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Toroidal Compactification of Closed Bosonic Strings

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Abstract

The boundary conditions of a free closed bosonic string propagating in a space where the compactified dimensions are toroidal limit the lattices defining the tori to be self dual, just as is required by modular invariance of the one-loop dual amplitudes. The spectra generated by the free-string oscillators for both Euclidean and Lorentzian lattices are described.

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The object of this talk is a simple explanation why the lattices that define the tori of the compactified dimensions for closed bosonic string must be self dual. This requirement is already apparent from the free string, and results from the closed-string boundary conditions. The work on Euclidean self-dual lattices is in [1], and the extension to self-dual Lorentzian lattices is being reported here [2].

It has been established that modular invariance of the "correction factor" [3] of the integrand of the one-loop string amplitude (which is accepted as a necessary constraint on unitary dual amplitudes) implies that the lattice defining the torus for the closed string is a self-dual lattice [4]. A similar result follows from the free string: for example, if we demand that all masses are integers (in units where the tachyon mass is -1) and internal symmetry is a finite dimensional Lie group whose generators are obtained by the Frenkel-Kac construction [5], then the "left" and "right" lattices must each be even self-dual Euclidean lattices, and can be chosen independently to be $E_8 \times E_8$ or $Spin(32)/Z_2$ for 16 compactified dimensions [1]. These results will be rederived here.

It has also been found that a lattice associated with GXG (G a Lie algebra with roots of equal length) is consistent with modular invariance [4], which is equivalent to dimensional reduction on a torus defined by an even self-dual Lorentzian lattice [6]. The dual model in this case has tachyons of at least two different mass values, with the fractional-mass tachyons in nontrivial representations of GXG. As stated in [6], the dual models of the Lorentzian case are modular invariant over a continuous range of parameters. Although some masses depend on the parameters, it is not possible to rid the bosonic theory of tachyons by a special choice of the parameters. The Lorentzian cases are also derived from the boundary conditions. An explicit example of a spectrum calculation will be given

Let us comment on the possible physical relevance of toroidal compactification. The stability of toroidal compactification of the bosonic string is questionable, especially if the tachyons neither decouple nor acquire non-negative masses. Several resolutions of this problem have been suggested. One is that extending the boundary conditions and the related spaces (such as to orbifolds [7]) might simply remove the tachyon(s) from the spectrum. Another is to impose symmetries that prohibit the tachyon; supersymmetric models have no tachyons, as is clearly seen in the formulation of the heterotic strings [8]. Finally, there are hints that in the correct ground state, even with toroidal compactification, the tachyons may decouple or become unphysical degrees of freedom. The group G associated with the lattice must be broken in a special way. Although the 26-dimensional bosonic theories appear to have tachyons and no fermions, G may be broken in a way where the 10- and maybe the 4-dimensional theory has chiral fermions and no tachyons. In this case a subgroup of G contributes to the Lorentz group in the smaller number of dimensions [4]. The closed string has more than enough states to give any one of the various superstring theories in 10 dimensions, including either heterotic strings, type IIA, or IIB, but it is not yet known whether the dynamics can actually decouple the unwanted states At present it is merely self consistent to assume these states decouple Already on the basis of this attractive possibility, I believe it worthwhile to analyze simple toroidal compactification.

The non-interacting closed bosonic string in light-cone gauge satisfies a two-dimensional free wave equation. The solution is [3]

$$X^i(\tau,\sigma) = x^i + 2\alpha' p^i \tau + 2N^i R\sigma + i\sqrt{\alpha'/2}\,\sum_{n\neq 0}(1/n)[a_n{}^i\, e^{-2in(\tau-\sigma)} + \tilde{a}_n{}^i\, e^{-2in(\tau+\sigma)}] \quad , \qquad (1)$$

where the coefficients are chosen so that in the first quantized theory, the corresponding operators satisfy canonical commutation relations, e.g.,

$$[x^i , p^j] = i\delta^{ij} \quad . \qquad (2)$$

The indices i, j, ... run from 1 to 24 and label the directions transverse to the light-cone components $X^{\pm}(\tau,\sigma)$, where $X^+(\tau,\sigma) = x^+ + 2\alpha' p^+ \tau$, and the dependent operator $X^-(\tau,\sigma)$ is gotten from the constraint equations.

As the parameter σ goes from 0 to π, the line traced out by $X^I(\tau,\sigma)$ is a closed curve, so $X^I(\tau,\pi)$ and $X^I(\tau,0)$ must designate the same point on the compactified space. (I, J, K = 1,...,D label the D components of $X^i(\sigma,\tau)$ in the compactified dimensions.) In principle the "radii" R^I may differ for each direction, but the generality of our arguments are only trivially affected by setting all of them equal to a common radius $R^I = R$.

In the first quantized theory N^I and some constant times $\sqrt{\alpha'}\, p^I$ are operators with integer eigenvalues, which, however, must satisfy the constraints on the system. It simplifies the analysis to make a canonical transformation of these zero-mode operators to a set where the constraints are diagonal. The $X^i(\tau,\sigma)$ can be separated into a "left-moving" piece (in the τ,σ space), which is a function of $\xi_L = \tau+\sigma$, and a "right-moving" piece, which is a function of $\xi_R = \tau-\sigma$ only:

$$X^i(\tau,\sigma) = X_L{}^i(\xi_L) + X_R{}^i(\xi_R) \quad , \tag{3}$$

where

$$X_L{}^i(\xi_L) = x_L{}^i + \alpha' p_L{}^i \xi_L + i\sqrt{\alpha'/2}\ \sum_{n\neq 0}(1/n)\, \tilde{a}_n{}^i\, e^{-2in\xi_L} \quad , \tag{4}$$

and an identical equation for $X_R{}^i(\xi_R)$, except that L is replaced by R and $\tilde{a}_n{}^i$ by $a_n{}^i$. With the conventions expressed in (4), the canonical commutation relations of the zero-mode operators in the compactified directions are

$$[\, x_L{}^I , p_L{}^J] = i\delta^{IJ} \quad , \qquad [x_R{}^I , p_R{}^J] = i\delta^{IJ} \quad , \tag{5}$$

and the remaining commutators are zero.

The canonical transformation between the "left" and "right" operators and the zero-mode operators in (1) is

$$x^I = x_L{}^I + x_R{}^I \quad , \qquad 2p^I = p_L{}^I + p_R{}^I \quad ,$$

$$\alpha'\tilde{x}^I = R(x_L{}^I - x_R{}^I) \quad , \qquad 2R\, N^I = \alpha'(p_L{}^I - p_R{}^I) \quad . \tag{6}$$

Thus, it is trivially confirmed that $[\tilde{x}^I , N^J] = i\delta^{IJ}$, etc., so $\tilde{x}^I$ is conjugate to N^I, although $\tilde{x}^I$ does not appear in (1). The factors of 2 are inserted in (6) so that (5) is canonical, in contrast to the conventions of the heterotic string [8]. Note that the winding number of a string can be transformed, for example, by self interactions, so it is an independent quantum operator unless further constraints on the theory prohibit such interactions.

We assume that the zero modes of the string completely describe the space-time in which the string propagates, so that the location of the string is its average position. (This simplifying assumption has usually been made in past treatments of the string, and will be needed in the details of our arguments. However, it has no physical basis; the nonzero modes could play a crucial role in the spatial compactification.) Then the term $2N^iR\sigma$ in (1) is interpreted as a "winding number" term and can be nonzero for the compactified dimensions. The unconstrained closed bosonic string has two sets of zero-mode operators for the compactified dimensions. Since the compactification refers just to the zero modes, the dimensionality of the torus of compactification is twice the number of compactified dimensions, so their number is actually twice that counted by the index I. The heterotic strings (with their constrained bosonic strings) have only one set of zero-mode operators for the compactified dimensions, so there the dimensionality of the torus is the same as the number of compactified dimensions [8].

The constraint equations are $(\dot{X} \pm X')^2 = 0$, which can also be written as

$$(\dot{X}_L)^2 = (\dot{X}_R)^2 = 0 \; . \qquad (7)$$

The zero-mode projections of (7) define the dependent operators p_L^- and p_R^- the σ translation invariance of the origin of the string leads to the definition of the mass operator,

$$\alpha' M^2 = (1/2)\, w_L^2 + \tilde{N} - 1 \quad , \qquad (8)$$

where the $\mathbf{w_L}$ and $\mathbf{w_R}$ lattices are related to the lattice of momentum eigenvalues by

$$w_L = \sqrt{\alpha'/2}\; \mathbf{p_L} \; , \qquad \mathbf{w_R} = \sqrt{\alpha'/2}\; \mathbf{p_R} \quad , \qquad (9)$$

and the number operators N and N are

$$\tilde{N} = \textstyle\sum_{n=1} \tilde{a}_{-n}^{\,i}\, \tilde{a}_n^{\,i} \quad , \quad N = \textstyle\sum_{n=1} a_{-n}^{\,i}\, a_n^{\,i} \quad , \qquad (10)$$

where the i sum is over compact and noncompact dimensions. The momentum vectors and number operators satisfy the constraint,

$$\tilde{N} + (1/2)\, \mathbf{w_L}^2 = N + (1/2)\, \mathbf{w_R}^2 \; . \qquad (11)$$

Only the finite subalgebra of the full affine algebra obtained by the Frenkel-Kac construction is a symmetry of the S-matrix [5]. The basic representation of the affine algebra is the Fock space obtained from $a_{-n}{}^{I}$, $\tilde{a}_{-n}{}^{I}$, and $\exp(i\mathbf{x}_L{}^{op}\cdot\mathbf{p}_L)$ and $\exp(i\mathbf{x}_R{}^{op}\cdot\mathbf{p}_R)$, where $\mathbf{p}_L$ and $\mathbf{p}_R$ are on root lattices (up to a normalization). The restriction (11) eliminates states from this irreducible representation, so the full infinite-dimensional affine algebra does not generate a symmetry of the S-matrix, nor does it commute with the Lorentz algebra.

The vectors of $\mathbf{w}_L$ and $\mathbf{w}_R$ each span a D-dimensional Euclidean sublattice; they can be viewed as sublattices of a 2D-dimensional lattice $\mathbf{W} = \{\mathbf{w}_L , \mathbf{w}_R\}$. This means we extend the vectors in $\mathbf{w}_L$ and $\mathbf{w}_R$ to be vectors in $\mathbf{W}$; this extension must include a prescription (consistent with the spatial and string periodicity discussed below) for recovering $(\mathbf{W})_L = \mathbf{w}_L$ and $(\mathbf{W})_R = \mathbf{w}_R$ from the vectors in $\mathbf{W}$; the sublattices $\mathbf{w}_L$ and $\mathbf{w}_R$ are Euclidean. As a nontrivial example, suppose that we define a scalar product on $\mathbf{W}$ where, in the L-R basis, the metric is diagonal and of the form,

$$\mathbf{U}\cdot\mathbf{V} = -\mathbf{u}_L\cdot\mathbf{v}_L + \mathbf{u}_R\cdot\mathbf{v}_R \ , \qquad (12)$$

for vectors $\mathbf{U}$ and $\mathbf{V}$ in $\mathbf{W}$. Then, in this basis $[(\mathbf{W})_L]^2 = \mathbf{w}_L{}^2$, so the P and L can be associated with the $(+,+,...,+)$ sector and $(-,-,...,-)$ sector, respectively, which, indeed, is the origin of the "chiral" notation. In this extension, the basis vectors can have nonzero components on both the left and right lattices. Equation (9) can be rewritten as

$$\mathbf{W} = \sqrt{\alpha'/2}\ \mathbf{P} = \sqrt{\alpha'/2}\ (\mathbf{p}_L , \mathbf{p}_R) \ . \qquad (13)$$

We first impose periodicity on the compactified <u>space</u>; for this we need the identification of the zero-mode operators and space-time. In particular, we assume that $\mathbf{P} = \{ \mathbf{p}_L, \mathbf{p}_R \}$ is the translation operator on the 2D-torus. The torus is defined by a lattice of points,

$$X^0 = \{ \mathbf{x}^0{}_L \ , \ \mathbf{x}^0{}_R \}. \tag{14}$$

The periodicity of the space is then guaranteed by restricting the eigenvalues of the operator $\mathbf{P}$ by [8]

$$\exp(i\mathbf{p}_R \cdot \mathbf{x}^0{}_R + i\mathbf{p}_L \cdot \mathbf{x}^0{}_L) \ |\text{physical}> \ = \ e^{2\pi i(\text{integer})}|\text{physical}> \ . \tag{15}$$

This is a translation in the 2D-dimensional lattice, not a scalar product on the 2D-dimensional lattice. This distinction is important because we will consider lattices where $\mathbf{W} \neq -\mathbf{W}$ in the sense of sets of lattice vectors.

Equation (15) is trivially solved by introducing the lattice dual to X^0; the eigenvalues of the operator $\mathbf{P}$ fall on the $\mathbf{W}$ lattice (13) if the lattice of the torus is defined by

$$\{ X^0{}_L, X^0{}_R \} \ = \ \sqrt{2\alpha'} \ \pi \ \{ \pm(\mathbf{W}^*)_L \ , (\mathbf{W}^*)_R \ \} \ , \tag{16}$$

where the sign corresponds with the choice of sign in (12) and

$$\mathbf{W}^\alpha \cdot \mathbf{W}^*{}_\beta \ = \ \delta^\alpha{}_\beta \ , \tag{17}$$

and the $\mathbf{W}^\alpha$ ($\alpha = 1,\ldots,2D$) are the basis vectors of the lattice and the $\mathbf{W}^*{}_\beta$ are the basis of its dual. Equation (16) is necessary for the space generated by the zero-mode operators to be a torus.

The only boundary condition we impose on the string itself is that it closes. As σ moves from 0 to π, $X^I(\tau,\sigma)$ must return to the same point. Of course X^I is defined modulo the lattice, so from (1) we find that for each vector in $\mathbf{x}^0{}_L + \mathbf{x}^0{}_R$, there exists a vector $2\pi R\,\mathbf{N}$ on a D-dimensional lattice. This requires finding a relative orientation of $\mathbf{x}^0{}_L$ and $\mathbf{x}^0{}_R$ in D-dimensional space. From (16) we can write this restriction as

$$\sqrt{2}\,\mathbf{N} = -(\sqrt{\alpha'}/R)\,[\pm(\mathbf{W}^*)_L + (\mathbf{W}^*)_R]\,, \qquad (18)$$

where $\sqrt{2}\,\mathbf{N}$ must be an even lattice. (In this and many of the following equations, "=" means equality of sets.) The order of taking the dual and L or R components can make a difference. With (18) and (6), we can write the boundary conditions as

$$\mathbf{W}_R - \mathbf{W}_L = (\mathbf{W}^*)_R \pm (\mathbf{W}^*)_L\,. \qquad (19)$$

We now investigate several solutions to (18) and (19). In the simplest solution, we assume that the basis vectors of $\mathbf{W}$ can be written in the L-R basis as [$\mathbf{w}_L$, $\mathbf{0}$] and [$\mathbf{0}$, $\mathbf{w}_R$], and that the left and right lattices are even so that $\alpha' M^2$ has integral eigenvalues only. For the Frenkel-Kac construction to be possible, the lattices $\mathbf{w}_L$ and $-\mathbf{w}_L$ are identical, so that (19) is solved with $\mathbf{W}$ self dual, $\mathbf{W} = \mathbf{W}^*$ with $\mathbf{w}_L = \mathbf{w}_L{}^*$ and $\mathbf{w}_R = \mathbf{w}_R{}^*$. Euclidean even self-dual lattices exist in 8n dimensions. The choice of lattices for $\mathbf{w}_L$ and $\mathbf{w}_R$ is restricted to E_8 for D = 8 compactified dimensions, and $E_8 \times E_8$ or Spin(32)/Z_2 for 16 dimensions. The next even self-dual lattices appear in 24 dimensions. The problem here is to construct from the $\mathbf{W}$ lattice an $\mathbf{N}$ lattice that satisfies (18).

We restrict our attention to the physically interesting case, D = 16, where 10 dimensions remain uncompactified. The simplest Euclidean even self-dual solution is where both $\mathbf{w}_L$ and $\mathbf{w}_R$ are $E_8 \times E_8$ root or Spin(32)/Z_2 weight lattices. Equation (18) is solved by superposing the two lattices so they exactly coincide in the D-dimensional space. Since both lattices are self conjugate, e.g., $\mathbf{w}_L = -\mathbf{w}_L$ (in the sense of sets) and $\mathbf{w}_L = \mathbf{w}_L - \mathbf{w}_R$, so $\sqrt{2}\,\mathbf{N}$ is even if $\alpha' = R^2$.

The symmetry of the states is G X G, where G = $E_8 \times E_8$ or SO(32).

The generators of this finite dimensional group are obtained from the Frenkel-Kac construction [5]. This construction is naturally extended to affine GXG, but restriction (11) breaks up the representation into a natural grading, according to the integers, $\tilde{N} + (1/2)\, w_L^2 - N - (1/2)\, w_R^2$. However, all the pieces of the affine representation do appear at different spins and masses, since the oscillators from the uncompactifed dimensions contribute to $\tilde{N}$ and N. This situation is, in fact, quite similar to the spectrum of the open string, which has no constraints, and whose spectrum decomposes into direct products of basic representations of the affine algebra times representations of the light-cone Lorentz group [9]. It looks hopeful that the representations of GXG times the Lorentz group (however it is constructed) for the closed bosonic string can be gathered into representations of some infinite-dimensional algebra.

To indicate the motivation for this optimism, it may help to display the first few levels of the affine $E_8 \times E_8$ basic representation, decomposed into $E_8 \times E_8$ irreducible representations, and the particle spectrum of the closed bosonic string reduced by 8 dimensions of the $E_8 \times E_8$ lattice. The components of the affine $E_8 \times E_8$ basic representation can be classifed by

$$N_+ = \tilde{N} + N + (1/2)\, w_L^2 + (1/2)\, w_R^2 \quad \text{(level)}$$

$$N_- = \tilde{N} - N + (1/2)\, w_L^2 - (1/2)\, w_R^2 \quad \text{(grade)}.$$

The first few levels of the basic represention of $E_8 \times E_8$ are then [9]:

$[N_+, N_-]$	$E_8 \times E_8$ representation
[0,0]	(1, 1)
[1,1]	(248, 1)
[1,-1]	(1, 248)

[2,2]	(1+248+3875, 1)
[2,0]	(248, 248)
[2,-2]	(1, 1+248+3875)
[3,3]	(1+248+248+3875+30380, 1)
[3,1]	(1+248+3875, 248)
[3,-1]	(248, 1+248+3875)
[3,-3]	(1, 1+ 248+248+3875+30380)
[4,4]	(1+1+248+248+248+3875+3875+30380+27000+147250, 1)
[4,2]	(1+248+248+3875+30380, 248)
[4,0]	(1+248+3875, 1+248+3875)
[4,-2]	(248, 1+248+248+3875+30380)
[4,-4]	(1, 1+1+248+248+248+3875+3875+30380+27000+147250)

The E_8 irreducible representations needed up to level 4 are conveniently and unambiguously labeled by their dimensionalities. This table is easily derived from the partition function for the basic representation of $E_8 X E_8$, $\Pi(1-x^k)^{-16}$, and the E_8 representation orbit decompositions [10]. More simply, the affine $E_8 \times E_8$ basic representation can be constructed from the affine E_8 basic representation, as is easily seen from the grading in the above table.

If there were no external space, then the constraint $N_- = 0$ of the closed string would yield a string spectrum of the form $\sum (\mathbf{r_n}, \mathbf{r_n})$, where $\mathbf{r_n}$ is the E_8 representation of the n-th level of the basic affine E_8 representation.

With the noncompactified 18 dimensions, the massless states are in representations of SO(16) and the massive states are in representations of

SO(17) in light-cone gauge. We carry our the construction with the full set of $\tilde{a}_{-n}^{1}$ and a_{-n}^{1}, while satisfying constraint (11).

$\alpha' M^2$	Spin (SO(16))	$E_8 X E_8$ representations
-1	(0000....)	(1,1) (Tachyon)
0	(2000...)	(1,1) (Graviton)
0	(0100...)	(1,1) (Antisymmetric tensor)
0	(1000...)	(1,248)+(248,1) (Yang-Mill vectors)
0	(0000...)	(1,1)+(248,248)
	Spin (SO(17))	
1	(4000...)	(1,1)
1	(2100...)	(1,1)
1	(0200...)	(1,1)
1	(3000...)	(1,248)+(248,1)
1	(1100...)	(1,248)+(248,1)
1	(0100...)	(1,1)+(248,248)
1	(2000...)	3(1,1)+(1,3875)+(3875,1)+(248,248)
1	(1000...)	2(1,248)+2(248,1)+(248,3875)+(3875,248)
1	(0000...)	2(1,1)+(1,3875)+(3875,1)+(248,248)+(3875,3875)

That the $\alpha' M^2=1$ states fall into massive representations is the miracle of Lorentz invariance. However, the full structure may be better seen in terms of the massless SO(16) representations. The following table lists the $E_8 \times E_8$ content of each massless representation for the $\alpha' M^2 = 1$ states, written in terms of the [level,grade] notation used in the affine basic representation:

(4000...) [0,0]

(2100...) [0,0]

(0200...) [0,0]

(3000...) ([1,1]+[1,-1]+2[0,0])

(1100...) ([1,1]+[1,-1]+2[0,0])

(0100...) [2,0] + ([1,1]+[1,-1]+2[0,0])

(2000...) [2,2]+[2,0]+[2,-2] + ([1,1]+[1,-1]+2[0,0])+2[0,0]

(1000...) [3,1]+[3,-1] + 3([1,1]+[1,-1]+2[0,0])

(0000...) [4,0]+[2,2]+[2,0]+[2,-2] + ([1,1]+[1,-1]+2[0,0])+[0,0]

Thus, we expect that the algebra that commutes with the light-cone generator is the subalgebra of the affine $E_8 \times E_8$ that commutes with the grading operator.

Several technical results are useful in deriving these results. The representations have been labeled with the Dynkin highest weights. The tensor product of SO(n) representations (n>6) needed are (2000...)X(1000...) = (3000...)+(1100...)+(1000...) and (2000...)X(2000...) = (4000...)+(2100...) +(0200...)+(2000...)+(0100...)+(0000....). The not-quite obvious branching rules for SO(n+1) to SO(n) representations are: (0100...)=(0100...)+(1000...); (0200...) = (0200...)+(1100...)+(2000...); (1100...) = (1100...)+(2000....) +(0100...)+(1000...); and (2100...) = (2100...)+(3000...)+(1100...)+(2000...) +(0100...) +(1000...).

We now turn to the case where $\mathbf{w_L}$ is an $E_8 \times E_8$ lattice and $\mathbf{w_R}$ is a Spin(32)/Z_2 lattice. The basis vectors of both $\mathbf{w_L}$ and $\mathbf{w_R}$ can be written in an SO(16) X SO(16) basis without distorting either weight lattice since SO(16) X SO(16) is a regular subalgebra of both $E_8 \times E_8$ and SO(32). The branching rule for the adjoint of $E_8 \times E_8$ is **(120,1)** + **(1,120)** + **(128,1)** + **(1,128)** of SO(16) X SO(16), where **120** is the adjoint and **128** is one of the spinors of SO(16). Similarly for Spin(32)/Z_2, the lowest lying representations with weights on the lattice include the adjoint **496** = **(120,1)** + **(1,120)** + **(16,16)** and one of the spinors **32768** = **(128,128)** + **(128',128')**, where **128'** is the other SO(16) spinor. We now construct $\sqrt{2}\mathbf{N}$ by adding vectors of $\mathbf{w}_L$ to $\mathbf{w}_R$; all that is needed to determine α'/R^2 from the lengths of the vectors of $\mathbf{w}_L - \mathbf{w}_R$ are the SO(16) X SO(16) congruency classes of the final set

SO(16) has 4 congruency classes, which we label a (adjoint), s (spinor), s' (other spinor) and v (vector). The addition rules for the weights of one SO(16) are those of $Z_2 X Z_2$, where a $\approx$ (0,0), s $\approx$ (1,0), s' $\approx$ (0,1) and v $\approx$ (1,1). Thus, the congruency classes of the $E_8 \times E_8$ lattice are (a,a) + (a,s) + (s,a) + (s,s), which closes on itself under addition of lattice vectors. The SO(16) X SO(16) congruency classes of the Spin(32)/Z_2 lattice are (a,a) + (s,s) + (s',s') + (v,v), which again closes under addition. The sum of an $E_8 \times E_8$ and Spin(32)/Z_2 lattices falls into SO(16) X SO(16) classes,

$$(a,a) + (s,s) + (s',s') + (v,v) + (s,a) + (a,s) + (s',v) + (v,s') \quad . \qquad (20)$$

The length squared of all spinor and adjoint weights are even integers. However, <u>all</u> vector weights are odd, so the lattice vectors in the last two classes of (20) are odd. Thus, $\mathbf{w}_L - \mathbf{w}_R$ has vectors of length 2,3,4... Then, $\sqrt{2}$ **N** is even if $\alpha' = 2R^2$. The zero-mass scalars are in [Adj($E_8 X F_8$),Adj(SO(32))]

A somewhat more complicated set of solutions that admit a natural left-right separation is the set of Lorentzian lattices based on any simple GXG, where rank(G) = D [4,6]. Suppose we place $\mathbf{w}_L$ and $\mathbf{w}_R$ on weight lattices so that $\alpha' M^2$ may have fractional eigenvalues The Frenkel-Kac construction of the GXG generators is unchanged (the roots are on the lattice), but several twisted versions of the basic representation with the same structure as the basic representation appear in the spectrum, in the same sense as we found an affine representation before, i.e., ignoring the grading constraint or the external spin.

The crucial point here is to note that the basis vectors of $\mathbf{W}$ do not have to be factorable into vectors with components, $(\mathbf{w}_L,\mathbf{0})$ and $(\mathbf{0},\mathbf{w}_R)$. Although $\mathbf{w}_L$ by itself is not self dual, it is possible to make an even self-dual lattice by doubling the space $\mathbf{w} = [\mathbf{w}_L, \mathbf{w}_R]$ and imposing a Lorentzian metric. Our explanation here on how this works will be an example. We look at the reduction by D=2 dimensions with the 4-dimensional Lorentzian lattice having metric (+,+,-,-) and containing certain weights of SU(3)XSU(3). The generalization for specific cases is obvious.

Let a_1 and a_2 be the basis vectors, which correspond to the highest weights of the $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations, respectively. [In the Dynkin basis, a_1 has components (1,0) and a_2 has components (0,1).] Then we can form a basis of this 4-dimensional lattice with the vectors,

$$v_1 = (a_1 , a_2-a_1) \quad , \quad v_2 = (a_2, a_1-a_2)$$

$$v_3 = (a_1-a_2 , -a_1) \quad , \quad v_4 = (a_1-a_2 , a_2) \tag{21}$$

These vectors are all null vectors and they are also self dual, i.e., there are a set of vectors v_i^* satisfying

$$v_i^* \cdot v_j = \delta_{ij} \tag{22}$$

It is easily confirmed that the dual vectors are $v_1^* = v_2$, $v_2^* = v_1$, $v_3^* = v_4$, and $v_4^* = v_3$. Note that for each $\mathbf{v_i}$, $(\mathbf{v_i})_L - (\mathbf{v_i})_R$ is a root of SU(3), $\pm(2a_1-a_2)$ or $\pm(2a_2-a_1)$.

This is an even lattice under the Lorentzian scalar product, which we now confirm. The most general vector is

$$v = \begin{vmatrix} (n_1+n_3+n_4)\, a_1 + (n_2-n_3-n_4)\, a_2 \\ (-n_1+n_2-n_3)\, a_1 + (n_1-n_2+n_4)\, a_2 \end{vmatrix} , \tag{23}$$

where the metric for the scalar product $a_i \cdot a_j$ (in the $\mathbf{w_L}$ or the $\mathbf{w_R}$ subspace) is

$$\frac{1}{3} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} ,$$

which is just of inverse of the SU(3) Cartan matrix. The norms of the left and right components are,

$$w_L^2 = (1/3)\,(n_1,n_2,n_3,n_4) \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & -1 & -1 \\ 1 & -1 & 2 & 2 \\ 1 & -1 & 2 & 2 \end{vmatrix} \begin{vmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{vmatrix} , \tag{24}$$

and the metric for $\mathbf{w_R}^2$ is

$$(1/3) \begin{vmatrix} 2 & -2 & 1 & 1 \\ -2 & 2 & -1 & -1 \\ 1 & -1 & 2 & -1 \\ 1 & -1 & -1 & 2 \end{vmatrix} .$$

Thus, the norm of a vector $\mathbf{w}$ is

$$w^2 = 2\, n_1 n_2 + 2 n_3 n_4 , \tag{25}$$

and the lattice is even and self dual. In order to satisfy that $\sqrt{2}$ **N** is even, we compute $\mathbf{v_R} - \mathbf{v_L}$ from (23),

$$\mathbf{v_R} - \mathbf{v_L} = (n_1+n_3)(2a_1-a_2) + (n_2-n_4)(2a_2-a_1), \qquad (26)$$

which is on the SU(3) root lattice, so $\alpha'=R^2$. Thus, we see that $\mathbf{v_R}$ and $\mathbf{v_L}$ are always in the same congruency (triality) class.

We now construct the spectrum of states of the closed bosonic string, reduced by two dimensions on this lattice. The calculation is a Fock space calculation of the same kind as we have often done. The mass operator is defined in (8) and the constraint in (11). The spectrum of states is:

$\alpha' M^2$	Spin	SU(3) X SU(3) representations	
-1	(0000...)	(1,1)	(the usual tachyon)
-2/3	(0000...)	$(3,3) + (\bar{3},\bar{3})$	(the new tachyon)
-1/3	Nothing		
0	(2000...)	(1,1)	(graviton)
	(1000...)	(1,8)+(8,1)	(Y. M. vectors)
(massless)	(0100...)	(1,1)	(Antisymmetric tensor)
	(0000...)	(1,1)+(8,8)	
	Massive representations		
1/3	(2000...)	$(3,3)+(\bar{3},\bar{3})$	
	(0100...)	$(3,3)+(\bar{3},\bar{3})$	
	(1000...)	$(6,\bar{3})+(\bar{6},3)+(\bar{3},6)+(3,\bar{6})$	
	(0000...)	$(6,6)+(\bar{6},\bar{6})+(3,3)+(\bar{3},\bar{3})$	
2/3	Nothing		
1	(4000...)	(1,1)	

(3000...)	(1,8)+(8,1)
(2100...)	(1,1)
(0200...)	(1,1)
(2000...)	3(1,1)+(1,8)+(8,1)+(8,8)
(1100...)	(1,8)+(8,1)
(0100...)	(1,1)+(8,8)
(1000...)	2[(1,8)+(8,1)+(8,8)]
(0000...)	2(1,1)+(1,8)+(8,1)+2(8,8)

We conclude with several comments on Lorentzian lattices. The lattice, $\Gamma_{p,q}$, which has a signature with p pluses and q minuses, can be even self dual only if $p = q + 8n$. Thus, their number appears to be greatly limited. Neverthess, if the GXG symmetry structure is not required, then it is possible to deform these lattices by a general SO(p,q) transformation. The independent parameters live in the coset SO(p,q)/SO(p)XSO(q); there are pq free parameters. An examination of $\Gamma_{1,1}$ shows how this works. The general prescription indicates that there should be a one-parameter family of solutions to the even self dual constraints. In a basis where the metric is a diagonal matrix with (1,-1), the basis vectors are

$$v_1 = (1/\sqrt{2})\, e^{x}\, (1, 1)$$
$$v_2 = (1/\sqrt{2})\, e^{-x}\, (1, -1) \quad . \qquad (27)$$

Note that v_1 and v_2 are null vectors, with their duals being v_2 and v_1, respectively. Thus the lattice is self dual

The general vector on this lattice is

$$v = (1/\sqrt{2})\ [\ n_1 e^{-x} + n_2 e^{x}\ ,\ n_2 e^{x} - n_1 e^{-x}\ ]\ . \qquad (28)$$

For two vectors labeled by integers n_1, n_2 and m_1, m_2, the scalar product is

$$u_1 \cdot u_2 = n_1 m_2 + n_2 m_1 \quad . \tag{29}$$

Thus, this lattice is even self dual for all values of x. However, $\mathbf{v}_L - \mathbf{v}_R$ is not a root of SU(2) unless x=0, since,

$$v_L - v_R = \sqrt{2}\, n_1\, e^{-x} \quad . \tag{30}$$

This result suggests that there is a continuum of consistent string theories. (The modular invariance of the one-loop amplitudes is checked in [4,6].)

The parameters take on critical values when the Lorentzian lattice is composed of the weights in GXG. For other values the symmetry is reduced. It is a subject of present investigation whether or not this freedom will allow for a discussion of symmetry breaking in the S-matrix framework.

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