Economic Model
("Long-Term Interindustry Transactions Model"): A Description

By
Edward A. Hudson
Dale W. Jorgenson

July 19, 1979

Work Performed Under Contract No. EM-78-C-01-4230

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ABSTRACT

The Hudson-Jorgenson Energy/Economic Model--formally known as the Long Term Interindustry Transactions Model (LITM)--is an econometric model of the structure of the U. S. economy. LITM integrates two separate models into one integrated system. These models are the Macroeconomic Model, a growth model incorporating the underlying trends of economic development, and the Interindustry Model, an endogenous coefficient input-output model of the structure of the economy incorporating patterns of expenditure, prices, and production on a sectoral basis. LITM emphasizes the energy system and its role within the economy. Applications of LITM have, therefore, focused on energy, the effect of energy changes on the economy, and the effect of econometric changes on the energy system. In addition, LITM can be used as framework for long term economic projection and structural analysis.

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I. INTRODUCTION

The Long Term Interindustry Transactions Model is a model of economic structure and economic growth. Its sectoral specification is oriented towards energy so that the model provides a framework for the analysis of interdependence within the energy system as well as interdependence between energy and the rest of the economy. In addition, the model provides a comprehensive basis for medium and long term economic and energy forecasting, and for the analysis of economic growth and structural change.

LITM integrates two models of economic activity. These models are the Macroeconomic and the Interindustry Models. Within these models, economic processes are separated into both national aggregate and sectoral activities. There are 10 domestic producing sectors, 6 of which cover energy extraction and processing, and 4 of which cover the main nonenergy producing sectors. Each producing sector is modeled in terms of price determination and input patterns. Final demand is separated into consumption, investment, government, and net exports. Consumption expenditure is based on a model of household behavior in which labor supply, consumption, and saving are all determined in a consistent manner. These final demands drive the production side of the model with an input-output system used to take account of the complex input requirements and interindustry flows required to sustain the final output of the economy. This whole production process is also subject to the limitations imposed by the availability of primary inputs, in particular, capital and labor. Over time, labor supply changes as a result of population and participation rate changes, while capital supply increases by investment net of depreciation. The LITM system places all these components within a consistent framework and produces a dynamic, general equilibrium model of the U.S. economy.

The LITM system has many innovative and powerful features. The two principal features are the endogenous coefficient, input-output models of producer behavior and the comprehensive, operational, general equilibrium system. The producer behavior models incorporate both price and quantity aspects of production in a single, consistent framework. This permits the consistent determination of output prices, output quantities, and input patterns (the endogenous input-output coefficients). In addition, household and other final demand behavior, input constraints, and input price determination are incorporated, and balance in interindustry transactions and input and output markets is imposed to derive a consistent, general equilibrium system.

Two types of simulation solutions may be performed. The first is the development of base case projections. In this mode, likely values of the exogenous variables are inserted and LITM is then solved to find the path and structure of economic and energy growth. The second type of solution concerns the impact on the energy and economic systems of changes in economic conditions or policies. These impacts are introduced through changes in the values of exogenous variables and coefficients. This can be used in sensitivity analysis form (what is the effect on the principal endogenous variables of some specified exogenous variable or equation specification?); it can be used in policy or contingency analyses (if conscious policy or uncontrolled events were to change in some specified way, what is the impact on key variables of concern?); or it can be used for uncertainty analysis (for example, what is the expected value of key variables given a probability distribution for critical, but uncertain, exogenous variables?). The LITM system is also set up in a framework which permits control types of analyses to be performed. This framework permits target values to be selected for endogenous variables and finds, as part of the model solution, the values of specified instruments needed to attain these targets. This permits control type experiments in the design of policy to be conducted and constraints on endogenous variables to be introduced into the system.

This report covers the theoretical specification and underpinning of the Long Term Interindustry Model system together with examples of its application to projecting the economy and policy analysis. Part II presents the theory of the model system in three components–the Macroeconomic Model (II.B), the Interindustry Model (II.C), and the integration of these two models into a unified system (II.D). Part III contains an example of the application of the system. Section III.A presents a base case projection of the economy and the energy system over the 1977 to 2000 period. Then, section III.B presents an analysis of a policy package of energy demand management and conservation policies.

II. THEORY

A. General

The major components of the Long Term Interindustry Transactions Model (LITM) are the Hudson-Jorgenson, Macroeconomic and Interindustry Models.[1] These two models are integrated into a consistent representation of economic structure and economic growth to form LITM. These two models, together with the integration procedure, are described in detail in this section, while applications of this system are illustrated in section III. While the Interindustry Model covers the entire economic system, it gives particular emphasis to the energy sectors. This provides the capability of linking detailed energy supply submodels to the model system, to produce a detailed representation of energy supply and demand and the economy as a whole. This potential has in fact been exploited in using LITM as an integrating framework for coal, oil, gas, and electricity supply models.[2]

The two major models in LITM are each self-contained models of the economic system and can be operated independently. Each model contains a representation of production activity. In the Macroeconomic Model, a production frontier relates outputs of consumption and investment goods to inputs of capital services, labor services, and productivity variables. The production structure of the Interindustry Model is an endogenous coefficient input-output structure in which production behavior by each of the domestic producing sectors is represented by a price possibility frontier. In the integrated LITM model, only one production representation can be used and the detailed Interindustry Model structure is employed.

For some applications, the Macroeconomic Model in stand-alone form may be an appropriate analytical tool. These are applications where only the aggregate structure and rate of economic growth are of concern. In this case, the basic exogenous variables are the time endowment of the household sector, the price of capital services, productivity trends, and government expenditures and tax rates. The Macroeconomic Model then simulates labor supply, consumption output, investment output, and capital stock, i.e., the main macroeconomic aggregates. For other

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[1] These models are presented in E. A. Hudson and D. W. Jorgenson, "U.S. Energy Policy and Economic Growth, 1975-2000," The Bell Journal of Economics and Management Science, Vol. 5, No. 2 (Autumn 1974a), pp. 461-514.

[2] The specific energy models included in this system are the Baughman-Joskow Regionalized Electricity Model, The Macavoy-Pindyck Model of Oil and Natural Gas, the NPC-Stitt Oil and Natural Gas Supply Model and The Zimmerman Coal Model.

applications, the Interindustry Model alone may provide the required analytical capability. These are applications where details of the economic structure at one time are the focus of concern. In this case, final demand expenditure aggregates and primary input supply prices and quantities are taken from a previous solution of the Macroeconomic Model. The Interindustry Model solves for sectoral output quantities and prices, input patterns, and final demand expenditure patterns.

For most analyses, however, it is important to consider production and spending activities in the Interindustry Model simultaneously and consistently with the macroeconomic picture. This requires that the Interindustry and Macroeconomic Models be run in integrated form. This is achieved by using the production relationships of the Interindustry Model as the production representation of the economy and by using only the household submodel, balance equations, and dynamic relationships from the Macroeconomic Model. Equations in existence in the original Macroeconomic Model relating to labor demand, production possibilities, and investment supply, are no longer used. The equations of the Macroeconomic Model are used only to provide the supply of capital and labor services and the demand for consumption goods in any given year, and to account for population, capital, and wealth accumulation over years. The remaining endogenous variables are obtained in the Interindustry Model. The system is in equilibrium when all final demands are equal to the supplies of final outputs and when all demands for inputs of primary factors are equal to the supplies of the primary factors.

B. The Macroeconomic Model

1. General Description

The Macroeconomic Model separates economic activity into four types of goods and services—output of consumption goods and services, output of investment goods, inputs of capital, and inputs of labor. The demand and supply for each of the commodities is calculated, and the equilibrium price and quantity patterns determined. In addition, the progression of this system over time is analyzed through capacity expansion resulting from capital growth, labor force growth, and technical improvement. The central component of this model is a macroeconometric production function, relating the output of consumption and investment goods to the inputs of capital and labor services and the level of production efficiency. A household sector submodel determines demand for consumption goods and the supply of labor. Variables relating to government expenditure, exports, and imports are exogenous in this model. The production function constrains the economy so that output cannot exceed capacity. Also, by determining the supply capacity of the economy, this function provides the means of introducing into the model the three fundamental growth producing forces—the supply of capital services, the supply of labor services, and the level of production efficiency. The capital services variable is obtained from the capital accumulation equation which expresses 1 year's capital in terms of the previous year's capital, depreciation, and new investment. The amount of investment is based on household preferences between present and future consumption, which determine the allocation of income between consumption and saving. Exogenous population growth, together with an endogenous work-leisure choice determine the growth of the labor force which then enters the production function through the labor input variable. Finally, disembodied, capital-embodied, and labor-embodied technical progress are incorporated directly into the production function. (The rates of each type of progress are based on extrapolations of past trends.) In this way, the basic dynamic causal factors of economic growth are included in the model.

The next step is to introduce submodels of producer and household behavior to determine demand and supply for each type of product. The producer submodel generates output supply and input demand functions on the basis of profit maximization in response to prices and within the constraint of the production function. This gives functions for the supply of consumption goods and of investment goods, and the demand for labor input and for capital input. Household behavior is modeled to simulate consumer response to market forces as consumption and labor decisions are made by the household in an attempt to maximize satisfaction. This generates consumption expenditure and labor supply functions that depend on prices, wealth, time, and preference parameters. The remaining elements of demand and supply are government and rest of world net demands. These are introduced, exogenously, to give a demand and supply function for each aggregate good—consumption goods, investment goods, labor services, and capital services.

These different demand and supply components are then reconciled within a simulated market process. There are two types of market clearing requirements. First, quantity demanded and supplied in each market must be brought into equality (with the exception that the labor market does not clear, this is recognized by the incorporation of an unemployment variable). Second, the value of expenditure on each type of good must equal the total value of receipts derived from the sale of the good. Market clearing conditions of both types are included in the model. They result in prices, and hence quantities, adjusting until equilibrium is achieved. In this way both prices and quantities for each of consumption goods output, investment goods output, labor input, and capital input are generated.

Once these basic production and price variables are determined, the national income accounting framework can be used to determine aggregate variables such as GNP, real GNP, the GNP price deflator, and the components of each of these variables.

2. Theories of Production and Household Behavior

This section briefly outlines the theory of production and household behavior underlying the macroeconomic growth model. The approach used in the macro-model is based on the neoclassical theory of economic growth in which inputs are related to output through a production function; household behavior determines the split of output between consumption and saving; and capital is augmented by net investment. (See for example Solow[3] for an elaboration of the one sector growth model and Burmeister and Dobell[4] for a discussion of two sector growth models.) The model used, however, is more detailed and complex than the theoretical growth models. First, the model has two inputs and two outputs and incorporates both prices and quantities for each; second, it introduces government and rest-of-world variables into the analysis; third, it introduces a household behavioral submodel that covers both the consumption-saving and labor-leisure choices; and fourth, the model is estimated econometrically to derive parameters based on the course of U.S. economic growth from 1947-1971.

The production sections of the growth model involve two inputs, capital and labor services, and two outputs, consumption and investment goods, which are linked by the production possibility frontier. This production possibility frontier is assumed to be separable between inputs and outputs and so can be arranged as:

$$f(CS, IS) = g(KD, LD)$$

where CS and IS are the output quantities of consumption goods and services, and of investment, and where KD and LD are the input quantities of capital and labor services, respectively. The actual specification of the production frontier is chosen to be of the most straight-forward, admissible form. The input function is taken to be Cobb-Douglas. For the output function, however, the Cobb-Douglas form is inadmissible as the resulting production frontier is not concave. Therefore, the next simplest form, the constant elasticity of transformation function, is used. The functional forms permit the production frontier to be specified as:

$$(a * CS^b + (1-a) * IS^b)^{\frac{1}{b}} = A * KD^d * LD^{1-d}$$

[3] Solow, R. M., "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, Vol. 70, February 1956, pp. 65-94.

[4] Burmeister, E. and A. R. Dobell, Mathematical Theories of Economic Growth, New York: Collier-MacMillan, 1970.

where a, b and d are parameters of the frontier and where A is an index of aggregate input to output productivity. The production frontier can then be expressed in translog form (following Christensen, Jorgenson, and Lau (1973))[5] to give:

$$\frac{1}{b} \ln(a * CS^b + (1\text{-}a) * IS^b) = \ln A + d * \ln KD + (1\text{-}d) * \ln LD$$

The left-hand side can be written as:

$$\frac{1}{b} * \ln(a * e^{b * \ln CS} + (1\text{-}a) * e^{b * \ln IS})$$

If a second order Taylor's series (around lnCS = 0, lnIS = 0) is taken, in terms of the logarithmic variables lnCS and lnIS, this expression becomes

$$a * \ln CS + (1\text{-}a) * \ln IS + \tfrac{1}{2}ba(1\text{-}a) * \ln^2 CS + \tfrac{1}{2}ba(1\text{-}a) * \ln^2 IS$$

$$- ba(1\text{-}a) * \ln CS * \ln IS$$

Therefore, the production frontier can be written as

$$a \bullet \ln CS + (1\text{-}a) * \ln IS + \tfrac{1}{2}ba(1\text{-}a) * (\ln CS - \ln IS) ** 2$$

$$= \ln A + d * \ln KD + (1\text{-}d) * \ln LD$$

For convenience in estimation, the parameters are altered slightly to obtain the equivalent form:

$$(1 + C1 - C2) * \ln CS + C2 + \ln IS + 0.5 * C3 * (\ln CS - \ln IS) ** 2$$

$$= \ln KD + C1 * \ln LD + (1 + C1) * \ln A$$

where C1, C2 and C3 are parameters to be estimated. The maximization of net revenue, subject to the production frontier, corresponds to a saddle point of the Lagrangean:

$$L = PCS * CS + PIS * IS - PKD * KD - PLD * LD - m((1 + C1 - C2)$$

$$\bullet \ln CS + C2 * \ln IS + 0.5 * C3 * (\ln CS - \ln IS) ** 2 - \ln KD$$

$$- C1 * \ln LD - (1 + C1) * \ln A).$$

[5]Christensen, L. R., D. W. Jorgenson and L. J. Lau, "Transcedental Logarithmic Production Frontiers," Review of Economics and Statistics, Vol. 55, No. 1, February 1973, pp. 28-45.

where m is the Lagrangean multiplier.

The saddle point is characterized by the following first order conditions:

$$PCS - m((1 + C1 - C2)/CS + C3 * (\ln CS - \ln IS)/CS) \quad 0$$

$$PIS - m(C2/IS - C3 * (\ln CS - \ln IS)/IS) = 0$$

$$-PKD + m/KD = 0$$

$$-PLD + m * C1/LD = 0$$

$$(1 + C1 - C2) * \ln CS + C2 * \ln IS + 0.5 * C3 *$$

$$(\ln CS - \ln IS) ** 2 - \ln KD - C1 * \ln LD - (1 + C1$$

$$* \ln A = 0$$

These can be re-expressed to give

(1) $$\frac{PCS * CS}{PKD * KD} = (1 + C1 - C2) + C3 * (\ln CS - \ln IS)$$

(2) $$\frac{PIS * IS}{PKD * KD} = C2 - C3 * (\ln CS - \ln IS)$$

(3) $$\frac{PLD * LD}{PKD * KD} = C1$$

(4) $$(1 + C1 - C2) * \ln CS + C2 * \ln IS + 0.5 * C3 * (\ln CS - \ln IS) ** 2$$

$$= \ln KD + C1 * \ln LD + (1 + C1) * \ln A$$

Equations (1)-(4) characterize the behavior of the production sector. They are econometrically fitted to time series data by three stage least squares. The estimated equations can then be used to depict production behavior in the growth model.

Household behavior is based on an intertemporal model of the household sector. In any given year, the household must decide how to allocate its resources between present consumption and future consumption, or equivalently, how much to spend during the year, and also how to allocate the pattern of consumption spending within the year. These two decisions are interrelated and, therefore, are treated as part of the same decision problem. This decision problem can be viewed as that of maximizing an intertemporal utility function subject to an intertemporal budget constraint. The household objective function used in the model of household behavior is:

$$U = \sum_{t=0}^{\infty} (1 + r)^{-t} \, (u * \ln C_t + v * \ln LJ_t)$$

i.e., the utility level is the total present value of the utility derived in each year from the consumption of goods and services, C, and from the consumption of leisure, LJ. The household plans over an infinite horizon, and 100r% is the household's rate of time p reference. The labor-leisure choice is explicitly included because actual household behavior over time reveals that the consumption of additional leisure is one of the principal ways in which higher living standards have been enjoyed.

The household sector formulates, based on the information available at the initial period, an optimal consumption and leisure plan for the entire future period. The first year of this plan is implemented. If external circumstances do not change from those expected, the following years of the plan would also be implemented. However, these external circumstances do change so the household sector formulates a succession of optimal plans. What is observed, then, is a sequence of initial years of successive optimal plans. Since a new maximization takes place in each time period, the current period is denoted by time index zero.

The expenditure choice is subject to a budget constraint, specifying that the present value of expenditure cannot exceed the present value of income and assets. The total budget comprises the value of wealth, time resources and unearned, or transfer, income; these elements make up the right-hand, or asset, side of the budget constraint. Expenditure comprises leisure, i.e., the direct consumption of time resources, and purchases of consumer goods and services; these items make up the left-hand side of the budget constraint. The budget constraint can now be expressed as:

$$\sum_{t=0}^{\infty} (1 + s)^{-t} \, (PC_t * C_t + PL_t * LJ_t)$$
$$= WL + \sum_{t=0}^{\infty} (1 + s)^{-t} \, (PL_t * LH_t + EL_t)$$

where s is the discount rate used, PC * C is expenditure on consumption of goods and services, PL * LJ is expenditure on leisure, WL is the initial wealth, PL * LH is the value of time resources, and EL is the value of net transfer receipts. The household decisions may be characterized by maximizing U subject to the budget constraint. This corresponds to a saddle point on the Lagrangean:

$$L = \sum_0^\infty (1 + r)^{-t} (u * \ln C_t + v * \ln LJ_t) - n(\sum_0^\infty (1 + s)^{-t} (PC_t * C_t$$
$$+ PL_t * LJ_t) - (WL + \sum_0^\infty (1 + s)^{-t} (PL_t * LH_t + EL_t))$$

where n is the Lagrangean multiplier.

The first order conditions are:

$$(1 + r)^{-t} * u * C_t^{-1} - n * (1 + s)^{-t} * PC_t = 0$$

$$(1 + r)^{-t} * v * LJ_t - n (1 + s) * PL_t = 0$$

$$WL + \sum_0^\infty (1 + s)^{-t} (PL_t * LH_t + EL_t) = \sum_0^\infty (1 + s)^{-t} (PC_t * C_t + PL_t * LJ_t)$$

The value of n can be found from the first order conditions by summing the first two equations over infinity and adding them together.

$$\frac{1}{n} = \frac{1}{m+v} * \frac{1}{\sum_0^\infty (1+r)^{-t}} * [\sum_0^\infty (1 + s)^{-t}(PC_t * C_t + PL_t * LJ_t)]$$

The first order conditions may be solved for the current consumption and leisure time for time t that is equal to zero.

$$PC_o * C_o = \frac{u}{n}$$

$$PL_o * LJ_o = \frac{v}{n}$$

which after substitution of 1/n is

$$PC_o * C_o = u/(u+v) * \frac{1}{\sum_0^\infty (1+r)^{-t}} * \sum_0^\infty (1+s)^{-t} (PC_t * C_t * PL_t * LJ_t)$$

$$PL_o * LJ_o = v/u+v * \frac{1}{\sum_0^\infty (1+r)^{-t}} * \sum_0^\infty (1+s)^{-t} (PC_t * C_t + PL_t * LJ_t)$$

at the aggregate level, PL_t and EL_t are expected to increase at constant rates. Thus, we may rewrite the demand functions for consumption goods and leisure in a simpler form for estimation. First, we substitute the budget constraint, and second, we consider the discount rate to be an unknown that will be estimated. Thus, we have

(5) $PC_o * C_o = C4 * WL + C5 * (PL * LH + EL)$

(6) $PL_o * LJ_o = C6 * WL + C7 * (PL * LH + EL)$

3. Specification of the Macroeconomic Model

The theory of production behavior outlined above generates the following equations characterizing the input and output preferred by producers:

$$\frac{PLD * LD}{PKD * KD} = C1$$

$$\frac{PIS * IS}{PKD * KD} = C2 - C3 * (\ln CS - \ln IS)$$

These equations can be viewed as defining the input demand and output supply functions of the production sector. The first equation specifies that the demand for labor services relative to capital input depends inversely on the ratio of the prices of the two inputs. The second equation specifies that the supply of investment goods depends on the price of investment goods, the input of capital services, and the volume of output already being devoted to supply of consumption goods.

Production must satisfy two further relations. First, there is the accounting identity between the value of output and value of inputs:

$$PCS * CS + PIS * IS = PLD * LD + PKD * KD$$

Given any three value components, the fourth is implied from this identity. In the model, the value of consumption goods supply is derived by this identity from the values of investment output and of capital and labor input. Second, there is the production possibility frontier which enforces the constraint that the quantity of total output is limited by the input quantities. Thus, output, whether of consumption or of investment goods, is limited by inputs, comprising capital and labor services together with the level of efficiency or productivity. The production frontier is, from the previous section, equation (4):

$$(1 + C1 - C2) * lnCS + C2 * lnIS + 0.5 * C3 * (lnCS - lnIS) **2$$

$$= lnKD + C1 * lnLD + (1 - C1) * lnA$$

The theory of household behavior provides equations predicting household behavior under specified economic conditions. These equations are the demand functions for the two consumption components: goods and services, and leisure. These demand functions are:

$$(5) \quad PC * C = C4 * WL + C5 * (PL * LH + EL)$$

$$(6) \quad PL * LJ = WL + C8 * PL * LH + C9 * EL +$$

$$C6 * WL + C7 * (PL * LH + EL)$$

The value of household expenditure on each type of consumption is determined by lagged household wealth, WL, the time resources of the household, PL * LH, and the net transfer income of households, EL. (The prices in these equations are net of tax, so PC is the price of consumption goods and services including any sales taxes or subsidies, and PL is the price received for labor services, net of income and payroll taxes.) The demand for leisure equation can alternatively be used to obtain the supply function for labor services. This equivalence is found by introducing the time constraint for the household sector:

$$LH = L + LJ$$

i.e., time allocated to labor plus leisure time must exactly match the total available amount of time, LH.

Equilibrium in any market process is characterized by the conditions that supply equals demand. This must be true in both quantity terms and in value terms. In quantity terms, the total sales in a market must equal the total purchases and, in value terms, the total value of receipts must equal the total value of expenditure. These conditions lead to a set of market balance equations. In each market, there is a quantity and a value balance equation. For consumption goods and services these equations are:

$$CS + CE = C + CG + CI + CR$$

$$(1 + TC) * PCS * CS + PCE * CE = PC * C + PCG * CG + PCI * CI + PCR * CR$$

Supply of consumption goods is from private producers (CS) or from government enterprises (CE). The demand is from households (C), government (CG), net exports (CR), and with changes in inventories (CI) absorbing any imbalance. The value equation is similar, except that sales taxes and subsidies must be incorporated because these absorb a portion of expenditure and do not accrue as receipts to producers. Similar relations hold for the market in investment goods:

$$IS + CI = I + IG + IR$$

$$(1 + TI) * PIS * IS + PCI * CI = PI * I + PIG * IG + PIR * IR$$

The supply of investment goods is from new production and from existing inventories, while the demand is from private purchases, government and net exports. In value terms, sales taxes are incorporated as a wedge between the price paid by the purchaser and the price received by the producer.

The demand for labor services must match the supply:

$$L = LD + LE + LG + LR + LU$$

where demand is from the private domestic sector, government enterprises, general government, net foreign demand, and unemployment is the exogenous remainder. All demands are exogenous except demand from the private sector. The corresponding value equation is:

$$PL * L = (1 - TL) * (PLD * LD + PLE * LE + PLG * LG + PLR * LR)$$

Taxes form a wedge between the price paid and the price received for labor services. Also, there is no direct payment to the unemployed. Income to the unemployed is from transfers and so are included in net transfer payment EL, which is incorporated into the household budget constraint.

The capital stock is, during any given year, fixed and so the supply of capital services is generated by the capital stock, rather than determined in the market. This process is summarized by:

$$KD = AK * KL$$

where KD are capital services and KL is the capital stock at the beginning of the year. The investment process is the means whereby the amount of capital is adjusted over time. This process operates through the dynamic relation:

$$K = AI * I + (1 - M) * KL$$

where K is the end of year capital stock. This relation specifies that the new capital stock is gross investment plus the depreciated size of the previous stock. The flow if income to capital is:

$$(1 - TK) * (PKD * KD - TP * PIL * AWL * KL) = N * PIL * AWL$$

$$* KL + D * PI * AL * KL - (PI * AL * KL - PIL * AWL * KL)$$

This specifies that income for capital services, less property taxes paid on the capital stock, and less income taxes paid on capital income, must equal the various components of capital income. These components, on the right-hand side of this equation, are the nominal yield to owners of the capital stock, plus depreciation on the stock, less capital gains.

The financial aggregate of savings and wealth can be constructed as:

$$S = PI * I + PG * (G - GL) + PR * (R - RL)$$

$$W = PI * AW * K + PG * G + PR * R$$

The first of these equations specifies that the value of savings must equal the value of investment which, in turn, is the value of private domestic investment together with new government and foreign investment. The second equation determines private domestic wealth as the value of private domestic capital together with net private claims on government and on the rest of the world.

The data base for the growth model is derived from the U.S. National Income and Product Accounts (NIPA).[6] These accounts are modified and extended to derive a full treatment of output and input flows and asset variables.[7] The

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[6]The National Income and Product Accounts are described in: The National Income and Product Accounts of the United States, 1929-1965, Statistical Tables, Bureau of Economic Analysis, U.S. Department of Commerce, Washington, D.C.

[7]On measurement of prices and quantities of input and output see: L. R. Christensen and D. W. Jorgenson, "U.S. Real Product and Real Factor Input, 1929-1967", Review of Income and Wealth, Series 16, No. 1, March 1970.

TABLE 1

COEFFICIENTS OF THE MACROECONOMIC MODEL

Coefficient	Value	Standard Error	t Statistic
C1	1.562814	.0186	84.02
C2	1.154118	.0163	70.80
C3	0.476591	.0173	27.55
C4	0.00280285	.00118	2.37
C5	0.146139	.00270	54.20
C6	0.0161712	.00682	2.37
C7	0.843158	.0156	54.20

output variables are defined so that consumption excludes purchases of consumer durables (these purchases are an addition to capital stock and so are included in investment) and includes the imputed value of services of household capital stock (the NIPA accounts include only the imputed value of services from owner-occupied residential structures). Investment comprises gross private domestic investment and includes purchases of new housing and other consumer durable equipment. Labor input is separated into price and quantity components with manhours-worked forming the basis for calculating the quantity of labor input. Capital income is separated from the NIPA accounts and treated as capital services to the existing capital stock. This procedure permits the price and quantity of capital services, as well as capital stock and rates of return on capital, to be identified. The capital stock data are developed independently through a cumulative net investment process for individual capital components which are then summed by Divisia aggregation. The capital accounts employ a methodology developed by Christensen and Jorgenson.[8] Finally, the income accounts are linked to a consistent set of wealth accounts to obtain estimates of wealth, rates of return on wealth, private sector claims on the government, and private sector claims on the rest of the world.[9]

4. Linkage Equations to the National Income Accounts

The variables of the growth model presented in the previous section are based on the Jorgenson system of economic accounting. This system is designed to measure the inputs to and outputs from the production process. As such, it provides an appropriate accounting framework for the analyses of economic growth. It differs from the NIPA in having a systematic accounting of the inputs into production, including both capital and labor inputs. The Jorgenson accounting system is well suited to the analysis of economic growth because it explicitly includes the main causal factors involved. For example, inputs of capital services and of labor services are explicitly measured, as well as the capital stock, the investment-capital dynamic linkage, and indices of aggregate technical progress.

Many uses of economic projections are based upon the standard NIPA definitions. Therefore, a set of linkage equations between the growth model and the NIPA GNP aggregates have been included in the growth model. Table 2 presents the parameter values for these linkage equations.

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[8] On the capital stock and capital input accounts see: L. R. Christensen and D. W. Jorgenson, "The Measurement of U.S. Real Capital Input, 1929-1967", Review of Income and Wealth, Series 15, No. 4, December 1969.

[9] An overview of the accounting system including income and wealth accounts is given in: L. R. Christensen and D. W. Jorgenson, "Measuring Economic Performance in the Private Sector", in M. Moss (ed.), The Measurement of Economic and Social Performance, Studies in Income and Wealth, No. 37, New York, Columbia University Press, 1973 and in: L. R. Christensen and D. W. Jorgenson, "U.S. Income, Saving and Wealth, 1929-1969", Review of Income and Wealth, Series 19, No. 4, December 1973.

TABLE 2

COEFFICIENTS OF THE LINKAGE EQUATIONS

Coefficient	Value
C8	-8.321
C9	0.2877
C10	-32.228
C11	0.4386
C12	0.366938
C13	0.364938

Two changes must be made so that the growth model output variables may be mapped into the NIPA output variables. The first concerns the imputed value of services of household sector capital (or consumers' durables) which are included in the Jorgenson measure of output but which are not, apart from imputed value of services of owner-occupied residential property, included in the NIPA output variables.[10] The second change concerns the purchases of new consumers' durables which are classified as investment in the Jorgenson accounts but as consumption in NIPA.

The imputed value of services of consumers' durables is one component of the value of services generated by the total private capital stock. This component can be modeled econometrically in terms of the total value of services of capital:

$$CDIM = C8 + C9 * PKD \cdot KD$$

where CDIM is the imputed value of services of consumers' durables and PKD * KD is the total value of capital services. A similar relation holds for these services in constant dollar terms:

$$CDIMQ = C10 + C11 * KD$$

where CDIMQ is the real value of imputed services of consumers' durables and KD is the real value of imputed services from total private capital.

Purchases of consumers' durables are included, in the growth model, in gross private domestic investment. The proportion of total investment that is made up of consumers' durable purchases can be explained by:

$$CDI = C12 * PI * I$$

where CDI is expenditure on consumers' durables and PI * I is total private investment. The corresponding relationship in real terms is:

$$CDIQ = C13 * I$$

where CDIQ is constant dollar expenditure on consumers' durables.

These equations permit the growth model variables to be expressed in terms of the standard NIPA variables. Personal consumption expenditure is equal to

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[10]The inputed value of services of household sector capital includes the services of owner occupied residential structures, as given in the national income and product accounts, and the implicit value of services of other consumer's durables.

the value of consumption in the growth model, less the imputed value of services of consumers' durables, and including expenditure on new consumers' durables:

$$CNIA = PC * C - CDIM + CDI$$

The same relationship holds in constant dollar terms:

$$CNIAQ = C - CDIMQ + CDIQ$$

From these, the implicit deflator for personal consumption expenditures can be obtained:

$$PCNIA = CNIA/CNIAQ.$$

Gross private domestic investment as defined in NIPA can be constructed from the growth model, as the total value of fixed investment plus inventory changes less the value of purchases of consumers' durables:

$$INIA = PI * I + PCI * CI - CDI$$

and, in constant dollars:

$$INIAQ = I + CI - CDIQ$$

The implicit price deflator for investment is then found as:

$$PINIA - INIA/INIAQ.$$

The government purchases component of GNP is equal to the value of government purchases of consumption goods and services, investment goods, and labor services:

$$GNIA = PCG * CG + PIG * IG + PLG * LG.$$

The corresponding equation in constant dollars is:

$$GNIAQ = CG + IG + LG$$

which permits the implicit price deflator for government purchases to be found:

$$PGNIA = GNIA/GNIAQ.$$

Net exports is the sum of net exports of consumption goods, investment goods and labor services. An adjustment is necessary to allow for the fact that CR in the growth model is net of income originating in the rest of the world. Therefore, this net income component must be added back into the net exports figure to obtain an estimate for the net exports of goods and services:

$$XNIA = PCR * CR + PIR * IR + PLR * LR + XW$$

The constant dollar form of the net export equation is:

$$XNIAQ = CR + IR + LR + XWQ.$$

Gross National Product (GNP) can be constructed directly from the components defined above. GNP is the sum of personal consumption expenditures, gross private domestic investment, government purchases of goods and services, and net exports. Therefore, current dollar Gross National Product is:

$$GNPNIA = CNIA + INIA + GNIA + XNIA$$

and constant dollar, or real, Gross National Product is:

$$GNPNAQ = CNIAQ + INIAQ + GNIAQ + XNIAQ$$

so that the aggregate price index, the GNP price deflator, is:

$$PGNPNA = GNPNIA/GNPNAQ.$$

5. Equations of the Macroeconomic Model

a. Production and Household Behavior

Investment supply:

$$\frac{PIS * IS}{PKD * KD} = C2 - C3 * (\ln CS - \ln IS)$$

Labor demand:

$$\frac{PLD * LD}{PKD * KD} = C1$$

Production possibility frontier:

$$\ln KD + C1 * \ln LD + (1 + C1) * \ln A$$

$$= (1 + C1 - C2) * \ln CS + C2 * \ln IS + 0.5 * C3 * (\ln CS - \ln IS) ** 2$$

Consumption demand:

$$PC * C = C4 * WL + C5 * (PL * LH + EL)$$

Leisure demand:

$$PL * LJ = C6 * WL + C7 * (PL * LH + EL)$$

b. Accounting Identities

Capital stock accumulation:

$$K = AI * I + (1 - M) * KL$$

Supply of capital services:

$$KD = AK * KL$$

Values of input and output:

$$PIS * IS + PCS * CS = PKD * KD + PLD * LD$$

Value of consumption goods supply and demand:

$$(1 + TC) * PCS * CS + PCE * CE = PC * C + PCG * CG + PCI * CI + PCR * CR$$

Value of investment goods supply and demand:

$$(1 + TI) * PIS * IS + PCI * CI = PI * I + PIG * IG + PIR * IR$$

Value of capital services:

$$(1 - TK) * (PKD * KD - TP * PIL * AWL * KL) = N * PIL * AWL * KL + D * PI * AL * KL - (PI * AL * KL - PIL * AWL * KL)$$

Value of labor services:

$$(1 - TL) * (PLD * LD + PLE * LE + PLG * LG + PLR * LR) = PL * L$$

c. Market Balance Equations

Consumption goods supply and demand:

$$CS + CE = C + CG + CI + CR$$

Investment goods supply and demand:

$$IS + CI = I + IG + IR$$

Allocation of time:

$$LH = L + LJ$$

Labor supply and demand:

$$L = LD + LE + LG + LR + LU$$

d. Financial Aggregates

Savings:

$$S = PI * I + PG * (G - GL) + PR * (R - RL)$$

Wealth:

$$W = PI * AW * K + PG * G + PR * R$$

Transfers:

$$EL = BL * (PLD * LD + PLE * LE + PLG * LG + PLR * LR)$$

e. Linkage Equations to National Income Accounts

Imputed services from consumers' durables:

$$CDIM = C12 + C13 * PKD * KD$$

$$CDIMQ = C14 + C15 * KD$$

Purchases of consumers' durables:

$$CDI = C16 * PI * I$$

$$CDIQ = C17 * I$$

Personal consumption expenditure aggregates:

$$CNIA = PC * C - CDIM + CDI$$

$$CNIAQ = C - CDIMQ + CDIQ$$

$$PCNIA = CNIA/CNIAQ$$

Gross private domestic investment aggregates:

$$INIA = PI * I + PCI * CI - CDI$$

$$INIAQ = I + CI - CDIQ$$

$$PINIA = INIA/INIAQ$$

Government purchases aggregates:

$$GNIA = PCG * CG + PIG * IG + PLG * LG$$

$$GNIAQ = CG + IG + LG$$

$$PGNIA = GNIA/GNIAQ$$

Net export aggregates:

$$XNIA = PCR * CR + PIR * IR + PLR * LR + XW$$

$$XNIAQ = CR + IR + LR + XWQ$$

Gross National Product aggregates:

$$GNPNIA = CNIA + INIA + GNIA + XNIA$$

$$GNPNAQ = CNIAQ + INIAQ + GNIAQ + XNIAQ$$

$$PGNPNA = GNPNIA/GNPNAQ$$

f. Lagged dependent variables

KL = K(-1)

PIL = PI(-1)

WL = W(-1)

g. Parameter values

C1 = 1.562814

C2 = 1.154118

C3 = 0.476591

C4 = 0.00280285

C5 = 0.146139

C6 = 0.0161712

C7 = 0.843158

C8 = -8.321

C9 = 0.2877

C10 = -32.228

C11 = 0.4386

C12 = 0.366938

C13 = 0.364988

6. Variables in the Macroeconomic Model

a. Endogenous Variables

(1) Growth Model Variables

Variable Name	Definition
C	Personal consumption expenditures, including imputed value of services of consumers' durables. (Billions of 1958 dollars.)
CS	Supply of consumption goods and services, including services of consumers' durables, by private enterprises. (Billions of 1958 dollars.)
EL	Government transfer payments to persons, other than social insurance funds. (Billions of dollars.)
I	Gross private domestic investment, including purchases of consumers' durables. (Billions of 1958 dollars.)
IS	Supply of investment goods by private enterprises, including supply of consumers' durables. (Billions of 1958 dollars.)
K	Capital stock, private domestic economy. (Billions of 1958 dollars, end of year value.)
KD	Services from the private domestic capital stock. (Billions of 1958 dollars.)
L	Supply of labor services.
LD	Purchases of labor services by the private domestic production sector.
LJ	Consumption of leisure time by the household sector.
N	Nominal rate of return on private domestic tangible assets.
PC	Implicit price deflator, personal consumption expenditures, including services of durables. (1958 = 1.)

Variable Name	Definition
PCS	Implicit price deflator, supply of consumption goods and services, including consumers' durables, by private enterprise.
PI	Implicit price deflator, gross private domestic investment, including purchases of consumers' durables. (1958 = 1.)
PIS	Implicit price deflator, supply of investment goods, including consumers' durables, by private enterprises.
PL	Implicit price deflator, supply of labor services by the household sector.
PLD	Implicit price deflator, purchases of labor services by the private domestic production sector. (1958 = 1.)
S	Gross private national saving. (Billions of dollars.)
W	Private national wealth. (Billions of dollars.)

(2) GNP Aggregates

Variable Name	Definition
CNIA	Personal consumption expenditures, NIPA definition. (Billions of dollars.)
CNIAQ	Real personal consumption expenditures, NIPA definition. (Billions of 1958 dollars.)
GNIA	Government purchases of goods and services, NIPA definition. (Billions of dollars.)
GNIAQ	Real government purchases of goods and services, NIPA definition. (Billions of 1958 dollars.)
GNPNIA	Gross National Product, NIPA definition. (Billions of dollars.)
GNPNAQ	Real Gross National Product, NIPA definition. (Billions of 1958 dollars.)

Variable Name	Definition
INIA	Gross private domestic investment, NIPA definition. (Billions of dollars.)
INIAQ	Real gross private domestic investment, NIPA definition. (Billions of 1958 dollars.)
PCNIA	Implicit price deflator, personal consumption expenditures, NIPA definition. (1958 = 1.)
PINIA	Implicit price deflator, gross private domestic investment, NIPA definition. (1958 = 1.)
PGNIA	Implicit price deflator, government purchases of goods and services, NIPA definition. (1958 = 1.)
PGNPNA	Implicit price deflator, Gross National Product, definition. (1958 = 1.)
XNIA	Net exports of goods and services, NIPA definition. (Billions of dollars.)
XNIAQ	Real net exports of goods and services, NIPA definition. (Billions of 1958 dollars.)

(3) Linkage Variables

Variable Name	Definition
CDI	Purchases of consumers' durables. (Billions of dollars.)
CDIM	Imputed value of services from existing stock of consumers' durables. (Billions of dollars.)
CDIMQ	Real services from existing stock of consumers' durables. (Billions of 1958 dollars.)
CDIQ	Real purchases of consumers' durables. (Billions of 1958 dollars.)

b. Exogenous Variables

Variable Name	Definition
A	Aggregate input to output productivity index. (1958 = 1.)
AI	Aggregation variable, relating real investment to real capital stock.
AK	Aggregation variable, relating quantity of capital services to beginning of year capital stock.
AL	Aggregation variable, relating lagged price of capital stock to price of investment.
AW	Aggregation variable, relating asset price to investment price.
AWL	Variable AW, lagged 1 year.
BL	Rate of government transfer payments to persons, other than from social insurance funds, relative to the labor income tax base.
CE	Supply of consumption goods and services by government enterprises. (Billions of 1958 dollars.)
CG	Government purchases of consumption goods and services. (Billions of 1958 dollars.)
CI	Change in business inventories of consumption goods. (Billions of 1958 dollars.)
CR	Net exports of consumption goods and services, less income originating in the rest of the world. (Billions of 1958 dollars.)
D	Rate of depreciation for private domestic tangible assets.
G	Net private domestic claims on government, excluding social insurance funds. (Billions of 1958 dollars, end of year value.)

Variable Name	Definition
GL	Net private domestic claims on government, excluding social insurance funds. (Billions of 1958 dollars, end of year value.) (G lagged 1 year.)
IG	Government purchases of investment goods. (Billions of 1958 dollars.)
IR	Net exports of investment goods. (Billions of 1958 dollars.)
LE	Purchases of labor services by government enterprises.
LG	Purchases of labor services by general government.
LH	Total time available to the household sector.
LR	Net purchases of labor by the rest of the world sector.
LU	Quantity of labor unemployed.
M	Rate of replacement for private domestic tangible assets.
P	Population. (Millions.)
PCE	Implicit price deflator, supply of consumption goods and services by government enterprises. (1958 = 1.)
PCG	Implicit price deflator, government purchases of consumption goods and services. (1958 = 1.)
PCI	Implicit price deflator, change in business inventories of consumption goods. (1958 = 1.)
PCR	Implicit price deflator, net exports of consumption goods and services, less income originating in rest of world. (1958 = 1.)
PG	Implicit price deflator, net private domestic claims on government. (1958 = 1.)

Variable Name	Definition
PIG	Implicit price deflator, government purchases of investment goods. (1958 = 1.)
PIR	Implicit price deflator, net exports of investment goods. (1958 = 1.)
PKD	Implicit price deflator for services from private domestic capital.
PLE	Implicit price deflator, purchases of labor services by government enterprises. (1958 = 1.)
PLG	Implicit price deflator, purchases of labor services by general government. (1958 = 1.)
PLR	Implicit price deflator, purchases of labor services by the rest of the world. (1958 = 1.)
PR	Implicit price deflator, net claims on the rest of the world. (1958 = 1.)
R	Net private claims on the rest of the world. (Billions of 1958 dollars, end of year value.)
RL	Net private claims on the rest of the world. (Billions of 1958 dollars, beginning of year value). (R lagged 1 year.)
TC	Effective rate of sales taxes, less subsidies, on the value of consumption expenditures.
TI	Effective rate of sales taxes on investment expenditures.
TK	Effective rate of tax on income to capital services.
TL	Effective rate of tax on income for labor services.
TP	Effective rate of tax on the value of capital stock.

Variable Name	Definition
XW	Net income originating, rest of the world. (Billions of dollars.)
XWQ	Real net income originating, rest of the world. (Billions of 1958 dollars.)

C. The Interindustry Model

1. General Description

The Interindustry Model is an econometric model of interindustry transactions for 10 domestic sectors, the demand for primary inputs, and the final demand for outputs. Thus, through a series of models of producer and consumer behavior, each component of the conventional input-output model is constructed. The principal innovation of the Interindustry Model is that the input-output coefficients are endogenous variables rather than fixed coefficients, and the personal consumption expenditure bridge shares are endogenous rather than fixed. In addition, the market balance equations ensure that the demand is equal to the supply in physical terms for each type of energy. The Interindustry Model permits the structure of technology to be determined as a function of the prices of sector outputs, and of the prices of capital, labor, and competing imports.

The Interindustry Model incorporates an input-output accounting framework which permits the entire chain of production to be traced through the economy. Table 3 presents the 10 sector classification scheme for the 6 energy and 4 nonenergy sectors. Table 4 presents the direct links of the j^{th} producing sector with the rest of the economy. The j^{th} producing sector purchases primary inputs of capital services, labor services, and foreign imports as well as intermediate inputs from the 10 producing sectors. The goods and services produced by the j^{th} sector go partly as inputs into other producing sectors and partly to satisfy final demands, i.e., the demands of consumers, investors, government, and the rest of the world. This input-output framework permits the analysis of interdependence between the various producing sectors in the economy. In particular, interdependence between energy sectors and the rest of the economy and interdependence between sectors or fuels within the energy system are explicitly considered. Other types of interdependence, for example between imports and U.S. production and between final demand and sectoral output levels, are also represented. This organization scheme is presented in table 5 in the conventional input-output form. This matrix

TABLE 3

INTERINDUSTRY TRANSACTION STRUCTURE: SECTORAL DIVISIONS

Primary Input Sectors	Producing Sectors	Final Demand Categories
1. Imports	1. Agriculture, Non-fuel Mining and Construction	1. Personal Consumption of Goods and Services
2. Capital Services	2. Manufacturing, Excluding Petroleum Refining	2. Gross Private Domestic Investment
3. Labor Services	3. Transportation	3. Government Purchases of Goods and Services
	4. Communications, Trade and Services	4. Exports
	5. Coal Mining	
	6. Crude Petroleum Extraction	
	7. Petroleum Refining	
	8. Electric Utilities	
	9. Natural Gas Utilities	
	10. Natural Gas Extraction	

NOTE: In the computer model, the sectors providing inputs (i.e., the 10 producing sectors and the 3 primary input sectors), are numbered consecutively, 1-13. Similarly, the receiving sectors (the 10 producing sectors and the 4 final demand categories) are numbered 1-14. This is the numbering scheme used in table 6.

TABLE 4

INPUTS AND OUTPUTS OF SECTOR j.

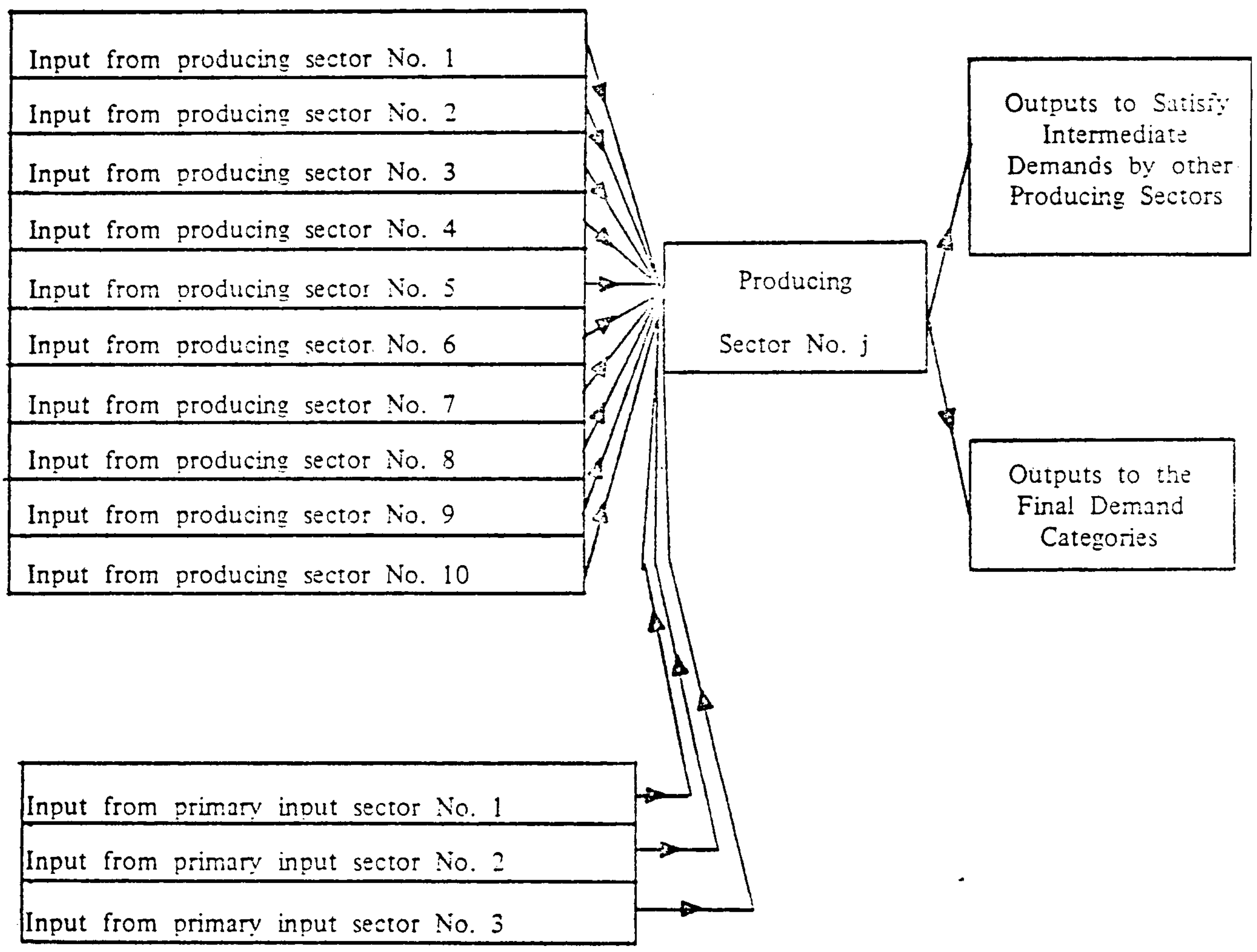

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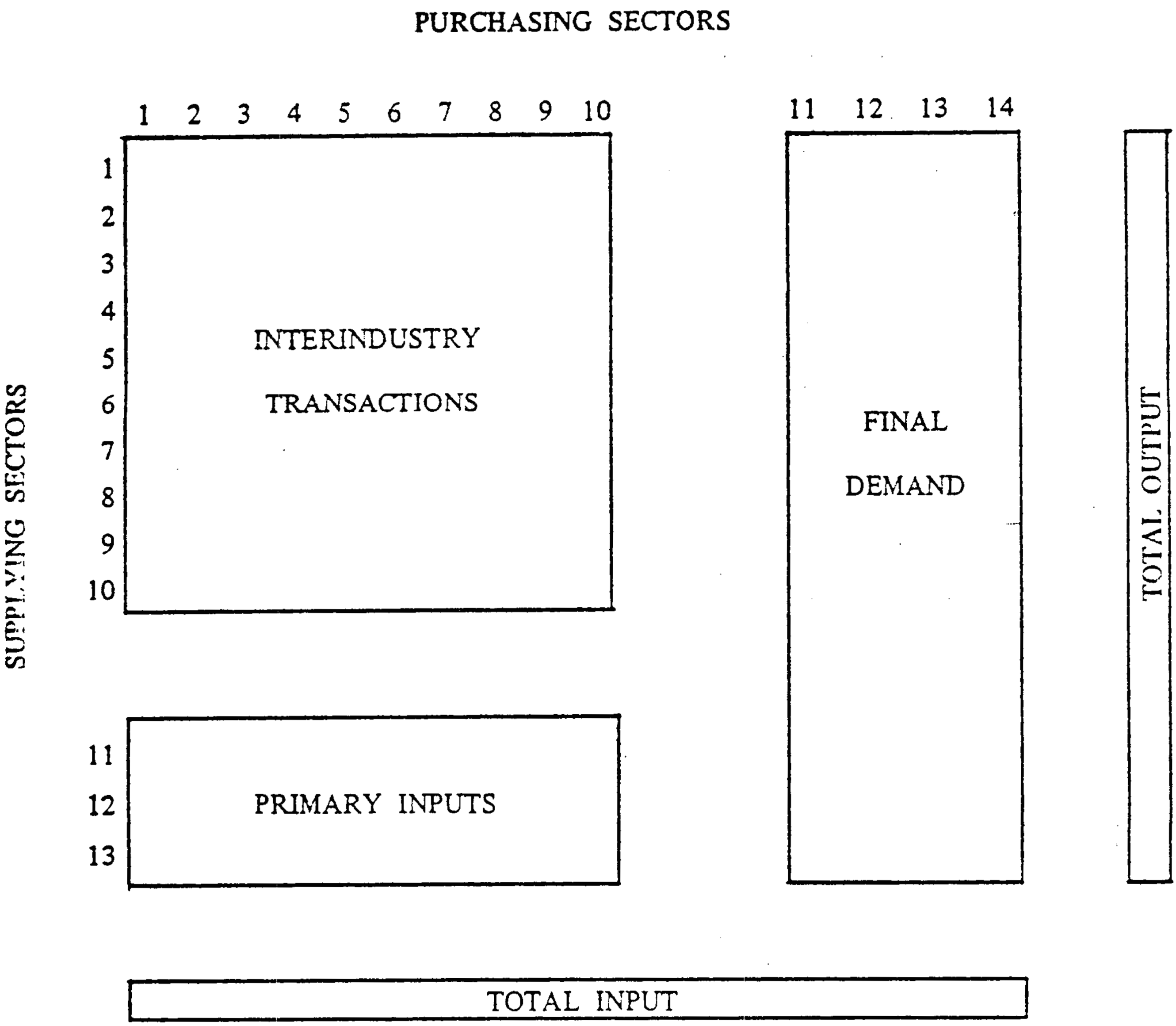
TABLE 5

INPUT-OUTPUT TRANSACTIONS TABLE



representation is also used in LITM as the organizational basis for the interindustry accounting system.

The specific role played by the input-output framework in the complete model is to link the primary, intermediate, and final sales in the economy into an accounting system so that these various parts of the economy form a consistent whole. Supply is the gross output of the domestic producing sector, and demand is intermediate input demands by all 10 producing sectors together with final demands by the personal consumption, investment, government purchases, and export sectors. Competitive imports of nonfuel goods are regarded as inputs into U.S. production and their levels are found endogenously. Imports of fuels are regarded as perfect substitutes for the domestic product and so enter directly to supplement the domestic supply in these sectors; fuel imports are exogenous to LITM. The model also includes accounting identities between the value of domestic availability of each type of product and the sum of the values of all purchases of inputs, including capital and labor, used in production.

The segments of the model that fit into this input-output framework come from two sources: detailed models of producer behavior and detailed models of final demand. The model of final demand is based on the expenditure components that are simulated in the Macroeconomic Model and has the function of allocating these expenditures among the supplying sectors. Producer behavior is modeled by 10 submodels, one for each producing sector. Each submodel relates sectoral output to inputs into the sector, incorporating information on input requirements and interrelationships derived from actual production data. These submodels are used, as described below, to determine the prices that producers must charge to cover costs, given primary input prices and given production constraints in each sector. Given the prices of each input, these submodels determine the efficient pattern of inputs into each sector, which minimizes unit costs of production. This efficient production pattern is represented by a set of input-output coefficients.

The input-output coefficients in each producing sector are endogenous which enables the model to capture a critically important feature of the modern economy—its flexibility in adapting to changes in relative prices. This flexibility permits producers to change input patterns, and to substitute within technical limits, the relatively less expensive or abundant input for the relatively expensive or scarce input. There is a comparable pattern of responsiveness of final demand patterns to relative prices in the models of final demand.

Once the production and final demand models have been solved, the Interindustry Model is equivalent to a conventional input-output model—it has a set of input-output coefficients, a final demand vector, and a fundamental equation relating gross output to intermediate demands for inputs and final demands in each sector. Also, prices are part of the solution so the resulting transactions matrix can be expressed in both current and constant dollars.

Once the transactions matrix has been found, further analysis of the energy sector is conducted. First, supply of each type of energy resource from U.S. production and exogenous fuel imports are determined. Demand for each fuel, less imports, then gives the quantity demanded from U.S. supplies. This demand is then compared to supply possibilities to determine whether there are any imbalances, given the ruling price of the fuel. If the Macroeconomic and Interindustry Models are used without any attached energy supply models, these supply possibilities are determined by judgmental estimates or by drawing upon governmental or industry supply analyses. If the model system includes the specific supply submodels, these submodels serve as supply functions, giving the quantity supplied at the ruling price. If the demand for U.S. supply is found to be infeasible at the ruling price, the price is altered and the entire solution sequence of both interindustry and supply models is repeated; this iteration process continues until resource demands are consistent with resource supplies.

The second type of analysis concerns the energy balance expression of energy information in terms of physical energy units. The interindustry transaction quantities are expressed in constant dollars, i.e., in the constant prices of the base year. These transactions are then expressed in terms of British Thermal Units by the use of BTU/constant dollar ratios, for each type of transaction, obtained from historical data. The application of other physical units to constant dollar ratios permits coal variables to be expressed in terms of tons, petroleum variables in barrels, natural gas in cubic feet, and electricity in kilowatt hours. The fuel prices used in the model are indices relative to the base year price. Multiplication of these indices by the base year price in dollars per physical unit enables the fuel prices to be expressed in terms of dollars per physical unit. Thus, the price information can be presented in terms of dollars per BTU, dollars per ton, dollars per barrel, dollars per cubic foot, and dollars per kilowatt hour.

2. Theory of Producer Behavior

The Interindustry Model is based on models of producer behavior for each of the 10 producing sectors.[11] The production relationships within each sector

[11] Production submodels are required for each of the 10 producing sectors. Historical interindustry data are available, however, for only 9 producing sectors; inputs into crude petroleum extraction and natural gas extraction are combined into a single crude petroleum and natural gas sector. Therefore, the pattern of inputs into crude petroleum and into natural gas extraction was taken to be the same, each being determined by a production model based on the combined data. There was one aspect in which these 2 submodels differed, however, and this was the level of total input-to-output productivity. This difference permits the different supply possibilities in each sector to move independently and permits the crude petroleum price to be independent of the natural gas price. For sectors other than crude petroleum and natural gas extraction, a full 10 sector breakdown of intermediate inputs is possible, as data is available on crude petroleum purchases and on crude natural gas purchases by each sector.

are represented by price possibility frontiers[12] which express the relationship between input prices and the sectoral output price. Although expressed in terms of prices, these frontiers embody the same information as the standard production function or production possibility frontier, i.e., information about input requirements, and substitutability and complementarity between inputs. These price frontiers determine the sectoral output prices as well as the demand for each input into each production sector. These demand functions are, in fact, the input-output coefficients which express demand for each input relative to the level of that sector's output. These sectoral output prices and input demand functions are independent of final demand, depending only on the price frontiers, the sectoral efficiency levels and the prices of competitive imports, capital services, and labor services. Once prices are known, final demands for each sector's output are calculated and the total level of output from each sector and the interindustry transactions are found by standard input-output techniques.

This separation of output pricing from the pattern of final demand draws on a set of nonsubstitution theorems that were first introduced by Samuelson.[13] These theorems state that when production takes place under constant returns and primary input prices are given, output prices are determined solely by supply consideration, independent of final demand. Figure 1 illustrates this theorem for a typical industry. The supply curve is horizontal because of the constant returns assumption and the additional assumption that the time frame is long enough to permit all short-run bottlenecks to be removed. In this case, given the supply curve, the demand curve determines the quantity produced and sold, but the price is determined by supply.

The price possibility frontiers of the production sectors define output price in terms of input prices and an efficiency index. The general form of these frontiers is:

$$P_i = F_i \, (P_{1i}, \ldots, P_{10i}, P_{Ki}, P_{Li}, P_{Ri}, t) \qquad i=1,10$$

where

P_i = average price of output from sector i;

P_{ji} = price of sector j output purchased by sector i;

[12] For a discussion of the price possibility frontier, see: L. R. Christensen, D. W. Jorgenson and L. J. Lau, "Transcendental Logarithmic Production Frontiers", Review of Economics and Statistics, Vol. 55, No. 1, February, 1973.

[13] See, for nonsubstitution theorems: P. A. Samuelson, The Collected Scientific Papers of Paul A. Samuelson, J. Stiglitz, ed., Cambridge MIT Press, 1966, pp. 513-536.

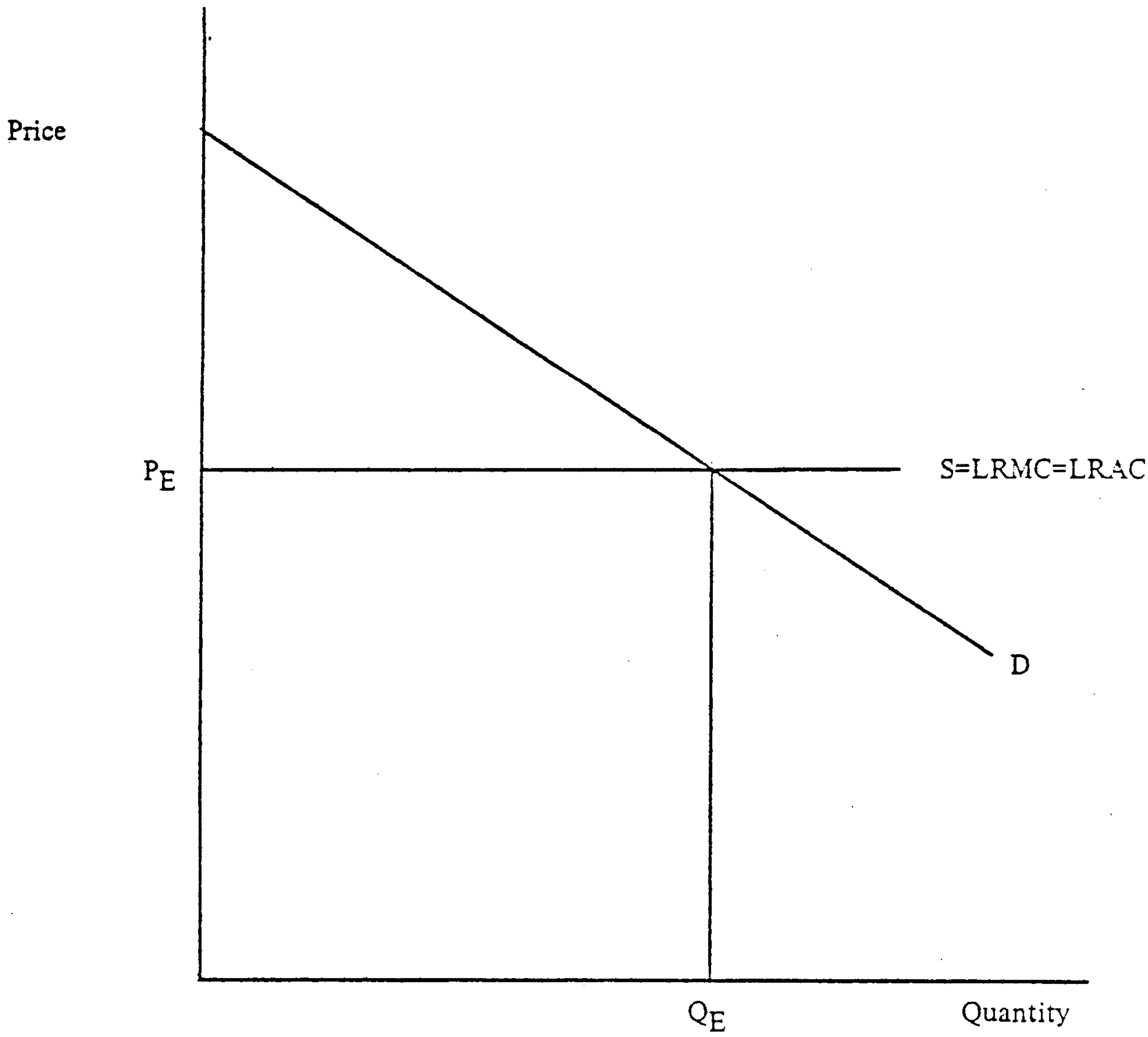


Figure 1. Industry Supply and Demand

PK_i = price of capital services purchased by sector i;

PL_i = price of labor services purchased by sector i;

PR_i = price of imports purchased by sector i;

t = time.

This general form specifies the output price in terms of the 13 input prices and time. The simplifying assumption is made that the productivity term is separable from the price arguments in the frontier. This means that efficiency or productivity changes affect all inputs equally, and it implies that the frontier can be expressed:

$$P_i = G_i\ (P_{1i}, ..., P_{10i,}\ P_{Ki}, P_{Li}, P_{Ri})*H_i\ (t) \qquad 1=1,10$$

where $H_i(t)$ is a multiplicative factor representing disembodied technical efficiency. This form can then be rearranged slightly by defining TE as the reciprocal of the function H so that:

$$P_i * TE_i = G_i\ (P_{1i}, ..., P_{10i}, P_{Ki}, P_{Li}, P_{Ri}) \qquad 1=1,10$$

TE is an index of input-to-output productivity, or of Hicks neutral technical efficiency. The greater the value of TE, the greater the quantity of output produced from a specified set of inputs and so the lower the price of output.

The general price possibility frontier can be expressed in the translog functional form. This form, introduced by Christensen, Jorgenson and Lau,[14] is a second order approximation to a general functional form and has the advantage of imposing no *a priori* restrictions on parameter values or on interrelationships between its included variables. In this form, the price frontiers become:

$$\ln P_i + \ln TE_i = \delta_{0i} + \sum_{j=1}^{13} \delta_{ji} \ln P_{ji} + ½ \sum_{j=1}^{13} \sum_{k=1}^{13} \beta_{jki} \ln P_{ji} \ln P_{ki} \qquad i=1,10$$

where the 13 input sectors comprise the 10 domestic producing sectors together with capital services, labor services, and imports.

[14] The translog functional form was introduced in: L. R. Christensen, D. W. Jorgenson, and L. J. Lau, "Conjugate Duality and the Transcedental Logarithmic Utility Function", Econometrica, Vol. 39, No. 4, July 1971.

An additional assumption is introduced on the form of these price frontiers in order to facilitate the econometric estimation. This simplification is necessary for three reasons. First, the data-base, a time series of input-output transactions matrices from 1947 to 1971, does not contain sufficient observations to estimate a large number of parameters. Second, the estimation of the frontiers must incorporate many restrictions over the parameters, i.e., for symmetry and concavity. Lastly, the estimation program must have the capability of handling simultaneously all 13 inputs into each sector.

In order to handle these requirements, the price possibility frontiers are constructed from a series of interdependent steps rather than direct estimation. The first step aggregates the six individual energy prices into an overall energy price index and the five intermediate materials prices into an overall materials price index. Both these aggregations are performed with translog functions. The second step is to construct the price possibility frontier itself in terms of the prices of capital services, labor services, the aggregate energy price, and the aggregate materials price. The price frontier remains a mapping between the individual input prices and the output price; the computations in this mapping are made through the intermediate step of aggregate energy and materials price indices.

Mathematically, this procedure may be viewed as follows. The price frontier is:

$$P_i * TE_i = G_i\ (P_{1i}, \ldots, P_{10i}, P_{Ki}, P_{Li}, P_{Ri})$$

This frontier is then expressed as:

$$P_i * TE_i = G(P_{Ki};\ P_{Li};\ P_{Mi}\ (P_{1i}, P_{2i}, P_{3i}, P_{4i}, P_{Ri})\ ;$$

$$P_{Ei}\ (P_{5i}, P_{6i}, P_{7i}, P_{8i}, P_{9i}, P_{10i})\ )$$

where P_{Mi} and P_{Ei} are functions of prices, the materials and energy prices, within the overall price frontier. The output price can then be found first by finding the values of P_{Mi} and P_{Ei} then, using P_{Ki}, P_{Li}, P_{Ei}, P_{Mi}, and TE_i, finding the value of P_i.

In specific terms, the output price index is computed from the price possibility frontier as follows.

(a) The prices of the energy and intermediate materials aggregates are expressed in terms of the individual prices:

$$\ln P_{Ei} = \sum_{j=5}^{10} \delta_{ji}^{E} \ln P_{ji} + \tfrac{1}{2} \sum_{j=5}^{10} \sum_{k=5}^{10} \beta_{jki}^{E} \ln P_{ji} \ln P_{ki}$$

$$\ln P_{Mi} = \sum_{j=1,2,3,4,R} \delta^{M}_{ji} \ln P_{ji} + \tfrac{1}{2} \sum_{j,k=1,2,3,4,R} \sum \beta^{M}_{jki} \ln P_{ji} \ln P_{ki} \quad \text{for } i=1,10$$

Thus, the aggregate energy price P_{Ei}, depends on the prices of the six types of energy—coal, crude petroleum, refined petroleum products, crude natural gas, gas from gas utilities, and electricity. Similarly, the aggregate price of intermediate materials, P_{Mi}, is defined in terms of the prices of the five nonenergy intermediate inputs—agriculture, manufacturing, transportation, services, and, for the nonenergy sectors, imports.

(b) The price of output as well as the Hicks neutral productivity index is expressed in terms of capital, labor, energy and intermediate materials prices:

$$\ln P_i + \ln TE_i = \delta_{0i} + \delta_{Ki} \ln P_{Ki} + \delta_{Li} \ln P_{Li} + \delta_{Ei} \ln P_{Ei} + \delta_{Mi} \ln P_{Mi}$$

$$+ \tfrac{1}{2} \sum_{j,k=K,L,E,M} \sum \beta_{jki} \ln P_{ji} \ln P_{ki} \quad \text{for } i=1,10$$

This partial separation of the price possibility frontier is a simplification imposed on the general translog function. The overall price possibility frontier is assumed to be separable and homogeneous in the inputs within each of the energy and the intermediate materials aggregates.[15] This is a sufficient condition for this tier structure to hold. The price possibility frontier is separable in the commodities within an aggregate if and only if the ratio of the relative shares of any two commodities within an aggregate is independent of the prices of commodities outside the aggregate.[16] For example, the six types of energy make up an appropriate aggregate if the relative value shares of any two types of energy depend only on the prices of energy and not on the prices of nonenergy intermediate inputs or the prices of capital services and labor services.

The price frontiers express output prices in terms of other output prices, primary input prices, technical efficiency indices, and the price frontier parameters. These frontiers must be simultaneously satisfied if the domestic production system is operating so that prices cover average costs of production in

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[15] For a discussion of the separability and homogeneity conditions see: L. R. Christensen, D. W. Jorgenson, and L. J. Lau, "Transcedental Logarithmic Production Frontiers".

[16] Separability is further discussed in: W. W. Leontief, "Introduction to a Theory of the Internal Structure of Functional Relationships", Econometrica, Vol. 157, No. 4, October 1947.

all 10 sectors. The econometrically estimated frontiers are used to solve for the domestic output prices by the following steps:

1. The technical efficiency parameters are inserted as exogenous data, based on extrapolations of recently observed data;

2. The primary input prices, i.e., price indices for competitive imports, capital services, and labor services are obtained from the Macroeconomic Model, simulated in conjunction with the Interindustry Model;

3. The prices at which interindustry transactions occur are expressed in terms of average sector output prices, i.e.,

$$P_{ji} = F_{ji} * P_i$$

where the markup factors F_{ji} are based on historical data;

4. Once the information 1.-3. has been inserted into the price possibility frontier, there remains a system of 10 equations in 10 variables—the output prices of the 10 domestic production sectors. The system is then solved for these output prices before the pattern of final demand is calculated.

Producers, organizing input purchases in a cost minimizing fashion, behave so that the value shares of inputs satisfy:

$$\frac{P_{ji} * X_{ji}}{P_i * X_i} = \frac{\partial \ln P_i}{\partial \ln P_{ji}}$$

i.e., the share of purchases of the j^{th} input in total purchases by the i^{th} sector is equal to the partial derivative of the logarithm of the output price of i with respect to the logarithm of the price of the j^{th} input into sector i. (This result is derived in appendix A.) The share equations for capital, labor, energy, and intermediate materials inputs are given by:

$$\frac{P_{Ki} * K_i}{P_i * X_i} = \alpha_{Ki} + \beta_{KKi} \ln P_{Ki} + \beta_{KLi} \ln P_{Li} + \beta_{KEi} \ln P_{Ei} + \beta_{KMi} \ln P_{Mi}$$

$$\frac{P_{Li} * L_i}{P_i * X_i} = \alpha_{Li} + \beta_{LKi} \ln P_{Ki} + \beta_{LLi} \ln P_{Li} + \beta_{LEi} \ln P_{Ei} + \beta_{LMi} \ln P_{Mi}$$

$$\frac{P_{Ei} * E_i}{P_i * X_i} = \alpha_{Ei} + \beta_{EKi} \ln P_{Ki} + \beta_{ELi} \ln P_{Li} + \beta_{EEi} \ln P_{Ei} + \beta_{EMi} \ln P_{Mi}$$

$$\frac{P_{Mi} * M_i}{P_i * X_i} = \alpha_{Mi} + \beta_{MKi} \ln P_{Ki} + \beta_{MLi} \ln P_{Li} + \beta_{MEi} \ln P_{Ei} + \beta_{MMi} \ln P_{Mi}$$

$$i = 1,10$$

Input quantities of capital services, labor services, energy, and intermediate materials are denoted by K, L, E, and M respectively, and X_i denotes the quantity of output from sector i.

The share equations within the energy and intermediate materials aggregates are analogous for sectors i = 1 to 10:

$$\frac{P_{ji} * X_{ji}}{P_{Ei} * E_i} = \alpha^E_{ji} + \sum_{k=5}^{10} \beta^E_{jki} \ln P_{ki} \qquad j = 5,\ 10$$

$$\frac{P_{ji} * X_{ji}}{P_{Mi} * M_i} = \alpha^M_{ji} + \sum_{k=1,2,3,4,R} \beta_{jki} \ln P_{ki} \qquad K = 1,2,3,4,R$$

These share equations are used in the Interindustry Model to derive the endogenous, input-output coefficients and are also used in the econometric estimation of the producer submodels. Since output prices have already been determined from the price possibility frontiers and the prices of capital, labor and imports, are determined in the Macroeconomic Model, the above equations can be solved for the numerical values of each share. The quantity demand functions, or input-output coefficients, can then be found as:

$$\frac{K_i}{X_i} = \frac{P_i}{PK_i} * (\alpha_{Ki} + \sum_{j=K,L,E,M} \beta_{Kji} \ln P_{ji}) \ ;$$

$$\frac{L_i}{X_i} = \frac{P_i}{P_{Li}} * (\alpha_{Li} + \sum_{j=K,L,E,M} \beta_{Lji} \ln P_{ji}) \ ;$$

$$\frac{X_{ji}}{X_i} = \frac{P_i}{P_{ji}} * (\alpha_{Ei} + \sum_{k=K,L,E,M} \beta_{Eki} \ln P_{ki}) (\alpha^E_{ji} + \sum_{k=5}^{10} \beta^E_{jki} \ln P_{ki})$$

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for the energy inputs, j = 5, 10; and

$$\frac{X_{ji}}{X_i} = \frac{P_i}{P_{ji}} (\delta_{Mi} + \sum_{k=K,L,E,M} \beta_{Mki} \ln P_{ki}) (\delta^M_{ji} + \sum_{k=1,2,3,4,R} \beta^M_{jki} \ln P_{ki})$$

for the intermediate materials inputs, j=1,2,3,4,R. These expressions determine the input-output coefficients endogenously, as functions of the prices which are already known.

3. Econometric Estimation of Production Submodels

The econometric estimation of the producer submodels is performed on the basis of the share equations presented on page 42. All the parameters in the price possibility frontiers, with the exception of the constant terms, are identified in the parameters of the share equations. The constant terms are identified from the base year values of the prices—as the price indices are, by definition, unity in the base year, the constant terms of the frontiers must be zero for the frontiers to apply in this year.

To be consistent with the economic theory of production, several restrictions must be imposed on the parameters of the price possibility frontiers. First, there is an accounting identity for each sector between the value of input and the value of output (profit is included in the price of capital services so that this identity always holds). This condition implies:

$$P_i * X_i = P_{Ki} * K_i + P_{Li} * L_i + P_{Ei} * E_i + P_{Mi} * M_i$$

for the aggregate price frontier, i = 1, 10; and

$$P_{Ei} * E_i = \sum_{j=5}^{10} P_{ji} * X_{ji}$$

$$P_{Mi} * M_i = \sum_{j=1,2,3,4,R} P_{ji} * X_{ji}$$

for the energy and intermediate materials aggregates. These identities mean that the relative shares must sum to unity, which implies the following parameter restrictions for the functions of aggregate output price:

$$\alpha_{Ki} + \alpha_{Li} + \alpha_{Ei} + \alpha_{Mi} = 1$$

$$\beta_{KKi} + \beta_{LKi} + \beta_{EKi} + \beta_{MKi} = 0$$

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$$\beta_{KLi} + \beta_{LLi} + \beta_{ELi} + \beta_{MLi} = 0$$

$$\beta_{KEi} + \beta_{LEi} + \beta_{EEi} + \beta_{MEi} = 0$$

$$\beta_{KMi} + \beta_{LMi} + \beta_{EMi} + \beta_{MMi} = 0 \qquad i = 1, 10$$

For the functions of the energy and intermediate goods aggregates, the implied parameter restrictions are:

$$\sum_{j=5}^{10} \alpha_{ji}^{E} = 1$$

$$\sum_{j=5}^{10} \beta_{jki}^{E} = 0 \qquad k = 5, 10$$

and

$$\sum_{j=1,2,3,4,R} \alpha_{ji}^{M} = 1$$

$$\sum_{j=1,2,3,4,R}^{10} \beta_{jki}^{M} = 0 \qquad k = 1,2,3,4,R$$

for i = 1, 10.

These restrictions permit one equation to be dropped from each set of share equations, and the parameters of this equation to be determined from the estimated parameters with these restrictions. In the aggregate (KLEM) submodels, the share equations for capital services, labor services, and energy are estimated with the materials share, determined from these three equations and the identity between the value of input and the value of output. For the share equations within energy input, the share of gas from gas utilities is determined from the other fitted shares and the identity. For the input of intermediate services into the nonenergy producing sectors, the import share is determined from the four fitted shares and the residual. For the energy sectors, imports are exogenous so only three intermediate material input shares are fitted. The fourth input share, services, is determined from the identity.

The second set of restrictions on the parameters of the price possibility frontiers is due to the theoretical result that, if the logarithm of the price possibility frontier is twice differentiable in the logarithms of the input prices, the

Hessian of the frontier should be symmetric. These restrictions place conditions on the second order parameters. For the aggregate (KLEM) submodel, the following parameter restrictions are implied:

$$\beta_{KLi} = \beta_{LKi} \quad , \quad \beta_{KEi} = \beta_{EKi} \quad ,$$

$$\beta_{LEi} = \beta_{ELi} \quad , \quad \beta_{KMi} = \beta_{MKi} \quad ,$$

$$\beta_{LMi} = \beta_{MLi} \quad , \quad \beta_{EMi} = \beta_{MEi} \quad . \qquad i = 1,\ 10$$

Analogous restrictions apply for the energy and intermediate materials submodels, i.e.,

$$\beta^{E}_{jki} = \beta_{kji} \qquad j,k = 5,10$$

$$\beta^{E}_{jki} = \beta_{kji} \qquad j,k = 1,2,3,4,R$$

for i = 1, 10.

The third set of theoretical restrictions is that the price possibility frontiers be convex. This convexity condition is imposed at the base year, using a method developed by Lau.[17] (This is detailed in appendix B.)

Another restriction is that some of the shares are identically zero. This is an empirical restriction arising from the fact that some sectors never purchase some types of inputs. For these cases, the corresponding parameters are set to zero and no estimation is required. The zero shares dealt with in this way are: crude petroleum and crude natural gas into transportation; coal mining and electric utilities; electricity into gas utilities; coal and gas from gas utilities into crude petroleum and crude natural gas.

The estimation of the producer submodels is based on a time series of input-output transactions data, in constant and current dollars, covering the period 1947 to 1971. The data are based on the Bureau of Economic Analysis input-output tables for 1947, 1958, and 1963 and also incorporate many other quantity, price, output and transactions time series and cross section data. The intermediate transactions components of the interindustry transactions matrices were

[17] The econometrics of the imposition of convexity at the base year are developed in: L. J. Lau, Econometrics of Monotonicity, Convexity and Quasiconvexity, Institute for Mathematical Studies in the Social Sciences, Stanford University, Technical Report No. 123, March 1974.

constructed by Jack Faucett Associates.[18] These data are based on the Bureau of Economic Analysis input-output tables in conjunction with time series for energy purchases (from the Bureau of Mines, Bureau of the Census, Federal Power Commission, American Petroleum Institute, and Edison Electric Institute) and with time series for sectoral output levels. The value added data in the Faucett transactions tables were edited and separated between labor and capital components in subsequent work by Data Resources, Inc. The capital service quantity data is based on Faucett capital stock data, and is a Divisia aggregate index of services from producers' durables equipment, non-residential structures, land and working capital in each industry. Developed by Christensen and Jorgenson, the capital services price index for each industry uses data from Jack Faucett Associates, the U.S. National Income and Product Accounts, and service price formulae for each asset.[19] The labor price and quantity data is based on a 67-industry data base aggregated into the 10 sector framework of this study. Chinloy, Gollop, and Jorgenson developed the labor data base.[20]

A simultaneous equation, estimation system was used to estimate the share equations. The estimation system had to accommodate the simultaneity and the nonlinearity of the equations. The method of "Iterative Minimum Distance" was used which is among the class of estimators that are consistent and uniformly asymptotically normal. This estimator has the advantage of taking into account information on all of the constraints. When non-linearities in the variables are not present, the "Iterative Minimum Distance" estimator is equivalent to three stage least

[18] The interindustry transactions data are developed and presented in: Jack Faucett Associates, Data Development for the I-O Energy Model, Final Report to the Energy Policy Project, Chevy Chase, Maryland, May 1973 and Jack Faucett Associates, Data Development for the I-O Energy Model: Comments and Revisions, Chevy Chase, Maryland, September 1973.

[19] The capital data in the interindustry transactions tables are based on: Jack Faucett Associates, Development of Capital Stock Series by Industry Sector, Chevy Chase, Maryland, March 1973; Jack Faucett Associates, Data Development for the I-O Energy Model, Final Report to the Energy Policy Project, Chevy Chase, Maryland, May 1973; Jack Faucett Associates, Data Development for the I-O Energy Model: Comments and Revisions, Chevy Chase, Maryland, September 1973; Jack Faucett Associates, Measures of Working Capital, Chevy Chase, Maryland, September 1973; L. R. Christensen and D. W. Jorgenson, "The Measurement of U.S. Real Capital Input, 1929-1967", Review of Income and Wealth, Series 15, No. 4, December 1969.

[20] The development of the labor input data for the interindustry transactions tables is presented in: F. M. Gollop, Modelling Technical Change and Market Imperfections: An Econometric Analysis of Manufacturing, 1947-1971, Unpublished Ph.D. thesis, Department of Economics, Harvard University, Cambridge, Massachusetts, July 1974. See also, F. M. Gollop and D. W. Jorgenson, "U.S. Total Factor Productivity by Industry, 1949-1973", paper presented at the NBER Conference on New Developments in Productivity Measurement, Williamsburg, Virginia, November 1975.

squares. The "Iterative Minimum Distance" estimator has been discussed by Jorgenson and Laffont and Malinvaud.[21,22]

Tables 6 to 8 present the estimated parameters of the translog price frontiers for the aggregate (KLEM) function and the energy and intermediate materials submodels for each of the 10 producing sectors. The asymptotic standard errors of the estimates of the parameter values are given in parentheses in these tables. For ease of interpretation of these tables, the list of producing sectors is repeated:

1. Agriculture, Non-fuel mining, Construction.
2. Manufacturing, excluding petroleum refining.
3. Commercial Transportation.
4. Services, Trade, Communications.
5. Coal Mining.
6. Crude Petroleum.
7. Petroleum Refining.
8. Electric Utilities.
9. Gas Utilities.
10. Natural Gas Extraction.

In addition, the nomenclature for the parameters and parameter subscripts in tables 6 to 8 are:

K. Input of capital services.
L. Input of labor services.
E. Input of energy.
M. Input of nonenergy intermediate materials.

Also, the subscript notation within the energy and materials submodels is:

Energy Input

1. Coal Mining.
2. Crude Petroleum.

[21] On the estimation of non-linear equation systems incorporating parameter systems and on the minimum distance estimator see: E. Malinvaud, Statistical Methods of Econometrics. Amsterdam, North-Holland, 1970; D. W. Jorgenson and J. J. Laffont, "Efficient Estimation of Non-Linear Simultaneous Equations with Additive Disturbances", Annals of Social and Economic Measurement, Vol. 3, No. 4, October 1974.

[22] The computational algorithm employed in the estimation is based on the Gauss method with variable step size. The computational algorithm is discussed in detail in: E. R. Berndt, B. H. Hall, R. E. Hall, and J. E. Hausman, "Estimation and Inference in Nonlinear Structural Models", Annals of Social and Economic Measurement, Vol. 3, No. 4, October 1974.

TABLE 6

ESTIMATED PARAMETERS OF THE TRANSLOG PRICE POSSIBILITY FRONTIER FOR THE AGGREGATE (KLEM) SUBMODEL FOR 10 PRODUCING SECTORS OF THE U.S. ECONOMY, 1947-71.

	Producing Sector									
Parameter	1	2	3	4	5	6	7	8	9	10
α_K	.1785 (.0019)	.1149 (.0020)	.1799 (.0031)	.2994 (.0039)	.1277 (.0039)	.5616 (.0110)	.0753 (.0020)	.3458 (.0130)	.3206 (.0084)	.5616 (.0110)
α_L	.2354 (.0081)	.2940 (.0063)	.4096 (.0043)	.4173 (.0030)	.4139 (.0119)	.0838 (.0044)	.1162 (.0028)	.1925 (.0043)	.1008 (.0038)	.0838 (.0044)
α_E	.0244 (.0009)	.0202 (.0063)	.0380 (.0017)	.0182 (.0004)	.1857 (.0105)	.0947 (.0012)	.5490 (.0090)	.2120 (.0136)	.4895 (.0062)	.0947 (.0012)
α_M	.6616 (.0121)	.5708 (.0089)	.3726 (.0069)	.2653 (.0056)	.2727 (.0158)	.2599 (.0142)	.2595 (.0084)	.2496 (.0071)	.0891 (.0115)	.2599 (.0142)
β_{KK}	.0851 (.0007)	.0590 (.0057)	.1018 (.0148)	.0595 (.0005)	.0280 (.0036)	.2462 (.0668)	.0402 (.0036)	.1330 (.0096)	.1224 (.0590)	.2462 (.0668)
β_{KL}	-.0366 (.0015)	.0030 (.0024)	-.0601 (.0170)	.0114 (.0002)	-.0357 (.0074)	-.0470 (.0293)	.0620 (.0569)	.0288 (.0097)	.0179 (.0048)	-.0470 (.0293)
β_{KE}	-.0052 (.0002)	-.0055 (.0087)	-.0137 (.0043)	.0011 (.0001)	.0099 (.0046)	-.0532 (.0128)	-.0706 (.0436)	-.1682 (.1109)	-.0975 (.3447)	-.0532 (.0128)
β_{KM}	-.0435 (.0007)	-.0565 (.0322)	-.0280 (.0304)	-.0719 (.0012)	-.0022 (.0012)	-.1460 (.1785)	-.0316 (.0149)	.0064 (.0037)	-.0429 (.2536)	-.1460 (.1785)
β_{LL}	.0287 (.0007)	.0737 (.0419)	.0582 (.0015)	.0848 (.0001)	-.0751 (.0964)	.0655 (.0488)	-.0674 (.2259)	-.0968 (.0385)	.0642 (.0742)	.0655 (.0488)
β_{LE}	.0023 (.0001)	.0054 (.0029)	-.0180 (.0065)	.0098 (.0003)	.1145 (.0380)	.0284 (.0288)	.0066 (.0075)	.0239 (.0200)	-.0806 (.0389)	.0284 (.0288)

TABLE 6

ESTIMATED PARAMETERS OF THE TRANSLOG PRICE POSSIBILITY FRONTIER FOR THE AGGREGATE (KLEM) SUBMODEL FOR 10 PRODUCING SECTORS OF THE U.S. ECONOMY, 1947-71. (Continued)

Parameter	Producing Sector 1	2	3	4	5	6	7	8	9	10
β_{LM}	.0056	-.0821	.0199	-.1059	-.0037	-.0469	-.0012	.0441	-.0015	-.0469
	(.0001)	(.0846)	(.0024)	(.0018)	(.0025)	(.0281)	(.0006)	(.0953)	(.0009)	(.0281)
β_{EE}	.0072	.0188	.0198	.0020	.0087	-.0319	.1846	.0638	.2128	-.0319
	(.0003)	(.0021)	(.0023)	(.0024)	(.0070)	(.0098)	(.1110)	(.0310)	(.1727)	(.0098)
β_{EM}	-.0044	-.0187	.0119	-.0129	-.1332	.0566	-.1205	.0805	-.0347	.0566
	(.0002)	(.0234)	(.0152)	(.0013)	(.0524)	(.0500)	(.1515)	(.1151)	(.2371)	(.0500)
β_{MM}	.0422	.1573	-.0038	.1907	.1392	.1362	.1533	-.1311	.0790	.1362
	(.0002)	(.2419)	(.0012)	(.0034)	(.0686)	(.2619)	(.2847)	(.1600)	(.0826)	(.2619)

TABLE 7

ESTIMATED PARAMETERS OF THE TRANSLOG PRICE POSSIBILITY FRONTIER FOR THE ENERGY (E) SUBMODEL FOR 10 PRODUCING SECTORS.

	Producing Sector									
Parameter	1	2	3	4	5	6	7	8	9	10
α_1^E	.0053	.2040	.0799	.1142	.8510	.0000	.0021	.3165	.0068	.0000
	(.0011)	(.0177)	(.0393)	(.0553)	(.0200)	(.0000)	(.0004)	(.0089)	(.0039)	(.0000)
α_2^E	.0021	.0002	.0000	.0051	.0000	.8885	.8104	.0000	.0000	.8885
	(.0002)	(.0000)	(.0000)	(.0016)	(.0000)	(.0041)	(.0044)	(.0000)	(.0000)	(.0041)
α_3^E	.8389	.3384	.8107	.3520	.0400	.0365	.1180	.1173	.0190	.0365
	(.0039)	(.0055)	(.0847)	(.0295)	(.0055)	(.0038)	(.0039)	(.0107)	(.0084)	(.0038)
α_4^E	.1212	.2858	.0406	.4136	.1062	.0750	.0057	.3829	.0000	.0750
	(.0074)	(.0062)	(.0065)	(.0096)	(.0654)	(.0052)	(.0004)	(.0091)	(.0000)	(.0052)
α_5^E	.0325	.1716	.0688	.1091	.0029	.0000	.0267	.1829	.5785	.0000
	(.0021)	(.0101)	(.0072)	(.0052)	(.0014)	(.0000)	(.0010)	(.0053)	(.0169)	(.0000)
α_6^E	.0000	.0000	.0000	.0060	.0000	.0000	.0371	.0000	.3958	.0000
	(.0000)	(.0000)	(.0000)	(.0019)	(.0000)	(.0000)	(.0002)	(.0000)	(.0057)	(.0000)
β_{11}^E	.0052	.1624	.0735	.1011	-.0118	.0000	.0017	.0762	.0068	.0000
	(.0032)	(.1844)	(.0930)	(.1942)	(.0433)	(.0000)	(.0013)	(.1069)	(.0115)	(.0000)
β_{12}^E	.0000	.0000	.0000	-.0006	.0000	.0000	-.0095	.0000	.0000	.0000
	(.0000)	(.0000)	(.0000)	(.0016)	(.0000)	(.0000)	(.0135)	(.0000)	(.0000)	(.0000)
β_{13}^E	-.0068	-.0690	-.0648	-.0402	.0220	.0000	.0067	.0833	-.0001	.0000
	(.0123)	(.0824)	(.0924)	(.0629)	(.0683)	(.0000)	(.0107)	(.0310)	(.0002)	(.0000)
β_{14}^E	.0011	-.0583	-.0032	-.0472	-.0100	.0000	.0009	-.0578	.0000	.0000
	(.0023)	(.0755)	(.0024)	(.0601)	(.0138)	(.0000)	(.0054)	(.0386)	(.0000)	(.0000)

TABLE 7

ESTIMATED PARAMETERS OF THE TRANSLOG PRICE POSSIBILITY FRONTIER FOR THE ENERGY (E) SUBMODEL FOR 10 PRODUCING SECTORS. (Continued)

Parameter	Producing Sector 1	2	3	4	5	6	7	8	9	10
β^E_{15}	.0005 (.0012)	-.0350 (.0585)	-.0055 (.0075)	-.0125 (.0048)	-.0002 (.0003)	.0000 (.0000)	.0003 (.0005)	-.1017 (.0136)	-.0040 (.0081)	.0000 (.0000)
β^E_{16}	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	-.0007 (.0019)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	-.0027 (.0091	.0000 (.0000)
β^E_{22}	.0010 (.0011)	-.0029 (.0029)	.0000 (.0000)	.0047 (.0054)	.0000 (.0000)	.0239 (.0186)	-.0254 (.0281)	.0000 (.0000)	.0000 (.0000)	.0239 (.0186)
β^E_{23}	.0007 (.0112)	-.0001 (.0003)	.0000 (.0000)	-.0018 (.0022)	.0000 (.0000)	-.0249 (.0180)	.0389 (.1329)	.0000 (.0000)	.0000 (.0000)	-.0249 (.0180)
β^E_{24}	-.0007 (.0010)	.0073 (.0292)	.0000 (.0000)	-.0040 (.0112)	.0000 (.0000)	.0010 (.0246)	.0126 (.0102)	.0000 (.0000)	.0000 (.0000)	.0010 (.0246)
β^E_{25}	-.0010 (.0028)	-.0043 (.0172)	.0000 (.0000)	-.0010 (.0790)	.0000 (.0000)	.0000 (.0000)	-.0165 (.0075)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)
β^E_{26}	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0027 (.0031)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)
β^E_{33}	-.0252 (.0696)	.2239 (2.2254)	.1534 (.2169)	.2281 (.5186)	-.0287 (.0311)	-.0232 (.0247)	-.0206 (.0668)	.0001 (.0001)	.0186 (.0384)	-.0232 (.0247)
β^E_{34}	.0128 (.0219)	-.0967 (.2632)	-.0329 (.0491)	-.1456 (.3877)	.0064 (.0128)	.0481 (.0100)	-.0164 (.1125)	-.0966 (.0211)	.0000 (.0000)	.0481 (.0100)
β^E_{35}	.1854 (.2266)	-.0581 (.0612)	-.0557 (.1172)	-.0384 (.5297)	.0003 (.0005)	.0000 (.0000)	-.0086 (.0439)	.0162 (.0129)	-.0110 (.0212)	.0000 (.0000)

TABLE 7

ESTIMATED PARAMETERS OF THE TRANSLOG PRICE POSSIBILITY FRONTIER FOR THE ENERGY (E) SUBMODEL FOR 10 PRODUCING SECTORS. (Continued)

Parameter	Producing Sector									
	1	2	3	4	5	6	7	8	9	10
β^E_{36}	.0000	.0000	.0000	-.0021	.0000	.0000	.0000	.0000	-.0075	.0000
	(.0000)	(.0000)	(.0000)	(.0118)	(.0000)	(.0000)	(.0000)	(.0000)	(.0169)	(.0000)
β^E_{44}	-.0410	.1868	.0389	.2425	.0055	-.0490	.0038	.2077	.0000	-.0490
	(.4766)	(.1566)	(.2080)	(.0974)	(.0059)	(.0162)	(.0043)	(.1358)	(.0000)	(.0162)
β^E_{45}	.0278	-.0390	-.0028	-.0451	-.0019	.0000	-.0008	-.0502	.0000	.0000
	(1.2038)	(3.8738)	(.0210)	(2.5537)	(.1668)	(.0000)	(.1103)	(.1910)	(.0000)	(.0000)
β^E_{46}	.0000	.0000	.0000	-.0006	.0000	.0000	.0000	.0000	.0000	.0000
	(.0000)	(.0000)	(.0000)	(.0017)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)
β^E_{55}	-.0158	.1364	.0640	.0972	.0018	.0000	.0257	.1357	.0604	.0000
	(.9129)	(3.7713)	(1.1127)	(2.2070)	(.0257)	(.0000)	(.0578)	(.0146)	(.1072)	(.0000)
β^E_{56}	.0000	.0000	.0000	-.0002	.0000	.0000	.0000	.0000	-.0455	.0000
	(.0000)	(.0000)	(.0000)	(.0002)	(.0000)	(.0000)	(.0000)	(.0000)	(.1031)	(.0000)
β^E_{66}	.0000	.0000	.0000	.0009	.0000	.0000	.0000	.0000	.0557	.0000
	(.0000)	(.0000)	(.0000)	(.0010)	(.0000)	(.0000)	(.0000)	(.0000)	(.1149)	(.0000)

TABLE 8

ESTIMATED PARAMETERS OF THE TRANSLOG PRICE POSSIBILITY FRONTIER FOR THE INTERMEDIATE MATERIALS (M) SUBMODEL FOR 10 PRODUCING SECTORS.

	Producing Sector									
Parameter	1	2	3	4	5	6	7	8	9	10
α_1^M	.2578 (.0275)	.1348 (.0167)	.1221 (.0138)	.0819 (.0048)	.0193 (.0032)	.0738 (.0097)	.0448 (.0076)	.1134 (.0044)	.0909 (.0031)	.0738 (.0097)
α_2^M	.3777 (.0225)	.5933 (.0148)	.1373 (.0149)	.2548 (.0194)	.4270 (.0295)	.1464 (.0027)	.2131 (.0293)	.1046 (.0062)	.1408 (.0087)	.1464 (.0027)
α_3^M	.0653 (.0035)	.0472 (.0022)	.1932 (.0116)	.0532 (.0043)	.0675 (.0055)	.0894 (.0180)	.2289 (.0366)	.1200 (.0101)	.0940 (.0073)	.0894 (.0180)
α_4^M	.2674 (.1948)	.1643 (.0141)	.4382 (.0192)	.5774 (.0204)	.4839 (.0345)	.6904 (.0369)	.5132 (.0611)	.6620 (.0133)	.6743 (.0310)	.6904 (.0369)
α_5^M	.0318 (.0027)	.0603 (.0026)	.1091 (.0083)	.0327 (.0015)	.0023 (.0017)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)
β_{11}^M	.0799 (.0653)	.0376 (.0333)	.1072 (.1129)	-.0454 (.2979)	.0190 (.0100)	-.0557 (.3647)	-.0165 (.0238)	.0170 (.0367)	-.0752 (.1472)	-.0557 (.3647)
β_{12}^M	-.1012 (.1332)	-.0200 (.0263)	-.0168 (.0220)	.0848 (.0536)	-.0083 (.0203)	.1098 (.0503)	.0483 (.0621)	.0755 (.0210)	.1850 (.4780)	.1098 (.0503)
β_{13}^M	.0629 (.0814)	.0043 (.0091)	-.2369 (.5707)	-.0094 (.0038)	-.0013 (.0028)	-.0255 (.0271)	.0005 (.0009)	-.0292 (.0372)	-.0192 (.0171)	-.0255 (.0271)
β_{14}^M	-.0672 (.0534)	-.0571 (.0641)	-.0535 (.0432)	-.0806 (.0149)	-.0094 (.0218)	-.0286 (.0203)	-.0323 (.0308)	-.0633 (.0202)	-.0906 (.0221)	-.0286 (.0203)
β_{15}^M	.0256 (.0207)	.0352 (.0459)	-.0133 (.0286)	.0506 (.0533)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)

TABLE 8

ESTIMATED PARAMETERS OF THE TRANSLOG PRICE POSSIBILITY FRONTIER FOR THE INTERMEDIATE MATERIALS (M) SUBMODEL FOR 10 PRODUCING SECTORS. (Continued)

Parameter	Producing Sector									
	1	2	3	4	5	6	7	8	9	10
β^M_{22}	.2349 (.1767)	.1958 (.1225)	.1185 (.1177)	.0973 (.0726)	.2447 (.1098)	.0077 (.0292)	.1113 (.2720)	.0023 (.0070)	-.1416 (.4130)	.0077 (.0292)
β^M_{23}	-.0219 (.0316)	-.0361 (.0188)	-.0265 (.1009)	-.0091 (.0109)	-.0288 (.1071)	.0053 (.0143)	-.0592 (.0852)	.0037 (.0054)	.0155 (.2054)	.0053 (.0143)
β^M_{24}	-.1009 (.1833)	-.0710 (.1647)	-.0602 (.0995)	-.1179 (.0687)	-.2066 (.0988)	-.1228 (.1095)	-.1003 (.0913)	-.0815 (.0224)	-.0889 (.0481)	-.1228 (.1095)
β^M_{25}	-.0109 (.0069)	-.0607 (.0695)	-.0150 (.0155)	-.0550 (.0414)	-.0010 (.0071)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)
β^M_{33}	.0039 (.0191)	.0435 (.0232)	.1559 (.1135)	.0502 (.0849)	.0629 (.1119)	-.0418 (.3994)	.0680 (.2141)	.1027 (.0641)	-.0553 (.0753)	-.0418 (.3994)
β^M_{34}	-.0187 (.0214)	-.0030 (.0035)	-.0847 (.2363)	-.0321 (.0159)	-.0327 (.0378)	.0620 (.5086)	-.0092 (.0119)	-.0773 (.0383)	.0289 (.0237)	.0620 (.5086)
β^M_{35}	-.0263 (.0201)	-.0087 (.0107)	-.0211 (.0256)	.0005 (.0010)	-.0002 (.0101)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)
β^M_{44}	.1959 (.1069)	.1218 (.1080)	.2462 (.2749)	.2348 (.0774)	.2497 (.1295)	.0894 (.1003)	.1418 (.1450)	.2221 (.0563)	.1505 (.0568)	.0894 (.1003)
β^M_{45}	-.0090 (.0194)	.0093 (.0089)	-.0478 (.0420)	-.0042 (.0038)	-.0011 (.0109)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)
β^M_{55}	.0206 (.0247)	.0329 (.0208)	.0972 (.1134)	.0081 (.0064)	.0023 (.0115)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)	.0000 (.0000)

3. Refined Petroleum Products.
4. Electric Utilities.
5. Gas Utilities.
6. Natural Gas Extraction.

Materials Input

1. Agriculture, Non-fuel mining, Construction.
2. Manufacturing, excluding petroleum refining.
3. Commercial Transportation.
4. Services, Trade, Communication.
5. Imports.

4. Final Demand

The next step in determining the interindustry transactions flows is to find the final demand for the output of the 13 supplying sectors—the 10 domestic producing sectors and the 3 primary input sectors: capital services, labor services, and imports. These demands are based upon the total expenditures on each of the final demand components—personal consumption, investment, and government purchases—taken from the accompanying macroeconomic model together with export demand. The final demand submodels in the Interindustry Model serve to disaggregate these totals to determine demand for the output of each supplying sector.

The total value of domestic private investment, from the macro-model, is allocated over the supplying sectors in fixed proportions. Total investment includes the value of consumers' purchases of durable goods. These durables yield benefits in the form of a flow of services rendered to consumers over future years and are treated as additions to the capital stock of the economy. This imputed service flow is included in consumption, exactly analogous to the conventional treatment of owner-occupied residential structures in the National Income and Product Accounts. Government purchases of goods and services are also allocated in fixed proportions over the supplying sectors. The export components of final demand are entered as exogenous variables.

The investment and government final demand components are computed as

$$I_i = \frac{SI_i * PI * I}{PI_i}$$

$$G_i = \frac{SG_i * (PCG * CG + PIG * IG + PLG * LG + PLE * LE)}{PG_i} \qquad i=1,13$$

Where I_i and G_i are the real expenditures on commodity i; SI_i and SG_i are the shares of commodity i purchases in the total value of investment and government purchases; and PI_i and PG_i are the prices of commodity i to investment and government purchases. The remaining terms in these expressions are the value of investment and the total value of government purchases, both of which are taken from the Macroeconomic Model.

Personal consumption expenditure is directed towards 8 of the 10 domestic producing sectors, as well as towards capital services and imports. There are no consumption purchases of crude oil or gas since these products must pass through the petroleum refining sector or the gas utilities sector before they can be consumed. There are no consumption purchases of labor services since such purchases are classified as part of the output of the services sector. Most imports into the U.S. are of a competitive nature, i.e., they are imported and sold in direct competition to domestic production: these imports are classified as imports into the producing sectors and not as imports directly into final consumption. Some imports however, are of a noncompetitive nature, e.g., spending by American tourists in other countries, and these imports are classified as a direct purchase by consumers. The personal consumption sector contains a substantial stock of durable goods and these goods render services over a period of time. The imputed value of these services is included as a purchase by consumers from capital services. Correspondingly, new purchases of consumer durables are included not in consumption but in investment, as they add to the capital goods stock.

The allocation of personal consumption expenditure over the supplying sectors is based on a submodel of consumer behavior. This submodel incorporates constant own and cross price elasticities of demand for the goods and services available to consumers. The implied behavior can be represented by equations giving the quantity demanded of each type of product in terms of the prices facing the household sector. These demand functions are of the form:

$$C_i = \left(\frac{SC_i * PC * C}{PC_i}\right) * PC_i^{\,1-e_i} * \prod_{\substack{j=1,13 \\ j \neq i}} PC_j^{\,e_{ji}} * Z \qquad \text{for } i=1,13$$

where SC_i is an expenditure share parameter, the e's are elasticity parameters, and Z is an aggregation variable which ensures that total consumption expenditure across all goods is equal to PC * C. The own price elasticity of demand for good i is e_i, the elasticity of demand for good i with respect to the price of good j is e_{ji}. The present specification of the consumer demand models uses a structure in which the cross price elasticities are assumed to be zero and the own price elasticities are assumed to be unity except for petroleum (elasticity 0.6), electricity (elasticity 0.85), and natural gas (elasticity 0.9). In solving the consumption submodel, the value of PC * C is taken from the Macroeconomic Model, and the PC_i are based on the output prices of the producing sectors and the prices of capital services, labor services, and imports.[23] The values of the share parameters SC_i are the 1971 consumption expenditure shares and are shown in table 9. (This table also shows the values of the corresponding parameters for investment and government purchases, SI_i and SG_i.)

The final demand submodels generate a matrix of final demands for the output of each of the domestic producing sectors. These are then summed to derive the vector of total real final demand, Y.

$$Y_i = C_i + I_i + G_i + EX_i$$

5. Interindustry Transactions and Energy Flows

The Interindustry Model has now generated real final demands, a complete set of input-output coefficients, and the output prices of the domestic producing sectors. The final demands and the input-output coefficients together can now be treated as a conventional input-output system and used to determine the gross sectoral outputs and the inputs into production. This determination is based on a set of balance equations relating real output from each sector to real demand for that output. These are:

$$X_i = \sum_{j=1}^{10} X_{ij} + Y_i \qquad i = 1, 10$$

where X_i is the gross output from sector i and X_{ji} is the input from sector i to sector j and Y_i is real final demand for commodity i. The input-output coefficients are:

$$a_{ij} = \frac{X_{ij}}{X_j}$$

[23]The solution of prices prior to the calculation of final demand makes use of the non-substitution theorem of Samuelson, See footnote 13 on page 37.

TABLE 9

EXPENDITURE SHARE PARAMETERS FOR CONSUMPTION, INVESTMENT, AND GOVERNMENT PURCHASES

	Supplying Sector	Share Parameters For		
		Consumption (SC_i)	Investment (SI_i)	Government (SG_i)
1.	Agriculture, Non-fuel Mining, Construction	0.0135	0.3356	0.1992
2.	Manufacturing	0.1621	0.6185	0.2287
3.	Transportation	0.0238	0.0041	0.0164
4.	Services	0.5811	0.0400	0.5232
5.	Coal Mining	0.0001	0.0001	0.0001
6.	Crude Petroleum	0.0000	0.0000	0.0000
7.	Refined Petroleum	0.0230	0.0003	0.0064
8.	Electricity	0.0151	0.0003	0.0022
9.	Gas Utilities	0.0086	0.0000	0.0022
10.	Crude Natural Gas	0.0000	0.0000	0.0000
11.	Imports	0.0162	0.0012	0.0217
12.	Capital Services	0.1565	0.0000	0.0000
13.	Labor Services	0.0000	0.0000	0.0000

so that

$$X_i = \sum_{j=1}^{10} a_{ij} X_j + Y_i \qquad i = 1,10$$

In matrix notation, this system is:

$$X = AX + Y$$

This can be solved for the gross outputs:

$$X = (I - A)^{-1} Y$$

These are the gross outputs that ensure that exactly enough is produced by each sector to meet the real final demands and the intermediate demands for its product. In addition, there are balance equations between the value of inputs into each sector and the value of its output:

$$P_i * X_i = \sum_{j=1}^{10} P_{ji} * X_{ji} + PK_i * K_i + PL_i * L_i + PR_i * R_i \qquad i=1,10$$

The transactions matrices can now be calculated. The constant dollar transactions are found from the input-output coefficients and the levels of total output for each sector. The constant dollar sales from sector i to sector j is:

$$X_{ij} = a_{ij} * X_j \qquad i=1,10 \text{ and } K,L,R$$

$$j=1,10.$$

The prices of these transactions are based on the price of output from each sector. The transactions prices are calculated as:

$$P_{ij} = F_{ij} * P_i \qquad i=1,10 \text{ and } K,L,R$$

$$j=1,10.$$

This gives the transactions price as a multiple of the average price of output from the sector. The multipliers F_{ij} are exogenous variables that are unity in 1971 and reflect any systematic difference in the price of one type of sale from the average price of output from the sector. The value of transactions can now be found as the product of the price and quantity of each type of transaction. Thus, the current dollar transactions matrix is found as:

$$\text{Value of input from i to j} = P_{ij} * X_{ij} \qquad i=1,10 \text{ and } K,L,R$$

$$j=1,10.$$

This information on intermediate sales in the producing sectors, along with the final demand variables already found, permits the entire transactions matrix to be constructed in both current and constant dollar terms.

Fuel prices are part of the solution of the Interindustry Model, although these prices are expressed in terms of price indices. The conversion from price indices to dollar prices is simply a matter of scaling. Thus, as the price of each fuel is known relative to its price in the base year, the multiplication of these price indices by the actual base year dollar price gives the simulated price in terms of dollars per physical unit. Likewise, fuel outputs are simulated in terms of base year dollars. By multiplying fuel outputs by the number of physical units of fuel that could be purchased for one dollar in the base year, the output of each fuel can be re-expressed in physical units--tons of coal, barrels of petroleum, kilowatt hours of electricity, and cubic feet of gas, as well as in terms of British Thermal Units for each of those fuels.

The interindustry transactions matrix also gives the constant dollar inputs of each fuel into each producing sector and each final demand sector. These inputs are expressed in BTU's per dollar in the base year. This gives a 6 by 14 matrix of energy flows, i.e., input from each energy form into each type of use. Finally, the energy flows are expressed in terms of the Bureau of Mines energy accounting convention[24] by mapping the 14 use classification into the conventional 4 use system, the 4 energy uses being household and commercial, industrial, transportation, and electrical generation.

[24]On the nature of the Bureau of Mines energy accounting system, see for example, W. G. Dupree Jr. and J. A. West, United States Energy Through the Year 2000. U.S. Department of the Interior, Washington, D.C., December 1972 and Walter Dupree et al., Energy Perspectives 2, U.S. Department of the Interior, Washington, D.C., June 1976.

6. Equations of the Interindustry Model

a. Production Behavior

Price Possibility Frontiers: (i = 1, 10)

$$\ln P_i + \ln TE_i = \sum_{\substack{j= \\ K,L,E,M}} \alpha_{ji} \ln P_{ji} + ½ \sum_{\substack{j,k= \\ K,L,E,M}}\sum \beta_{jki} \ln P_{ji} \ln P_{ki}$$

$$\ln P_{Ei} = \sum_{j=5,10} \alpha^E_{ji} \ln P_{ji} + ½ \sum_{\substack{j,k= \\ 5,10}}\sum \beta^E_{jki} \ln P_{ji} \ln P_{ki}$$

$$\ln P_{Mi} = \sum_{\substack{j= \\ 1,2,3,4,R}} \alpha^M_{ji} \ln P_{ji} + ½ \sum_{\substack{j,k= \\ 1,2,3,4,R}}\sum \beta^M_{jki} \ln P_{ji} \ln P_{ki}$$

Input-Output Coefficients: (j = 1, 10)

$$a_{Kj} = \frac{P_j}{P_{Kj}} * (\alpha_{Kj} + \sum_{i=K,L,E,M} \beta_{Kij} \ln P_{ij})$$

$$a_{Lj} = \frac{P_j}{P_{Lj}} * (\alpha_{Lj} + \sum_{i=K,L,E,M} \beta_{Lij} \ln P_{ij})$$

$$a_{ij} = \frac{P_j}{P_{ij}} * (\alpha_{Ej} + \sum_{\substack{k= \\ K,L,E,M}} \beta_{Ekj} \ln P_{kj}) (\alpha^E_{ij} + \sum_{\substack{k= \\ 5,10}} \beta^E_{ikj} \ln P_{kj}) \qquad \text{for } i = 5,\ 10$$

$$a_{ij} = \frac{P_j}{P_{ij}} * (\alpha_{Mj} + \sum_{\substack{k= \\ K,L,E,M}} \beta_{Mkj} \ln P_{kj}) (\alpha^M_{ij} + \sum_{\substack{k=1, \\ 2,3, \\ 4,R}} \beta^M_{ikj} \ln P_{kj}) \qquad \text{for } i=1,2,3,4,R.$$

b. Final Demand Behavior

Household Consumption Demand: (i = 1, 13)

$$C_i = (\frac{SC_i * PC * C}{PC_i}) * PC_i^{1-e_i} * \prod_{\substack{j=1,13 \\ j \neq i}} PC_j^{e_{ji}} * Z$$

Investment Demand: $(i = 1, 13)$

$$I_i = \frac{SI_i * PI * I}{PI_i}$$

Government Purchases: $(i = 1, 13)$

$$G_i = SG_i * \frac{(PCG * CG + PIG * IG + PLG * LG + PLE * LE)}{PG_i}$$

c. Balance Equations

Interindustry Balance: $(i = 1, 10)$

$$X_i = \sum_{j=1}^{10} X_{ij} + Y_i$$

Value Balance: $(j = 1, 10)$

$$P_j * X_j = \sum_{i=1}^{13} P_{ij} * X_{ij}$$

d. Identities

Real Final Demand: $(i = 1, 10)$

$$Y_i = C_i + I_i + G_i + EX_i$$

Interindustry Input:

$$X_{ij} = a_{ij} * X_j \qquad (i=1,13;\ j=1,10)$$

Interindustry Prices:

$$P_{ij} = F_{ij} * P_i \qquad (i=1,10;\ j=1,10)$$

$$P_{11j} = F_{11j} * P_R \qquad (j=1,10)$$

$$P_{12j} = F_{12j} * P_K \qquad (j=1,10)$$

$$P_{13j} = F_{13j} * P_L \qquad (j=1,10)$$

Final Demand Prices:

$$PC_i = F_{Ci} * P_i \qquad (i=1,13)$$

$$PI_i = F_{Ii} * P_i \qquad (i=1,13)$$

$$PG_i = F_{Gi} * P_i \qquad (i=1,13)$$

Primary Input Demand:

$$R^D = \sum_{j=1}^{10} X_{11j} + C_{11} + I_{11} + G_{11}$$

$$K^D = \sum_{j=1}^{10} X_{12j} + C_{12} + I_{12} + G_{12}$$

$$L^D = \sum_{j=1}^{10} X_{13j} + C_{13} + I_{13} + G_{13}$$

NOTE: For subscript definitions, refer to table 6. Subscript i refers to supplying sector i (for i=1,...,10) while i=11, 12, 13 refers to input of capital services, labor services, and imports, respectively. Subscript j refers to purchases by producing sector j (for j=1,...,10) and to purchases by consumers (j=11), investment (j=12), government (j=13), and exports (j=14).

7. Variables in the Interindustry Model

a. Endogenous Variables

a_{ij}	Input-output coefficient; input of i into j.
C_i	Real personal consumption of good i.
G_i	Real government purchases of good i.
I_i	Real investment demand for good i.
K^D	Total real demand for capital services.
L^D	Total real demand for labor services.
P_i	Price of output received by sector i.

P_{ij}	Price of good i to sector j.
P_{Ei}	Price of energy input to sector i.
P_{Mi}	Price of intermediate materials input to sector i.
PC_i	Price of good i to consumers.
PG_i	Price of good i to government.
PI_i	Price of good i to investment purchases.
R^D	Total real demand for imports.
X_i	Total real output from sector i.
X_{ij}	Real input of good i to sector j.
Y_i	Total real final demand for good i.

b. Exogenous Variables

F_{ij}	Ratio of price of good i paid by purchaser j to the producers' price of good i.
F_{Ci}	Ratio of price of good i to consumers to the producers' price of good i.
F_{Ii}	Ratio of price of good i to investment purchasers to the producers price of good i.
F_{Gi}	Ratio of price of good i to government purchasers to the producers' price of good i.
P_R	Price index of imported goods and services.
SG_1	Share of commodity i purchases in the total value of government purchases.
SC_i	Share of commodity i purchases in the total value of commodity purchases.

SI_i	Shares of commodity i purchases in the total value of investment purchases.
TE_i	Index of input-to-output productivity in producing sector i.

c. Variables from Macroeconomic Model

C	Real personal consumption expenditure.
CG	Government purchases of consumption goods and services.
I	Gross private domestic investment.
IG	Government purchases of investment goods.
LE	Purchases of labor services by government enterprises.
LG	Purchases of labor services by general government.
PC	Price deflator for personal consumption expenditure.
PCG	Price deflator for government purchases of consumption goods and services.
PI	Price deflator for gross private domestic investment.
PIG	Price deflator for government purchases of investment goods.
PK	Price of capital services.
PL	Price of labor services.
PLE	Price deflator for government enterprises purchases of labor services.
PLG	Price deflator for general government purchases of labor services.
Z	Aggregation variable for consumption.

D. Integration of the Macroeconomic and Interindustry Models

1. Introduction

The next step in the development of the model system is to integrate the Macroeconomic and Interindustry Models so that a single complete and consistent equilibrium solution may be found. This model solution incorporates the detailed analysis of production at the sectoral level, including price formation and the determination of input-output coefficients; the determination of final demands and GNP output aggregates; the determination of the supply of capital and of labor and the growth of these over time; and ensures consistency between factor demands and factor supplies and between output demands and output supplies. In a theoretical sense, this integrated model corresponds to a Walrasian dynamic, general equilibrium model of the economy, implemented on an operational basis.[25] In a practical sense, this integrated model provides the capability of tracing out the direct and indirect effects, in both price and quantity terms, and including dynamic effects, of changes affecting the economic structure. This offers a very valuable tool for forecasting, policy analysis, and contingency analysis in the economic field in general and the energy field in particular.

Integration of the Macro-economic and Interindustry models requires that the information flows from one model to the other are inserted and that any areas of overlap are resolved. The principal information flows from the Macroeconomic Model into the Interindustry Model are: final demands by consumption, investment and government purchases, and the supplies of capital and labor inputs. The information flowing in the reverse direction is the demand, by the producing sectors, for capital, labor and imported inputs, and the supplies and prices of output from the producing sectors.

The two models, in stand-alone form, have areas of overlap, and it is necessary to remove these overlaps or at least to reconcile the two representations if consistency is to be achieved. The main area of overlap between the two models is in the characterization of production because each has its own function relating factor inputs to production outputs—the Macroeconomic Model has an aggregate production function, and the Interindustry Model has 10 sectoral price possibility frontiers. This overlap is resolved by removing the macroeconomic production function, and its implied factor demand equations. The detailed sectoral price frontiers, with their embodied information about all 10 intermediate inputs, 3 primary inputs, and all interdependencies between inputs, is used as the production representation of the integrated model.

[25] See L. Walras, Elements of Pure Economics, Translated by W. Jaffe, Homewood, Irwin, 1954.

2. Design

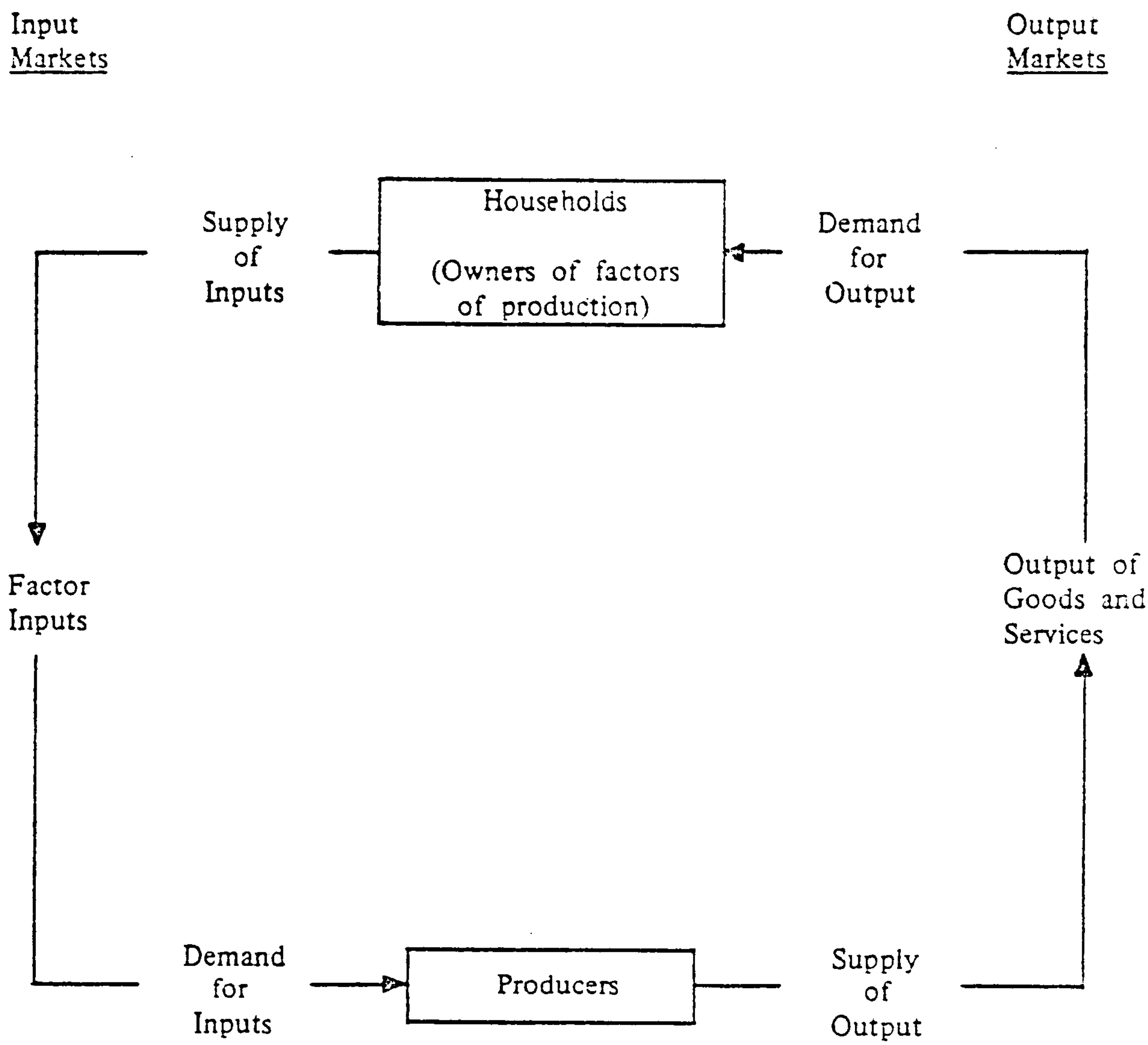
The economy can be conceptualized as a circular flow in which the decision-makers are households, as owners of factors of production, and producers. This flow, in its simplest form, can be represented as in figure 2. Households supply labor services or capital that they own directly or through the intermediaries of corporate and non-corporate business institutions. These factor services are purchased by producers as inputs into production. Producers then offer finished goods and services which are purchased by households directly as consumption or indirectly, through business institutions, as investment (another indirect form of final demand is government purchases). In the reverse direction to these real flows, there is an equal value of financial flows: outlays on final demand form producers' receipts which are used to purchase inputs which then form factor income. To represent the economy in this equilibrium system, four analytical components are required:

1. Demand for final output.
2. Supply of factor input.
3. Supply of final output.
4. Demand for factor input.

In addition, balance conditions are required to ensure consistency between these components in the circular economic flow. The household sector of the Macroeconomic Model simulates decisions on the consumption-saving choice which generates demand or expenditure on consumer goods and services. Demand for saving, by the household sector as well as by government and rest of world sectors, represents the demand to transfer resources into the future and is equivalent to a demand for investment. Government purchases are introduced exogenously. These items together provide the required elements of component (1), demand for final output.

Labor supply is modeled in the household sector, and the supply of capital services is generated by the existing capital stock. In turn, capital stock changes over time by net investment, i.e., private domestic capital formation net of depreciation. Thus, the supply of inputs, or component (2), is generated by the Macroeconomic Model. (The production segment of the Macroeconomic Model, the production possibility frontier and the implied output supply and input demand equations, are not used in the integrated model; this information is derived for the Interindustry Model.)

The Interindustry Model provides the production structure that yields components (3) and (4): the supply of final output and the corresponding demand for factor inputs. The final demand totals are introduced into the Interindustry



⟶ is the flow of inputs and of goods and services. There is a financial flow, of incomes and of revenues, in the opposite direction.

Figure 2. Overview of Economic Flows

Model where they are allocated over commodities to derive the required net output from each domestic producing sector. These outputs are related to the production structure, represented by the price possibility frontiers and associated input-output coefficients, to find the required input of factor services.

It remains to bring these components into equilibrium by introducing market balance equations and quantity balance equations that ensure that demands are equal to supply n both output and input markets. These equations are provided by the balance equations in the two models together with a further set of conditions introduced as part of the model integration. The result is a dynamic, equilibrium model of the economy, its structure, and its growth.

The actual structure of the integration system is presented in figure 3. This relates the circular flow structure of figure 1 to the actual model components involved, and also summarizes the price and quantity links in the integration. The household sector and the balance equations for consumption output, investment output, labor input, and capital input are used from the Macroeconomic Model. These equations, incorporating lagged capital, lagged wealth, input prices, and output prices generate consumption and investment expenditures, and labor and capital supply. The price possibility frontiers from the Interindustry Model are used to solve for output prices and then for the input-output coefficients, based on the prices of capital, labor, and imports. These output prices, together with final demand expenditures, are used, in the final demand allocation sections of the Interindustry Model, to solve for the vector of real final demands. These, with the input-output coefficients, generate the required input of capital services, labor services, and imports. Finally, demands and supplies of capital and labor are compared and, if unequal, used as a basis for changing these factor prices, after which the solution is repeated until the entire system is consistent.

3. Solution of the Integrated System

The approach is equivalent to the solution of the circular flow system just outlined, but the need to achieve full consistency within this complex system requires, for computational tractability, the adoption of an iterative solution procedure. This iterative procedure is, in essence, based on the prices of capital services and labor services. An equilibrium economic system can only solve for relative prices so one price must, therefore, be chosen as a numeraire. The price of capital services is used for this purpose so that PKD (Macroeconomic Model) or P_K (Interindustry Model) are set exogenously. PL, the price of labor services, is solved for, iteratively using the net demand for labor as the criterion function.

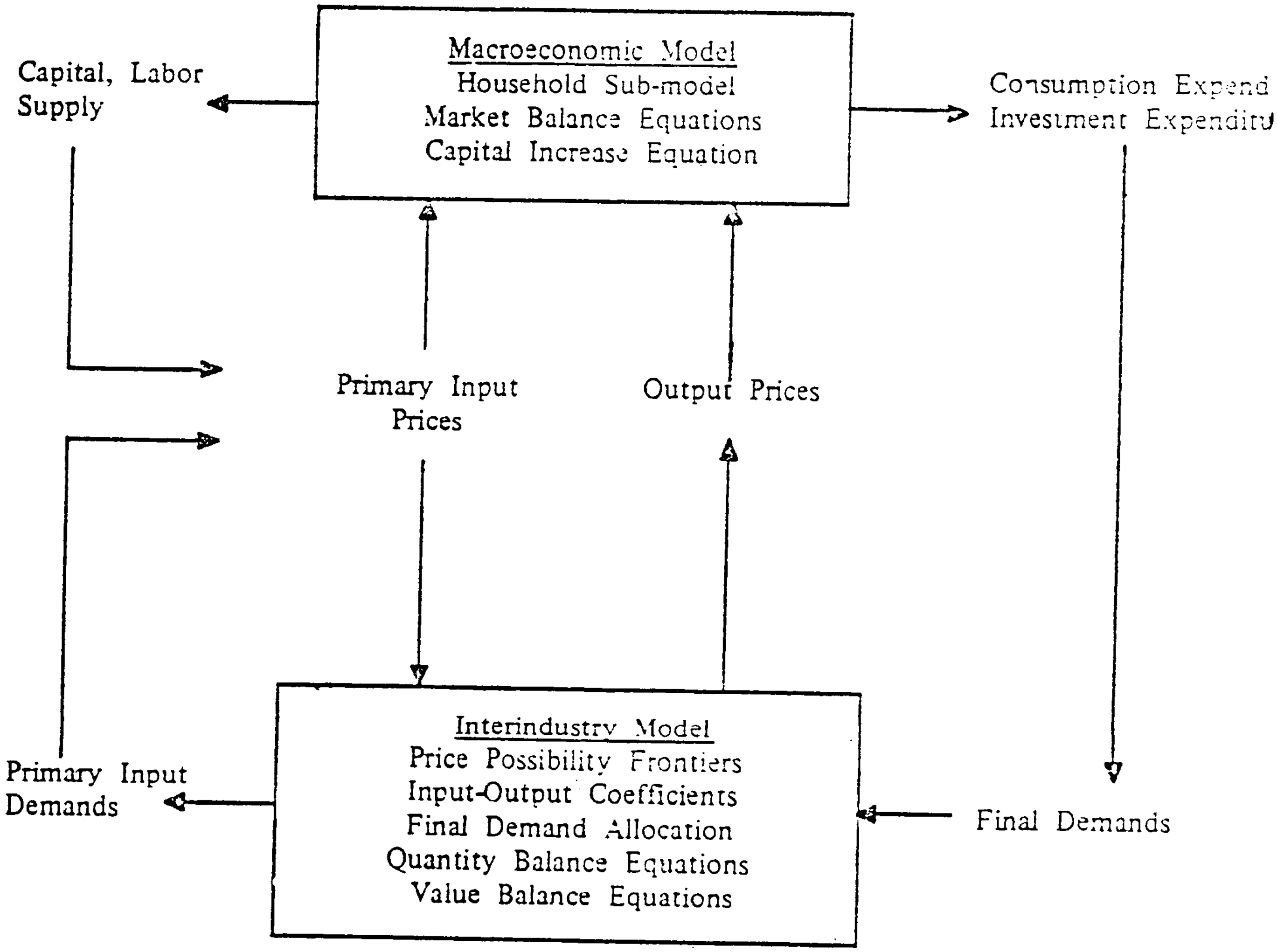


Figure 3. Overview of Economic Flows in the Integrated Model

Taking PKD nad PL as known, the Macroeconomic Model is processed as follows to find aggregate final demand expenditure and aggregate factor supplies. The supply of labor is:

$$L = LH - LJ$$

$$PL * L = PL * LH - PL * LJ$$

$$= -C6 * WL + (1 - C7) * PL * LH - C7 * EL$$

using the household demand for leisure equation. The value balance equation for the labor market gives:

$$PLD * LD + PLE * LE + PLG * LG + PLR * LR = \frac{PL * L}{1\text{-}TL}$$

Transfers to persons are given by:

$$EL = BL * (PLD * LD + PLE * LE + PLG * LG + PLR * LR)$$

so that:

$$EL = BL * \frac{PL * L}{1\text{-}TL}$$

Therefore,

$$EL = \frac{BL}{1\text{-}TL} * (-C6 * WL + (1\text{-}C7) * PL * LH - C7 * EL$$

and

$$EL = \frac{BL}{1\text{-}TL} * (-C6 * WL + (1\text{-}C7) * PL * LH / (1 + C7 * \frac{BL}{1\text{-}TL})$$

This permits labor supply to be found from the previous equation for L. The labor market value balance equation then allows the value of labor services available to the private domestic sector to be found:

$$PLD * LD = \frac{PL * L}{1\text{-}TL} - PLE * LE - PLG * LG - PLR * LR$$

Also, the household sector consumption demand equation permits the value of consumption expenditure to be found:

$$PC * C = C4 * WL + C5 * PL \bullet LH + C6 * EL$$

This expenditure, with the value balance equation for consumption goods and services, yields the value of the required output of consumption goods to be calculated:

$$PCS * CS = (PC * C + PCG * CG + PCI * CI + PCR * CR - PCE * CE) / (1 + TC)$$

The supply of capital services depends on the existing capital stock:

$$KD = AK * KL$$

The national income accounting identity between output and input values then implies:

$$PIS * IS = PKD * KD + PLD * LD - (PCS * CS)$$

where the left-hand side is the value of output of investment goods. Finally, the value balance equation for investment goods is used to find the value of expenditure on gross private domestic investment:

$$PI * I = (1+TI) * (PIS * IS) * PCI * CI - PIG * IG - PIR * IR$$

At this point, the Macroeconomic Model has yielded the information required to interface with the Interindustry Model:

- supply and price of capital services (KD, PKD).
- supply and price of labor services (L, PL).
- personal consumption expenditure (PC * C).
- private domestic investment (PI * I).
- government purchases (PCG * CG + PIG * IG + PLE * LE + PLG * LG).

In addition, the quantity of exports and the price of imports are exogenous.

The solution procedure now enters the Interindustry Model which is summarized in figure 4. The following steps are important:

1. The price possibility frontiers, in conjunction with the prices of capital services, labor services, and imports, are used to compute the prices of sectoral output. These output prices then allow the price indices for consumption, investment, and government purchases to be calculated.

2. The price possibility frontiers, with the output prices just derived, are used to calculate the input-output coefficients.

3. The final demand allocation model, using the output prices, allocate personal consumption expenditure, investment expenditure, and government purchases over the domestic producing sectors. These, together with export demand, are summed to obtain the vector of total real final demand.

4. The balance equations for each sector's output are used, with the real final demands and the input-output coefficients, to compute the gross output required from each sector. This permits the inputs into each sector to be found, including the demands for imports, capital services, and labor services.

5. The demands for imports, capital services, and labor services by each of the producing sectors as well as by each of the final demand sectors are now known. These are summed to derive total demand for each of imports, capital services, and labor services.

The solution algorithm now introduces some additional integration conditions to derive the criterion function on which to base the price of labor, PL. In the Macroeconomic Model, the national income accounting identity ensures that the value of primary inputs is equal to the value of final output. The Macroeconomic Model's final demands are inserted directly into the Interindustry Model. In turn, the value balance property of the interindustry transactions ensures that the gross value of input equals the gross value of output and, therefore, that the value of primary input in the Macroeconomic Model is equal to the value of primary input in the Interindustry Model. Since the solution procedure also enforces equality between the value of net exports between the two models, this input value equality reduces to the equality between the value of capital and labor services in each model, or:

$$PKD * KD + PL * (L - LU) = P_K * K^D + P_L * L^D$$

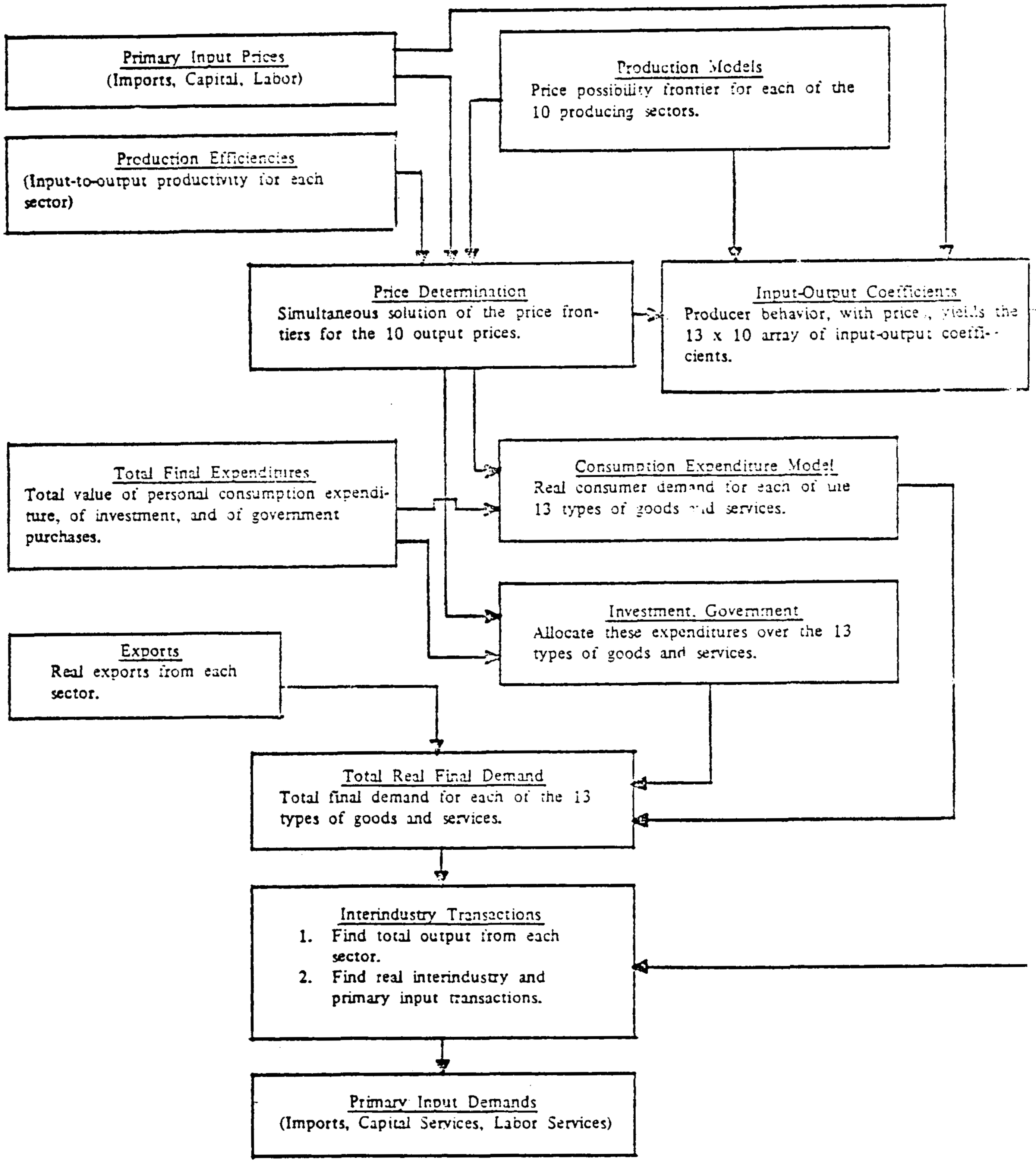


Figure 4. Solution Sequence for the Interindustry Model

In this equation, the left-hand side refers to the Macroeconomic Model while the right-hand side refers to the Interindustry Model. The left-hand side is the value of capital and labor supply, where the exogenous level of unemployment is deducted from L to obtain effective labor supply. The right-hand side is the value of demand for capital and labor services. The two sets of prices are equal by definition, only the notation is different, so:

$$PKD * KD + PL * (L - LU) = PKD * K^D + PL * L^D$$

Now, the criterion function for the price of labor is the net demand for labor services:

$$f(PL) = L^D - (L - LU)$$

The iteration proceeds until PL assumes a value that yields zero net demand for labor services. At this stage,

$$L^D = L - LU$$

Therefore, using the above value balance equation, it must also be the case that

$$KD = K^D$$

i.e., the supply and demand for capital services are in balance. Finally, it can be noted that a balance of trade constraint is also added to the iteration procedure–real exports are adjusted until net exports are the same in each model.

This solution procedure is summarized in figure 5. The Macroeconomic Model, or that part of the model that is retained in the integrated system, and the Interindustry Model are linked sequentially within an overall iterative loop. This iterative approach to this solution renders the computational burden more manageable. Once the two models are consistent, i.e., once the core of the integrated system is solved, various sets of output information are computed. These include the macroeconomic aggregates, in both current and constant dollars, and the matrix of interindustry transactions, also in current and constant dollars. In addition, the energy flows in these interindustry transactions are converted to physical units–British Thermal Units, tons, barrels, cubic feet, and kilowatt hours–permitting energy prices and quantities to be presented in terms of these physical units.

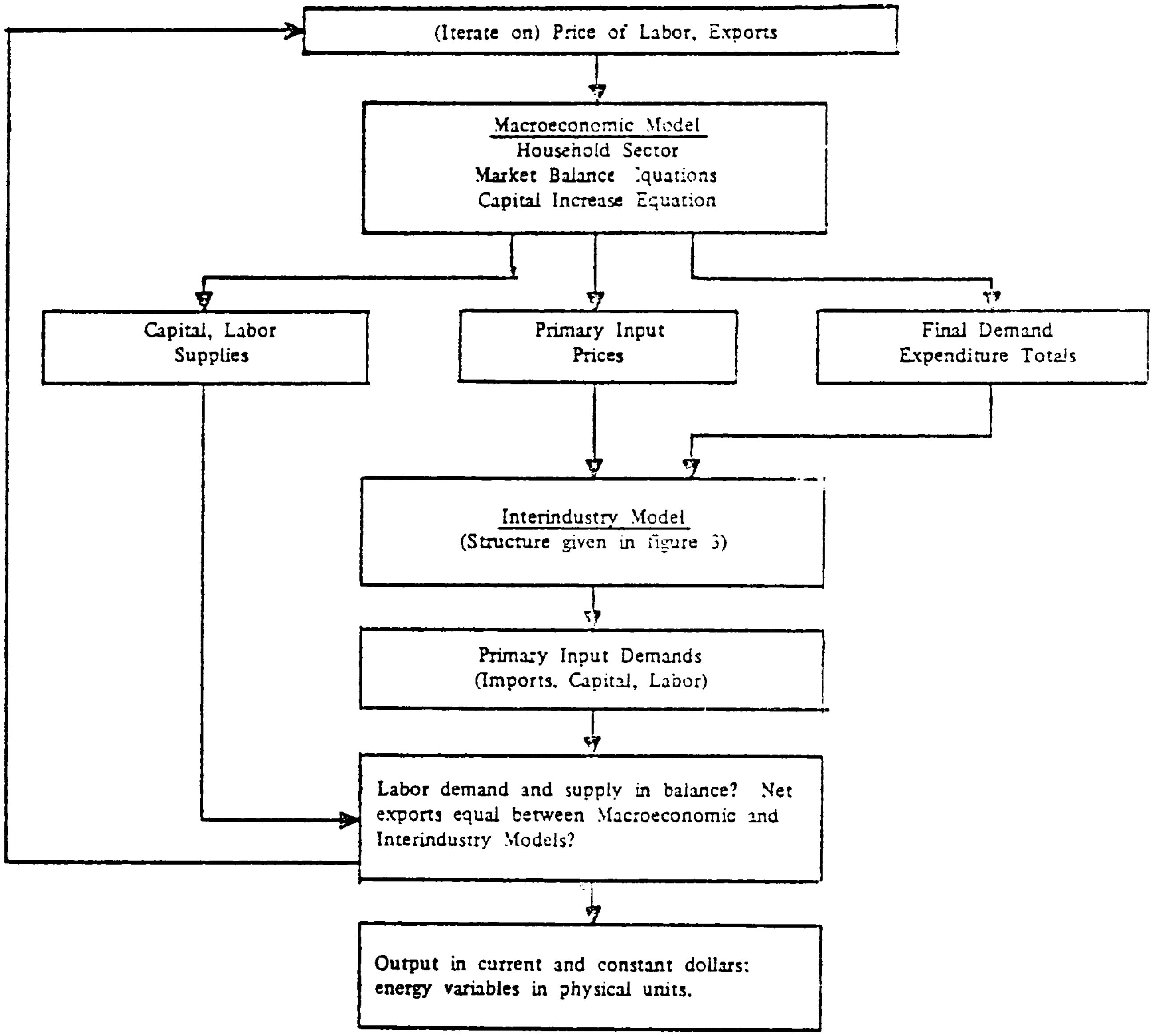


Figure 5. Solution Structure of the Integrated Model

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