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HADRONIC MULTIPLICITY DISTRIBUTIONS: THE NEGATIVE BINOMIAL AND ITS ALTERNATIVES

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ABSTRACT

We review properties of the negative binomial distribution, along with its many possible statistical or dynamical origins. Considering the relation of the multiplicity distribution to the density matrix for Boson systems, we re-introduce the partially coherent laser distribution, which allows for coherent as well as incoherent hadronic emission from the k fundamental cells, and provides equally good phenomenological fits to existing data. The broadening of non-single diffractive hadron-hadron distributions can be equally well due to the decrease of coherence with increasing energy as to the large (and rapidly decreasing) values of k deduced from negative binomial fits. Similarly the narrowness of e^+-e^- multiplicity distribution is due to nearly coherent (therefore nearly Poissonian) emission from a small number of jets, in contrast to the negative binomial with enormous values of k.

I. NEGATIVE BINOMIAL DISTRIBUTION

The negative binomial distribution

$$P_n^k = \frac{(n+k-1)!}{n!(k-1)!} \frac{(\bar{n}/k)^n}{(1+\bar{n}/k)^{n+k}} \tag{1}$$

with $\bar{n}$ (the average multiplicity) and k as the parameters, has been found to give an excellent account of hadronic multiplicity distributions[1]. In particular the recent fit to non-single diffractive data from all energies by the UA5 group is especially interesting in that the parameter k is required to decrease rather rapidly with energy. Put differently, the scaling form of (1) (a special case of the gamma distribution)

$$\bar{n}P_n^k \sim \psi_k(z) \equiv \frac{k^k}{(k-1)!} z^{k-1} e^{-kz} \tag{2}$$

$$z \equiv n/\bar{n}$$

does not exhibit KNO scaling, as would be the case were k constant. These facts, in particular the energy dependence of k have inspired many theoretical conjectures about the meaning of (1) and its physical meaning.

The negative binomial distributions occurs in many physical and mathematical contexts. Here we mention of few examples, referring to standard mathematical texts[3] and our forthcoming review article[4] for more information on this and related distributions.

(1) P_n^k is a <u>generalized Bose-Einstein distribution</u> composed of k (integral) cells of equal average occupancy $\bar{n}/k$ (see Knox[1] and ref. 3).

(2) P_n^k is the <u>superposition of Poisson distributions</u> for the particular case of the <u>Poisson transform</u>[4,5] of the weight f(x)

$$P_n = \int_0^\infty dx\ f(x) \frac{(x\bar{n})^n e^{-x\bar{n}}}{n!} \tag{3}$$

with the weight $f(x) = \psi_k(x)$ given by Eq. (2). This formula suggests the possibility[6-8] that the observed broad hadronic distributions are the consequence of an average (in the event sample) over varying inelasticities or equivalently impact parameters, for continuous k.

(3) P_n^k corresponds to the counting distribution characteristic of a <u>Gaussian field ensemble</u>, whether in semiclassical photocount theory[2,9] or in the representation of the oscillator by the diagonal coherent state representation[9,10]. For applications to hadronization, k would be the average number of effective cells (or emitters). Although there is no fundamental basis for k, most people imagine that the number of emitters should increase with energy.

(4) P_n^k can be derived as a <u>composite Poisson-logarithmic distribution</u>[11-13] in which clusters produced with a Poisson distribution decay into the final hadrons via a logarithmic distribution. This picture (as do the others mentioned here) requires further elaboration to become compelling.

(5) P_n^k is the solution of various[3,4,14-17] <u>probability evolution equations</u> in the parameter $\bar{n}$. In these cases the mathematics is more clear than the physical processes allowing the reduction of the many-body problem to a few degrees of freedom obeying the appropriate equations.

(6) P_n^k is the long time distribution whose time dependent Poisson kernel $f(x,t)$ obeys a suitable <u>stochastic differential equation</u>. At least two cases are known[6,18].

(7) P_n^k can result from a <u>Cantor set type of cascade structure</u> (including parallel or composite cascades) in which P_n^k is the fraction of a line occupied at the <u>nth</u> stage[19]. This interpretation is closely connected with the interpretation of the $k=1$ (Bose-Einstein) distribution as a "geometric" distribution[3].

The foregoing list in no way exhausts the rich variety of contexts in which the negative binomial distribution occurs in nature[20]. We have emphasized those which may have relevance to the particle physics multihadron production problem.

II. PROTOTYPE HADRONIZATION DENSITY MATRICES

The counting distribution P_n can in principle be obtained from the projection of the wave function $\psi(t)$ on the n particle states at $t\to\infty$, i.e. when the produced hadrons ...

$$P_n = \langle n|\rho_{out}|n\rangle$$

$$\rho_{out} = |\psi_{out}\rangle\langle\psi_{out}| \qquad (4)$$

for a so called "pure" state. The "in" density matrix $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$ is related to ρ_{out} via the S "matrix" $S = |\psi_{in}\rangle\langle\psi_{out}|$ by $\rho_{out} = S^{+}\rho_{in}S$, indicating the relation of (4) to the usual formulation in terms of phase space integrations over the squared S-matrix. Having said this, we admit that a dynamical evaluation of ρ_{out} is not easier than that of S. Nevertheless, one can make educated guesses[7] on the structure of ρ_{out} on the experimental results and accumulated from statistical physics, particularly the sophisticated results from quantum optics[9-10]. Such results can then provide well-formulated goals for more ambitious dynamical schemes such as jet calculus[21], dual topological models[22], etc. This framework suggests the merit of deriving the "stochastic essence" from the full exclusive event by searching for suitable probabalistic equations for inclusive variables. Although traditional in other branches of science, particle physics has heretofore made little use of these techniques.

The final hadrons are to good approximation described by a set of free Bose fields, whose creation and destruction operators (a,a^{+}) are nothing but free harmonic oscillators. ρ_{out} is therefore some function of the outgoing a's and a^{+}'s. Although the actual a and a^{+} variables are equipped with momenta and other degrees of freedom, it turns out to be fruitful to consider a prototype model with one (or a few) effective oscillators. The most popular oscillator states, the number states $|n\rangle = (a^{+})^{n}|0\rangle/(n!)^{\frac{1}{2}}$ have ill-defined phase and hence are only indirectly related to classical-like field motions. This does not matter for the incoherent <u>thermal ensemble</u>, whose (mixed) density matrix

$$\rho = \frac{\exp(-\beta H)}{\mathrm{Tr}\,\exp(-\beta H)} = \sum_{n=0}^{\infty} \frac{N^{n}}{(1+N)^{n+1}}|n\rangle\langle n| \qquad (5)$$

is diagonal in the number basis. Recall that in this case the occupation number N of the Bose-Einstein distribution is given by $N^{-1} = \exp(\beta\omega)-1$.

Suppose, however, we have the opposite case of an oscillator undergoing sinusoidal motion. In this case we expect a Poisson distribution for probabilities of the system being found in the $n\underline{th}$ excited state. As is well-known[4,9,10], the most suitable states in this case are the coherent states $|\alpha\rangle$ which can be defined (for any complex α) by

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_n \frac{\alpha^n}{(n!)^{\frac{1}{2}}}|n\rangle \tag{6}$$

The Poisson distribution follows immediately on identifying the mean multiplicity S with $|\alpha|^2$

$$\rho = |\alpha\rangle\langle\alpha|$$

$$P_n = \frac{S^n e^{-S}}{n!}, \quad S = |\alpha|^2 \tag{7}$$

The motion $\langle\alpha|x(t)|\alpha\rangle \sim |\alpha|^{\frac{1}{2}}\cos(\phi-\omega t)$ where $\phi = \arg\alpha$.) Although $|\alpha\rangle\langle\alpha|$ has many off-diagonal elements in the number basis, the counting process is not sensitive to them. Hence observation of (7) in no way implies that the actual physical system has the full classical-like phase structure.

Next suppose that instead of the pure state $|\alpha\rangle\langle\alpha|$ we have a mixed ensemble with real weight function $\Phi(\alpha)$

$$\rho = \int d^2\alpha\ \Phi(\alpha)|\alpha\rangle\langle\alpha| \tag{8}$$

This representation has considerable great generality than might be surmised from the foregoing. Moreover, as one can easily see the diagonal element $\langle n|\rho|n\rangle$ leads directly to the Poisson transform, Eq. (3), which thereby inherits this greater generality.

As our first example we note that a Gaussian weight leads to the Bose-Einstein distribution:

$$\Phi(\alpha) = \exp(-|\alpha|^2/N)/\pi N$$

$$P_n = \frac{N^n}{\ldots} \tag{9}$$

(For k modes the direct product $\exp(-\Sigma|\alpha_i|^2/(N/k))/(\pi N/k)^k$ leads directly to the negative binomial, Eq. (1). Note that these results are compatible with but do not require thermal equilibrium.

A very interesting generalization, which actually arises in a model of a single-mode laser[9], is the displaced Gaussian weight whose P_n interpolates between (7) and (9)

$$\Phi = \exp(-|\alpha-\beta|^2/N)/\pi N$$
$$P_n = \frac{N^n}{(1+N)^{n+1}} \exp - \frac{S}{1+N} \; L_n \; \frac{-S}{1+N} \qquad (10)$$

Here the average multiplicity is $\bar{n} = S+N$ with $S = |\beta|^2$; L_n is the usual Laguerre polynomial (positive for negative argument.)

The notation is chosen so that S measures the strength of the coherent signal and N the strength of the (Gaussian) noise. We shall refer to (10) and its generalization to k (equal strength) cells:

$$P_n^k = \frac{(N/k)^n}{(1+N/k)^{n+k}} \exp - \frac{S/N}{1+N/k} \; L_n^{k-1} \; \frac{-kS/N}{1+N/k} \qquad (11)$$

as the partially coherent laser distribution (PCLD). These formulas were originally derived for the photocount distribution for k-mode lasers whose emitting modes have a (common) signal and noise ratio as defined above. This suggests application to the description of hadron counting for emissions from a set of cells having to first approximation a common (S,N).

III. DESCRIPTION OF MULTIPLICITY DISTRIBUTIONS

The PCLD Eq. (11) was first used[5,24)] to describe hadronic multiplicities in 1983, although equivalent physics was assumed for moments (for finite rapidity differences) as early as 1978 by the Marburg[25)] group. We note that (11) depends on three parameters (S,N,k) as opposed to just two for the negative binomial, Eq. (1). We shall use the equivalent set ($\bar{n}$ = N+S, $m = (N/S)^{1/2}$,k). Note that as $N/S \to \infty$ (11) goes over to the negative binomial, while $N/S \to 0$ leads to the Poisson. We have used the noise to signal amplitude $m = (N/S)^{1/2}$ rather than the usual S/N ratio for the following reason. Near the Poisson limit the shape of the wings of the distribution is very sensitive to a small amount of noise. Hence m can be a few percent, in some sense very close to Poissonian, yet to the eye the curve looks quite different from Poissonian (see Fig. 1 of ref. 5 for illustration of this fact).

Since (11) has an extra parameter, it is not surprising that it leads to multiplicity fits as good or better than the negative binomial. What is not visible at first sight, however, is that one can trade an increasing k for an increasing S/N: either will narrow the distribution in a way which accomodates the data equally well from a X^2 criterion. Examples were given[26] by us at the Luna conference; more will be given elsewhere. Due to space limitations we here only assert again the result: <u>fits to multiplicity distributions alone cannot distinguish between the negative binomial from the partially coherent distribution with smaller k and non-zero S/N</u>. However, analysis of the $p\bar{p}$ forward-backward correlation[28] does constrain the parameters, indicating that N/S > 1 but not that (1) is really in force. What are the consequences of this ambiguity in the parameter space of (11)? The most important ones are:

(1) The large values of κ obtained by UA5 by fitting Eq. (1) to non-single-diffractive data can be eliminated by allowing coherent emission to be substantial at low energy, disappearing completely at higher energies (so that the energy dependence of KNO plot could stabilize at higher energies.) This point of view, stressed recently by the Marburg group[29], shows how the puzzling rapid decrease of k can be replaced by a more plausible increase of randomness of the emitting fields with increasing energy.

(2) Recent measurements of e^+-e^- annihilation to hadrons at 29 GeV have given precise charged multiplicity distributions. These data were very well described by (1) with very large values of k (ranging up to 100). In 1984 we claimed that existing data were almost Poissonian, the deviations being due to a small amount of noise superposed on almost coherent-state behavior for one or two quark jets (i.e. k = 1 or 2 is literally the number of sources). Since for $k\to\infty$ the negative binomial approaches the Poisson, these views are not very different mathematically event though we have no idea how to interpret k = 50 or

even 20, in a physical way. Recently[31] we have analyzed the data of ref. 30 to try to discriminate phenomenologically between these alternatives. It is very hard to distinguish, even with the aid of F/B correlation data and restricted rapidity intervals, although the nearly Poissonian limit looks somewhat better.

To summarize, the replacement of the negative binomial distribution (1) by the partially coherent distribution for both hadron-hadron and e^+-e^- multiplicity distributions. In each case the physical picture is intuitively simple. For h-h we have emissions from a small average number of effective cells (whose number could even be constant). The energy dependence of the KNO plot is then to be interpreted as the decrease of S/N; current collider results are nearly at the negative binomial limit. It is therefore tempting to speculate that the C_n moments will saturate beginning by Fermilab collider energies. For e^+-e^- hadronizations we can, as in ref. 5 continue to identify k as the number of jets (except at the lower energies), which is small. The narrowness of the distribution is due to the largeness of S/N. What is missing in this parametrization by (11) is any understanding of why KNO scaling should hold (as it seems to) in e^+-e^- annihilations. The pure Poisson does not scale, and we have neither a dynamical or statistical explanation of how N/S should be tuned to conform to the apparent experimental validity of KNO scaling in e^+-e^- annihilations. We hope that the next generation of experiments will shed light on this question.

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