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WAPD-TM-1612

Distribution Category UC-78

PROOF OF BREEDING IN THE LIGHT WATER BREEDER REACTOR

(LWBR Development Program)

WAPD-TM--1612

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DE88 005093

September 1987

Contract No. DE-AC11-76PN00014

Printed in the United States of America
Available from the
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22161

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FOREWORD

The Shippingport Atomic Power Station located in Shippingport, Pennsylvania was the first large-scale, central-station nuclear power plant in the United States and the first plant of such size in the world operated solely to produce electric power. This program was started in 1953 to confirm the practical application of nuclear power for large-scale electric power generation. It has provided much of the technology being used for design and operation of the commercial, central-station nuclear power plants now in use.

Subsequent to development and successful operation of the Pressurized Water Reactor in the Atomic Energy Commission (now Department of Energy, DOE) owned reactor plant at the Shippingport Atomic Power Station, the Atomic Energy Commission in 1965 undertook a research and development program to design and build a Light Water Breeder Reactor core for operation in the Shippingport Station.

The objective of the Light Water Breeder Reactor (LWBR) program has been to develop a technology that would significantly improve the utilization of the nation's nuclear fuel resources employing the well-established water reactor technology. To achieve this objective, work has been directed toward analysis, design, component tests, and fabrication of a water-cooled, thorium oxide-uranium oxide fuel cycle breeder reactor for installation and operation at the Shippingport Station. The LWBR core started operation in the Shippingport Station in the Fall of 1977 and finished routine power operation on October 1, 1982. After End-of-Life core testing was completed, the core was removed and the spent fuel shipped to the Naval Reactors Expended Core Facility for detailed examination to verify core performance including an evaluation of breeding characteristics.

In 1976, with fabrication of the Shippingport LWBR core nearing completion, the Energy Research and Development Administration, now DOE, established the Advanced Water Breeder Applications (AWBA) program to develop and disseminate technical information which would assist U.S. industry in evaluating the LWBR concept for commercial-scale applications. The AWBA program, which was concluded in September, 1982, explored some of the problems that would be faced by industry in adopting technology confirmed in the LWBR program. Information developed includes concepts for commercial-scale prebreeder cores which would produce uranium-233 for light water breeder cores while producing electric power, improvements for breeder cores based on the technology developed to fabricate and operate the Shippingport LWBR core, and other information and technology to aid in evaluating commercial-scale application of the LWBR concept.

All three development programs (Pressurized Water Reactor, Light Water Breeder Reactor, and Advanced Water Breeder Applications) have been conducted under the technical direction of the Office of the Deputy Assistant Secretary for Naval Reactors of DOE.

Technical information developed under the Shippingport, LWBR, and AWBA programs has been and will continue to be published in technical memoranda, one of which is this present report.

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	xiii
I. INTRODUCTION	1
II. DESCRIPTION OF LWBR CORE	5
A. Fuel Rods	5
B. Fuel Modules	5
C. Reactivity Control and Axial Zoning of Fuel	8
D. Radial Zoning of Fuel	9
III. FUEL MANUFACTURE AND INITIAL CORE LOADING DETERMINATION	15
IV. PREDICTION OF FISSILE INVENTORY RATIO AND EXPENDED CORE LOADINGS	21
A. The LWBR Calculational Model	21
B. Use of the Calculational Model to Predict the Fissile Inventory Ratio	23
C. Prediction of Individual Expended Fuel Rod Loadings and Isotopics	25
D. Rod Fission Product Axial Distributions from CINDER	28
V. THE TWO-STAGE STRATIFIED RANDOM SAMPLING PLAN	32
A. Background and Objectives of the Sampling Plan	32
B. Simulation Studies Used to Develop the Sampling Plan	33
C. Selection of 12 Proof-Of-Breeding Modules	36
D. Selection of 500 Proof-of-Breeding Rods	37
1. Definition of Rod Strata	37
2. Allocation of Proof-Of-Breeding Rods to Strata	39
3. Rod Selection Method	41
E. Selection of 24 Additional Rods	42
F. Selection of 17 Rods for Destructive Assay	44

TABLE OF CONTENTS (Cont'd)

	<u>Page</u>
VI. NONDESTRUCTIVE AND DESTRUCTIVE ASSAY RESULTS	63
A. Nondestructive Assay Method	63
B. Nondestructive Assay Results, Seed Rods	66
C. Nondestructive Assay Results, Standard Blanket Rods	67
D. Nondestructive Assay Results, Power Flattening Blanket Rods	68
E. Nondestructive Assay Results, Reflector Rods	69
F. Destructive Assay Results	70
VII. ROD STRATUM AND MODULE LOADINGS: ESTIMATION MODELS AND RESULTS ..	96
A. Stratum Average Estimation	97
B. Linear Regression Estimation	99
1. The Basic Model	99
2. Additional Terms in the Linear Regression Model: Water Channel Effect	103
3. Detection of Water Channel Effects	107
C. Module Loading Estimation and Variance	108
D. Results for Proof-Of-Breeding Modules	109
E. Module-to-Module Variation and Core Symmetry	111
VIII. ESTIMATION OF TOTAL EXPENDED CORE LOADING AND FISSILE INVENTORY RATIO	125
A. Total Expended Core Loading Estimation	125
1. Loading Estimate for Unassayed Power Flattening Blanket Modules II-1 and II-3	126
2. Loading Estimate for Unassayed Power Flattening Blanket Modules III-1, III-3, III-4, and III-5	126
3. Loading Estimate for Unassayed Reflector Module IV-7	127

TABLE OF CONTENTS (Cont'd)

	<u>Page</u>
B. Variance in Total Expended Core Loading Calculation	128
1. Random Variance	128
2. Systematic Variance	130
a. Sources of Systematic Error	130
b. PIFAG-ANL Difference as a Measure of Systematic Error	133
C. Expended Core Loading - Results	134
D. Estimation of Fissile Inventory Ratio	135
E. Determination of the Lower 95% Confidence Bound	136
IX. SUMMARY AND DISCUSSION OF SIGNIFICANCE	141
X. ACKNOWLEDGMENTS	144
XI. REFERENCES	145
APPENDIX A1: The Stratum Average Module Loading Model	A1
APPENDIX A2: The Linear Regression Module Loading Model	A6
APPENDIX A3: The Module-to-Module Variance	A14
APPENDIX B: Comparison of Predicted Isotopic Distributions to Destructive Analysis	B1

LIST OF TABLES

<u>Table</u>	<u>Title</u>	<u>Page</u>
III-1.	Seed and Blanket Module Initial Thorium and Uranium Loadings ...	20
V-1.	Module Strata Present in LWBR Core	46
V-2.	Seed and Blanket Rod Strata	47
V-3.	Reflector Rod Strata	48
V-4.	Rods Selected for Destructive Assay	49
VI-1.	Nondestructive Assay Results for Proof-Of-Breeding Rods from Seed Module I-1	72
VI-2.	Nondestructive Assay Results for Proof-Of-Breeding Rods from Seed Module II-3	73
VI-3.	Nondestructive Assay Results for Proof-Of-Breeding Rods from Seed Module III-1	74
VI-4.	Nondestructive Assay Results for Proof-Of-Breeding Rods from Seed Module III-2	75
VI-5.	Nondestructive Assay Results for Standard Blanket Proof-Of- Breeding Rods from Blanket Module I-3	76
VI-6.	Nondestructive Assay Results for Standard Blanket Proof-Of- Breeding Rods from Blanket Module II-2	78
VI-7.	Nondestructive Assay Results for Standard Blanket Proof-Of- Breeding Rods from Blanket Module III-2	79
VI-8.	Nondestructive Assay Results for Standard Blanket Proof-Of- Breeding Rods from Blanket Module III-6	80
VI-9.	Nondestructive Assay Results for Power Flattening Blanket Proof-Of-Breeding Rods from Blanket Module II-2	81
VI-10.	Nondestructive Assay Results for Power Flattening Blanket Proof-Of-Breeding Rods from Blanket Module III-2	83
VI-11.	Nondestructive Assay Results for Power Flattening Blanket Proof-Of-Breeding Rods from Blanket Module III-6	85
VI-12.	Nondestructive Assay Results for Proof-Of-Breeding Rods from Type IVA Reflector Module IV-4	87

LIST OF TABLES (Cont'd)

<u>Table</u>	<u>Title</u>	<u>Page</u>
VI-13.	Nondestructive Assay Results for Proof-Of-Breeding Rods from Type IVA Reflector Module IV-9	88
VI-14.	Nondestructive Assay Results for Proof-Of-Breeding Rods from Type IVB Reflector Module IV-3	90
VI-15.	Nondestructive Assay Results for Proof-Of-Breeding Rods from Reflector Module V-4	91
VI-16.	Fissile Loadings and Uncertainties for Destructively Assayed Proof-Of-Breeding Rods	92
VI-17.	t-Test for ANL Destructive Assay Versus PIFAG Nondestructive Assay	93
VII-1.	Summary of Linear Regression Estimation Parameters	116
VII-2.	Significance of Water Channel Effect	117
VII-3.	Estimation Results for Module Loadings and Standard Deviations	118
VII-4.	Comparison of Calculated and Measured Module Loading Estimates	119
VII-5.	Comparison of Module Loadings for Symmetrically Located Modules	120
VIII-1.	Random Standard Deviation in Total Loading for Each Kind of Fuel and Whole Core	139
VIII-2.	Calibration Curve Standard Deviations	139
VIII-3.	PIFAG Correction Factor Standard Deviations	139
VIII-4.	Results for Expended Core Loading and Fissile Inventory Ratio	140
B-1.	ANL Measured Uranium Isotopic Contents Versus Predictions	B6
B-2.	ANL Measured Axial Distributions Versus Predictions for High Zone Seed Rod in Cell C10, Module I-1	B8
B-3.	ANL Measured Axial Distributions Versus Predictions for Low Zone Seed Rod in Cell P39, Module I-1	B8
B-4.	ANL Measured Axial Distributions Versus Predictions for High Zone Standard Blanket Rod in Cell E56, Module I-3	B9

LIST OF TABLES (Cont'd)

<u>Table</u>	<u>Title</u>	<u>Page</u>
B-5.	ANL Measured Axial Distributions Versus Predictions for Low Zone Standard Blanket Rod in Cell A49, Module I-3	B9
B-6.	ANL Measured Axial Distributions Versus Predictions for High Zone Power Flattening Blanket Rod in Cell F73, Module III-6 ...	B10
B-7.	ANL Measured Axial Distributions Versus Predictions for Reflector Rod in Cell B1, Module IV-3	B10

LIST OF FIGURES

<u>Figure</u>	<u>Title</u>	<u>Page</u>
II-1.	Cross-Sectional Diagram of LWBR Core, Showing Module Identification	11
II-2.	Cross-Sectional Diagram of a Type I Seed-Blanket Module Combination	12
II-3.	Cross-Sectional Diagram of a Type II Blanket Module, Showing Locations of Standard Blanket Rods (Region 1) and Power Flattening Blanket Rods (Region 2)	13
II-4.	Cross-Sectional Diagram of a Type III Blanket Module, Showing Locations of Standard Blanket Rods (Region 1) and Power Flattening Blanket Rods (Region 2)	14
II-5.	Vertical Cross Section of a Type I Seed-Blanket Module Combination, Showing Movable Fuel Control and "Stepped" Binary Fuel Distribution	15
II-6.	R-Z Schematic of a Type I Seed-Blanket Module Combination, Showing Radial and Axial Zoning of Fuel	16
IV-1.	Mesh Lines and Depletion Block Boundaries in PDQ for One-Sixth of a Type I Seed-Blanket Module Combination	30
IV-2.	Calculated Breeding in LWBR	31
V-1.	Diagram of LWBR Core, Showing Locations of Proof-Of-Breeding Modules (Shaded) and Boundaries of Sixth-Core Sectors (Dashed)	50
V-2.	Location of Proof-Of-Breeding Rods in Seed Module I-1	51
V-3.	Location of Proof-Of-Breeding Rods in Seed Module II-3	52
V-4.	Location of Proof-Of-Breeding Rods in Seed Module III-1	53
V-5.	Location of Proof-Of-Breeding Rods in Seed Module III-2	54
V-6.	Location of Proof-Of-Breeding Rods in Blanket Module I-3	55
V-7.	Location of Proof-Of-Breeding Rods in Blanket Module II-2	56
V-8.	Location of Proof-Of-Breeding Rods in Blanket Module III-2	57
V-9.	Location of Proof-Of-Breeding Rods in Blanket Module III-6	58

LIST OF FIGURES (Cont'd)

<u>Figure</u>	<u>Title</u>	<u>Page</u>
V-10.	Location of Proof-Of-Breeding Rods in Reflector Module IV-4 (A Type IVA Module)	59
V-11.	Location of Proof-Of-Breeding Rods in Reflector Module IV-9 (A Type IVA Module)	60
V-12.	Location of Proof-Of-Breeding Rods in Reflector Module IV-3 (A Type IVB Module)	61
V-13.	Location of Proof-Of-Breeding Rods in Reflector Module V-4	62
VI-1.	Fit of PIFAG Thermal Assay Loadings Versus PDQ Calculated Loadings for All Proof-Of-Breeding Rods from Reflector Module IV-9	94
VI-2.	Fit of PIFAG Epithermal Assay Loadings Versus PDQ Calculated Loadings for All Proof-Of-Breeding Rods from Reflector Module IV-9	95
VII-1.	Rods Included in Water Channel Effect for Reflector Type IV Module	96
VII-2.	Seed I-1 Thermal Assay Data Without Water Channel Effect	122
VII-3.	Seed I-1 Thermal Assay Data With Water Channel Effect	123
VII-4.	Deviation of Power from Average for Module Type, As Deduced from End-Of-Life Reactivity Worth Measurements	124

ABSTRACT

The Light Water Breeder Reactor (LWBR) was developed to demonstrate practical breeding while producing electrical energy in a commercial pressurized water reactor generating station. After the ^{233}U -thorium fueled core operated successfully for five years, expended fuel from the core was assayed to prove breeding — that is, production of more fissile fuel from thorium than was expended to produce energy. This demonstrates the existence of a vast source of electrical energy potentially capable of supplying the entire national need for many centuries using plentiful domestic thorium.

The LWBR operated for more than 29,000 effective full power hours in the Shippingport Atomic Power Station and contained more fissile fuel at the end of its five-year operational lifetime than at the beginning. Five hundred twenty-four fuel rods were randomly selected from the spent core for nondestructive assay to determine their final fissile fuel content. Two different nondestructive assay measurements were performed on each rod. Seventeen of these rods were subsequently assayed destructively to validate the nondestructive results. Two estimation methods, which agreed closely, were used to determine the fissile fuel content of the spent core. The more accurate method gave a value of 507.98 kg for the end-of-life fuel content of the core, with a total standard deviation of 0.14%. The fuel content of the beginning-of-life core was determined during core manufacture to be 501.02 kg, with a standard deviation of less than 0.04%. The increase in fissile fuel during core life was $1.39\% \pm 0.14\%$, in close agreement with prediction, thus demonstrating breeding in LWBR.

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PROOF OF BREEDING IN THE LIGHT WATER BREEDER REACTOR

(LWBR Development Program)

W. C. Schick, Jr., B. R. Beaudoin, W. J. Beggs,
L. B. Freeman, G. Tessler

I. INTRODUCTION

The Light Water Breeder Reactor (LWBR) was designed and built to demonstrate practical breeding while producing electrical energy in a commercial pressurized light-water reactor using the ^{233}U - ^{232}Th fuel cycle. The LWBR core was installed in the Shippingport Atomic Power Station at Shippingport, Pennsylvania, which was the Nation's first large scale commercial nuclear power plant. LWBR operated successfully for about five years from 1977 to 1982 with an average capacity factor of 65% and an average availability factor of 76%, generating more than 29,000 effective full power hours of energy. After final shutdown, the core was dismantled and parts of it were examined to evaluate fuel performance and to determine the breeding performance of the core.

This report describes the proof-of-breeding program which demonstrates that LWBR was indeed a breeder; that is, that the reactor core contained more fissile fuel at the end of its life than at the beginning. The program consists of five essential parts, as follows:

1. The measurement of the fissile fuel content of the beginning-of-life core, based on both destructive and nondestructive assay of individual fuel pellets during core manufacture.

2. The calculational prediction of the fissile fuel content of the end-of-life core and of each individual fuel rod in the core.

3. The selection and nondestructive assay of 524 fully representative expended fuel rods using the Production Irradiated Fuel Assay Gauge (PIFAG) to determine their final fissile fuel content to a precision of a few tenths of one percent.

4. The subsequent wet chemical destructive assay of 17 of these rods at Argonne National Laboratory to validate the precision and accuracy of the nondestructive assay results.

5. The use of the destructive and nondestructive assay results to determine the total fissile content of the expended core. The expended core fissile content was divided by the initial content to give the fissile inventory ratio. A value greater than 1.0 for the fissile inventory ratio indicates that breeding took place.

The proof-of-breeding program was a genuine measurement of breeding, not merely a qualification of the computer codes used to calculate breeding. Both the initial and final fissile content of the core were measured with sufficient precision and accuracy to demonstrate breeding with a high degree of confidence. During core manufacture the breeding gain was predicted to be 1.2%, corresponding to a fissile inventory ratio of 1.012 (Reference 1). To measure a breeding gain of this magnitude clearly required an assay precision substantially better than 1%.

The assay measurements performed during core manufacture determined the initial fissile content of each core fuel rod to a precision of 0.13% (one

standard deviation, corresponding to 0.25% at the 95% confidence level — see Reference 2). The nondestructive assay of spent fuel rods from the expended core was done to a precision of 0.2% to 0.5% for most rods. The destructive assay of 17 expended rods was carried out with a precision of better than 0.11%. Based on these measurements, the fissile content of the expended core is known to a precision of 0.14%. The assay results yield a best estimate fissile inventory ratio of 1.0139 ± 0.0014 , corresponding to a breeding gain of $1.39\% \pm 0.14\%$. (All uncertainties are one standard deviation.) The lower 95% confidence bound on the fissile inventory ratio is 1.0115. Thus, it has been demonstrated with a high degree of confidence that LWBR was a breeder.

The most significant result of the LWBR program is that it has demonstrated unequivocally that self-sustaining ^{233}U - ^{232}Th breeders can be built, operated in light water reactor plants to produce electrical energy, and breed more fissile fuel than they consume. This means that the plentiful domestic supply of low and moderate cost thorium represents an available resource potentially capable of providing about fifty times the amount of energy which could be produced using current design light water reactors and the domestic supply of low and moderate cost uranium. This light water breeder system could supply the entire electrical energy need of the United States for centuries. To realize this potential, however, would require developing and constructing automated and shielded facilities for fuel reprocessing and refabrication.

This report is organized as follows:

- a. Section II describes the LWBR core.
- b. Section III describes the determination of the initial fissile fuel content of the core, with more details in References 2 and 3.
- c. Section IV presents the calculational methods used to predict the fissile fuel content of the expended core.

- d. Section V describes the sampling plan used to select the specific expended fuel rods to be nondestructively and destructively assayed.
- e. Section VI discusses and presents results from the the nondestructive assay of 524 expended fuel rods, with more details of this work given in References 4 and 5. Section VI also presents a summary of results from the destructive assay work, which is described more fully in Reference 6.
- f. Section VII describes the two estimation models used to determine the fissile content of a fuel module based on the assay results from a sample of rods taken from that module, along with a summary of results for the assayed modules.
- g. Section VIII discusses the determination of the final fissile content of the whole core and the evaluation of the fissile inventory ratio.
- h. Section IX gives an overall summary and a discussion of the significance of the work.
- i. Section X contains the acknowledgments and Section XI the references.
- j. Appendices A1 through A3 give derivations of various equations used in the main body of the report.
- k. Appendix B presents a comparison of predicted isotopic distributions to the results from the destructive assay.

II. DESCRIPTION OF LWBR CORE

A. Fuel Rods

The LWBR core contained 17,290 fuel rods assembled into 39 modules. The fuel rods were Zircaloy tubes containing ceramic fuel pellets. The fissile fuel pellets were made of a mixture of uranium oxide (urania, UO_2) and thorium oxide (thoria, ThO_2) in solid solution. The uranium in the UO_2 was nearly pure (greater than 98% isotopic abundance) fissile ^{233}U . The thorium in the ThO_2 was natural thorium, which is essentially 100% ^{232}Th . This isotope is classified as fertile, in that a ^{232}Th nucleus can absorb a neutron to become fissile ^{233}U after beta decay. The mixed-oxide fuel is also known as "binary" fuel.

In addition to the binary fuel pellets, there were also pellets made of pure thoria. Every fuel rod in the central part of the core had a length of thoria pellets on either end of a stack of binary pellets. This arrangement provided fertile fuel above and below the fissile fuel. In the radial direction, there was on the periphery of the core a fertile region called the "reflector," in which all fuel rods contained only thoria pellets.

B. Fuel Modules

A cross-sectional diagram of the core, showing the placement of modules, is shown in Figure II-1. The small hexagons in the figure represent axially movable seed modules, of which there were twelve in the core. All seed modules were identical in construction; each contained 619 fuel rods 0.306 inch in diameter. Each seed module was surrounded by an annular stationary blanket module. The three innermost blanket modules had the shape of regular hexagons and were designated as Type I blanket modules. Each contained 443 fuel rods 0.572 inch in diameter. The arrangement of fuel rods in a Type I seed-blanket module combination is illustrated in Figure II-2.

The nine outer blanket modules were nonconcentric hexagonal annuli of two different shapes, respectively designated as Type II and Type III blanket modules. The fuel in these modules was of two types. The inner part of the modules (i.e., the part nearest to the center of the core) contained fuel rods similar to those found in the Type I blanket modules; these fuel rods were called standard blanket rods. The outer part of the Type II and Type III blanket modules contained fuel with higher fissile content than the standard blanket fuel. This was done to flatten the radial power distribution of this 60 MW(e) reactor, making its central region more representative of a larger commercial core generating more than 500 MW(e). The fuel rods in this outer region of the blanket were called power flattening blanket rods. (It may be noted that the term "regular blanket" has sometimes been used in other reports to refer to standard blanket rods, as has the term "blanket" alone. In this report, the term "blanket" alone will be used only in reference to a whole blanket module, or when standard and power flattening blanket fuel rods are considered collectively.) Aside from the higher fuel concentration, the power flattening blanket rods were also distinguished from the standard blanket rods in having a slightly smaller diameter (0.527 inch). Since the pitch was the same for both kinds of blanket rods, the smaller diameter of the power flattening blanket rods served to increase the amount of coolant in the corresponding region of the core. The resulting increase in moderation further flattened the power in this relatively small core. The arrangement of fuel rods in Type II and Type III blanket modules is shown in Figures II-3 and II-4, respectively. The standard blanket rods are located in Region 1 in the figures, and the power flattening blanket rods are located in region 2.

For the purpose of identification, the seed modules have been designated as Type I, Type II, or Type III according to the type of the surrounding blanket module. Although all seed modules had nearly identical fissile contents initially, the different locations in the core gave rise to differences in fissile content at end of life. Specifically, the Type II seeds were farther from core center and hence experienced somewhat less burnup than the Type I seeds, while the Type III seeds experienced somewhat less burnup

than the Type II seeds. Different seed modules of the same type, however, would be expected to have approximately the same final fissile content, provided that the core power was symmetrically distributed throughout life. (Core symmetry will be discussed in Section VII.E.)

Different blanket modules of the same type would, like the seed modules, be expected to have the same final fissile content, with one important exception: Two of the Type II blanket modules and two of the Type III blanket modules each had one fewer fuel rod than the other modules of the same type because of the presence of a flux well at one rod location. (The flux wells were used for neutron flux measurements for testing purposes only, not for core operation.) This difference among Type II and Type III blanket modules gave rise to small differences in fissile fuel content among modules of each type.

On the periphery of the core were located 15 reflector modules. These modules were designated Type IV or Type V according to their shape. The Type IV modules contained 228 fuel rods 0.832 inch in diameter (except that one Type IV module had only 227 rods because of the presence of a flux well at one rod location). The Type V modules contained 166 rods identical in construction to the rods in the Type IV modules. As noted above, the reflector rods contained only fertile thorium oxide fuel initially. During core life, however, ^{233}U fuel was created in the reflector through the capture by ^{232}Th nuclei of neutrons leaking from the central core. This was essential for successful breeding in this small demonstration core, since at end of life the reflector modules contained nearly 8% of the fissile fuel in the core.

The six Type V reflector modules were symmetrically placed in the core and thus would be expected to have approximately equal amounts of fissile fuel at end of core life. The nine Type IV reflector modules, on the other hand, were of two distinct subtypes: Modules IV-1, IV-2, and IV-3 had Type V modules on both sides, while modules IV-4 through IV-9 had a Type V module on one side and another Type IV on the other side (see Figure II-1). The first three were

denoted as Type IVB modules and were slightly farther from the core center than the last six, which were denoted as Type IVA modules. Because the neutron flux decreased with distance from core center, the expended IVB modules had slightly less fissile fuel than the expended IVA modules.

In terms of their final fissile contents, then, the modules in the LWBR core may be divided into nine distinct groups: three Type I seeds, three Type II seeds, six Type III seeds, three Type I blankets, three Type II blankets, six Type III blankets, six Type IVA reflectors, three Type IVB reflectors, and six Type V reflectors. These groups were called "module strata" in the sampling plan that was used to select the fuel rods to be nondestructively assayed, as will be discussed in Section V.

C. Reactivity Control and Axial Zoning of Fuel

The seed modules were moved vertically to control core reactivity. This is illustrated in Figure II-5 for a Type I seed-blanket combination. In the shutdown position, the seed modules were withdrawn below the blankets sufficiently far that the neutron coupling between modules was too weak for criticality. Criticality was achieved by raising all twelve seed modules in a uniform bank. The critical height increased during lifetime due to the accumulation of fission-product poisons and the changing fuel distribution caused by burnup and by conversion of ^{232}Th to ^{233}U .

Figure II-5 also illustrates the "stepped" axial distribution of fissile fuel used in seed and blanket modules. The rods towards the center of the seed modules and the rods near the outside of the blanket modules had a stack of binary fuel pellets 84 inches long, with roughly 10 inches of thoria pellets above and below the binary fuel. The rods near the outside of the seed modules and near the inside of the blanket modules had a binary fuel length of only 42 inches, with a correspondingly longer stack of thoria fuel above (in seed rods) or below (in blanket rods) the binary fuel. Located between the 84 and 42 inch binary rods were rods with 70 and 56 inch binary

pellet stacks. This arrangement led to the "staircase" effect evident in the aligned position of Figure II-5. Each "step" in the figure is 14 inches high. (It should be pointed out that Figure II-5 is not to scale, in that the width of the "steps" has been exaggerated for clarity. This makes the central part of the seed module, containing the 84-inch binary rods, appear smaller than it really was.)

This axial zoning of the fuel served to increase the reactivity worth of the movable seed modules, thereby eliminating the need for control rods or other neutron poison materials for reactivity control. In addition, the axial zoning also aided breeding by providing an extended thorium region in which a great deal of fissile fuel could be created. For example, the final fuel content of the 42-inch binary standard blanket rods was more than twice as great as the initial content, due primarily to creation of fissile fuel in the long thorium section of these rods.

D. Radial Zoning of Fuel

LWBR also employed radial fuel zoning to reduce power peaking in the vicinity of metal/water (M/W) channels. This is illustrated schematically in Figure II-6 for a Type I seed-blanket combination. The 84-inch binary rods in the central portion of the seed fuel contained a higher concentration of fissile uranium, 5.2 weight percent (w/o), than the shorter binary rods, which were 4.3 w/o. The standard blanket fuel had three different concentrations, indicated in Figure II-6 as low zone (1.2 w/o), medium zone (1.7 w/o), and high zone (2.0 w/o). Power flattening blanket fuel (present only in Type II and Type III blanket modules) also had three different concentrations designated as low zone (1.7 w/o), medium zone (2.0 w/o) and high zone (2.7 w/o). Different concentrations were not mixed in the same rod; all binary pellets in any given rod had the same concentration of fissile uranium.

For proof of breeding, it was necessary to assay a representative sample of fuel rods of each type present in the LWBR core. These fuel rod types were

called "rod strata" in the sampling plan that was used to select the rods to be assayed, as discussed in Section V. Among the different kinds of binary fuel (seed, standard blanket, and power flattening blanket), the different module types containing binary fuel (Types I, II, and III), the different binary lengths (42 to 84 inches), and the different concentrations (low, medium, and high), there were 37 binary fuel rod strata to be considered. In addition, the reflector rods, which contained only fertile fuel, were also divided into strata based on the amount of fissile fuel bred into the rods during core life. There were 13 reflector rod strata, making a total of 50 strata in all. These rod strata will be discussed further in Section V.

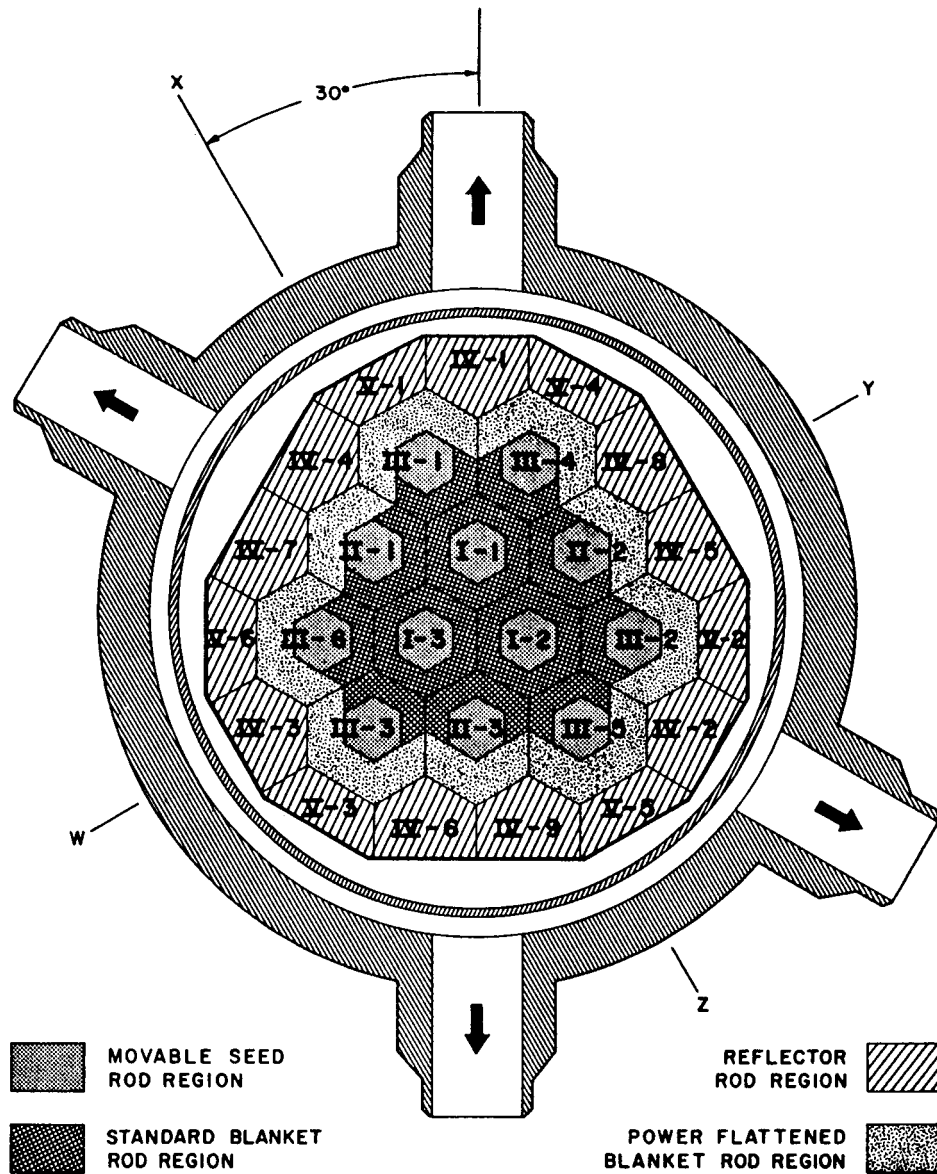


Figure II-1. Cross-Sectional Diagram of LWBR Core, Showing Module Identification

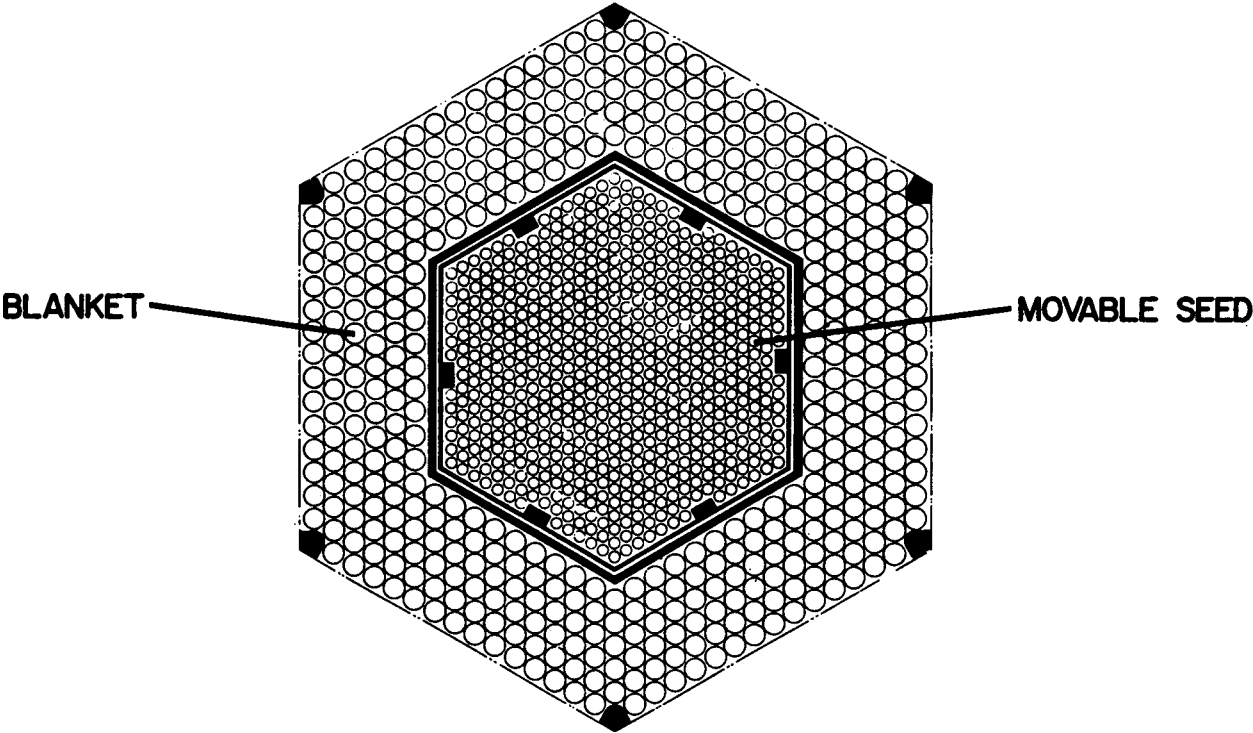


Figure II-2. Cross-Sectional Diagram of a Type I Seed-Blanket Module Combination

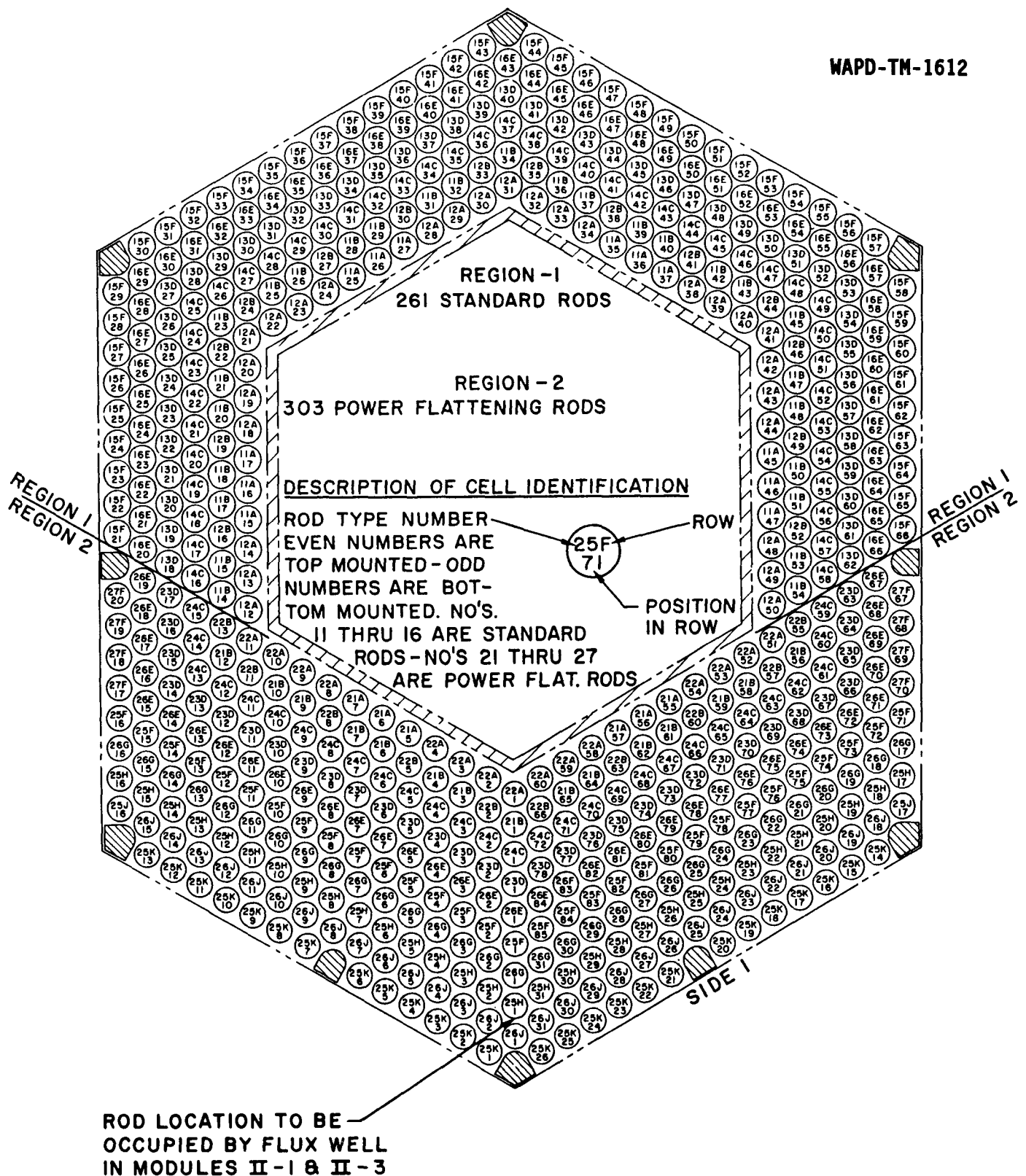


Figure II-3. Cross-Sectional Diagram of a Type II Blanket Module, Showing Locations of Standard Blanket Rods (Region 1) and Power Flattening Blanket Rods (Region 2)

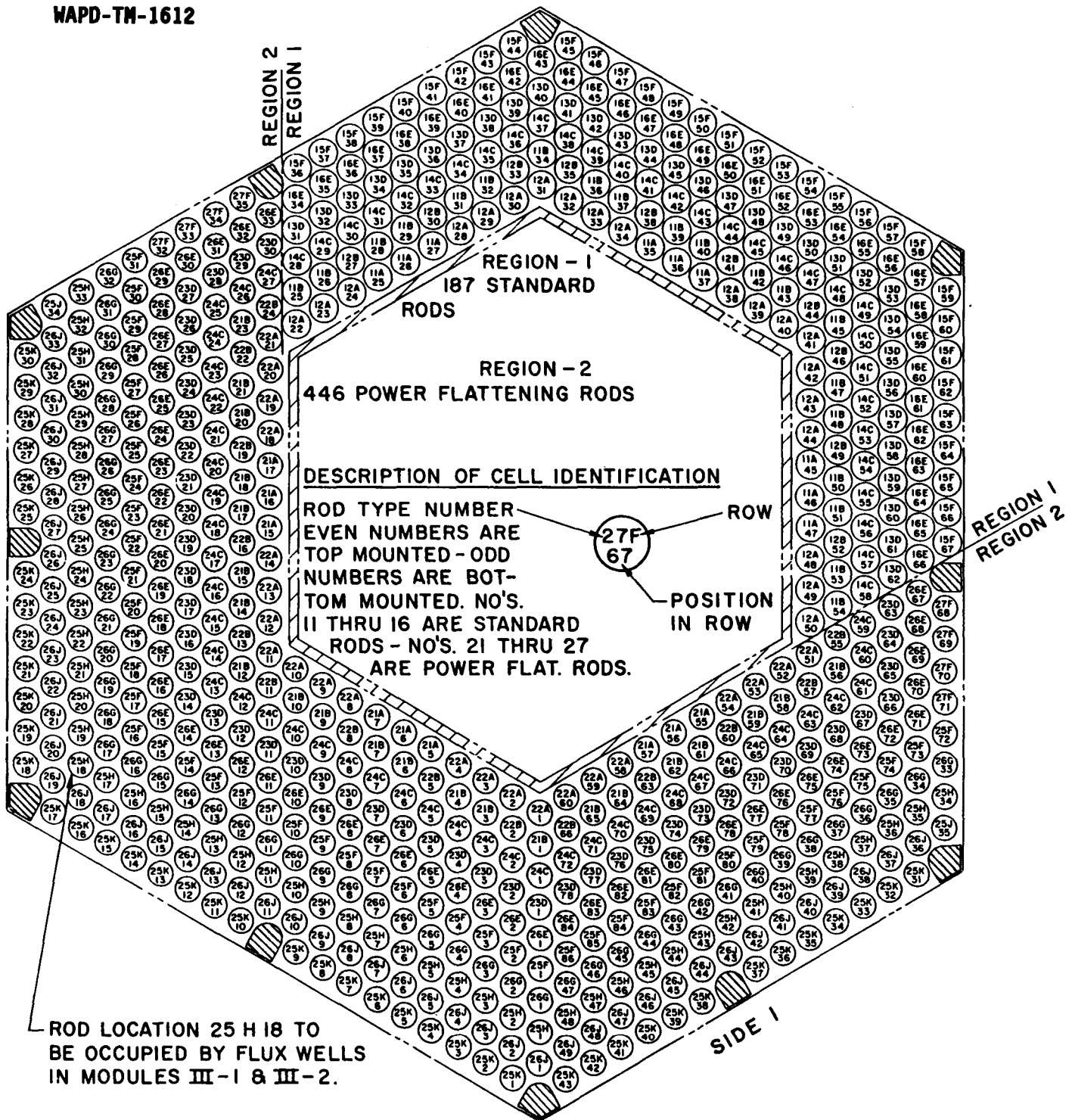


Figure II-4. Cross-Sectional Diagram of a Type III Blanket Module, Showing Locations of Standard Blanket Rods (Region 1) and Power Flattening Blanket Rods (Region 2)

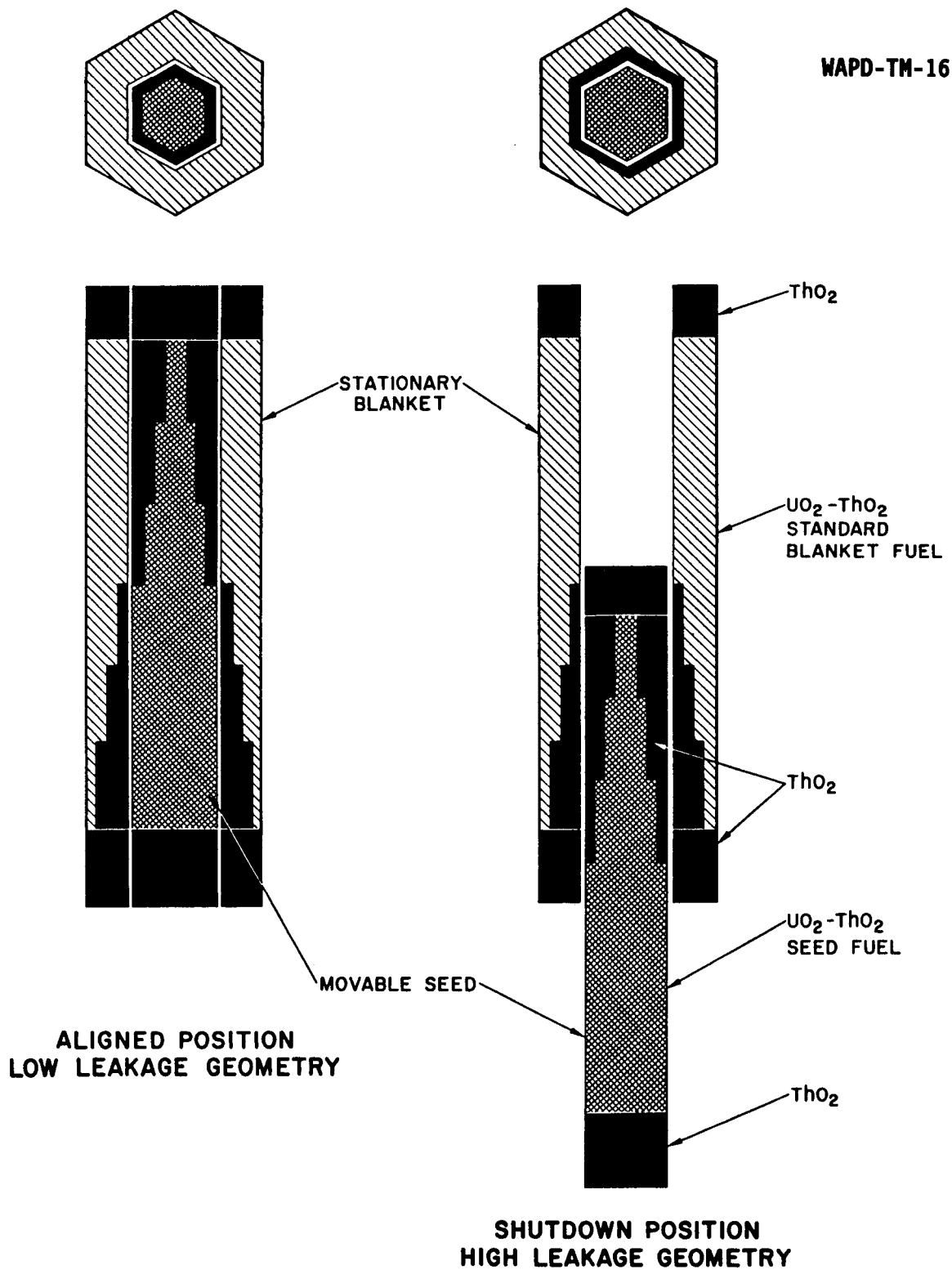


Figure II-5. Vertical Cross Section of a Type I Seed-Blanket Module Combination, Showing Movable Fuel Control and "Stepped" Binary Fuel Distribution

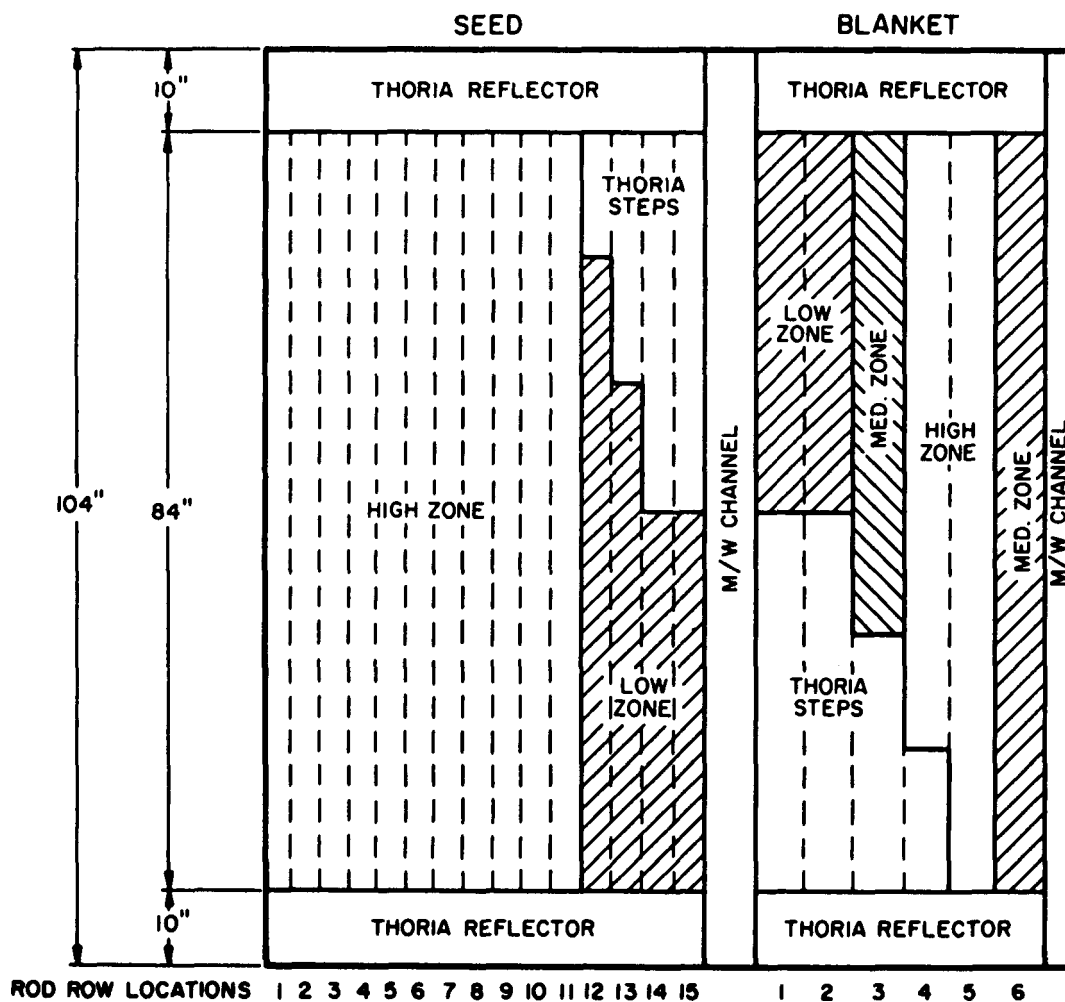


Figure II-6. R-Z Schematic of a Type I Seed-Blanket Module Combination, Showing Radial and Axial Zoning of Fuel. The Module Centerline is at Left Boundary of Figure.

III. FUEL MANUFACTURE AND INITIAL CORE LOADING DETERMINATION

The initial fissile content of the core was determined by assaying individual fuel pellets during core manufacture. The procedure for doing this was described in detail in References 2 and 3, and the method is summarized briefly here.

The fuel pellets were manufactured in lots called "blends." Each binary fuel blend was made by mixing specified amounts of thorium oxide and uranium oxide powder and pressing the mixture into pellets. The pellets were then sintered to increase the density to approximately 98% of the theoretical density for solid oxide material, producing a solid state solution. Each blend contained from 1000 to 5000 pellets that were used in the core. Approximately 900 blends were manufactured.

The binary pellet blends were divided into eight types, based on the kind of fuel (seed, standard blanket, or power flattening blanket) and the uranium concentration (low and high for seed fuel; low, medium, and high for both kinds of blanket fuel). One blend from each of the eight types was designated as a "calibration blend." Thirty pellets from each calibration blend were selected at random for nondestructive assay. For subsequent blends of each type, twenty pellets were randomly selected for nondestructive assay.

The nondestructive assay work was performed on a device called the pellet assay gauge. Pellets were irradiated one at a time in this gauge with neutrons from ^{252}Cf sources. The delayed neutrons emitted by the fuel after irradiation were counted in ^3He proportional counters. The gauge was initially calibrated by determining its response to the thirty pellets randomly selected from each calibration blend. Twenty of the thirty pellets from each calibration blend were subsequently assayed destructively using conventional wet chemistry techniques. The destructive and nondestructive assay results were used to establish both the response of the gauge (i.e., delayed neutron counts per gram of pellet weight versus weight percent of

total uranium in the pellet) and the sensitivity of the gauge to variations in pellet diameter, length, density, and isotopic mix. The ten remaining pellets selected from each calibration blend that were not destructively assayed were kept as "retainer samples." Retainer samples were used to normalize the assay results from other blends to the calibration measurements. A high count rate standard was used to assess the stability of the pellet assay gauge during routine operation. A final recalibration was performed after all assay work was completed, as described in Reference 3.

The twenty pellets randomly selected from each noncalibration blend were assayed in the gauge along with several retainer sample pellets. Five of the twenty pellets from each blend were subsequently assayed destructively as well. The destructive and nondestructive assay results were combined to determine the average weight percent of total uranium for the blend. The relative amounts of the uranium isotopes in each blend were measured by a mass spectrometer. The assay gauge results were used to determine the pellet-to-pellet variability. From this information on the blend (or blends) of pellets loaded into each fuel rod, along with the weight of the pellets, it was possible to determine the fissile fuel content ("loading") of each individual rod to a precision of $\pm 0.25\%$ or better at the 95% confidence level.

The fissile loadings of the individual rods of a given type were quite uniform. The rod-to-rod variability was typically a few tenths of one percent (one standard deviation). The total fissile loadings of the modules were similarly uniform, with module-to-module standard deviations typically about 0.1% (after correction for the fact that some modules had a flux well in place of one fuel rod). Table III-1 gives the relevant information. The standard blanket fuel and the power flattening blanket fuel are entered separately in this table for Type II and III blanket modules; this approach will be used generally throughout this report. It may also be seen from Table III-1 that the LWBR core initially contained about one-tenth percent as much ^{235}U as ^{233}U . The ^{233}U and ^{235}U were added together to give the total fissile

loadings of the modules and of the core as a whole. The initial core fissile inventory was 501.02 kilograms.

The overall precision in determining the total fissile content of the core at beginning of life is estimated to be $\pm 0.034\%$. The quality assurance methods used to achieve this high precision have been documented in Reference 3. Other relevant information about the as-built core may be found in Reference 7.

Some fuel rods were not loaded into the core after they were manufactured and inspected but were set aside to be used as calibration rods during the expended core assay campaign. In addition, several rods were manufactured specifically for use as calibration rods having different fissile concentrations than the fuel used in the core. Pellets from the special blends fabricated for these calibration rods were assayed destructively and nondestructively as were the pellets from the production blends. The special seed and blanket rods were constructed so that there would be calibration rods having the lower concentrations of fissile fuel expected to be found in the parts of expended fuel rods where the pure thorium fuel pellets were located initially. For reflector fuel, it was necessary to manufacture special calibration rods containing binary fuel because the reflector rods in the core contained only thorium pellets. The procedure of using rods prepared during core manufacture to calibrate the expended core assay device helped to minimize systematic errors in the determination of the breeding gain.

TABLE III-1. SEED AND BLANKET MODULE INITIAL THORIUM AND URANIUM LOADINGS

	Module	Rods	Thorium kgs	²³² U Grams	²³³ U Grams	²³⁴ U Grams	²³⁵ U Grams	²³⁶ U Grams	²³⁸ U Grams	U ^{fissile} Grams
Seed	I-1	619	433.61	0.12	16505.1	215.12	13.04	2.68	48.92	16518.1
	I-2	619	433.60	0.11	16506.9	218.02	14.95	3.21	48.66	16521.8
	I-3	619	433.91	0.10	16522.9	215.80	12.21	2.70	48.83	16535.1
	II-1	619	433.88	0.10	16529.4	215.21	11.79	2.57	48.55	16541.2
	II-2	619	433.66	0.10	16528.4	216.03	12.46	2.74	47.85	16540.8
	II-3	619	434.09	0.10	16568.7	215.20	11.49	2.49	45.85	16580.2
	III-1	619	433.57	0.11	16505.3	214.12	12.25	2.47	49.03	16517.5
	III-2	619	433.87	0.11	16545.4	214.16	11.01	2.33	47.44	16556.5
	III-3	619	434.07	0.11	16557.8	214.11	10.90	2.32	47.46	16568.7
	III-4	619	434.08	0.11	16552.1	214.04	10.85	2.28	47.34	16563.0
	III-5	619	434.11	0.10	16562.0	213.93	10.69	2.24	46.69	16572.7
	III-6	619	434.04	0.11	16557.2	214.96	11.40	2.45	47.39	16568.6
	Totals	7428	5206.55	1.34	198441.2	2580.75	143.10	30.53	574.04	198584.3
Std.	I-1	443	1299.45	0.13	16166.5	220.01	15.69	4.41	42.11	16182.2
Blkt.	I-2	443	1299.37	0.13	16163.9	218.54	14.85	4.07	42.19	16178.7
	I-3	443	1299.30	0.13	16161.5	217.06	13.97	3.73	42.24	16175.4
	II-1	261	765.65	0.07	9325.4	125.49	8.26	2.22	24.56	9333.6
	II-2	261	765.91	0.07	9324.0	126.84	9.05	2.53	24.39	9333.0
	II-3	261	765.71	0.07	9329.1	126.12	8.55	2.35	24.31	9337.6
	III-1	187	548.50	0.05	6619.9	90.10	6.42	1.81	17.20	6626.4
	III-2	187	548.69	0.05	6623.3	90.39	6.59	1.86	17.25	6629.9
	III-3	187	548.62	0.05	6618.4	90.50	6.67	1.90	17.18	6625.1
	III-4	187	548.69	0.05	6623.6	90.19	6.47	1.82	17.32	6630.1
	III-5	187	548.72	0.05	6626.4	90.26	6.49	1.83	17.28	6632.9
	III-6	187	548.46	0.05	6619.7	91.26	7.09	2.06	17.17	6626.7
	Totals	3234	9487.14	0.97	116201.6	1576.82	110.17	30.63	303.26	116311.7
Pwr.	II-1	302	741.39	0.11	15590.0	202.42	16.22	4.72	77.94	15606.3
Flat.	II-2	303	743.51	0.11	15644.8	198.72	14.85	3.92	90.47	15659.7
Blkt.	II-3	302	741.36	0.11	15588.4	192.90	13.25	3.44	95.46	15601.7
	III-1	445	1092.25	0.17	23155.6	291.57	21.14	5.68	132.64	23176.7
	III-2	445	1092.02	0.17	23131.0	305.97	26.29	7.57	117.14	23157.3
	III-3	446	1094.47	0.17	23212.3	289.36	20.53	5.33	141.33	23232.9
	III-4	446	1094.36	0.17	23197.7	310.19	27.32	7.93	112.01	23225.1
	III-5	446	1094.55	0.17	23201.8	303.67	25.16	7.16	120.02	23227.0
	III-6	446	1094.30	0.17	23210.8	295.43	22.61	6.00	136.16	23233.4
	Totals	3581	8788.26	1.39	185932.6	2390.28	187.41	51.80	1023.23	186120.1
Core										
	Totals	14243	23481.96	3.70	500575.4	6547.86	440.69	112.97	1900.53	501016.1
	(Excluding Reflector)									

IV. PREDICTION OF FISSILE INVENTORY RATIO AND EXPENDED CORE LOADINGS

The calculational model used in the analysis of the nuclear performance of the LWBR core has been described in References 7 and 8. The model is briefly summarized in this section, along with the application of the model to the prediction of the fissile inventory ratio and the estimation of the loading (fissile fuel content) of each fuel rod in the expended core.

A. The LWBR Calculational Model

The LWBR calculational model is based on the PDQ computer code (Reference 9). PDQ solves the diffusion-depletion equations for a reactor core in one, two, or three dimensions using various geometries and up to five energy groups. Four energy groups were used for the LWBR analysis, with the input parameters for the four groups calculated using the PAX computer program (References 10 and 11). Fission-product growth, decay, and burnup were explicitly included in PDQ for the three most important mass chains, while the remainder of the fission-product mass chains were calculated using the CINDER code (Reference 12) and then lumped together into a composite "residual" fission product for inclusion in PDQ burnup calculations. Temperature dependent effects (i.e., Doppler broadening, moderator thermal expansion, and scattering kernel) were included using the thermal feedback option in PDQ. The effects of xenon and iodine buildup and decay were incorporated using the xenon feedback option. Additional features of the model (see Reference 8) include an automated system of processing manufacturing data, an extensively analyzed nuclear data library, an accurate resonance integral calculation, space-energy corrections to infinite medium cross sections, a transport calculation for high energy neutrons, cladding diameter shrinkage and fuel pellet growth, and an extensive testing program against experiments and a highly developed analytical standard.

The geometry used to represent the LWBR core was hexagonal in the horizontal plane, representing the hexagonal shape of the seed and blanket

modules and the triangular pitch of the fuel rods. Most of the analyses were done explicitly in three dimensions on a one-sixth sector of the core, making use of the sixfold symmetry of LWBR. The horizontal cross-section of the sixth-core sector was represented by about 5000 solution points for neutron flux calculation. For depletion calculations, a coarser mesh was used; the sixth-core cross-section was divided into 1472 blocks, with the composition of the core taken to be uniform over the area of each block. Eighty-two of the blocks contained only structural materials and water and were assumed to have compositions that did not vary with depletion. The remaining 1390 blocks in the horizontal cross-section contained fuel and had compositions that were allowed to change with depletion. These blocks were called "depletion blocks." The 1390 depletion blocks were used to represent 2883 fuel rod locations in the sixth-core sector. The PDQ representation did not explicitly account for the existence of flux wells at a few rod locations; therefore, the number of fuel rods in the PDQ sector was slightly larger than one-sixth of the number of fuel rods actually present in the core. A correction was made to the PDQ prediction of fissile inventory ratio to account for this small deficiency in the model. The horizontal mesh and depletion blocks representing one-sixth of a Type I seed-blanket module combination are illustrated in Figure IV-1.

In the axial (vertical) dimension of the 3-D PDQ geometry, a uniform 3.5-inch mesh was used. Thus, the core was represented by a series of mesh planes 3.5 inches apart. These planes formed the upper and lower boundaries of the depletion blocks. Thirty blocks in the vertical direction were used to represent the total height of the fuel region of the core. The actual length of the fuel in the rods (which ranged from 103.8 to 104.9 inches for seed and blanket rods and from 101.1 to 103.1 inches for reflector rods) was slightly less than the 105 inches used in the PDQ description. Therefore, the density of the fuel material in the top and bottom blocks was adjusted as needed to give the correct total amount of fuel in the core at beginning of life. The number of depletion blocks used to represent the whole fuel region in the

one-sixth core sector was 1390 (horizontally) times 30 (vertically), or 41700 blocks in all.

The procedure for following core behavior over lifetime using PDQ was as follows:

1. Perform a spatial neutron flux calculation at a time t_1 , including both temperature feedback and xenon feedback.
2. Calculate core depletion and fission product growth over a "timestep" from t_1 to t_2 , assuming a constant neutron flux distribution as determined at time t_1 .
3. Adjust the position of the movable fuel, if necessary, to represent its actual position at time t_2 .
4. Perform another neutron flux calculation at time t_2 , using the depleted fuel composition and the new movable fuel position, and continue the above cycle throughout core life.

The timesteps used for most of core lifetime averaged about 600 effective full power hours (EFPH) in duration. Time in EFPH is clock time for the calculation multiplied by the fraction of full power at which the reactor operated during that time.

B. Use of the Calculational Model to Predict the Fissile Inventory Ratio

The fissile inventory ratio may be defined most generally (see Reference 7) as the ratio of the fissile fuel content of the core at any time in life to the initial fissile inventory, as illustrated in Figure IV-2. For the purpose of this report, however, it is the final value of the fissile inventory ratio that is of interest. As noted above, a final value greater than 1.0 means that breeding occurred.

One method of predicting the fissile inventory ratio is to take the ratio of the PDQ-calculated fissile content of the expended core to the measured initial fissile content. There is another method, however, that has been shown to be slightly more accurate, as discussed in Appendix B of Reference 7. This method makes use of the Conversion Ratio (CR), which is defined as the ratio of the rate of production of fissile atoms to the rate of destruction of fissile atoms at any instant of time, as plotted in Figure IV-2. The quantity, $(CR - 1)$ times the neutron absorption rate for fissile atoms, gives the instantaneous rate of change in the number of fissile atoms in the core. The integral of this quantity over core life gives the total change in the amount of fissile fuel present. The calculations were actually performed by evaluating the conversion ratio at the beginning and the end of each PDQ depletion timestep and then averaging the two values. The beginning and ending fissile atom neutron absorption rates were similarly averaged, and the average values were then used to calculate the change in the amount of fissile fuel during the timestep. The sum of the changes over all timesteps determined the total increase in the amount of fissile fuel in the core. This approach of using values averaged between the beginning and end of a timestep tended to compensate for the change in neutron flux during the timestep, in contrast to the PDQ depletion calculations in which the neutron flux distribution at the beginning of a timestep was used for the whole timestep.

The final value of fissile inventory ratio obtained from this procedure is 1.0135, as reported in Reference 13. (A value of 1.013, rounded to three decimal places, was reported in Reference 14, along with a detailed discussion of the calculations.) This is slightly larger than the value of 1.012 predicted when the core was designed, as reported in Reference 1. The reason is that the design value was based on the conservative assumption of continuous operation of the core at full power throughout life. Periods of shutdown for testing and station maintenance and operation at less than full power tended to increase the fissile inventory ratio. This is because neutron capture in ^{233}Pa is reduced during shutdown or operation at lower power. ^{233}Pa , which has a radioactive decay half-life of 27 days, is formed as an

intermediate step in the conversion of ^{232}Th to ^{233}U . A ^{233}Pa atom that does not capture a neutron will eventually become a ^{233}U atom after beta decay, but a ^{233}Pa atom that does capture a neutron will become a non-fissile atom of ^{234}U . This phenomenon imposes an upper limit on the allowable power density in a light water breeder reactor to retain breeding capability.

C. Prediction of Individual Expended Fuel Rod Loadings and Isotopics

The calculational model was used to estimate the final fissile loading of every rod in the expended LWBR core. In addition, the amount of ^{232}Th and each uranium isotope from ^{232}U to ^{236}U present in every expended core fuel rod was calculated, along with the total number of fissions that had taken place in the rod. This information was used for several purposes, one of which was to aid in the selection of the rods to be nondestructively assayed to measure the fissile content of the expended core.

It was noted above that the PDQ model used 1390 depletion blocks in the horizontal plane to represent the 2883 fuel rods in a sixth-core sector. Each depletion block included all or part of one or several rods. An example of the blocks used to represent the rods in one-sixth of a Type I seed-blanket module combination is shown in Figure IV-1. The PDQ output for each depletion block included the atomic density of ^{232}Th , the atomic densities of all uranium isotopes from ^{232}U to ^{236}U inclusive, and the number of fissions per unit volume. (The LWBR model used a special fission product chain in PDQ with a single isotope to serve as a counter for fissions, as discussed in Reference 8.) These atomic densities were corrected for the fact that PDQ assumed the fuel to be evenly distributed throughout the block, whereas it was actually contained within the fuel pellets, which occupied only part of the space inside the block. The atomic densities were then converted to grams of material per rod by multiplying by the atomic weight and the volume of fuel in a single rod within the block, and then dividing by Avagadro's number. The results were added over all 30 depletion blocks in the vertical direction used

to represent the total height of the fuel. (Note that the PDQ mesh had axial symmetry, so all depletion blocks in the vertical direction had exactly the same shape and were directly above or below each other.) This produced the number of grams of ^{232}Th and uranium isotopes per rod and the number of fissions per rod for each depletion block in the cross section of the core.

Next, an interpolation scheme was used to convert the results for each depletion block into data appropriate for the individual rods present in the core. The interpolation was performed by setting up a coordinate system in the horizontal plane and evaluating the coordinates of the center of each rod and of the centroid of each depletion block. The information for the depletion block was assigned to the position of the centroid of the block. The grams of thorium and uranium isotopes and the number of fissions for each actual core rod were then obtained by interpolating the information from the depletion blocks whose centroids were nearest to the center of the rod. Either one- or two-dimensional interpolation was used, depending on the region of the core. One-dimensional interpolation was used in the blanket modules and in the outer part of the seed modules, where each depletion block contained rods from only one rod row. In these regions, the numbers of grams of thorium and uranium isotopes and the number of fissions assigned to a rod were determined using the distance from the center of the rod to the centroids of the two nearest depletion blocks in the same rod row. That is,

$$N_r = (d_2 N_1 + d_1 N_2) / (d_1 + d_2)$$

where N_r is the number of grams (or fissions) calculated for the core rod, N_1 and N_2 are the numbers of grams (or fissions) per rod for the two nearest depletion blocks in the rod row, and d_1 and d_2 are the distances along the rod row from the center of the rod to the centroids of blocks 1 and 2, respectively.

A two-dimensional interpolation scheme was required in the reflector modules and in the inner part of the seed modules, since in these regions the

depletion blocks generally included rods from more than one rod row. (In fact, the depletion blocks sometimes included only part of a rod row in cases where a block boundary cut through a row of rods. An example of this may be seen in the fifth rod row from the bottom of Figure IV-1.) In these regions the three depletion blocks whose centroids were nearest to the center of a rod were used to determine the number of grams of thorium and uranium isotopes and the number of fissions for the rod. The three centroid points were used to define a plane in a three-dimensional (x,y,z) space in which the z-coordinate was grams of material or number of fissions. The z-coordinate of the rod was then determined using standard algebraic equations, as tabulated for example in Reference 15.

The calculated fissile loading of each core fuel rod was defined to be the sum of the number of grams of ^{233}U and ^{235}U obtained using the above interpolation procedure. A normalization factor was applied to the data from each rod to require the sum of the loadings of all the rods in the expended core to be equal to the total core loading as determined from the integration of the conversion ratio. This was done because the integral of the conversion ratio gave a more reliable estimate of the expended core loading than the direct calculation by PDQ, as explained in the previous section. The value of the normalization factor was 0.9963. (This normalization had the effect of including a portion of the ^{232}U content of the expended core in the calculated fissile loading of each rod. A discussion of the treatment of ^{232}U will be given in Section IV.A.)

The isotopic and fission product information for the proof-of-breeding core rods was used in the analysis of the data from the nondestructive assay gauge, as described in Reference 5. In addition, the predictions of loading of all the expended core rods aided in the selection of the specific rods to be assayed nondestructively, as will be discussed in Section V.

D. Rod Fission Product Axial Distributions from CINDER

The fission-product poisons present in the expended fuel rods affected the response of the nondestructive assay gauge. To correct for this effect, it was necessary to have a measure of the axial distribution of fission products along each assayed rod. The axial distribution of one fission product, ^{137}Cs , was determined experimentally by gamma-ray scanning as part of the assay procedure for each rod. The distribution of the other fission products was inferred from the ^{137}Cs distribution using depletion calculations performed by the CINDER computer code (Reference 12).

CINDER used the flux history for a PDQ depletion block to perform growth and depletion calculations for 84 atomic mass chains, including 186 individual fission products. Separate calculations were performed for each of the 30 depletion blocks along the length of 60 selected proof-of-breeding rods. These 60 rods included at least one rod from each of the 50 strata into which the proof-of-breeding rods were grouped, as described in Section V.D.1. CINDER calculations were performed for two or three rods from those seed and blanket strata having a wide range of depletion and for each destructively assayed rod. For those proof-of-breeding rods for which no CINDER calculations were performed, the CINDER data for a similar rod were used. In all cases, this similar rod was one from the same rod stratum having similar depletion. The data were normalized for the differences in depletion using the total number of fissions in the rods as calculated by PDQ.

The CINDER calculations gave the distribution of fission products at 3.5 inch intervals along the rod. In addition, they also provided the distribution of various uranium isotopes. This information permitted corrections to be made to the PIFAG data for the absorption of source neutrons in ^{234}U and ^{235}U as well as fission products. These corrections depended on the neutron absorption cross sections for the uranium isotopes and fission products, as well as on the atomic densities of these nuclides. The cross sections were calculated as a function of depletion using the RCP01 Monte Carlo code

(Reference 16) by the procedure described in Appendix C of Reference 5. Since the cross sections depended on depletion, they were considered to be quantities that varied with position along the rod, as did the atomic densities of fission products and uranium isotopes.

After the atomic densities and cross sections were determined as functions of position along the rod using the gamma scan data, the CINDER calculations, and the RCP01 calculations, this information was included in the equations that determined the amount of fissile fuel in the rod from the PIFAG (nondestructive assay gauge) data as described in Reference 5, thereby correcting the PIFAG data for the effect of absorption of neutrons in the fission products and uranium isotopes.

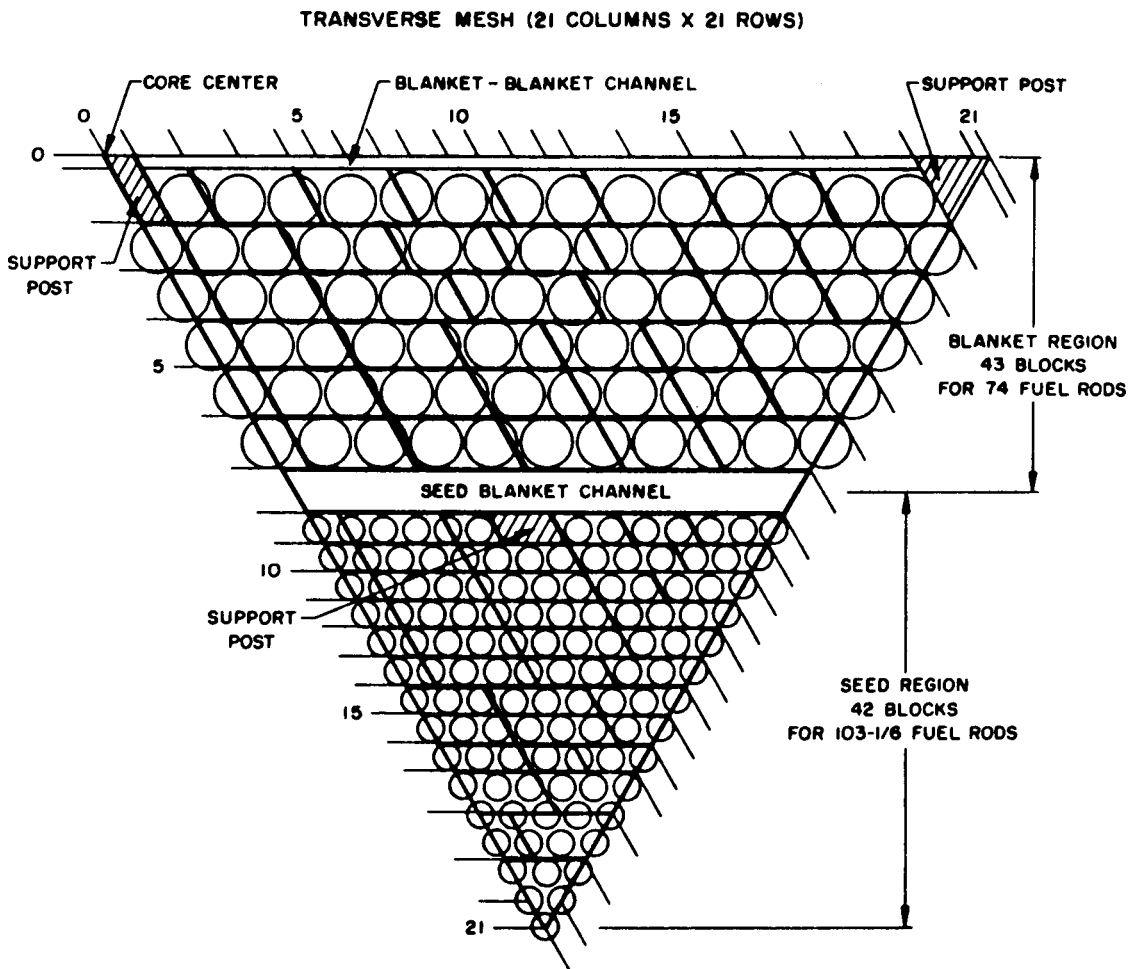


Figure IV-1. Mesh Lines and Depletion Block Boundaries in PDQ for One-Sixth of a Type I Seed-Blanket Module Combination

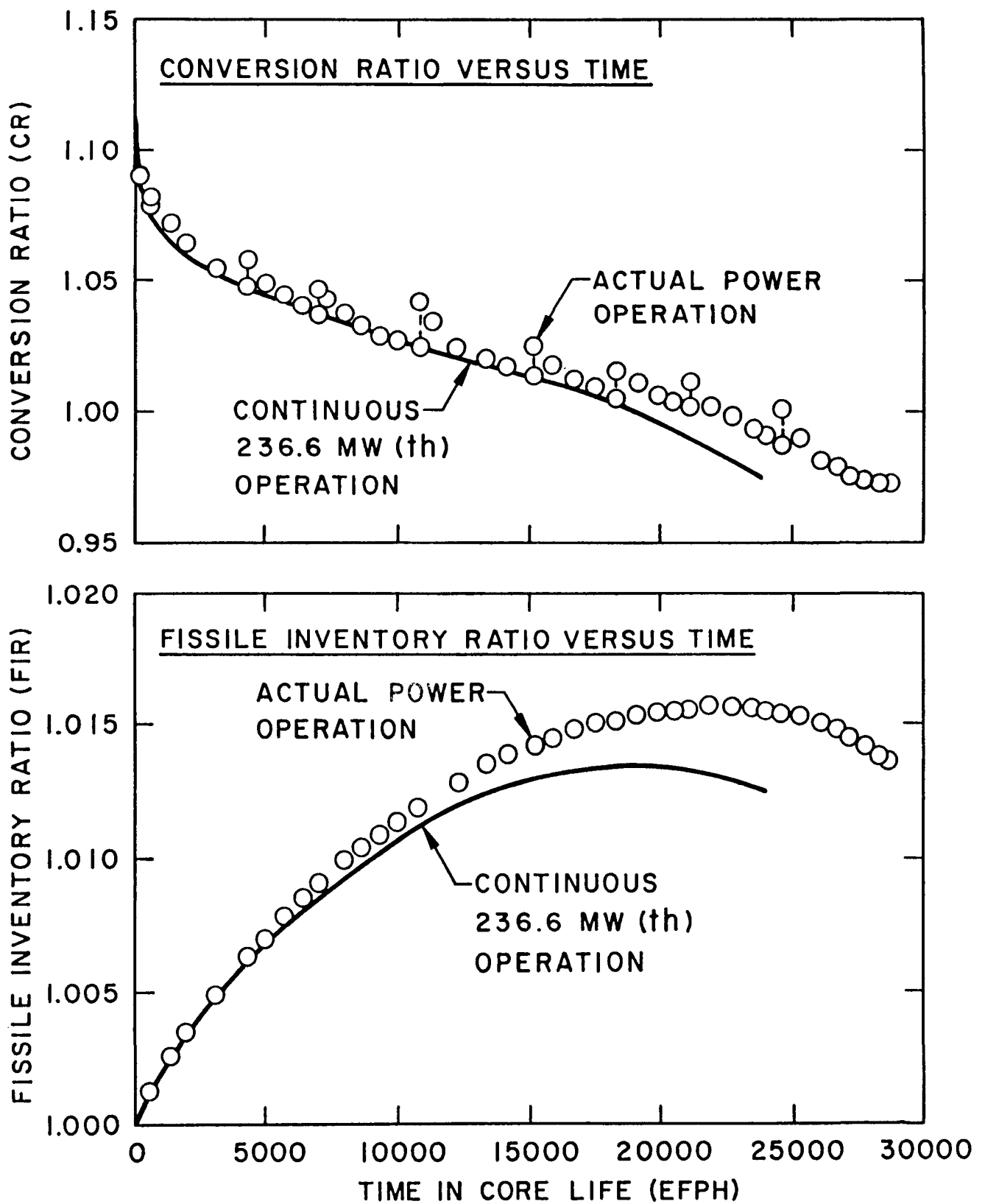


Figure IV-2. Calculated Breeding in LWBR

V. THE TWO-STAGE STRATIFIED RANDOM SAMPLING PLAN

A. Background and Objectives of the Sampling Plan

Upon completion of power operation, the LWBR core was shut down and dismantled. All 39 fuel modules were removed from the reactor vessel and shipped to the Naval Reactors Expended Core Facility in Idaho for processing. Twelve of the modules subsequently had their end plates cut off so that individual fuel rods could be removed. Some of the rods that were removed were nondestructively assayed to determine their fissile fuel loadings. The assayed rods were called "proof-of-breeding" (POB) rods, and the 12 modules from which they were taken were called POB modules.

This section describes the sampling plan that was used to select the POB rods and modules. The sampling plan had two stages, the first of which was the selection of the POB modules. These modules included at least one from each of the nine different types, or strata, of modules in the core. The second stage of the plan was the selection of the POB rods from the POB modules. The rods in the core were also grouped into strata, and a representative sample of POB rods was chosen from each rod stratum. The selection procedure for both rods and modules was random within each stratum; hence the name "two-stage stratified random sampling plan."

The primary objective was to establish a random sampling plan such that the assay results for the sampled fuel rods could be used to estimate the total loading of all the rods in the expended core with sufficient precision and accuracy to demonstrate that breeding had occurred. A related objective was to allocate the POB rods to the different rod and module strata in a way that would fairly and adequately represent the total population of rods in the core and would minimize the sampling variance in the core loading estimate, which is the uncertainty in the determination of the total fissile content of all 17,290 rods in the core based on measurements performed on a sample of these rods. Sufficient rods were to be chosen from each rod stratum to permit

an evaluation of the degree of variability in loading among the different rods of a given type. Similarly, replicate modules were to be chosen from some module strata so that information would be obtained on the module-to-module variation in loading (i.e., on the symmetry of the fuel distribution throughout the core). Finally, the sampling plan was intended to provide high confidence that breeding could be demonstrated even if the fissile inventory ratio happened to be at the low end of the uncertainty in its predicted value.

B. Simulation Studies Used to Develop the Sampling Plan

Prior to the start of the assay campaign, it was established that a selection of 500 fuel rods chosen from 12 modules according to the two-stage stratified random sampling plan could satisfy the above objectives. This conclusion was based on computer simulation studies carried out using the predicted final loadings of all rods in the expended core, obtained from PDQ calculations as described in Section IV.C. These studies used a Monte-Carlo approach to simulate the selection of the POB rods according to various sampling schemes, the assay of these rods yielding results subject to random experimental error, the estimation of the total core loading from the experimental results, and the comparison of this estimate to the "true" core loading determined by adding a random error term (and in some cases a bias error as well) to the PDQ-calculated loadings. The calculations were repeated a large number of times with all random quantities recalculated independently on each repetition, so that statistical methods could be used to analyze the results. This section describes how the simulation studies were carried out.

The simulation studies were based on the premise that the PDQ-calculated loadings of the core rods were subject to random error and, in some cases, a bias error. The "true" loading of each rod in a POB module was simulated by adding to the PDQ-calculated loading an error term. The random part of the error term was a normally distributed random number, calculated independently for each rod in the module, with an average value of zero and a standard deviation that was typically 2% to 3% of the PDQ loading for seed and blanket

rods. Reflector rods, for which the PDQ calculations were expected to be less accurate, were assigned random error terms with standard deviations ranging from 5% (for the rods with the most fuel) to 9% (for the rods with the least fuel) of the rod loading. Reflector rods were also assigned a non-random bias error. A typical bias error was a quadratic function of loading that tended to make the "true" loadings greater than the PDQ calculations for the rods with the smallest and largest amounts of fuel, and less than calculation for rods with intermediate amounts of fuel. This nonlinear bias error term was introduced to investigate the ability of the core loading estimation models (discussed in Section VII) to deal with a situation where the PDQ calculations differed from truth in a nonlinear fashion. In some cases, nonlinear error terms were also applied to rods from certain seed and blanket strata as well as from the reflector.

A "true" loading was calculated for each rod in a POB module, and these loadings were added to give the "true" loading of the whole module. The process was repeated for all POB modules in the core. The "true" loading of each non-POB module was determined by adding to the PDQ module loading a random error term. This error term was a normally distributed random number, calculated independently for each module, with a mean of zero and a standard deviation typically 0.5% of the module loading. In addition, a systematic module-to-module variation in loading, such as would have been caused by a $\pm 1\%$ asymmetry in power across the core during its operational lifetime, was introduced into some of the simulation studies. (Physics testing performed during core lifetime gave evidence of a small asymmetry in core power of less than $\pm 1\%$, as discussed in Reference 17.) The "true" loadings of all the modules, including the random and systematic error contributions, were added to give the "true" loading of the core.

Next, the POB rods were selected randomly according to the sampling plan being investigated in the simulation study. The assay results for these rods were simulated by adding a random experimental error to the "true" loading to give the "measured" loading. The experimental error was a normally

distributed random number with a mean of zero and a standard deviation equal to 0.5% of the loading for seed and blanket rods and 1% to 3% of the loading for reflector rods. These were considered to be reasonable estimates of the precision expected to be attained during the assay campaign. (The precision actually attained when the assays were performed was somewhat better than this, as will be shown in Section VI.) The "measured" loadings for the POB rods were then used to estimate the loading of the whole core using the two estimation methods to be discussed below. The estimated core loading was then compared to the "true" loading to determine the error in the estimate. The whole procedure was repeated typically 200 times, with the random quantities recalculated in each repetition while all other quantities remained fixed. The average difference between the estimated and "true" loadings for the repeated cases was taken as a measure of the bias error in the estimated core loading. The standard deviation in the difference between estimated and true loadings was taken as a measure of the random error in the estimate; the square of this random error is the "sampling variance" mentioned above.

A goal of $\pm 0.18\%$ was established for the allowable random error in the core loading determination, consistent with the objective of having high confidence that breeding could be demonstrated even if the measured fissile inventory ratio proved to be at the low end of prediction. The simulation studies were used to investigate the number of POB modules and rods needed to achieve this goal. Sampling plans using different numbers of POB rods and modules were investigated, and different ways of allocating the rod selection among the different rod strata were studied. The magnitudes of the various error terms were changed in some studies to investigate the sensitivity of the results to the assumptions made about the precision of the PDQ calculations and the experimental measurements. The studies showed that 500 POB rods, appropriately allocated among 50 rod strata and selected from 12 appropriately chosen POB modules, could meet the goal of $\pm 0.18\%$ random error. The selection of the POB modules and rods in accordance with the adopted sampling plan is discussed in the following sections.

C. Selection of 12 Proof-Of-Breeding Modules

The first stage of the two-stage stratified random sampling plan was the selection of 12 POB modules from the nine different module strata present in the core. The module strata were previously identified in Section II.B, and further information about them is given in Table V-1. One POB module was selected from each of six strata, and two POB modules were chosen from each of the remaining three large-population strata: Type III seed, Type III blanket, and Type IVA reflector. Inclusion of a second POB module from these strata provided a measure of the variation in final loading among modules within a given stratum. For all strata except the Type V reflector, the number of POB modules selected was one-third of the number of modules in the core. The Type V reflector modules contained so little fuel and contributed so little to the variance in the total core loading estimate that one module out of six was considered to be an adequate sample.

The selection of POB modules was governed by the following geometrical constraints, which were imposed to ensure that the chosen modules would come from all parts of the core:

1. For those module strata from which two modules were selected, the chosen modules should not be adjacent to each other.

2. The number of seed and blanket modules chosen from each sixth-core sector should not be less than one-half nor more than two and one-half. (It may be deduced from Figure V-1 that each sixth-core sector contained one-half of a Type I seed-blanket combination, one-half of a Type II combination, and one Type III combination. The total number of seed and blanket modules selected was eight, so the average number chosen per sector was one and one-third.)

3. The total number of modules (seed, blanket, and reflector) chosen from each sixth-core sector should not be less than one nor more than three.

4. The number of seed and blanket modules chosen from each half-core should not be less than three nor more than five. (Note that there were three ways of dividing the LWBR core into symmetric halves. This requirement applied to each of these divisions.)

5. The total number of modules chosen from each half-core should not be less than four and one-half nor more than seven and one-half. (Again, the requirement applied to each way of dividing the core into halves.)

A random selection was made from the 276,426 different combinations of 12 modules satisfying the above constraints. One change was subsequently made in the selected combination: The original choice of seed module I-3 was replaced by seed module I-1. This substitution was made because three of the four randomly-selected seed modules had generally higher than average power during operation of the core, as determined from physics tests performed at various times throughout core life (Reference 17). With seed I-1 replacing seed I-3, the average power for the POB seed modules was very close to the average power for all the seed modules in the core. Thus, a small deviation from a purely random module selection was made so that the physical properties of the core could be represented more accurately. This change was consistent with the geometrical constraints listed above.

The POB module selection is shown in Figure V-1.

D. Selection of 500 Proof-Of-Breeding Rods

1. Definition of Rod Strata

The second stage in the sampling plan was the selection of 500 POB rods from the 12 POB modules. The selection was based on the division of the core rods into different strata. All rods in a given stratum had the same physical properties and had essentially identical initial loadings. It was

pointed out in Section II.D that the binary fuel rods (i.e., the rods in the seed and blanket modules, which contained pellets made of thorium oxide and enriched ^{233}U oxide) could be divided naturally into 37 strata based on properties such as fissile concentration and length of the binary fuel pellet stack. For reflector rods, however, there was no such logical division into strata, since all reflector rods were constructed in the same way and all had zero fissile fuel initially. Therefore, it was decided to define the reflector strata based on the PDQ-predicted final (end-of-life) rod loadings. Rods from the Type IVA and IVB reflector modules were divided into five strata each, and rods from the Type V reflector modules were divided into three strata. Thus, there were 13 reflector rod strata and a total of 50 rod strata for the whole core. The definitions of the strata and information about them are given in Tables V-2 and V-3. Figures V-2 through V-13 show cross sections of the 12 POB modules with the boundaries between rod strata indicated by heavy lines and the POB rod locations indicated by shading.

The number of reflector rod strata and the ranges of final rod loadings used to define them were chosen to minimize the random and bias errors in the core loading estimate as determined from the simulation studies described previously in Section V.B. For the Type V reflector POB module, Figure V-13 shows that the strata formed a simple geometrical arrangement of three groups of parallel rod rows. For the Type IVA and IVB modules, however, Figures V-10 through V-12 show jagged boundaries between strata. These jagged boundaries are a consequence of the staggered rod configuration and the more complex distribution of rod loadings in the Type IV modules. Note that the stratum boundaries in the Type IVA modules do not exhibit left-right symmetry. This is because one side of each IVA module was slightly closer to core center and therefore saw a higher neutron flux during core life than the other side, which led to an asymmetric rod loading distribution. The Type IVB modules, on the other hand, were symmetrically located and hence had symmetric stratum boundaries.

2. Allocation of Proof-Of-Breeding Rods to Strata

The next step in the sampling plan was to decide how many of the POB rods to allocate to each rod stratum. The optimum allocation would be the scheme that produced the smallest sampling variance in the expended core loading estimate. The size of the sampling variance depended on the method used to deduce the total loading for all the rods in a stratum from the assay results for the POB rods. As will be described in more detail in Section VII, two methods were employed to do this. In the first method, called the stratum average method, the average measured loading for the POB rods was taken to represent the average loading of all the rods in the stratum. Therefore, in this method the estimate of the total stratum loading was given by the product of the average measured loading of the POB rods times the number of rods in the stratum. It can be shown (see Reference 18) that for the stratum average method the optimum allocation is given by the equation

$$n_i = 500 \times (N_i \sigma_i) / (\sum N_i \sigma_i)$$

where n_i is the theoretically optimum number of POB rods for the i^{th} stratum, N_i is the total number of rods in the core in the i^{th} stratum, σ_i is the standard deviation in final loading among the rods in the i^{th} stratum, and 500 is the total number of POB rods. Estimates of the standard deviation σ_i for each stratum based on the PDQ-calculated rod loadings are given in Tables V-2 and V-3. It may be noted that the standard deviations are generally larger for the reflector strata than for the seed or blanket strata. This is a consequence of the fact that the reflector rods had a very wide range of final loadings, in contrast to the much more uniform seed and blanket rod loadings. Using the estimates of σ_i from Tables V-2 and V-3 in the above equation indicates that for the stratum average method the optimum allocation scheme would allot more than 37% of the POB rods to the reflector strata, even though the reflector modules were predicted to have less than 8% of the fuel in the core.

The second method used to infer stratum loadings was called the linear regression method. In this method, a comparison was made between the PDQ predictions of the POB rod loadings and the measured loadings. It was assumed that the relationship between prediction and measurement could be satisfactorily represented by a straight line. The slope of this straight line was used to apply a correction to the average measured loading of the POB rods before multiplying by the number of rods in the stratum to get the estimated total stratum loading. This approach mitigated the effect of the large standard deviations among the reflector rod loadings, thereby allowing the stratum loadings to be determined with smaller uncertainty than the stratum average method, while using fewer reflector POB rods. However, it was recognized that the linear regression method could introduce biases into the loading calculations if the relationship between prediction and measurement proved to be nonlinear, or if the random error in the predicted rod loadings proved to be larger than expected so that the slope of the fitted straight line was poorly defined. Therefore, it was decided that a choice between the stratum average and linear regression methods should be deferred until a sufficient amount of assay data was available to determine how successfully the two methods would work in practice. Since the optimum allocation schemes are different for the two methods, a compromise plan was sought that would be as near as possible to optimum for both methods.

The optimum allocation of rods to strata for the linear regression method cannot easily be evaluated analytically. Therefore, the simulation studies described in Section V.B were used instead. As noted in that section, the simulation studies included a nonlinear bias error term for reflector rod loadings; this was done to test the ability of the linear regression method to deal with a situation where the relationship between predicted and measured rod loadings differed significantly from a straight line. The studies indicated that the linear regression method would work satisfactorily provided that at least five rods were chosen from each stratum. Based on the simulation studies, a plan for allocation of reflector POB rods was obtained that achieved near minimum variance for both methods without introducing

significant bias into the linear regression module loading estimates. This plan assigned 117 POB rods, or 23.4% of the total, to the reflector.

After the number of POB rods to be assigned to the reflector strata had been decided upon, the remaining POB rods were assigned to the seed and blanket strata through the use of the stratum average optimum allocation equation given above. A stipulation was made, however, that each stratum should have at least two POB rods. This plan allocated fewer POB rods to the reflector strata than the theoretically optimum allocation for the stratum average method, thereby allowing a larger number of rods to be allocated to the seed and blanket strata having the largest amounts of fuel. The simulation studies showed that this allocation scheme would yield a sampling variance for the total core loading estimate using the stratum average method that was only slightly larger than the theoretical minimum for that method. Use of the linear regression method for the reflector strata, and perhaps for some seed and blanket strata as well, would result in a substantially lower variance for the core loading. The simulation studies gave high confidence that the linear regression method could be used successfully, even if the PDQ predictions of the rod loadings turned out to be poorer than expected.

As will be shown in Section VII.B, the assay results demonstrated that the linear regression method could indeed be applied to the reflector strata and to the seed and blanket strata as well. Use of the linear regression estimation for all modules resulted in a much lower core loading variance than the stratum average method.

The numbers of rods allocated to the strata in the sampling plan are given in Tables V-2 and V-3.

3. Rod Selection Method

The final step in the sampling plan was to identify the specific rods to be assayed. These rods were selected randomly, subject to certain

geometrical constraints to ensure that all parts of each module would be adequately represented in the sample. As an example of the types of constraints applied, consider POB blanket module I-3 as shown in Figure V-6. The two innermost rod rows in this module, rows A and B, constitute one rod stratum — the stratum of 42" binary length, low zone, Type I standard blanket rods. Eleven POB rods were selected from this stratum. It was required that at least five rods be chosen from each of the two rod rows. It was further required that at least one rod be chosen from each of the six sides of the module. (For the purpose of this requirement, a rod in a corner position, such as the rod at position 21 in row A, was counted as half of a rod in each of the two adjacent sides.) The remaining rod rows C through F each constitute one rod stratum. Five to seven rods were selected from each of these rows. It was required that not more than two rods be chosen from any one side of the module from any one row. Among the four outer rows as a whole, at least two rods were required to be chosen from each side of the module.

Constraints similar to these were applied to the rod selection for all modules. The locations of the selected POB rods are shown in Figures V-2 through V-13. Each cell (rod location) in these figures is identified by a one or two digit number, followed by a letter, followed by another one or two digit number. The first number identifies the rod type for manufacturing purposes. This number is not relevant to the discussion in the present report, so it will be suppressed when rod cells are identified henceforth. The letter denotes the rod row and the following number denotes the position of the cell in the row. The boundaries between rod strata are identified by solid lines in the figures.

E. Selection of 24 Additional Rods

The results obtained during the nondestructive assay campaign tended to support the assumptions made in the rod selection process and showed that a good selection had been made. In particular, the assumption of a linear

relationship between measurement and PDQ prediction that is essential to the linear regression method was found to be valid, as will be shown in Section VII.B. Also, the degree of symmetry in the core was well within expectations (as will be shown in Section VII.E), and the number of POB rods allocated to each stratum was found to be satisfactory. However, one unexpected discovery made during the campaign was that the reflector modules had 3% to 5% more fuel than predicted by PDQ. The individual rods having the largest excess in fuel content above the amount predicted were the rods in the outermost rod rows, on the periphery of the core. Only a few such rods had been included in the sampling plan, because they contained very small amounts of fuel and made a small collective contribution to the total loading of the core. (The PDQ calculations predicted that all the rods in the outermost stratum of all the reflector modules would together contain only about 1% of the total fissile fuel in the expended core.) But when the assay results showed that some of these rods had up to 65% more fuel than predicted, it was decided that some additional reflector rods should be assayed to allow their properties to be characterized more precisely. Accordingly, twenty additional rods selected from the outer rod rows of reflector module IV-9 were assayed. The results for these additional rods were consistent with the results for the rods in the original sampling plan. The total fissile content of module IV-9 deduced from all the assay data, including that from the additional 20 rods, was only slightly different from the result obtained using just the rods in the original sampling plan. Thus, the assay of the additional rods demonstrated the effectiveness of the original sampling plan in providing a fair representation of the rods in the core, even where the assay results were somewhat different than predicted.

The PIFAG was used to process a few fuel rods that were included in the core examination program (Reference 19). It was decided to perform a full-fledged assay on four of these rods, which were included in the core examination program because they had particularly high fuel burnup. Although these rods were not part of the POB sampling plan, the data obtained in

assaying them were used in determining the respective module loadings using the linear regression method only.

Thus, the total number of nondestructively assayed rods was 524. Five hundred of these were from the original sampling plan, 20 were added to aid in the analysis of the reflector data, and four were included from the core examination work.

F. Selection of 17 Rods for Destructive Assay

Seventeen of the POB rods were sent to Argonne National Laboratory (ANL) for destructive assay after they had been nondestructively assayed in the PIFAG. Five of these rods were seed rods, four were standard blanket rods, six were power flattening blanket rods, and two were reflector rods. These numbers are in roughly the same proportion as the distribution of fuel in the expended core. The seed rods came from seed module I-1, the standard blanket rods from blanket module I-3, the power flattening blanket rods from blanket module III-6, and the reflector rods from Type IVB reflector module IV-1. In each case the module selected was the first POB module available containing rods of the corresponding kind. The objective was to obtain the destructive assay results as early in the POB campaign as possible, so that any problems or discrepancies between the destructive and nondestructive results could be dealt with in a timely fashion.

The destructively assayed rods were selected to give wide representation to the different rod strata. At least one rod was chosen having each of the four binary lengths (42", 56", 70", and 84") for seed, standard blanket, and power flattening blanket rods; likewise, each uranium concentration (low and high for seed rods; low, medium, and high for standard blanket and power flattening blanket rods) was represented by at least one rod in the sample. Two 84" high zone seed rods and three 84" high zone power flattening blanket rods were included because of the large number of rods of these types in the core, and because of the large total amount of fuel contained in them. (One

may deduce from Table V-2 that 84" high zone seed rods contained about one-fifth of the total fuel in the expended core and 84" high zone power flattening blanket rods about one-fourth.) One of the two selected 84" high zone seed rods had the highest depletion of any POB rod of that type, while the other had a low depletion. The three selected 84" high zone power flattening blanket rods included one with high depletion, one with low depletion, and one with average depletion. Similarly, the other seed and blanket rods selected for destructive assay included some with relatively high, some with low, and some with average depletion relative to other rods of the same type in the core. Finally, the two reflector rods selected included one with the highest depletion of any POB rod in the module and one with moderate depletion.

Thus, the rods selected for destructive assay included a representative sample of the different rod types and depletions in the expended core. These rods are tabulated in Table V-4.

TABLE V-1. MODULE STRATA PRESENT IN LWBR CORE

Stratum Index	Module Type	Number of Modules in Core	Number of Modules Sampled	Number of Rods Per Module	Sampled Rods Per Module	Predicted Fissile Grams/Mod	Predicted Fissile Grams Tot
1	Seed I	3	1	619	32	12572	37715
2	Seed II	3	1	619	34	12942	38826
3	Seed III	6	2	619	34	13277	79661
4	Blkt I	3	1	444	36	19711	59132
5	Blkt II	3	1	564*	63	25786#	77359#
6	Blkt III	6	2	633*	75	29617#	177701#
7	Ref1 IVA	6	2	228*	28	3180#	19078#
8	Ref1 IVB	3	1	228	27	2911	8734
9	Ref1 V	6	1	166	34	1663	9977
Core Totals		39	12				508183#

* Some modules had one fewer fuel rod due to presence of a flux well.

Not corrected for the missing flux well fuel rods.

TABLE V-2. SEED AND BLANKET ROD STRATA

		Number Of Rods		Sampled Rods		Avg. Init. Ldg. Per Rod	Calculated Final Loading (Grams)			
Stratum Type*		Per Mod.	In Core	Per Mod.	Total		Average Per Rod	Std. Dev.	Total Per Mod.	Core Total
Seed	I H-84	331	993	24	24	34.57	24.75	0.79	8192.7	24578.1
	I L-70	66	198	2	2	23.92	17.54	0.11	1157.9	3473.7
	I L-56	72	216	2	2	19.14	15.62	0.08	1124.8	3374.5
	I L-42	150	450	4	4	14.33	13.98	0.25	2096.4	6289.2
	II H-84	331	993	25	25	34.57	25.71	0.81	8510.6	25531.8
	II L-70	66	198	2	2	23.92	18.04	0.21	1190.5	3571.6
	II L-56	72	216	2	2	19.14	15.88	0.11	1143.0	3429.1
	II L-42	150	450	5	5	14.33	13.99	0.26	2097.8	6293.5
	III H-84	331	1986	25	50	34.57	26.57	0.80	8794.0	52763.8
	III L-70	66	396	2	4	23.92	18.50	0.29	1221.2	7327.0
	III L-56	72	432	2	4	19.14	16.11	0.14	1160.3	6961.6
	III L-42	150	900	5	10	14.33	14.01	0.26	2101.4	12608.5
Std.	I L-42	126	378	11	11	16.45	35.93	0.45	4527.8	13583.3
Blkt.	I M-56	72	216	5	5	30.34	41.34	0.35	2976.6	8929.9
	I H-70#	78	234	6	6	45.47	47.93	0.20	3738.9	11216.6
	I H-84	84	252	7	7	54.66	53.00	0.11	4451.6	13354.9
	I M-84	84	252	7	7	45.66	47.81	0.18	4015.7	12047.1
	II L-42	80	240	8	8	16.45	34.77	0.60	2781.7	8345.1
	II M-56	43	129	4	4	30.34	40.60	0.50	1745.8	5237.5
	II H-70	45	135	4	4	45.47	47.56	0.34	2140.1	6420.4
	II H-84	47	141	4	4	54.66	52.85	0.21	2483.9	7451.8
	II M-84	46	138	4	4	45.66	47.60	0.24	2189.5	6568.6
	III L-42	59	354	6	12	16.45	33.72	0.50	1989.4	11936.2
	III M-56	31	186	3	6	30.34	39.90	0.42	1236.9	7421.4
	III H-70	32	192	3	6	45.47	47.22	0.31	1511.0	9065.8
	III H-84	33	198	3	6	54.66	52.75	0.21	1740.8	10444.8
	III M-84	32	192	3	6	45.66	47.38	0.19	1516.2	9097.0
Pwr.	II L-42	46	138	4	4	18.96	30.48	0.38	1402.2	4206.6
Flat.	II M-56	29	87	3	3	30.74	35.12	0.32	1018.6	3055.9
Blkt.	II H-70	33	99	3	3	52.56	46.13	0.35	1522.3	4567.0
	II H-84#	187	561	27	27	63.06	54.35	1.43	10163.3	30489.9
	II M-84	8	24	2	2	46.40	42.37	0.10	338.9	1016.8
	III L-42	67	402	6	12	18.96	29.29	0.66	1962.5	11774.9
	III M-56	41	246	3	6	30.74	34.47	0.48	1413.4	8480.6
	III H-70	46	276	4	8	52.56	46.60	0.53	2143.5	12861.0
	III H-84#	284	1704	42	84	63.06	55.51	1.58	15764.0	94583.8
	III M-84	8	48	2	4	46.40	42.40	0.10	339.2	2035.2

* Strata are identified by module type (I, II, or III), uranium concentration (H=high, M=medium, L=low), and binary fuel stack length in inches.

Data for this stratum are not corrected for the flux well fuel rod missing from some modules.

TABLE V-3. REFLECTOR ROD STRATA

Stratum Type*	Number Of Rods		Sampled Rods		Calculated Final Loadings (Grams)			
	Per Mod.	In Core	Per Mod.	Total	Average Per Rod	Std. Dev.	Total Per Mod.	Core Total
IVA								
> 27.5	22	132	5	10	32.49	3.54	714.8	4288.7
19.5 - 27.5	35	210	5	10	23.57	2.15	825.0	4949.9
13.5 - 19.5	42	252	5	10	16.59	1.81	696.7	4180.4
7.5 - 13.5	56	336	6	12	10.34	1.74	579.0	3473.8
< 7.5#	73	438	7	14	4.99	1.36	364.2	2185.5
IVB								
> 27.0	19	57	5	5	31.06	3.18	590.2	1770.5
19.0 - 27.0	34	102	5	5	22.45	2.04	763.4	2290.2
12.3 - 19.0	48	144	6	6	15.32	1.98	735.5	2206.5
7.0 - 12.3	51	153	5	5	9.36	1.50	477.4	1432.2
< 7.0	76	228	6	6	4.54	1.29	344.8	1034.5
V								
> 14.0	41	246	12	12	19.16	3.26	785.6	4713.6
7.5 - 14.0	49	294	10	10	10.46	1.86	512.5	3075.2
< 7.5	76	456	12	12	4.80	1.48	364.6	2187.8

* Strata are identified by reflector module type (IVA, IVB, or V) and range of final rod loadings (in grams) as predicted by the LWBR calculational model.

Data for this stratum are not corrected for the flux well fuel rod missing from one module.

TABLE V-4. RODS SELECTED FOR DESTRUCTIVE ASSAY

Module	Rod Cell	Rod Stratum	Depletion	Calculated Final Loading (Grams)	
				Rod	Stratum Average
Seed I-1	C10	84" High Zone	Low	25.570	24.751
	L29	84" High Zone	High	23.185	24.751
	M49	70" Low Zone	Med.	17.608	17.544
	N63	56" Low Zone	Med.	15.638	15.623
	P39	42" Low Zone	Low	14.143	13.976
Blkt. I-3 (Std. Blkt. Rods)	A49	42" Low Zone	High	36.430	35.935
	C3	56" Med. Zone	Low	41.001	41.342
	D24	70" High Zone	Low	47.732	47.934
	E56	84" High Zone	High	52.860	52.996
Blkt. III-6 (Pwr. Flat. Blkt. Rods)	B62	42" Low Zone	Med.	29.357	29.291
	C13	56" Med. Zone	Low	34.042	34.474
	D29	70" High Zone	High	45.753	46.598
	E31	84" High Zone	High	52.120	55.507
	F73	84" High Zone	Med.	53.436	55.507
	H1	84" High Zone	Low	57.460	55.507
Ref1. IV-3	B1	Over 27.0 g	High	35.659	31.061
	E3	19.0 - 27.0 g	Med.	23.692	22.453

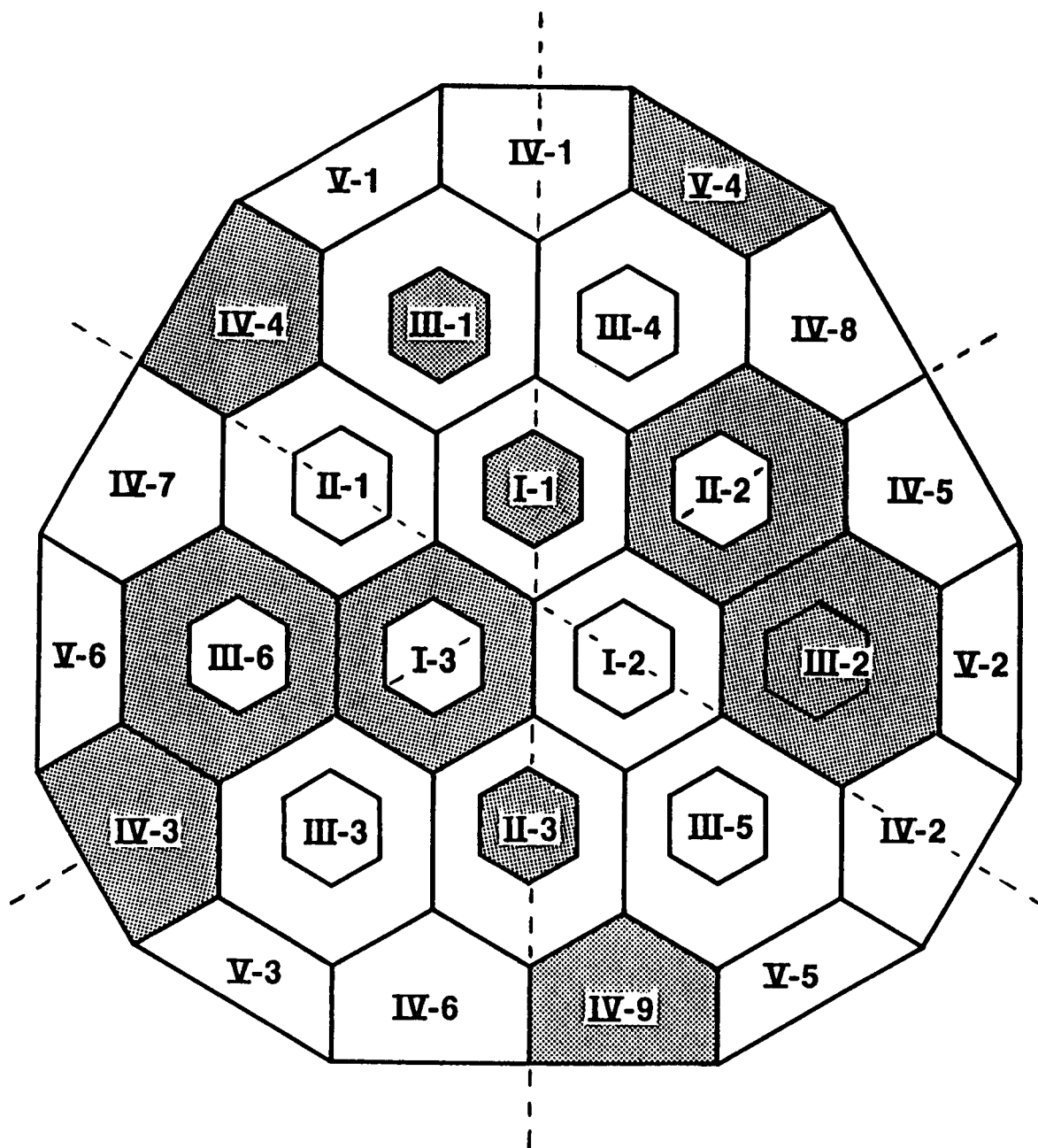
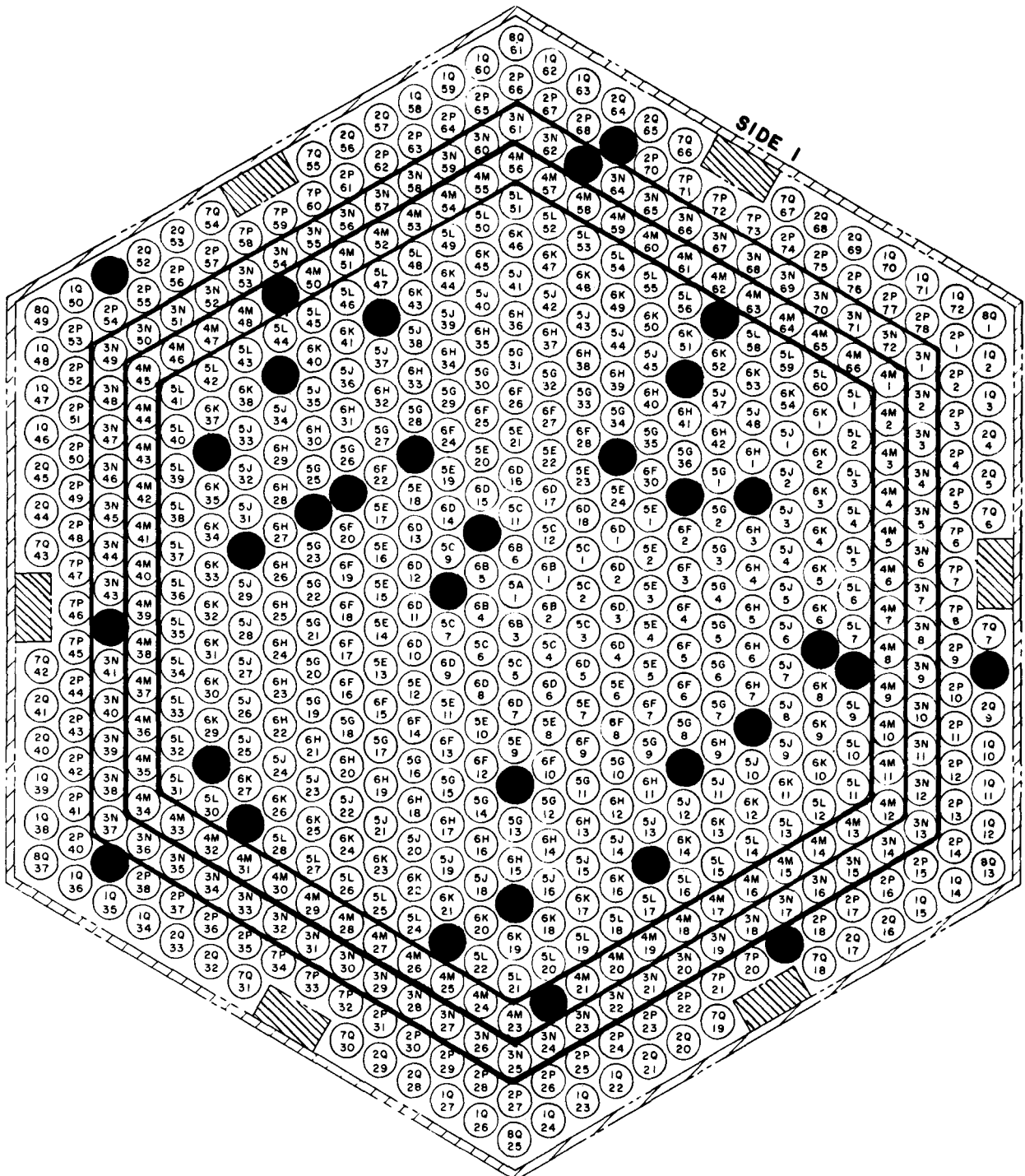
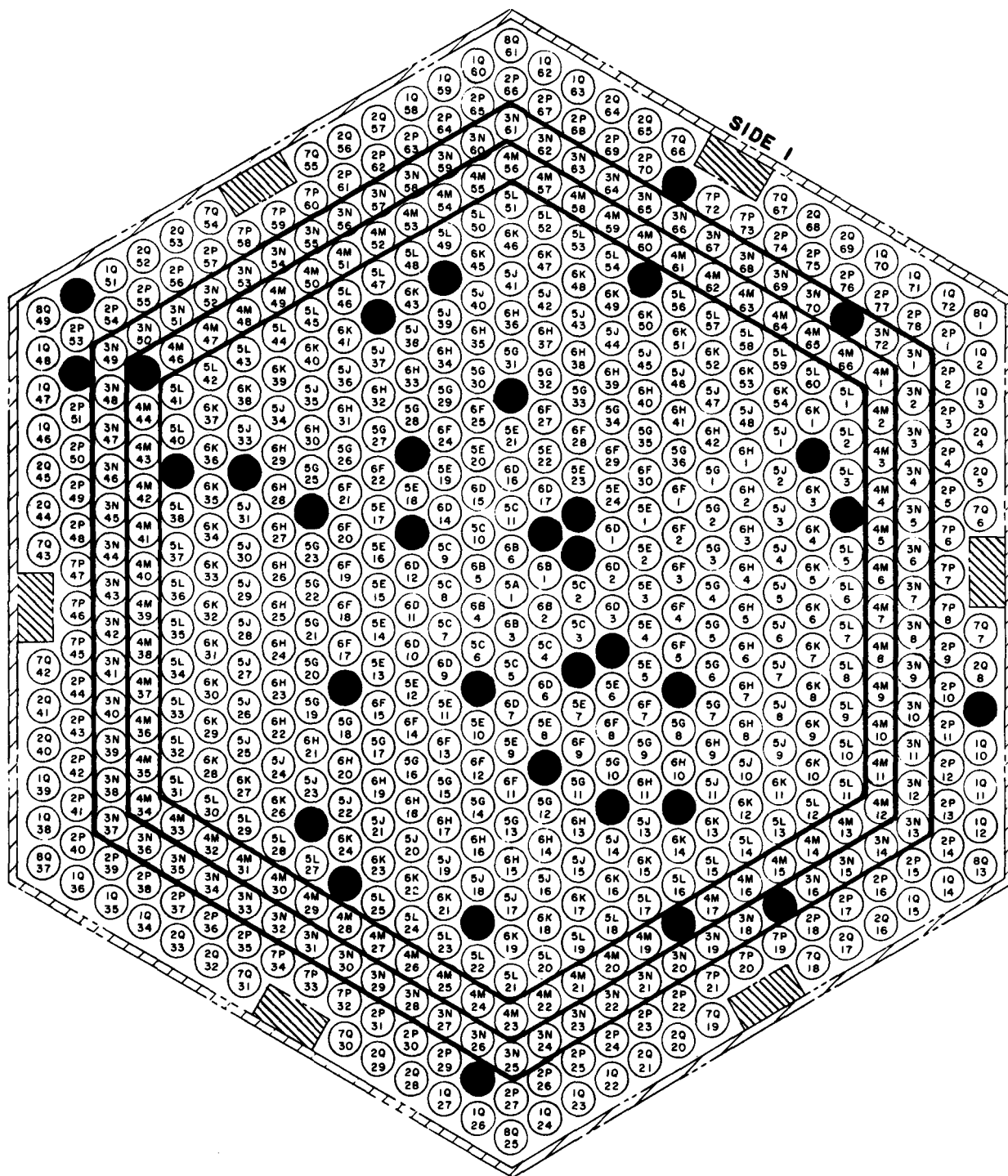


Figure V-1. Diagram of LWBR Core, Showing Locations of Proof-Of-Breeding Modules (Shaded) and Boundaries of Sixth-Core Sectors (Dashed)





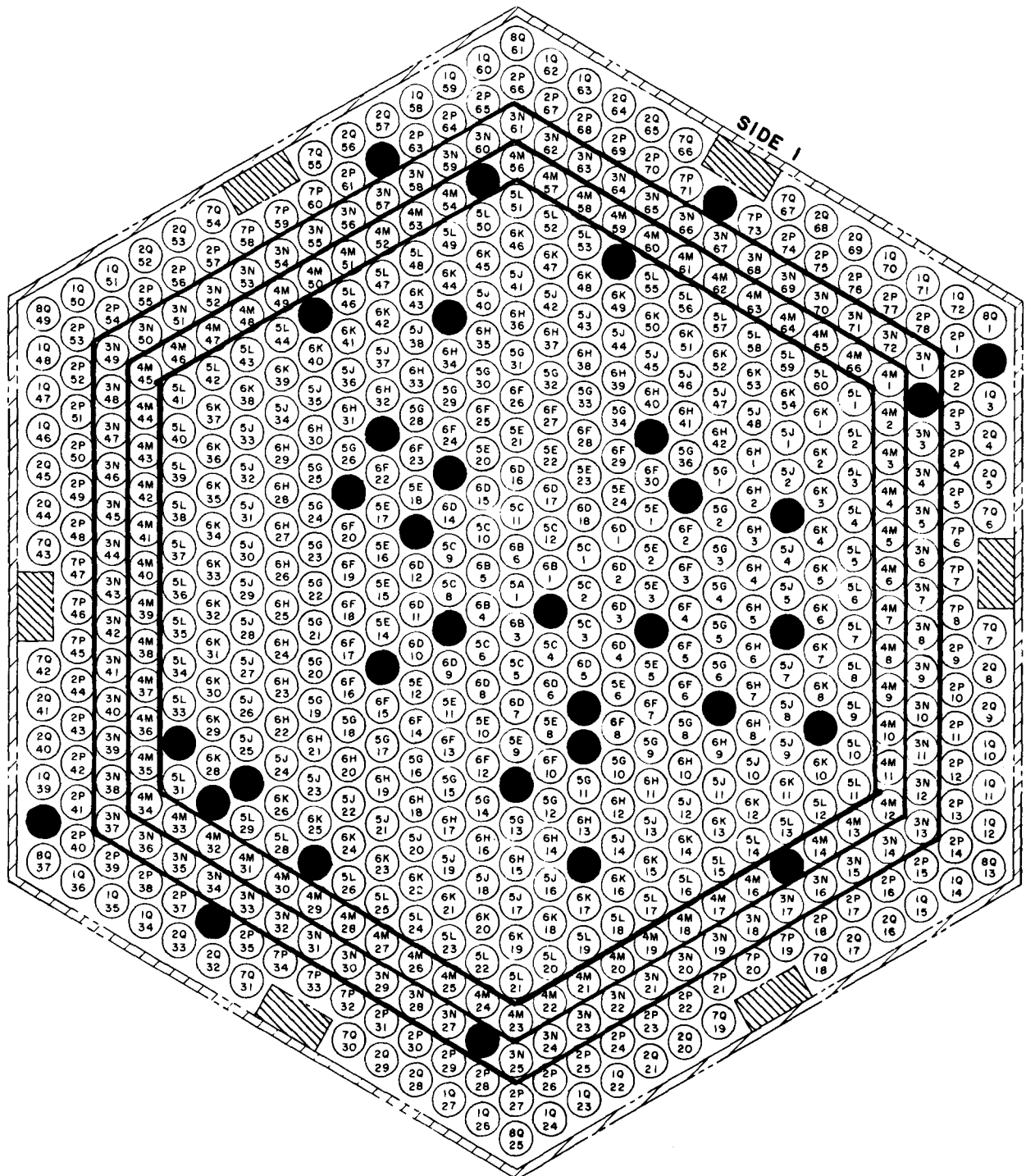


Figure V-4. Location of Proof-of-Breeding Rods in Seed Module III-1

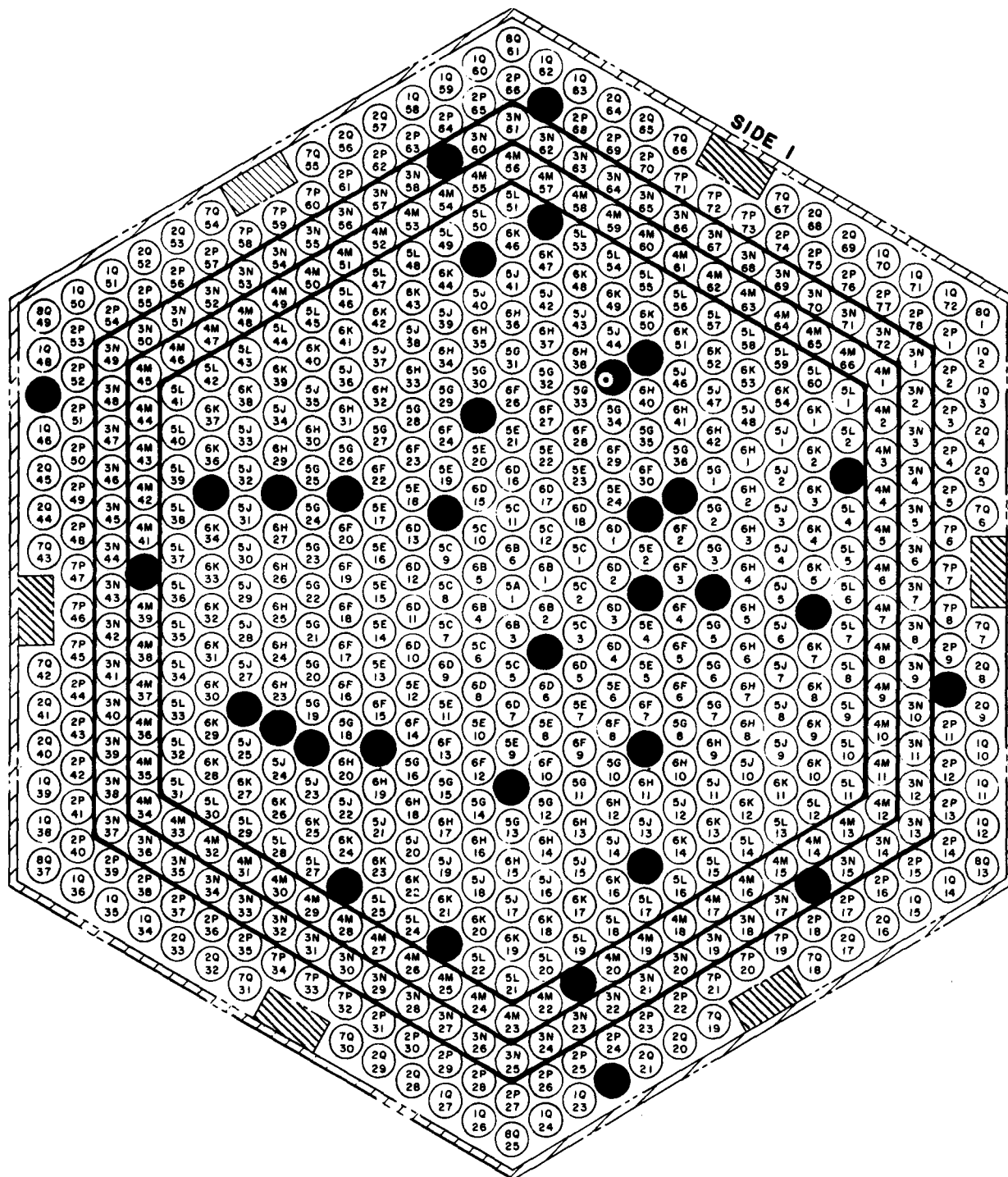


Figure V-5. Location of Proof-of-Breeding Rods in Seed Module III-2

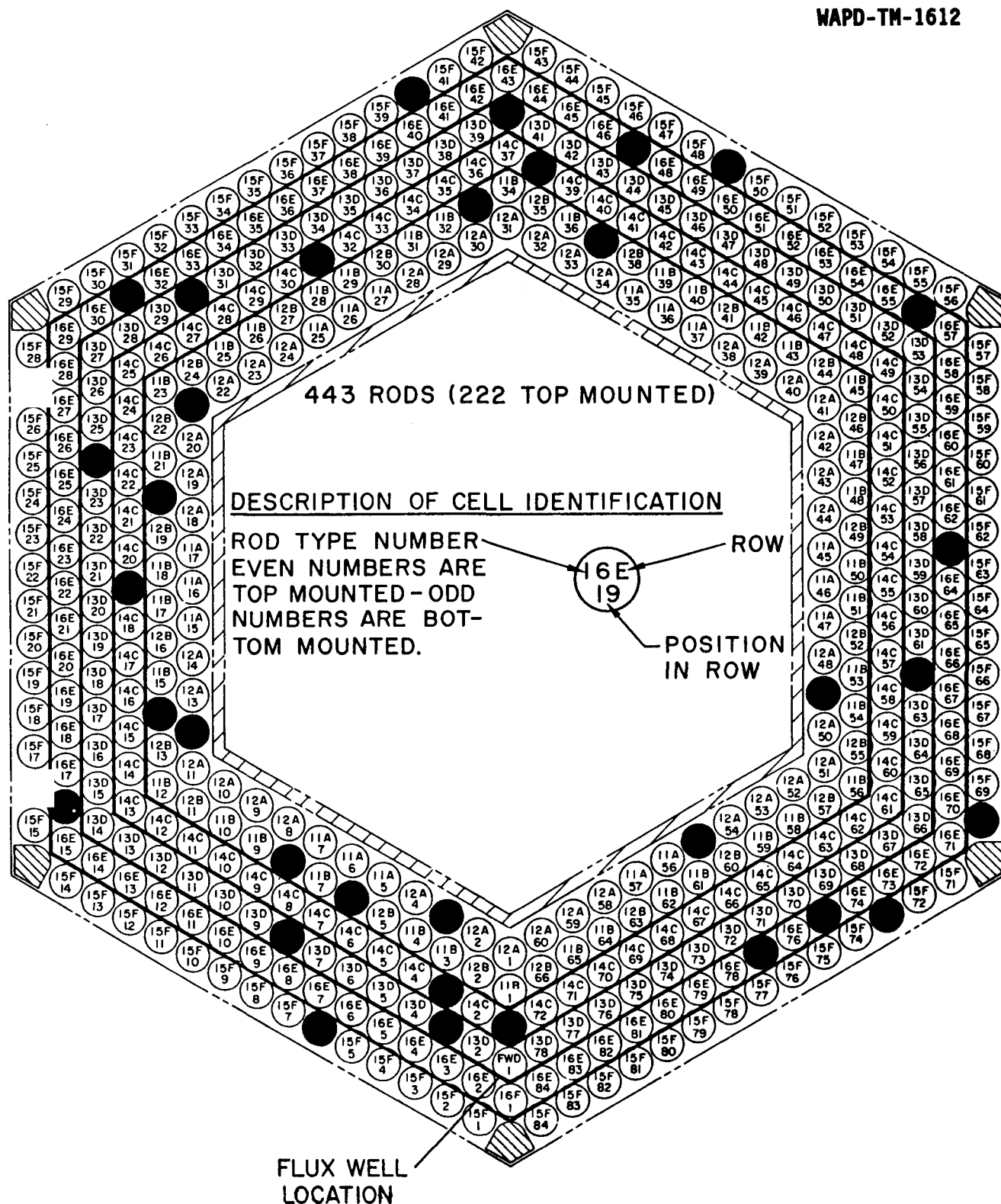


Figure V-6. Location of Proof-of-Breeding Rods in Blanket Module I-3

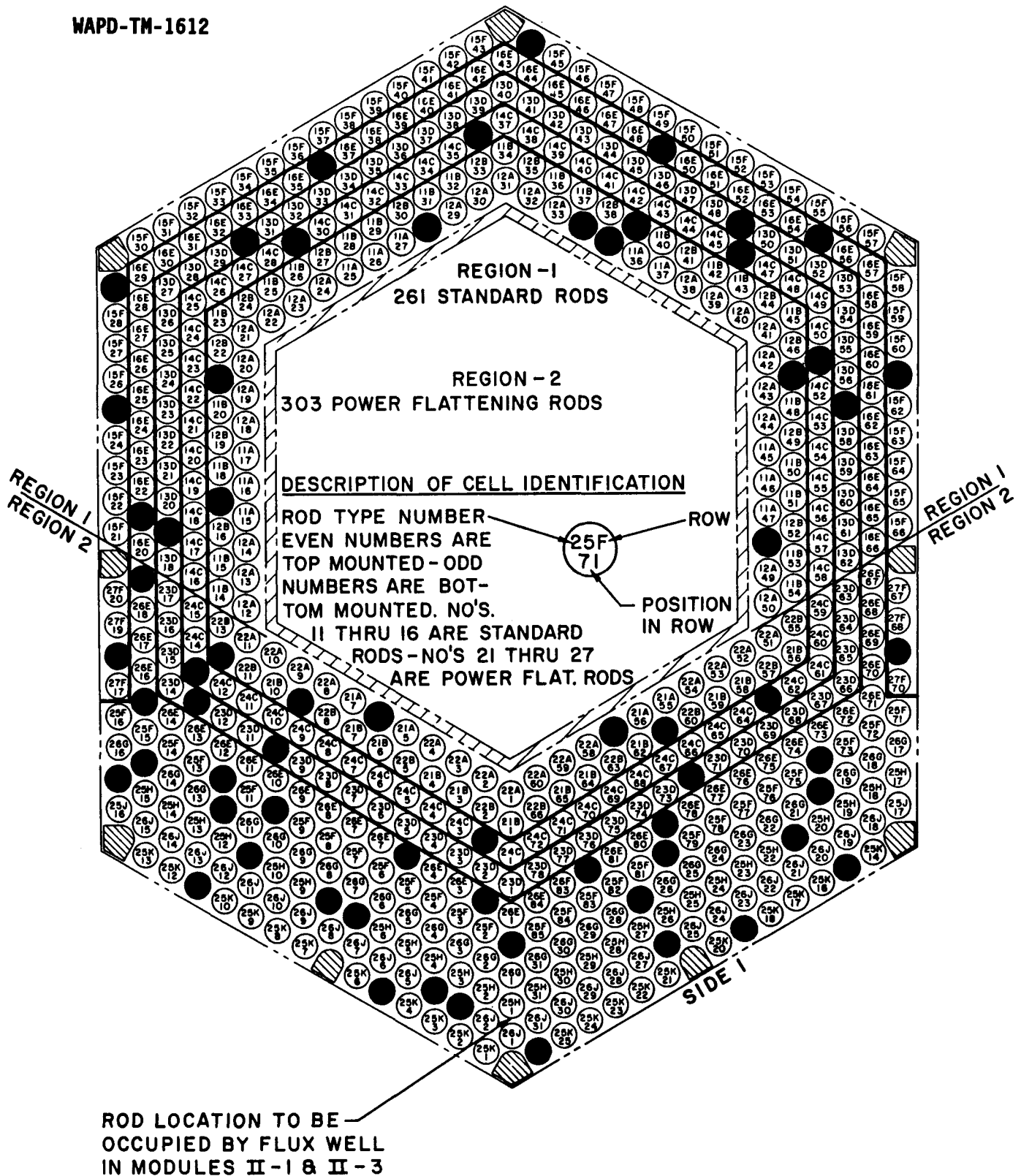


Figure V-7. Location of Proof-of-Breeding Rods in Blanket Module II-2

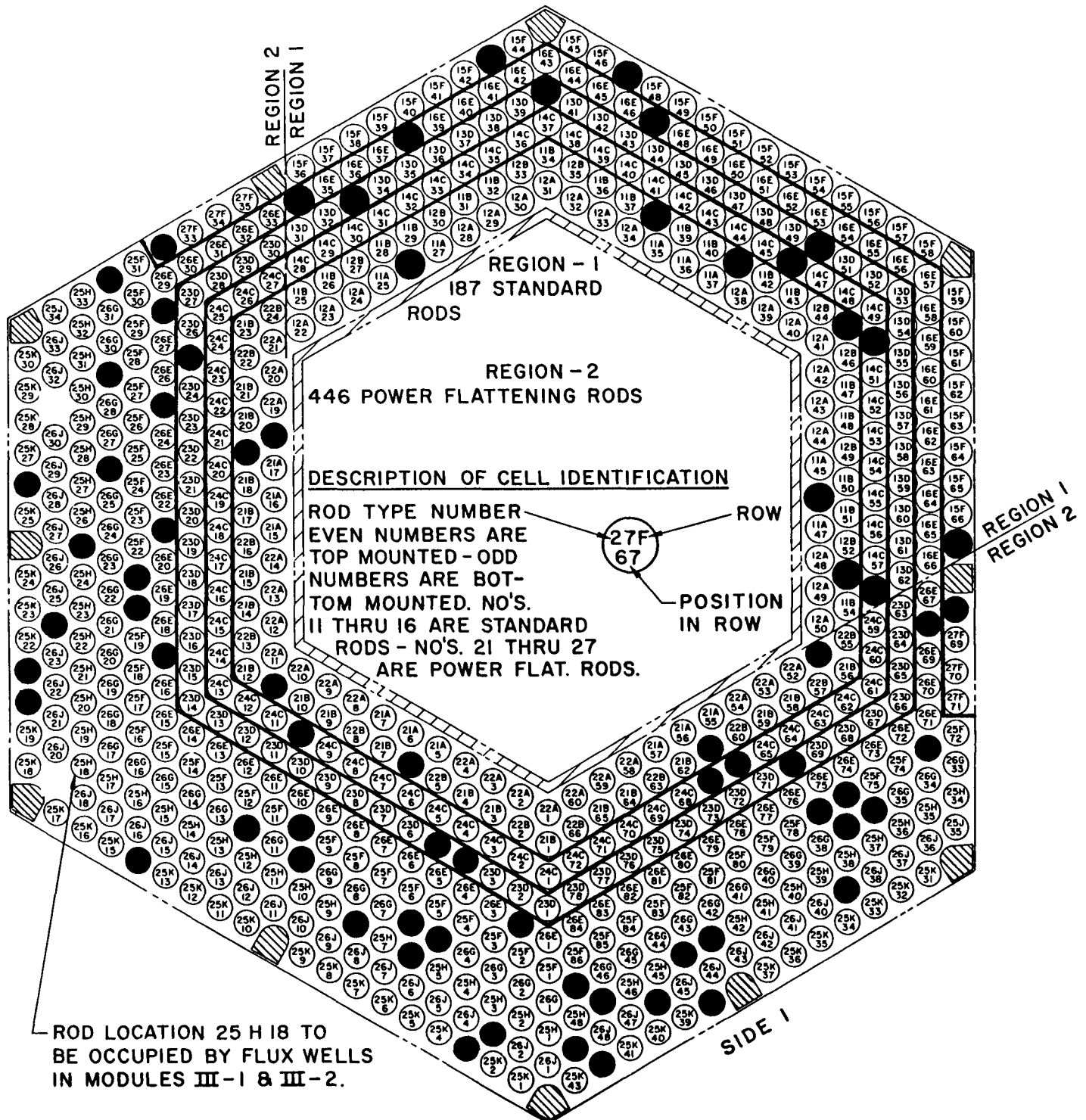


Figure V-8. Location of Proof-of-Breeding Rods in Blanket Module III-2

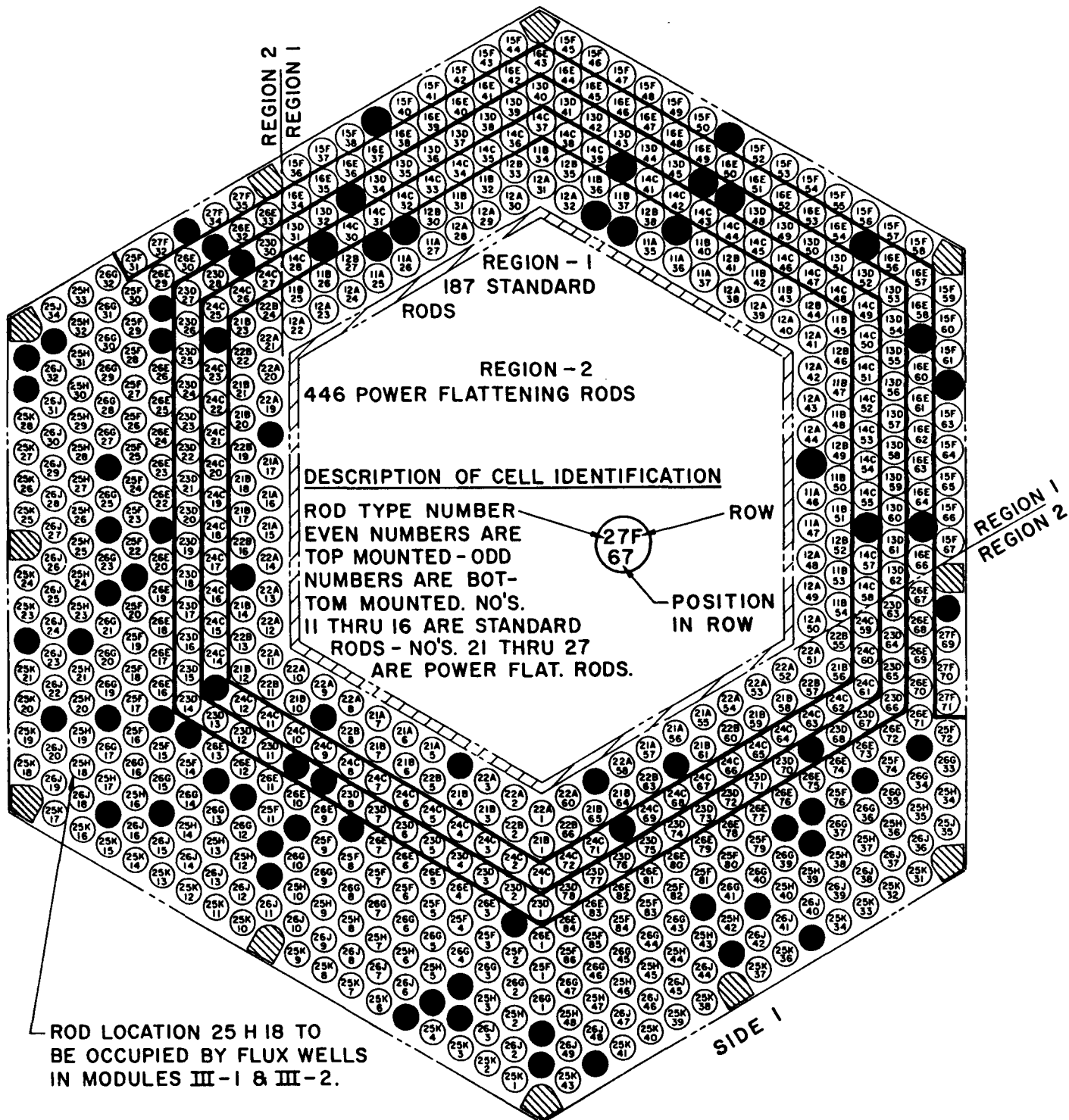
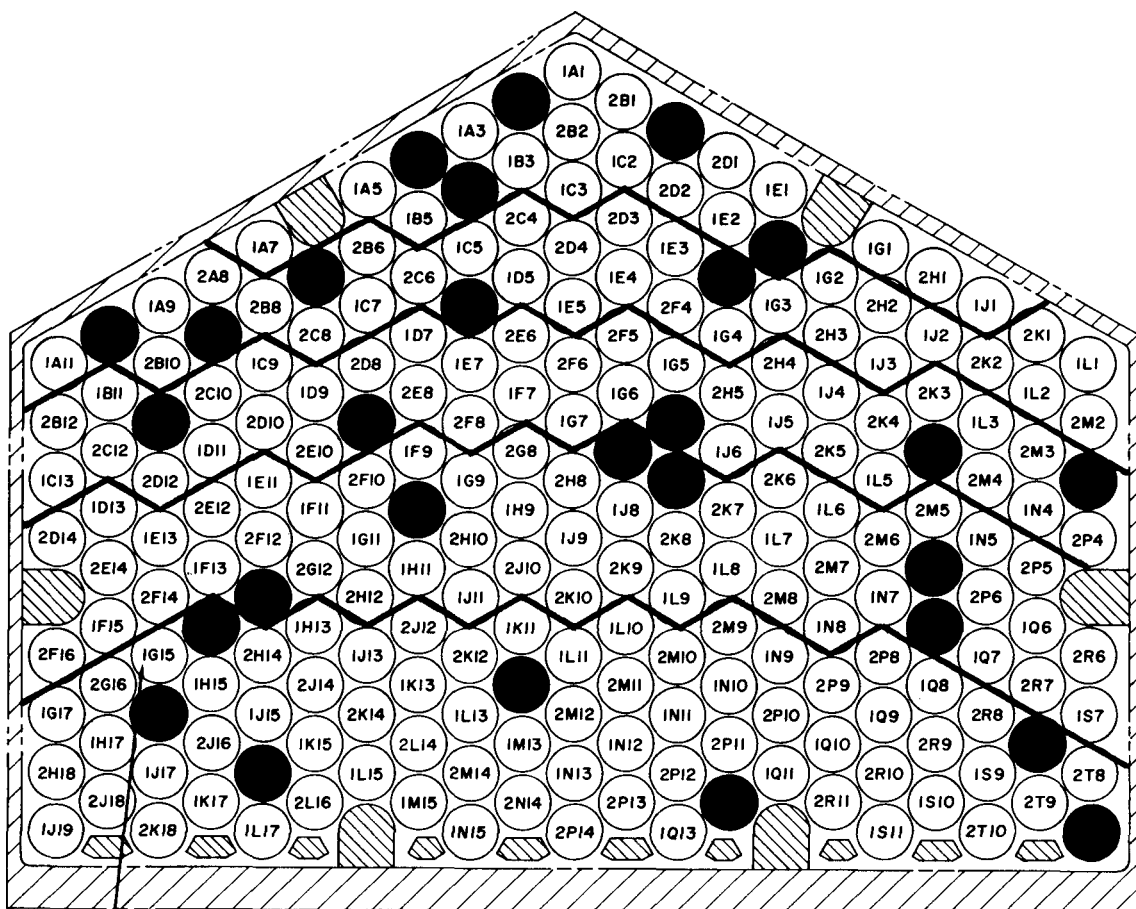


Figure V-9. Location of Proof-of-Breeding Rods in Blanket Module III-6



ROD LOCATION 1G15 TO BE OCCUPIED
BY FLUX WELL IN MODULE IV-7

IDENTIFICATION LEGEND
EXAMPLE:

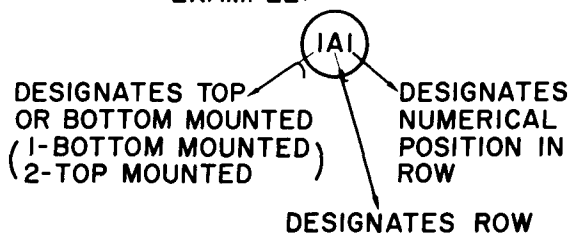
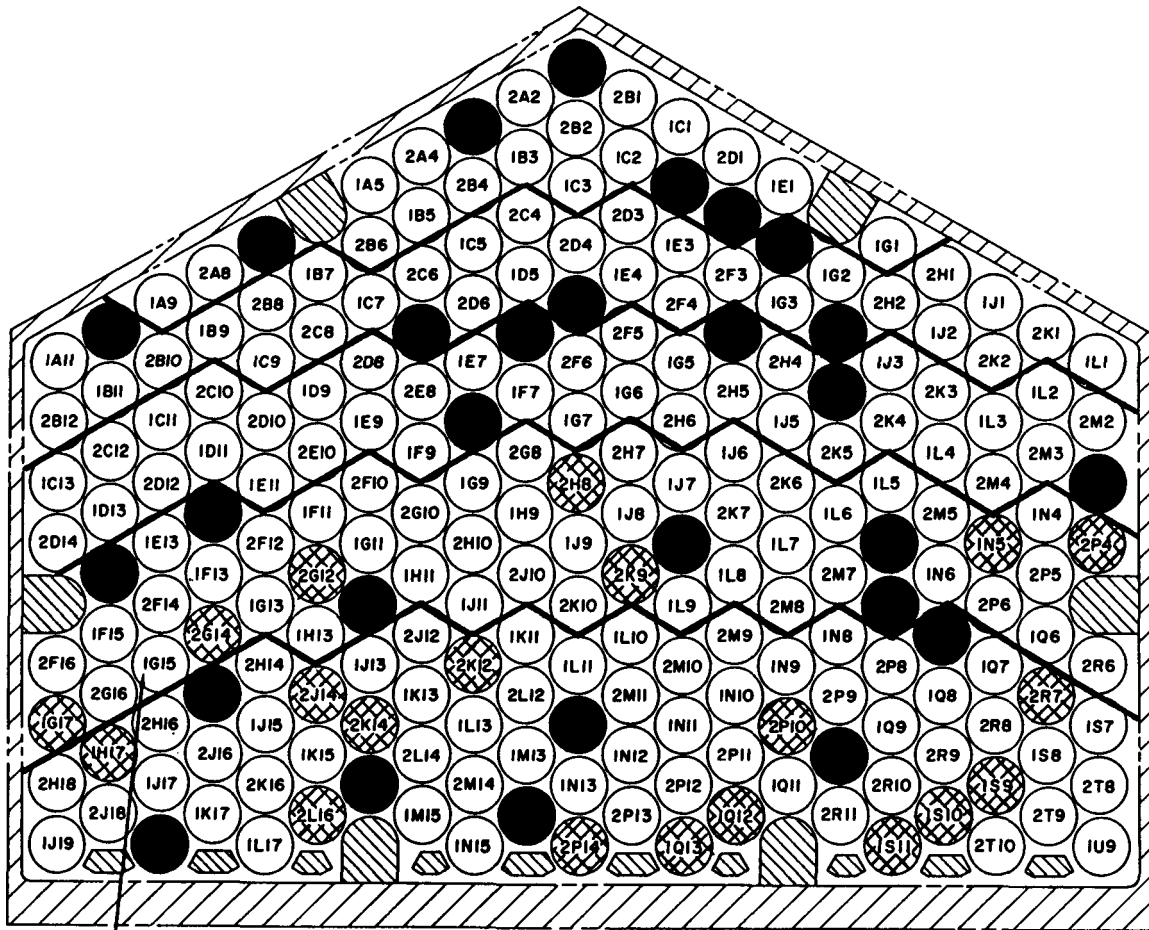


Figure V-10. Location of Proof-of-Breeding Rods in
Reflector Module IV-4 (A Type IVA Module)



ROD LOCATION 1G15 TO BE OCCUPIED
BY FLUX WELL IN MODULE IV-7

IDENTIFICATION LEGEND
EXAMPLE:

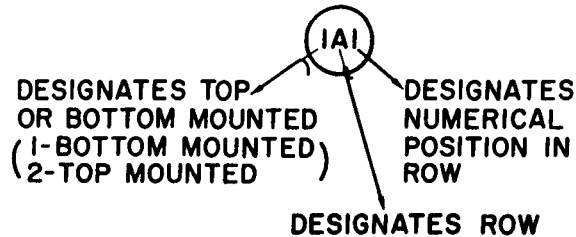
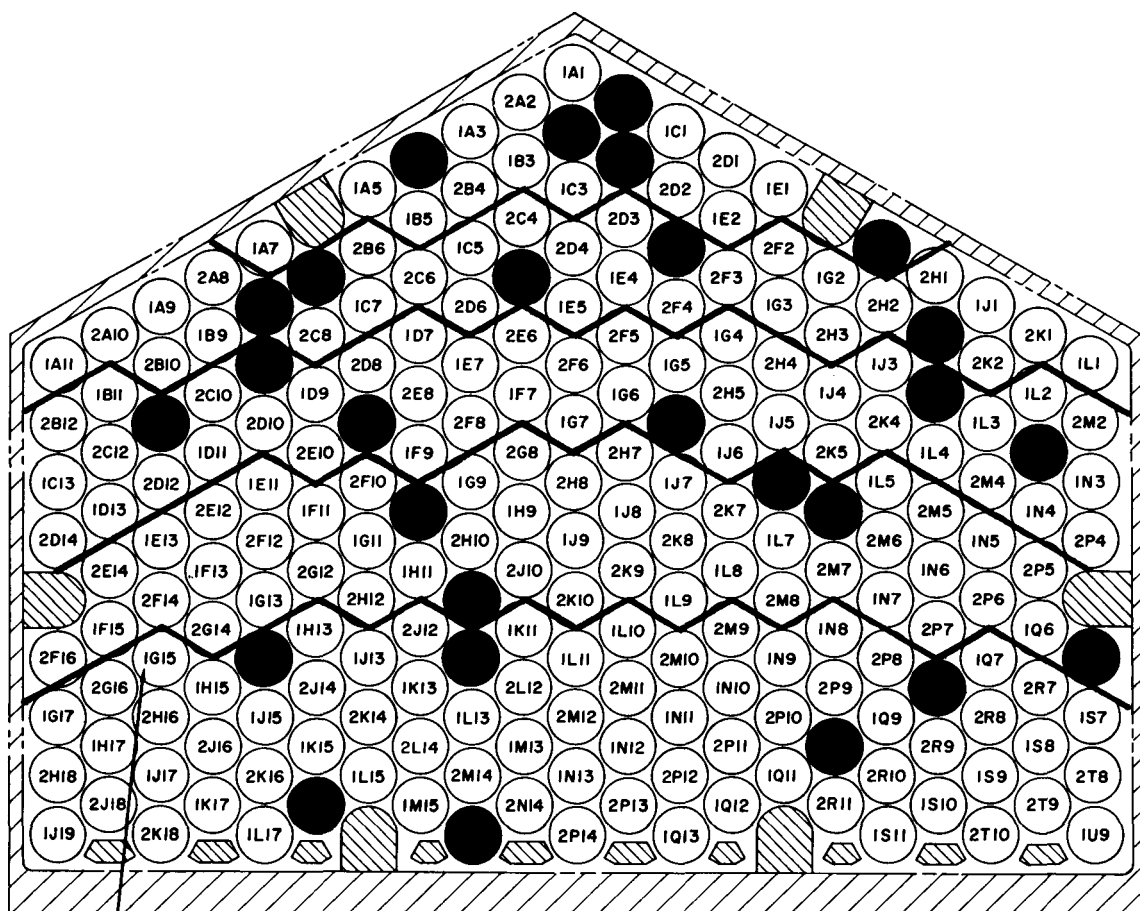


Figure V-11. Location of Proof-of-Breeding Rods in Reflector Module IV-9 (A Type IVA Module). Twenty Additional Rods Not Part of Original POB Sample Are Indicated by Cross Hatching.



ROD LOCATION 1G15 TO BE OCCUPIED
BY FLUX WELL IN MODULE IV-7

IDENTIFICATION LEGEND EXAMPLE:

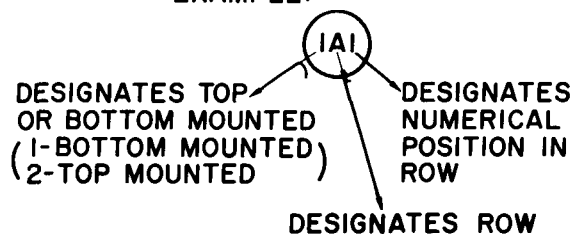
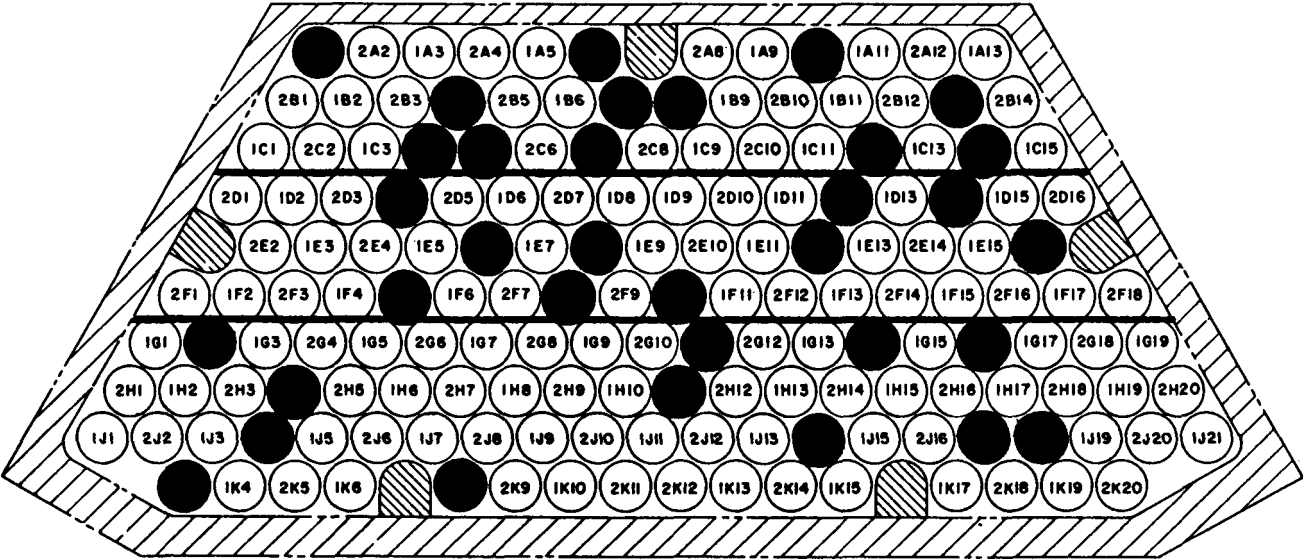


Figure V-12. Location of Proof-of-Breeding Rods in
Reflector Module IV-3 (A Type IVB Module)



IDENTIFICATION LEGEND
EXAMPLE:

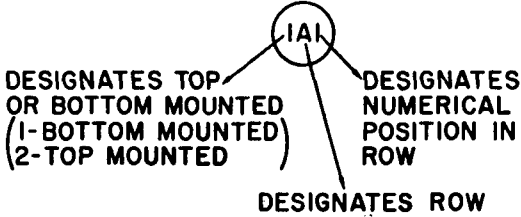


Figure V-13. Location of Proof-of-Breeding Rods in Refelctor Module V-4

VI. NONDESTRUCTIVE AND DESTRUCTIVE ASSAY RESULTS

A. Nondestructive Assay Method

The nondestructive assay of the POB fuel rods has been described in detail in Reference 5. A brief overview of the method is presented in the following paragraphs.

The nondestructive assay device, called the Production Irradiated Fuel Assay Gauge (PIFAG), operated by drawing the rod being assayed past a set of ^{252}Cf neutron sources, which induced fissions in the fissile fuel. Delayed neutrons subsequently emitted by the fissioning fuel atoms were detected by a set of boron-lined proportional counters. The number of delayed neutrons detected was a measure of the amount of fuel in the rod. Because the absorption of the irradiation neutrons in the fuel was sufficiently great to cause self-shielding, the response of the PIFAG was nonlinear; that is, the number of delayed neutrons detected was not directly proportional to the amount of fuel in the rod. Therefore, it was necessary to calibrate the response of the gauge by assaying calibration rods having different fissile linear densities (grams of fuel per inch of rod length). Four sets of calibration rods were used, one for each of the four kinds of rods in the core (seed, standard blanket, power flattening blanket, and reflector). The calibration sets included unirradiated core rods that had been set aside for this purpose during core manufacture and also specially fabricated rods having different linear densities of fuel than were present in rods manufactured for the core, as was described in Section III.

The materials between the ^{252}Cf sources and the fuel rod were chosen to give the irradiation neutrons the desired energy spectrum. Two different energy spectra were used, one predominately thermal and the other predominately epithermal. The thermal irradiation spectrum yielded a much higher delayed neutron count and therefore better statistical precision than the epithermal spectrum. However, the thermal spectrum was also more highly

self-shielded and hence gave rise to a more nonlinear response, thereby requiring larger corrections to be made for the self-shielding. In general, the corrections that were made to the delayed neutron data were significantly different for the two spectra. Complete calibrations were performed using both spectra, and all core rods were assayed with both. This provided two different, and nearly independent, measurements of the amount of fuel in each rod. The consistency between the two measurements was an indication of the adequacy of performance of the gauge. In the following sections both the thermal and epithermal results will be presented for all rods, and separate estimates of the total fissile content of each module and of the whole core will be made based on the thermal and epithermal results.

The loading (fissile fuel content) of an expended core fuel rod was defined to be the sum of the ^{233}U and ^{235}U content plus 0.61 times the amount of ^{232}U present. Part of the ^{232}U was included in the loading because this isotope has a significant fission cross section for thermal and epithermal neutrons; the factor of 0.61 is the ratio of fission to absorption in ^{232}U averaged over the whole LWBR core at several times in life. Inclusion of this isotope had a very small effect on the rod loadings, since the total amount of ^{232}U in the expended core was less than 0.1% of the inventory of ^{233}U , as deduced from PDQ calculations (Section IV) and from destructive isotopic assay of expended LWBR core rods at ANL (Reference 6). (Note that ^{232}U was not counted in determining the initial loadings of the core rods because the isotopic abundance of ^{232}U in the material from which the rods were fabricated was negligibly small — less than 8 parts per million on average.) ^{235}U , on the other hand, was a significant part of the rod loadings since the final ^{235}U inventory of the expended core was about 1% of the ^{233}U inventory. The relative response of the PIFAG to the different fissile nuclides — specifically, the ratio of the number of delayed neutron counts per gram of ^{232}U or ^{235}U to the counts per gram of ^{233}U — was calculated as indicated in Appendix B of Reference 20. The number of PIFAG delayed neutron counts for each rod was adjusted to account for the amount of ^{232}U and ^{235}U present and for the difference in response before the rod loading was determined.

No credit was taken in the rod loadings for the trace amount of fissile ^{239}Pu and ^{241}Pu in the expended core. These isotopes were created from neutron captures in the small amount of ^{238}U (less than 0.5% of the total uranium) in the material from which the fuel was manufactured, and together they constituted a few hundredths of one percent of the total fissile inventory of the expended core. Since there is no guarantee that plutonium would be recovered during fuel reprocessing, it was decided not to include any plutonium in determining the final loadings of the expended LWBR fuel rods. However, a correction was made to the PIFAG delayed neutron data to subtract off the small fraction of delayed neutron counts that came from ^{239}Pu and ^{241}Pu .

Several other corrections were applied to the PIFAG data. The reader is referred to Reference 5 for details.

Great care was taken to maintain the long-term stability of the PIFAG. A calibration rod having a large amount of fuel (and therefore providing good counting statistics) was assayed immediately before or immediately after each core rod. The cumulative data from this rod were analyzed for evidence of changes in PIFAG behavior with time. The stability of the gauge and the accuracy of calibration were sufficiently good to permit the loading of most seed and blanket rods to be determined with a precision between $\pm 0.2\%$ and $\pm 0.5\%$ (one standard deviation). The assay precision was not as good for reflector rods, especially for the epithermal assays, because of the relatively poor counting statistics due to the small amounts of fuel in many of the reflector rods.

The nondestructive assay results for all 524 POB rods are presented in the following sections, arranged by rod type.

B. Nondestructive Assay Results, Seed Rods

The nondestructive assay results for all 134 POB seed rods are presented by module in Tables VI-1 through VI-4. The rods are arranged by stratum, identified by binary pellet stack length (in inches) and fissile concentration (high zone or low zone). The second column in the tables gives the rod cell, which is identified by a letter giving the rod row followed by a number giving the position of the rod in the row. This information can be used to locate the rods in Figures V-2 through V-5. The third column gives the predicted final loading of the rod, calculated as described previously in Section IV.C. The next columns give the measured PIFAG thermal and epithermal loadings with their uncertainties (standard deviations). The last three columns give the percentage differences between the thermal measurement and calculation, between the epithermal measurement and calculation, and between the epithermal and thermal measurements, respectively. The last line of each table gives the total calculated loading, PIFAG thermal loading, and PIFAG epithermal loading for all the POB rods in the module, along with the percentage differences among these totals.

For the majority of the seed rods the measured loadings (both thermal and epithermal) agree with calculation to better than one percent. This provides confirmation of the validity of the LWBR calculational model. The differences between measurement and calculation show a definite pattern. The measured loadings tend to be slightly less than predicted for nearly all rods except those from the central part of the module, rod rows B through E, and those from the outermost rod row, which is row Q. Rods in row Q were adjacent to the water channel between the seed and blanket modules. Nearly all of these rods had at least 1% more fuel than predicted. (Blanket rods located next to water channels also had significantly more fuel than predicted, as will be shown in the following sections.)

The agreement between the thermal and epithermal measurements is quite good, within 1% for all but two of the 134 seed rods. Nevertheless, there is

a slight bias between the two types of measurement in that the epithermal values tend to be higher than the thermal values. The average difference for the seed rods is 0.27%.

Nearly all seed rods had less fuel at end of life than at beginning of life. (Recall that at beginning of life all rods in a stratum had nearly equal loadings. These initial loadings are tabulated in Table V-2 for seed and blanket rods.) The decrease was greatest for the 84 inch high zone rods, which had the higher concentration of fissile fuel initially. The 42 inch low zone rods, which were built with a long stack of pure thoria pellets in which fissile fuel could be created, generally had only a slight decrease in loading during core life. Among the different module types, the Type I seed rods, which had the highest depletion, experienced the largest decrease in loading; whereas the Type III seeds, which had the lowest depletion among the seed modules, had the smallest decrease in loading.

C. Nondestructive Assay Results, Standard Blanket Rods

The nondestructive assay results for all 96 standard blanket rods are presented by module in Table VI-5 through VI-8 in the same manner as for seed rods. The agreement between measurement and calculation is again good, although not quite as good as for seed rods in that a majority of the standard blanket rods have greater than 1% difference between measurement and calculation. As before, the PIFAG values tend to be slightly smaller than calculation for nearly all rods except those from the innermost and outermost rod rows, in this case rows A and F. Both of these rod rows were adjacent to water channels bounding the annular shape of the blanket modules.

The agreement between the thermal and epithermal PIFAG results is again quite good; for only one standard blanket rod do the measurements disagree by more than 1%. There is, nevertheless, again a small bias between thermal and epithermal results with the epithermal values being larger by 0.13% on the average.

The standard blanket rods, unlike the seed rods, tended to have more fuel at end of life than initially. This is most strikingly true for the 42 inch low zone rods, for which the fissile content more than doubled during lifetime. Only the 84 inch high zone standard blanket rods experienced a net loss of fuel during core life. The difference between seed and standard blanket rods in this regard is a consequence of the higher initial fissile concentration in the seed rods and the more closely spaced rod lattice for the standard blanket fuel.

D. Nondestructive Assay Results, Power Flattening Blanket Rods

The nondestructive assay results for all 155 power flattening blanket rods are presented by module in Tables VI-9 through VI-11. (Note that only three of the POB modules contained power flattening blanket rods, while four modules contained standard blanket rods.) As before, the agreement between measurement and calculation is good, within 1% in a substantial majority of cases. Again, the measured values tend to be slightly lower than calculation except for rods that were adjacent to water channels, for which the measured values are substantially greater than calculation. The power flattening blanket rods adjacent to water channels are those from rows A and K and the 84 inch medium zone rods from row F. The latter rods may be located in the module II-2 diagram in Figure V-7 on either side of the module immediately below the support post at the boundary between regions 1 and 2. There are eight 84 inch medium zone power flattening blanket rods in the module, four on each side. Similarly, in the module III-2 and III-6 diagrams shown in Figures V-8 and V-9, respectively, the 84 inch medium zone power flattening blanket rods are the four rods along each of the two sides of the module extending away from the interface between region 1 (the standard blanket region) and region 2 (the power flattening blanket region).

Among the power flattening blanket rods there are no cases where the epithermal and thermal measurements disagree by as much as 1%. Again,

however, the epithermal results tend to be higher than the thermal, with the average difference being 0.17%.

E. Nondestructive Assay Results, Reflector Rods

The nondestructive assay results for all 139 reflector rods are presented by module in Tables VI-12 through VI-15. Note that the rod strata are identified by the range of final loadings predicted by the PDQ code. It may be observed that the measured loadings are generally less than predicted for the rods with the highest loadings, but the measured loadings become progressively greater than predicted as the loading decreases, with the largest percentage differences occurring for the most lightly loaded rods. This is illustrated in Figure VI-1, which is a plot of PIFAG thermal loading versus prediction for all POB rods from reflector module IV-9. The straight line through the origin is the line on which all the data would lie if the PIFAG measurements agreed exactly with predictions. The other line is a least-squares fit to the data. The fit is quite good. (An even better fit is obtained if the data are corrected for the "water channel effect" as will be discussed in Section VII.B.) The PIFAG epithermal data for this module produce a very similar plot, as shown in Figure VI-2. Similar results were obtained for the other reflector POB modules.

It was noted above that the linear regression method of evaluating the total loading of all rods in a stratum from the nondestructive assay results for the POB rods assumes that the relationship between measurement and prediction can be satisfactorily represented by a straight line. Figures V-1 and V-2 demonstrate that this assumption is fulfilled for the reflector POB rods.

The precision of the PIFAG measurements was considerably poorer for reflector rods than for seed or blanket rods. This is a consequence of two factors. First, most reflector rods had substantially less fissile fuel than the seed or blanket rods, so the number of PIFAG delayed neutron counts was

smaller and hence the relative precision was not as good. Second, reflector rods contained much more ^{232}Th than seed or blanket rods. The contribution to the delayed neutron count from ^{232}Th (caused primarily by fast neutrons from the ^{252}Cf sources reaching the fuel rod with sufficient energy — greater than about 1.2 MeV — to cause fission in ^{232}Th) was correspondingly greater for reflector rods. This was especially true for the epithermal assay. For the reflector rods having the smallest amount of fissile fuel, the counts from ^{232}Th amounted to nearly 90% of the total delayed neutron counts in the epithermal system.

F. Destructive Assay Results

The destructive assay of 17 POB rods at Argonne National Laboratory (ANL) has been documented in Reference 6. A comparison of the destructive assay results with the PIFAG thermal and epithermal data for the same rods is presented in Table VI-16. (It should be noted that the quoted PIFAG results are based on the use of calibration rod data and calculational information only, and do not make use of the ANL data in any way.) The agreement is excellent. In most cases the measurements agree within 0.25%, and the maximum discrepancy between the destructive and nondestructive assay results (either thermal or epithermal) is only slightly greater than 0.5%. The PIFAG thermal values are higher than the ANL values in about half the cases and lower in about half. However, the PIFAG epithermal values are predominately higher than the ANL values, as indicated by the fact that 15 of the 17 numbers in the last column in the table are negative. This suggests that there may be a significant, albeit small, bias between the two. A statistical t-test indicates that this is indeed the case, as shown in Table VI-17. The test is applied to seed, standard blanket, power flattening blanket, and reflector rods separately and also to the whole sample of rods. The test indicates no significant difference between the ANL values and the PIFAG thermal values at the 5% significance level for any individual type of rod or for the sample as a whole. Between ANL and PIFAG epithermal values, however, there is a significant difference both for the standard blanket rods and for the whole

sample. The average difference for all rods is small, under 0.16%. This average difference is the best estimate of the overall bias between the two types of measurement.

Since the known measurement uncertainties are smaller for the destructive assay than for the nondestructive assay, and since the PIFAG thermal and ANL results are in agreement, it is appropriate to ascribe the source of the bias to error in the PIFAG epithermal measurements rather than in the destructive assay measurements. The size of the error is quite small, however, and does not compromise the use of the epithermal assay results to demonstrate breeding in LWBR. The effect of the bias on the measurement of fissile inventory ratio using the PIFAG epithermal data will be discussed in Section VIII.

TABLE VI-1. NONDESTRUCTIVE ASSAY RESULTS FOR PROOF-OF-BREEDING
RODS FROM SEED MODULE I-1

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
84" High Zone	C8	25.549	25.682±0.075	25.712±0.095	+0.52	+0.64	+0.11
	C10*	25.570	25.598±0.071	25.665±0.045	+0.11	+0.37	+0.26
	F1	25.478	25.481±0.072	25.372±0.065	+0.01	-0.42	-0.43
	F11	25.331	25.224±0.073	25.245±0.070	-0.42	-0.34	+0.08
	F21	25.331	25.202±0.075	25.336±0.070	-0.51	+0.02	+0.53
	F23	25.444	25.133±0.074	25.182±0.069	-1.22	-1.03	+0.19
	F29	25.502	25.353±0.074	25.317±0.064	-0.58	-0.73	-0.14
	G24	25.262	25.153±0.080	25.296±0.070	-0.43	+0.13	+0.57
	H2	25.218	24.741±0.073	24.787±0.068	-1.89	-1.71	+0.18
	H8	25.170	24.830±0.074	24.827±0.072	-1.35	-1.36	-0.01
	H10	25.189	24.837±0.072	24.813±0.069	-1.40	-1.49	-0.10
	J17	24.704	24.240±0.075	24.291±0.071	-1.88	-1.67	+0.21
	J30	24.763	24.476±0.075	24.494±0.070	-1.16	-1.09	+0.07
	J46	24.955	24.802±0.072	24.871±0.065	-0.61	-0.34	+0.28
	K7	24.496	24.069±0.080	24.141±0.072	-1.74	-1.45	+0.30
	K15	24.419	24.115±0.075	24.245±0.079	-1.25	-0.71	+0.54
	K28	23.876	23.389±0.077	23.387±0.065	-2.04	-2.05	-0.01
	K36	24.127	23.720±0.077	23.825±0.065	-1.69	-1.25	+0.44
	K39	24.281	23.883±0.087	23.902±0.069	-1.64	-1.56	+0.08
	K42	24.465	24.098±0.076	24.120±0.082	-1.50	-1.41	+0.09
	L8	23.551	23.426±0.078	23.431±0.072	-0.53	-0.51	+0.02
	L23	23.201	23.042±0.083	23.106±0.070	-0.68	-0.41	+0.28
	L29*	23.185	22.949±0.078	22.964±0.050	-1.02	-0.95	+0.07
	L57	23.648	23.510±0.078	23.668±0.080	-0.59	+0.08	+0.67
70" Low Zone	M22	17.374	17.282±0.066	17.346±0.069	-0.53	-0.16	+0.37
	M49*	17.608	17.449±0.065	17.523±0.043	-0.90	-0.48	+0.42
56" Low Zone	N42	15.624	15.498±0.061	15.556±0.069	-0.81	-0.44	+0.38
	N63*	15.638	15.635±0.057	15.684±0.039	-0.02	+0.30	+0.32
42" Low Zone	P19	14.252	14.166±0.049	14.225±0.059	-0.61	-0.19	+0.42
	P39*	14.143	13.910±0.050	13.993±0.037	-1.65	-1.06	+0.60
	P69	14.238	14.219±0.048	14.231±0.062	-0.13	-0.05	+0.09
	Q8	13.751	14.012±0.055	13.995±0.054	+1.90	+1.78	-0.12
	Q51	13.714	13.969±0.055	13.927±0.054	+1.86	+1.56	-0.30
Totals		729.057	723.093	724.477	-0.82	-0.63	+0.19

* These rods were also assayed destructively.

TABLE VI-2. NONDESTRUCTIVE ASSAY RESULTS FOR PROOF-OF-BREEDING
RODS FROM SEED MODULE II-3

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
84" High Zone	C1	26.401	26.377±0.071	26.468±0.103	-0.09	+0.25	+0.34
	C12	26.466	26.544±0.061	26.584±0.079	+0.29	+0.45	+0.15
	D4	26.291	26.357±0.069	26.514±0.087	+0.25	+0.85	+0.59
	D5	26.313	26.488±0.068	26.687±0.099	+0.67	+1.42	+0.75
	D8	26.417	26.381±0.067	26.557±0.086	-0.14	+0.53	+0.67
	D13	26.686	26.779±0.075	26.880±0.086	+0.35	+0.73	+0.38
	D18	26.417	26.545±0.068	26.555±0.087	+0.48	+0.52	+0.04
	F6	26.064	25.999±0.075	26.182±0.084	-0.25	+0.45	+0.70
	F10	26.176	26.159±0.069	26.182±0.091	-0.06	+0.03	+0.09
	F16	26.450	26.406±0.077	26.393±0.088	-0.17	-0.21	-0.05
	F23	26.646	26.371±0.064	26.283±0.082	-1.03	-1.36	-0.33
	F26	26.450	26.420±0.076	26.464±0.090	-0.11	+0.06	+0.17
	G24	26.620	26.428±0.059	26.549±0.084	-0.72	-0.27	+0.46
	H12	25.875	25.744±0.077	25.728±0.121	-0.51	-0.57	-0.06
	J12	25.458	25.120±0.070	25.277±0.111	-1.33	-0.71	+0.62
	J32	26.212	25.919±0.060	26.092±0.084	-1.12	-0.46	+0.67
	K2	24.896	24.651±0.070	24.680±0.085	-0.98	-0.87	+0.12
	K20	24.938	24.535±0.069	24.502±0.096	-1.62	-1.75	-0.13
	K25	25.402	25.092±0.071	25.226±0.084	-1.22	-0.69	+0.54
	K42	25.836	25.597±0.064	25.804±0.084	-0.93	-0.12	+0.81
	K44	25.646	25.528±0.062	25.566±0.084	-0.46	-0.31	+0.15
	L4	24.038	23.616±0.076	23.599±0.085	-1.75	-1.82	-0.07
	L26	24.477	24.279±0.071	24.419±0.102	-0.81	-0.24	+0.58
	L39	24.905	24.693±0.065	24.744±0.083	-0.85	-0.65	+0.20
	L55	24.535	24.321±0.065	24.345±0.084	-0.87	-0.77	+0.10
70" Low Zone	M18	17.934	17.829±0.070	17.860±0.071	-0.58	-0.41	+0.18
	M45	18.070	17.980±0.052	18.057±0.067	-0.49	-0.07	+0.43
56" Low Zone	N17	15.801	15.670±0.070	15.773±0.080	-0.83	-0.18	+0.66
	N71	15.836	15.734±0.057	15.748±0.078	-0.64	-0.55	+0.09
42" Low Zone	P28	14.209	14.065±0.046	14.181±0.067	-1.01	-0.20	+0.83
	P52	14.086	14.056±0.045	14.174±0.078	-0.21	+0.63	+0.84
	P71	14.308	14.208±0.044	14.267±0.105	-0.70	-0.29	+0.41
	Q9	13.789	14.023±0.051	14.099±0.069	+1.70	+2.25	+0.54
	Q50	13.511	13.762±0.042	13.805±0.062	+1.85	+2.17	+0.31
Totals		783.153	779.675	782.244	-0.44	-0.12	+0.33

TABLE VI-3. NONDESTRUCTIVE ASSAY RESULTS FOR PROOF-OF-BREEDING
RODS FROM SEED MODULE III-1

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
84" High Zone	B2	27.210	27.188±0.076	27.182±0.083	-0.08	-0.10	-0.02
	C7	27.266	27.191±0.072	27.298±0.077	-0.28	+0.12	+0.40
	D13	27.462	27.617±0.063	27.667±0.071	+0.57	+0.75	+0.18
	E4	27.027	26.741±0.059	26.776±0.067	-1.06	-0.93	+0.13
	E7	26.960	26.978±0.061	27.026±0.067	+0.07	+0.25	+0.18
	E13	27.189	27.241±0.074	27.382±0.067	+0.19	+0.71	+0.52
	E19	27.509	27.444±0.058	27.603±0.068	-0.23	+0.34	+0.58
	F1	27.149	27.024±0.055	26.982±0.067	-0.46	-0.62	-0.16
	F9	26.854	26.729±0.057	26.714±0.067	-0.46	-0.52	-0.06
	F11	26.770	26.672±0.058	26.615±0.110	-0.37	-0.58	-0.21
	F21	27.463	27.316±0.057	27.487±0.067	-0.53	+0.09	+0.63
	G7	26.597	26.329±0.070	26.314±0.070	-1.01	-1.06	-0.06
	G27	27.491	27.309±0.063	27.542±0.066	-0.66	+0.19	+0.85
	G35	27.251	27.157±0.056	27.302±0.066	-0.34	+0.19	+0.54
	J3	26.522	26.270±0.060	26.288±0.067	-0.95	-0.88	+0.07
	J6	26.355	26.275±0.068	26.325±0.068	-0.30	-0.11	+0.19
	J15	26.087	25.703±0.061	25.825±0.078	-1.47	-1.01	+0.47
	J39	27.205	26.928±0.058	27.020±0.066	-1.02	-0.68	+0.34
	K9	25.546	25.321±0.064	25.360±0.063	-0.88	-0.73	+0.15
	K27	25.892	25.451±0.065	25.666±0.103	-1.70	-0.87	+0.85
	L27	25.080	24.827±0.064	24.860±0.067	-1.01	-0.88	+0.13
	L30	25.042	24.681±0.068	24.784±0.079	-1.44	-1.03	+0.41
	L32	25.066	25.104±0.062	25.078±0.067	+0.15	+0.05	-0.10
	L45	26.139	26.276±0.057	26.391±0.081	+0.52	+0.96	+0.44
	L54	25.889	25.975±0.059	26.012±0.061	+0.33	+0.48	+0.14
70" Low Zone	M15	18.178	18.124±0.056	18.102±0.060	-0.30	-0.42	-0.12
	M55	18.792	18.763±0.047	18.774±0.057	-0.16	-0.10	+0.06
56" Low Zone	N2	16.046	15.917±0.049	15.983±0.054	-0.80	-0.39	+0.42
	N26	15.864	15.778±0.050	15.785±0.055	-0.54	-0.50	+0.04
42" Low Zone	P36	14.353	14.196±0.037	14.224±0.039	-1.09	-0.90	+0.20
	P62	14.277	14.276±0.031	14.316±0.052	-0.01	+0.27	+0.28
	P72	14.193	14.208±0.039	14.076±0.050	+0.10	-0.83	-0.93
	Q2	13.724	13.835±0.041	13.982±0.051	+0.80	+1.87	+1.06
	Q38	13.590	13.765±0.039	13.681±0.052	+1.29	+0.67	-0.62
Totals		804.035	800.608	802.420	-0.43	-0.20	+0.23

TABLE VI-4. NONDESTRUCTIVE ASSAY RESULTS FOR PROOF-OF-BREEDING
RODS FROM SEED MODULE III-2

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
84" High Zone	C4	27.132	27.282±0.057	27.455±0.066	+0.55	+1.19	+0.63
	D14	27.486	27.507±0.063	27.579±0.066	+0.08	+0.34	+0.26
	E1	27.228	27.287±0.063	27.462±0.061	+0.22	+0.86	+0.64
	E3	27.093	27.086±0.060	27.065±0.057	-0.03	-0.10	-0.08
	F1	27.149	27.055±0.086	27.146±0.067	-0.35	-0.01	+0.34
	F11	26.770	26.585±0.075	26.785±0.067	-0.69	+0.06	+0.75
	F21	27.463	27.347±0.055	27.430±0.065	-0.42	-0.12	+0.30
	F25	27.531	27.465±0.054	27.576±0.083	-0.24	+0.16	+0.40
	G4	26.881	26.706±0.062	26.820±0.057	-0.65	-0.23	+0.43
	G9	26.712	26.747±0.062	26.881±0.061	+0.13	+0.63	+0.50
	G17	26.920	26.832±0.095	27.093±0.061	-0.33	+0.64	+0.97
	H21	26.770	26.632±0.057	26.666±0.061	-0.52	-0.39	+0.13
	H22	26.843	26.526±0.057	26.671±0.076	-1.18	-0.64	+0.54
	H28	27.331	27.157±0.055	27.244±0.065	-0.64	-0.32	+0.32
	H39	27.307	27.225±0.058	27.423±0.069	-0.30	+0.42	+0.73
	J26	26.594	26.484±0.061	26.552±0.053	-0.41	-0.16	+0.25
	J45	27.028	26.925±0.055	26.960±0.066	-0.38	-0.25	+0.13
	K6	25.962	25.860±0.061	25.774±0.072	-0.39	-0.73	-0.34
	K15	25.681	25.312±0.062	25.410±0.066	-1.44	-1.06	+0.39
	K35	26.649	26.257±0.056	26.464±0.089	-1.47	-0.70	+0.79
	K45	26.745	26.671±0.052	26.735±0.064	-0.27	-0.04	+0.24
	L3	25.159	25.012±0.067	25.121±0.060	-0.58	-0.15	+0.43
	L23	24.681	24.567±0.073	24.543±0.067	-0.46	-0.56	-0.10
	L26	25.018	24.768±0.065	24.758±0.078	-1.00	-1.04	-0.04
	L52	25.763	25.727±0.075	25.775±0.064	-0.14	+0.04	+0.19
70" Low Zone	M21	18.123	18.064±0.056	18.072±0.049	-0.33	-0.28	+0.04
	M40	18.758	18.835±0.050	18.915±0.053	+0.41	+0.83	+0.42
56" Low Zone	N16	16.008	15.842±0.051	15.879±0.054	-1.04	-0.80	+0.23
	N59	16.289	16.359±0.041	16.442±0.076	+0.43	+0.94	+0.50
42" Low Zone	P10	14.305	14.220±0.042	14.379±0.051	-0.59	+0.52	+1.12
	P67	14.132	14.255±0.032	14.199±0.049	+0.87	+0.48	-0.39
	Q22	13.841	14.087±0.046	14.081±0.051	+1.78	+1.73	-0.04
	Q47	13.680	14.000±0.040	14.018±0.045	+2.34	+2.47	+0.13
Totals		791.030	788.685	791.370	-0.30	+0.04	+0.34

TABLE VI-5. NONDESTRUCTIVE ASSAY RESULTS FOR STANDARD BLANKET
PROOF-OF-BREEDING RODS FROM BLANKET MODULE I-3

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
42" Low Zone	A3	35.761	36.219±0.091	36.221±0.110	+1.28	+1.29	+0.00
	A12	35.446	36.058±0.090	35.997±0.103	+1.73	+1.56	-0.17
	A21	35.631	35.899±0.088	35.847±0.104	+0.75	+0.61	-0.14
	A49*	36.430	36.884±0.097	36.796±0.081	+1.25	+1.00	-0.24
	A55	36.272	36.848±0.098	36.829±0.112	+1.59	+1.54	-0.05
	B6	35.452	35.122±0.081	35.387±0.101	-0.93	-0.18	+0.76
	B8	35.367	34.852±0.078	34.910±0.086	-1.46	-1.29	+0.16
	B14	35.147	34.967±0.082	34.787±0.092	-0.51	-1.02	-0.51
	B20	35.417	34.980±0.081	35.239±0.083	-1.23	-0.50	+0.74
	B33	35.895	35.341±0.083	35.469±0.094	-1.54	-1.19	+0.36
	B37	36.251	35.875±0.087	35.874±0.096	-1.04	-1.04	-0.00
56" Med. Zone	C1	41.020	40.877±0.091	40.902±0.128	-0.35	-0.29	+0.06
	C3 *	41.001	40.711±0.091	40.756±0.072	-0.71	-0.60	+0.11
	C19	40.978	40.043±0.099	40.404±0.133	-2.28	-1.40	+0.90
	C31	41.371	40.477±0.105	40.538±0.122	-2.16	-2.01	+0.15
	C38	41.614	41.183±0.106	41.345±0.109	-1.04	-0.65	+0.39
70" High Zone	D3	47.756	47.179±0.121	47.226±0.136	-1.21	-1.11	+0.10
	D8	47.683	47.107±0.118	47.177±0.140	-1.21	-1.06	+0.15
	D24*	47.732	47.355±0.128	47.384±0.113	-0.79	-0.73	+0.06
	D30	47.857	47.397±0.128	47.430±0.132	-0.96	-0.89	+0.07
	D40	48.079	47.463±0.121	47.648±0.142	-1.28	-0.90	+0.39
	D62	48.148	47.749±0.149	47.709±0.166	-0.83	-0.91	-0.08
84" High Zone	E16	52.969	52.386±0.123	52.576±0.148	-1.10	-0.74	+0.36
	E31	52.956	52.526±0.178	52.704±0.148	-0.81	-0.48	+0.34
	E47	53.138	52.512±0.152	52.636±0.159	-1.18	-0.94	+0.24
	E56*	52.860	52.514±0.154	52.471±0.146	-0.65	-0.74	-0.08
	E63	53.128	52.542±0.138	52.705±0.158	-1.10	-0.80	+0.31
	E75	53.066	52.576±0.136	52.812±0.150	-0.92	-0.48	+0.45
	E77	53.037	52.579±0.133	52.602±0.180	-0.86	-0.82	+0.04

* These rods were also assayed destructively.

TABLE VI-5 (Continued)

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
84" Med. Zone	F6	47.646	48.379±0.121	48.446±0.123	+1.54	+1.68	+0.14
	F16	47.609	47.928±0.117	47.957±0.118	+0.67	+0.73	+0.06
	F27	47.566	48.188±0.116	48.155±0.142	+1.31	+1.24	-0.07
	F40	47.880	48.422±0.142	48.437±0.126	+1.13	+1.16	+0.03
	F49	48.070	48.682±0.133	48.779±0.144	+1.27	+1.48	+0.20
	F70	47.890	48.491±0.129	48.279±0.127	+1.26	+0.81	-0.44
	F73	47.880	48.463±0.152	48.439±0.126	+1.22	+1.17	-0.05
Totals		1592.001	1586.775	1588.869	-0.33	-0.20	+0.13

TABLE VI-6. NONDESTRUCTIVE ASSAY RESULTS FOR STANDARD BLANKET
PROOF-OF-BREEDING RODS FROM BLANKET MODULE II-2

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ---- Thermal - Calc. Epith. - Calc. Epith. - Ther.		
42" Low Zone	A28	35.577	36.098±0.089	35.880±0.094	+1.46	+0.85	-0.60
	A34	35.577	35.977±0.091	35.804±0.086	+1.13	+0.64	-0.48
	A35	35.548	35.854±0.095	35.784±0.108	+0.86	+0.67	-0.19
	A48	34.072	34.293±0.076	34.260±0.100	+0.65	+0.55	-0.10
	B17	33.995	33.513±0.070	33.608±0.116	-1.42	-1.14	+0.28
	B21	34.438	34.134±0.078	33.948±0.109	-0.88	-1.42	-0.54
	B39	35.321	34.882±0.081	34.959±0.121	-1.24	-1.02	+0.22
	B47	34.438	33.962±0.075	33.954±0.109	-1.38	-1.41	-0.02
56" Med. Zone	C29	40.897	39.917±0.099	39.980±0.120	-2.40	-2.24	+0.16
	C36	41.199	40.829±0.105	40.960±0.159	-0.90	-0.58	+0.32
	C46	40.835	40.012±0.098	40.032±0.145	-2.01	-1.97	+0.05
	C51	40.444	39.462±0.092	39.319±0.117	-2.43	-2.78	-0.36
70" High Zone	D19	46.887	46.165±0.110	46.457±0.179	-1.54	-0.92	+0.63
	D30	47.718	46.937±0.121	47.344±0.160	-1.64	-0.78	+0.87
	D49	47.729	47.133±0.140	47.683±0.192	-1.25	-0.09	+1.17
	D57	47.383	46.736±0.108	46.968±0.163	-1.37	-0.88	+0.50
84" High Zone	E21	52.488	51.750±0.112	52.121±0.136	-1.41	-0.70	+0.72
	E36	52.998	52.624±0.133	52.883±0.157	-0.70	-0.22	+0.49
	E49	53.009	52.492±0.132	52.784±0.182	-0.98	-0.42	+0.56
	E55	52.928	52.628±0.131	52.831±0.166	-0.57	-0.18	+0.39
84" Med. Zone	F25	47.421	47.766±0.112	47.806±0.112	+0.73	+0.81	+0.09
	F29	47.387	48.090±0.122	47.943±0.140	+1.48	+1.17	-0.31
	F44	47.676	48.454±0.133	48.376±0.176	+1.63	+1.47	-0.16
	F61	47.464	47.729±0.116	47.829±0.135	+0.56	+0.77	+0.21
Totals		1033.427	1027.434	1029.514	-0.58	-0.38	+0.20

TABLE VI-7. NONDESTRUCTIVE ASSAY RESULTS FOR STANDARD BLANKET
PROOF-OF-BREEDING RODS FROM BLANKET MODULE III-2

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
42" Low Zone	A26	33.419	33.860±0.076	33.778±0.103	+1.32	+1.07	-0.24
	A46	33.747	34.105±0.076	34.072±0.104	+1.06	+0.96	-0.10
	B38	34.174	33.727±0.070	33.774±0.103	-1.31	-1.17	+0.14
	B41	34.246	33.824±0.113	33.868±0.103	-1.23	-1.11	+0.13
	B45	33.951	33.741±0.090	33.527±0.101	-0.62	-1.25	-0.63
	B53	33.109	32.532±0.075	32.497±0.100	-1.74	-1.85	-0.11
56" Med. Zone	C46	40.319	39.894±0.091	39.849±0.112	-1.05	-1.17	-0.11
	C50	40.239	39.566±0.091	39.716±0.103	-1.67	-1.30	+0.38
	C58	39.194	38.283±0.078	38.356±0.107	-2.32	-2.14	+0.19
70" High Zone	D33	46.824	46.101±0.109	46.110±0.125	-1.55	-1.52	+0.02
	D40	47.410	46.852±0.110	46.775±0.132	-1.18	-1.34	-0.16
	D50	47.473	47.082±0.115	47.246±0.163	-0.83	-0.48	+0.35
84" High Zone	E34	52.085	51.499±0.110	51.608±0.133	-1.12	-0.92	+0.21
	E38	52.787	52.264±0.116	52.389±0.138	-0.99	-0.75	+0.24
	E47	52.926	52.743±0.125	52.512±0.142	-0.35	-0.78	-0.44
84" Med. Zone	F43	47.355	47.829±0.125	47.660±0.172	+1.00	+0.64	-0.35
	F47	47.552	48.186±0.117	48.227±0.148	+1.33	+1.42	+0.08
	F67	46.988	47.058±0.106	47.190±0.122	+0.15	+0.43	+0.28
Totals		763.799	759.145	759.154	-0.61	-0.61	+0.00

TABLE VI-8. NONDESTRUCTIVE ASSAY RESULTS FOR STANDARD BLANKET
PROOF-OF-BREEDING RODS FROM BLANKET MODULE III-6

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
42" Low Zone	A33	34.362	34.555±0.084	34.721±0.108	+0.56	+1.05	+0.48
	A34	34.410	34.673±0.086	34.612±0.119	+0.76	+0.58	-0.18
	A45	33.591	34.117±0.079	34.134±0.127	+1.57	+1.62	+0.05
	B28	33.426	32.896±0.070	32.999±0.092	-1.59	-1.28	+0.31
	B29	33.576	33.108±0.076	32.994±0.094	-1.39	-1.73	-0.34
	B39	34.252	33.843±0.077	33.829±0.104	-1.19	-1.24	-0.04
56" Med. Zone	C29	39.399	38.748±0.085	38.943±0.110	-1.65	-1.16	+0.50
	C40	40.319	39.888±0.092	39.845±0.130	-1.07	-1.18	-0.11
	C56	39.255	38.704±0.082	38.891±0.129	-1.40	-0.93	+0.48
70" High Zone	D33	46.985	46.296±0.108	46.276±0.129	-1.47	-1.51	-0.04
	D46	47.434	46.917±0.118	47.065±0.169	-1.09	-0.78	+0.32
	D47	47.429	46.880±0.118	46.912±0.172	-1.16	-1.09	+0.07
84" High Zone	E55	52.903	52.461±0.127	52.625±0.147	-0.84	-0.53	+0.31
	E59	52.861	52.333±0.135	52.640±0.178	-1.00	-0.42	+0.59
	E65	52.445	51.791±0.113	52.031±0.138	-1.25	-0.79	+0.46
84" Med. Zone	F39	47.344	47.751±0.109	47.770±0.117	+0.86	+0.90	+0.04
	F51	47.580	48.214±0.120	48.080±0.159	+1.33	+1.05	-0.28
	F62	47.306	47.736±0.111	47.815±0.125	+0.91	+1.07	+0.17
Totals		764.876	760.911	762.181	-0.52	-0.35	+0.17

TABLE VI-9. NONDESTRUCTIVE ASSAY RESULTS FOR POWER FLATTENING
BLANKET PROOF-OF-BREEDING RODS FROM BLANKET MODULE II-2

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
42" Low Zone	A6 #	30.598	31.138±0.086	31.054±0.131	+1.76	+1.49	-0.27
	A57	30.481	31.004±0.085	30.808±0.150	+1.72	+1.07	-0.63
	B9	30.675	30.632±0.086	30.606±0.096	-0.14	-0.22	-0.08
	B12	30.868	30.799±0.085	30.723±0.106	-0.22	-0.47	-0.25
	B61	30.502	30.416±0.079	30.415±0.087	-0.28	-0.29	-0.01
56" Med. Zone	C2	34.685	34.446±0.076	34.588±0.104	-0.69	-0.28	+0.41
	C13	35.466	35.286±0.109	35.228±0.101	-0.51	-0.67	-0.16
	C63	35.290	34.955±0.094	34.785±0.094	-0.95	-1.43	-0.49
70" High Zone	D10	45.995	45.815±0.129	45.864±0.147	-0.39	-0.29	+0.11
	D13	45.947	46.012±0.116	45.904±0.117	+0.14	-0.09	-0.23
	D72	46.063	45.919±0.110	46.031±0.128	-0.31	-0.07	+0.24
84" High Zone	E2	55.478	54.709±0.111	54.921±0.118	-1.39	-1.00	+0.39
	E5	54.650	54.060±0.115	54.309±0.128	-1.08	-0.62	+0.46
	E15	52.653	52.424±0.127	52.433±0.132	-0.43	-0.42	+0.02
	E19#	52.402	51.687±0.135	51.714±0.128	-1.36	-1.31	+0.05
	E79	54.327	53.790±0.112	53.901±0.162	-0.99	-0.78	+0.21
	E82	54.857	53.904±0.107	53.962±0.118	-1.74	-1.63	+0.11
	F1	56.566	56.303±0.111	56.602±0.122	-0.46	+0.06	+0.53
	F10	54.924	54.553±0.121	54.547±0.121	-0.68	-0.69	-0.01
	F12	54.634	54.104±0.120	54.521±0.129	-0.97	-0.21	+0.77
	F74	54.335	53.928±0.124	54.027±0.137	-0.75	-0.57	+0.18
	F80	55.298	54.772±0.111	54.851±0.126	-0.95	-0.81	+0.14
	G12	54.936	54.475±0.113	54.791±0.113	-0.84	-0.26	+0.58
	G15	53.301	53.451±0.123	53.416±0.103	+0.28	+0.21	-0.07
	G20	54.685	54.400±0.120	54.706±0.122	-0.52	+0.04	+0.56
	G27	55.869	55.501±0.118	55.697±0.122	-0.66	-0.31	+0.35
	H7	55.619	55.335±0.138	55.206±0.133	-0.51	-0.74	-0.23
	H8	55.407	54.896±0.110	55.166±0.141	-0.92	-0.43	+0.49
	H11	54.987	54.750±0.121	54.721±0.111	-0.43	-0.48	-0.05
	H16	52.645	52.554±0.144	52.529±0.121	-0.17	-0.22	-0.05
	H21	54.825	54.576±0.114	54.554±0.129	-0.45	-0.50	-0.04

Rods selected for core examination study, not part of POB sample.

TABLE VI-9 (Continued)

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
84" High Zone (Cont.)	J3	55.714	55.858±0.113	56.096±0.116	+0.26	+0.69	+0.42
	J4	55.656	55.354±0.106	55.359±0.123	-0.54	-0.54	+0.01
	J26	54.931	54.594±0.107	54.851±0.175	-0.61	-0.15	+0.47
	K5	53.221	53.976±0.105	54.223±0.115	+1.42	+1.88	+0.46
	K11	51.606	52.891±0.120	52.853±0.145	+2.49	+2.42	-0.07
	K15	51.316	52.225±0.125	52.152±0.118	+1.77	+1.63	-0.14
	K19	51.846	52.704±0.105	52.600±0.143	+1.65	+1.45	-0.20
	K26	53.453	54.818±0.102	54.822±0.116	+2.55	+2.56	+0.01
84" Med.	F18	42.261	42.750±0.121	42.695±0.144	+1.16	+1.03	-0.13
	F69	42.261	42.474±0.120	42.392±0.116	+0.50	+0.31	-0.19
Totals		2001.230	1998.237	2000.622	-0.15	-0.03	+0.12

TABLE VI-10. NONDESTRUCTIVE ASSAY RESULTS FOR POWER FLATTENING
BLANKET PROOF-OF-BREEDING RODS FROM BLANKET MODULE III-2

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
42" Low Zone	A18	29.863	30.067±0.080	29.982±0.094	+0.68	+0.40	-0.28
	A51	30.476	30.548±0.079	30.503±0.117	+0.24	+0.09	-0.15
	B6	28.611	28.529±0.059	28.627±0.095	-0.29	+0.06	+0.34
	B11	28.260	28.105±0.059	28.079±0.085	-0.55	-0.64	-0.09
	B19	29.671	29.381±0.069	29.419±0.090	-0.98	-0.85	+0.13
	B61	29.711	29.609±0.070	29.338±0.090	-0.34	-1.25	-0.92
56" Med. Zone	C10	33.987	33.718±0.072	33.824±0.145	-0.79	-0.48	+0.32
	C66	34.682	34.312±0.078	34.432±0.111	-1.07	-0.72	+0.35
	C67	34.581	34.402±0.075	34.488±0.123	-0.52	-0.27	+0.25
70" High Zone	D4	47.056	46.680±0.092	46.821±0.110	-0.80	-0.50	+0.30
	D5	46.989	46.507±0.088	46.857±0.142	-1.03	-0.28	+0.75
	D25	46.173	45.950±0.105	46.003±0.118	-0.48	-0.37	+0.12
	D70	46.162	45.692±0.104	45.736±0.105	-1.02	-0.92	+0.10
84" High Zone	E2	56.446	55.998±0.097	56.247±0.114	-0.79	-0.35	+0.44
	E17	56.083	55.581±0.127	55.758±0.118	-0.90	-0.58	+0.32
	E21	55.058	54.587±0.107	54.585±0.150	-0.86	-0.86	-0.00
	E25	54.464	53.799±0.111	53.862±0.118	-1.22	-1.10	+0.12
	E28	53.660	53.403±0.117	53.741±0.143	-0.48	+0.15	+0.63
	E68	52.165	51.301±0.162	51.570±0.125	-1.66	-1.14	+0.53
	F10	56.789	56.244±0.100	56.122±0.142	-0.96	-1.17	-0.22
	F20	56.400	55.952±0.106	56.040±0.172	-0.79	-0.64	+0.16
	F21	56.182	55.740±0.101	56.166±0.116	-0.79	-0.03	+0.76
	F73	53.094	53.149±0.121	53.258±0.113	+0.10	+0.31	+0.20
	F76	54.874	54.400±0.130	54.738±0.131	-0.86	-0.25	+0.62
	F77	55.085	54.651±0.111	54.895±0.187	-0.79	-0.34	+0.45
	G5	57.311	56.980±0.126	57.207±0.157	-0.58	-0.18	+0.40
	G6	57.229	57.018±0.100	57.018±0.113	-0.37	-0.37	-0.00
	G10	57.164	56.721±0.103	57.033±0.114	-0.78	-0.23	+0.55
	G12	57.290	56.947±0.100	57.369±0.120	-0.60	+0.14	+0.74
	G26	55.813	55.324±0.106	55.636±0.108	-0.88	-0.32	+0.56
	G29	55.149	54.431±0.110	54.882±0.118	-1.30	-0.48	+0.83
	G32	52.894	52.721±0.112	52.865±0.088	-0.33	-0.05	+0.27

TABLE IV-10 (Continued)

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
84" High Zone (Cont.)	G36	54.872	54.487±0.113	54.784±0.118	-0.70	-0.16	+0.54
	G37	55.179	54.942±0.110	55.158±0.110	-0.43	-0.04	+0.39
	G47	57.395	57.100±0.134	57.408±0.129	-0.51	+0.02	+0.54
	H6	57.287	56.768±0.098	57.185±0.113	-0.91	-0.18	+0.73
	H8	57.132	57.030±0.096	57.153±0.114	-0.18	+0.04	+0.22
	H25	56.078	55.888±0.099	55.743±0.108	-0.34	-0.60	-0.26
	H43	56.153	55.733±0.101	56.028±0.126	-0.75	-0.22	+0.53
	H44	56.443	56.010±0.100	56.290±0.114	-0.77	-0.27	+0.50
	H47	57.168	57.001±0.093	56.995±0.113	-0.29	-0.30	-0.01
	J3	56.943	57.091±0.098	57.314±0.126	+0.26	+0.65	+0.39
	J19	56.750	56.345±0.109	56.311±0.111	-0.71	-0.77	-0.06
	J24	56.053	56.077±0.119	56.214±0.104	+0.04	+0.29	+0.25
	J31	54.354	54.302±0.106	54.530±0.132	-0.10	+0.32	+0.42
	J39	54.282	54.418±0.103	54.699±0.116	+0.25	+0.77	+0.52
	J46	56.197	55.818±0.101	56.172±0.104	-0.68	-0.05	+0.63
	J49	56.515	57.039±0.094	57.217±0.113	+0.93	+1.24	+0.31
	K3	55.356	56.423±0.092	56.449±0.192	+1.93	+1.97	+0.04
	K14	55.478	56.277±0.097	56.379±0.112	+1.44	+1.62	+0.18
	K20	54.906	55.722±0.095	55.710±0.096	+1.49	+1.46	-0.02
	K21	54.599	55.428±0.119	55.488±0.112	+1.52	+1.63	+0.11
	K26	52.535	53.904±0.115	53.670±0.106	+2.61	+2.16	-0.43
	K38	53.567	54.478±0.156	54.417±0.112	+1.70	+1.59	-0.11
	K42	54.782	55.902±0.098	55.869±0.111	+2.04	+1.98	-0.06
84" Med. Zone	F32	42.423	42.939±0.110	43.143±0.112	+1.22	+1.70	+0.47
	F68	42.539	43.035±0.138	42.845±0.117	+1.16	+0.72	-0.44
Totals		2884.355	2878.603	2886.272	-0.20	+0.07	+0.27

TABLE VI-11. NONDESTRUCTIVE ASSAY RESULTS FOR POWER FLATTENING
BLANKET PROOF-OF-BREEDING RODS FROM BLANKET MODULE III-6

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
42" Low Zone	A4	28.804	29.110±0.063	29.154±0.088	+1.06	+1.21	+0.15
	A18	30.059	30.389±0.087	30.264±0.088	+1.10	+0.68	-0.41
	A59	29.029	29.275±0.071	29.282±0.090	+0.85	+0.87	+0.02
	B9	28.548	28.354±0.062	28.304±0.087	-0.68	-0.85	-0.18
	B15	29.083	28.964±0.092	28.820±0.094	-0.41	-0.90	-0.50
	B62*	29.357	29.367±0.056	29.318±0.049	+0.04	-0.13	-0.17
56" Med. Zone	C13*	34.042	33.914±0.054	34.018±0.053	-0.38	-0.07	+0.31
	C24	35.132	34.791±0.082	34.891±0.096	-0.97	-0.69	+0.29
	C70	34.185	33.952±0.078	34.003±0.093	-0.68	-0.53	+0.15
70" High Zone	D9	46.945	46.515±0.086	46.712±0.102	-0.92	-0.50	+0.42
	D10	46.989	46.629±0.093	46.415±0.108	-0.77	-1.22	-0.46
	D29*	45.753	45.364±0.104	45.350±0.077	-0.85	-0.88	-0.03
	D69	46.210	45.734±0.106	45.975±0.114	-1.03	-0.51	+0.53
84" High Zone	E2	56.526	56.216±0.128	56.189±0.120	-0.55	-0.60	-0.05
	E8	55.844	55.488±0.118	55.482±0.119	-0.64	-0.65	-0.01
	E14	56.446	55.995±0.108	55.896±0.187	-0.80	-0.97	-0.18
	E15	56.375	56.536±0.109	56.587±0.121	+0.29	+0.38	+0.09
	E21	54.906	54.296±0.111	54.418±0.123	-1.11	-0.89	+0.23
	E27	53.821	53.214±0.124	53.525±0.125	-1.13	-0.55	+0.58
	E28	53.368	53.165±0.115	53.424±0.112	-0.38	+0.10	+0.49
	E31*	52.120	51.575±0.115	51.624±0.082	-1.04	-0.95	+0.09
	F10	56.753	56.424±0.108	56.650±0.112	-0.58	-0.18	+0.40
	F12	56.877	56.710±0.120	56.610±0.151	-0.29	-0.47	-0.18
	F13	56.976	56.679±0.108	56.744±0.146	-0.52	-0.41	+0.11
	F21	56.043	55.472±0.119	55.650±0.120	-1.02	-0.70	+0.32
	F73*	53.436	53.670±0.101	53.752±0.076	+0.44	+0.59	+0.15
	F75	54.788	54.631±0.115	54.650±0.113	-0.29	-0.25	+0.04
	F77	55.305	54.755±0.129	54.876±0.121	-0.99	-0.78	+0.22
	F78	55.477	54.944±0.110	55.122±0.122	-0.96	-0.64	+0.32
	G11	57.165	56.311±0.137	56.632±0.163	-1.49	-0.93	+0.57
	G18	57.395	56.691±0.105	56.722±0.153	-1.23	-1.17	+0.06
	G22	56.446	55.919±0.136	56.187±0.187	-0.93	-0.46	+0.48

* These rods were also assayed destructively.

TABLE VI-11 (Continued)

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
84" High Zone (Cont.)	G24	55.987	55.776±0.150	55.753±0.122	-0.38	-0.42	-0.04
	G26	55.603	55.066±0.112	55.312±0.128	-0.97	-0.52	+0.45
	G38	55.651	55.213±0.113	55.442±0.124	-0.79	-0.38	+0.41
	G42	56.363	56.010±0.109	56.179±0.113	-0.63	-0.33	+0.30
	H1 *	57.460	57.295±0.073	57.440±0.070	-0.29	-0.03	+0.25
	H4	57.500	57.413±0.104	57.527±0.149	-0.15	+0.05	+0.20
	H11	57.132	56.808±0.104	57.021±0.119	-0.57	-0.19	+0.38
	H15	57.458	57.485±0.125	57.519±0.113	+0.05	+0.11	+0.06
	H22	56.716	56.389±0.119	56.655±0.125	-0.58	-0.11	+0.47
	H41	55.847	55.376±0.108	55.429±0.120	-0.84	-0.75	+0.10
	J1	56.750	56.680±0.104	57.041±0.110	-0.12	+0.51	+0.64
	J4	57.057	56.988±0.142	57.173±0.121	-0.12	+0.20	+0.32
	J5	57.052	57.207±0.102	57.224±0.152	+0.27	+0.30	+0.03
	J17	56.943	57.336±0.117	57.287±0.134	+0.69	+0.60	-0.08
	J21	56.588	56.533±0.186	56.878±0.132	-0.10	+0.51	+0.61
	J33	52.962	53.224±0.116	53.606±0.124	+0.49	+1.22	+0.72
	J43	55.305	55.019±0.107	55.047±0.145	-0.52	-0.47	+0.05
	K5	55.390	56.656±0.140	56.308±0.129	+2.28	+1.66	-0.61
	K22	54.025	54.956±0.100	55.134±0.158	+1.72	+2.05	+0.32
	K29	51.472	52.943±0.116	52.874±0.120	+2.86	+2.72	-0.13
	K30	51.099	52.467±0.115	52.478±0.120	+2.68	+2.70	+0.02
	K35	52.535	54.042±0.110	53.700±0.133	+2.87	+2.22	-0.63
	K42	54.994	56.730±0.116	56.300±0.167	+3.16	+2.38	-0.76
84" Med. Zone	F33	42.266	42.732±0.125	42.852±0.117	+1.10	+1.39	+0.28
	F68	42.583	42.883±0.119	42.726±0.116	+0.71	+0.34	-0.37
Totals		2882.935	2880.276	2884.152	-0.09	+0.04	+0.13

* This rod was also assayed destructively.

TABLE VI-12. NONDESTRUCTIVE ASSAY RESULTS FOR PROOF-OF-BREEDING RODS
FROM TYPE IVA REFLECTOR MODULE IV-4

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
Over 27.5 Gms.	A1 #	40.243	39.323±0.063	39.343±0.175	-2.28	-2.24	+0.05
	A2	37.541	36.574±0.057	36.275±0.173	-2.58	-3.37	-0.82
	A4	34.047	33.092±0.059	32.948±0.170	-2.80	-3.23	-0.44
	B4	29.679	29.115±0.050	28.943±0.142	-1.90	-2.48	-0.59
	C1	36.579	35.794±0.055	36.000±0.140	-2.14	-1.58	+0.58
	F2	28.268	28.963±0.057	29.081±0.164	+2.46	+2.88	+0.41
19.5 to 27.5 Gms.	A10	23.297	22.792±0.039	22.544±0.135	-2.17	-3.23	-1.09
	B7	25.228	25.674±0.049	25.442±0.158	+1.77	+0.85	-0.90
	B9	21.335	21.224±0.033	21.144±0.133	-0.52	-0.89	-0.38
	D6	21.054	21.800±0.062	21.662±0.135	+3.54	+2.89	-0.63
	F3	24.509	24.727±0.046	24.703±0.175	+0.89	+0.79	-0.09
13.5 to 19.5 Gms.	C11	16.484	16.343±0.055	16.177±0.143	-0.86	-1.86	-1.01
	E9	14.724	15.663±0.028	15.612±0.122	+6.38	+6.03	-0.33
	H6	14.939	16.087±0.069	15.999±0.215	+7.69	+7.09	-0.55
	L4	15.891	16.326±0.024	16.181±0.178	+2.74	+1.82	-0.89
	N3	18.154	18.584±0.035	18.640±0.226	+2.37	+2.68	+0.30
7.5 to 13.5 Gms.	G10	10.820	11.718±0.029	11.544±0.124	+8.30	+6.68	-1.49
	G13	8.093	8.855±0.027	8.798±0.139	+9.41	+8.71	-0.64
	H7	13.409	14.517±0.027	14.286±0.133	+8.26	+6.55	-1.59
	J7	12.358	13.375±0.054	12.999±0.123	+8.23	+5.19	-2.81
	N6	10.514	11.319±0.030	11.100±0.115	+7.66	+5.57	-1.94
	P7	8.555	9.260±0.024	9.204±0.121	+8.23	+7.58	-0.60
Under 7.5 Gms.	G14	7.426	8.124±0.026	8.093±0.120	+9.40	+8.98	-0.39
	H16	5.507	6.148±0.024	6.225±0.131	+11.64	+13.03	+1.25
	K16	3.817	4.920±0.018	4.925±0.127	+28.88	+29.03	+0.11
	L12	5.574	6.385±0.025	6.398±0.112	+14.56	+14.77	+0.19
	Q12	3.493	5.239±0.024	5.242±0.115	+49.97	+50.07	+0.07
	S8	6.301	6.905±0.033	6.893±0.119	+9.60	+9.40	-0.17
	U9	4.288	6.206±0.026	6.051±0.117	+44.74	+41.14	-2.49
Totals		502.126	515.053	512.453	+2.57	+2.06	-0.50

Rod selected for core examination study, not part of POB sample.

TABLE VI-13. NONDESTRUCTIVE ASSAY RESULTS FOR PROOF-OF-BREEDING RODS
FROM TYPE IVA REFLECTOR MODULE IV-9

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
Over 27.5 Gms.	A1	40.243	39.550±0.074	39.687±0.147	-1.72	-1.38	+0.35
	A3	36.579	35.777±0.060	35.766±0.158	-2.19	-2.22	-0.03
	A7	31.441	31.837±0.061	31.806±0.168	+1.26	+1.16	-0.10
	D2	29.679	29.191±0.053	29.033±0.135	-1.65	-2.18	-0.54
	E2	28.238	27.846±0.050	28.005±0.179	-1.39	-0.82	+0.57
19.5 to 27.5 Gms.	A10	26.710	26.358±0.046	26.405±0.134	-1.32	-1.14	+0.18
	D7	20.354	21.170±0.031	21.113±0.179	+4.01	+3.73	-0.27
	E5	20.836	21.654±0.040	21.668±0.120	+3.93	+3.99	+0.06
	F2	26.736	27.276±0.050	27.068±0.134	+2.02	+1.24	-0.76
	H3	20.355	20.679±0.046	20.643±0.136	+1.59	+1.42	-0.18
13.5 to 19.5 Gms.	E6	19.332	20.296±0.035	20.313±0.128	+4.98	+5.07	+0.08
	F8	14.939	16.077±0.054	15.821±0.117	+7.62	+5.91	-1.59
	G4	19.419	20.216±0.044	19.939±0.127	+4.10	+2.68	-1.37
	J4	16.527	17.233±0.038	17.126±0.190	+4.28	+3.63	-0.62
	N3	15.271	15.744±0.032	15.768±0.116	+3.10	+3.25	+0.15
7.5 to 13.5 Gms.	E12	13.152	13.737±0.031	13.600±0.129	+4.44	+3.41	-0.99
	E14	12.800	13.188±0.028	12.958±0.190	+3.03	+1.23	-1.74
	G12†	9.776	10.658±0.042	10.599±0.123	+9.02	+8.41	-0.56
	G14†	8.555	9.222±0.029	9.013±0.163	+7.80	+5.35	-2.27
	G17†	7.968	8.692±0.025	8.662±0.119	+9.08	+8.72	-0.33
	H8 †	12.067	13.133±0.047	12.926±0.136	+8.84	+7.12	-1.58
	H12	8.549	9.390±0.028	9.234±0.136	+9.84	+8.02	-1.65
	K8	9.755	10.652±0.026	10.701±0.119	+9.19	+9.70	+0.46
	K9 †	8.715	9.696±0.024	9.651±0.151	+11.26	+10.74	-0.47
	M6	10.040	10.854±0.029	10.688±0.113	+8.12	+6.46	-1.53
	N5 †	10.897	11.361±0.030	11.151±0.119	+4.26	+2.33	-1.85
	N7	8.093	8.823±0.027	8.577±0.116	+9.01	+5.97	-2.78
	P4 †	12.907	13.716±0.033	13.562±0.129	+6.27	+5.07	-1.13

† Additional rods not part of original POB sample.

TABLE VI-13 (Continued)

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
Under 7.5 Gms.	H15	6.806	7.593±0.026	7.334±0.148	+11.57	+7.76	-3.41
	H17†	6.301	6.937±0.019	6.730±0.116	+10.09	+6.82	-2.98
	J14†	6.153	7.104±0.026	7.174±0.135	+15.45	+16.59	+0.99
	K12†	6.531	7.359±0.021	7.383±0.120	+12.69	+13.06	+0.33
	K14†	5.337	6.344±0.020	6.440±0.135	+18.88	+20.67	+1.51
	K18	3.691	5.490±0.017	5.584±0.136	+48.76	+51.28	+1.69
	L15	4.105	5.590±0.028	5.423±0.114	+36.17	+32.12	-2.98
	L16†	3.561	5.464±0.027	5.208±0.120	+53.43	+46.26	-4.67
	M12	5.015	5.890±0.018	5.547±0.126	+17.44	+10.61	-5.82
	N14	3.384	4.827±0.022	4.878±0.139	+42.63	+44.13	+1.05
	P7	7.426	8.088±0.027	7.948±0.117	+8.91	+7.02	-1.73
	P10†	4.884	5.819±0.021	5.757±0.158	+19.15	+17.87	-1.07
	P14†	2.872	4.648±0.022	4.616±0.114	+61.85	+60.76	-0.68
	Q10	4.337	5.425±0.023	5.219±0.116	+25.08	+20.32	-3.80
	Q12†	3.264	4.955±0.023	4.782±0.135	+51.81	+46.49	-3.50
	Q13†	2.732	4.713±0.017	4.574±0.114	+72.53	+67.45	-2.94
	R7 †	6.761	7.305±0.026	7.198±0.117	+8.04	+6.46	-1.47
	S9 †	4.074	5.140±0.031	5.136±0.170	+26.19	+26.09	-0.08
	S10†	3.357	4.736±0.030	4.613±0.114	+41.07	+37.41	-2.60
	S11†	2.732	4.647±0.022	4.688±0.114	+70.14	+71.63	+0.88
Totals		593.253	632.102	627.718	+6.55	+5.81	-0.69

† Additional rods not part of original POB sample.

TABLE VI-14. NONDESTRUCTIVE ASSAY RESULTS FOR PROOF-OF-BREEDING RODS FROM TYPE IVB REFLECTOR MODULE IV-3

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
Over 27.0 Gms.	A1 #	37.489	37.346±0.059	37.163±0.135	-0.38	-0.87	-0.49
	A4	32.645	31.984±0.048	31.939±0.143	-2.02	-2.16	-0.14
	B1 *	35.659	34.845±0.052	34.934±0.111	-2.28	-2.03	+0.26
	B2	31.713	31.203±0.075	31.172±0.129	-1.61	-1.70	-0.10
	C2	29.859	29.237±0.069	29.396±0.119	-2.08	-1.55	+0.54
	G1	28.073	28.490±0.034	28.380±0.124	+1.49	+1.09	-0.39
19.0 to 27.0 Gms.	B7	24.318	24.718±0.037	24.696±0.139	+1.65	+1.55	-0.09
	B8	22.606	22.434±0.038	22.296±0.147	-0.76	-1.37	-0.61
	D5	21.274	21.798±0.046	21.509±0.156	+2.46	+1.11	-1.32
	E3 *	23.692	23.874±0.031	23.687±0.102	+0.77	-0.02	-0.78
	J2	20.683	20.452±0.077	20.283±0.119	-1.12	-1.93	-0.82
12.3 to 19.0 Gms	C9	18.189	18.418±0.031	18.428±0.126	+1.26	+1.31	+0.05
	C11	15.976	15.912±0.038	15.749±0.114	-0.40	-1.42	-1.03
	E9	14.001	14.928±0.028	14.651±0.113	+6.62	+4.64	-1.86
	H6	13.529	14.452±0.031	14.261±0.103	+6.82	+5.41	-1.32
	K3	16.757	17.014±0.040	16.886±0.127	+1.53	+0.77	-0.75
	M3	15.300	15.086±0.022	15.002±0.115	-1.40	-1.95	-0.56
7.0 to 12.3 Gms.	G10	10.158	11.024±0.026	10.711±0.111	+8.53	+5.44	-2.85
	J11	7.201	8.131±0.035	8.034±0.103	+12.90	+11.57	-1.18
	K6	11.375	12.298±0.033	12.402±0.131	+8.11	+9.03	+0.85
	L6	10.385	11.386±0.024	11.173±0.131	+9.64	+7.59	-1.87
	R6	8.080	9.089±0.027	8.719±0.119	+12.49	+7.91	-4.07
Under 7.0 Gms.	H14	6.133	6.904±0.024	6.785±0.099	+12.56	+10.63	-1.72
	K12	5.767	6.566±0.020	6.380±0.105	+13.85	+10.62	-2.83
	L16	2.978	4.513±0.021	4.405±0.118	+51.51	+47.89	-2.39
	N15	2.466	4.088±0.021	3.999±0.133	+65.79	+62.18	-2.18
	Q8	5.604	6.260±0.018	6.117±0.120	+11.69	+9.15	-2.28
	Q10	4.072	5.013±0.020	4.765±0.124	+23.09	+17.00	-4.95
Totals		475.982	487.463	483.921	+2.41	+1.67	-0.73

* These rods were also assayed destructively.

Rod selected for core examination study, not part of POB sample.

TABLE VI-15. NONDESTRUCTIVE ASSAY RESULTS FOR PROOF-OF-BREEDING RODS FROM REFLECTOR MODULE V-4

Rod Stratum	Rod Cell	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	---- % Differences ----		
					Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
Over 14.0 Gms.	A1	22.691	21.094±0.038	21.027±0.134	-7.04	-7.34	-0.32
	A6	24.829	24.197±0.044	24.192±0.138	-2.55	-2.56	-0.02
	A10	23.439	21.984±0.040	22.044±0.166	-6.21	-5.95	+0.27
	B4	19.220	18.093±0.035	17.828±0.131	-5.86	-7.24	-1.46
	B7	20.381	20.238±0.027	20.090±0.126	-0.70	-1.43	-0.73
	B8	20.270	20.200±0.040	20.184±0.134	-0.34	-0.42	-0.08
	B13	18.320	17.077±0.048	16.754±0.164	-6.78	-8.55	-1.89
	C4	15.497	15.186±0.028	15.116±0.129	-2.01	-2.46	-0.46
	C5	15.945	15.451±0.044	14.926±0.164	-3.10	-6.39	-3.39
	C7	16.488	16.225±0.035	15.850±0.133	-1.60	-3.87	-2.31
	C12	15.227	14.973±0.035	14.998±0.128	-1.67	-1.50	+0.17
	C14	14.822	14.459±0.028	14.319±0.121	-2.45	-3.40	-0.97
7.5 to 14.0 Gms.	D4	12.535	12.657±0.031	12.656±0.152	+0.98	+0.97	-0.01
	D12	12.653	12.730±0.030	12.667±0.164	+0.61	+0.11	-0.49
	D14	12.136	12.152±0.022	12.133±0.123	+0.14	-0.02	-0.16
	E6	10.599	10.825±0.036	10.201±0.135	+2.13	-3.76	-5.76
	E8	10.889	11.359±0.041	11.581±0.190	+4.32	+6.35	+1.95
	E12	10.481	10.806±0.029	10.819±0.123	+3.10	+3.23	+0.13
	E16	10.156	10.487±0.029	10.517±0.123	+3.26	+3.55	+0.28
	F5	8.296	8.724±0.027	8.657±0.121	+5.16	+4.34	-0.77
	F8	8.879	9.330±0.029	9.030±0.120	+5.07	+1.70	-3.21
	F10	8.907	9.403±0.027	9.201±0.166	+5.57	+3.30	-2.15
Under 7.5 Gms.	G2	6.514	6.947±0.021	6.994±0.168	+6.65	+7.37	+0.68
	G11	7.230	7.723±0.025	7.581±0.118	+6.83	+4.86	-1.84
	G14	6.717	7.309±0.025	7.279±0.113	+8.82	+8.36	-0.42
	G16	6.359	6.958±0.033	6.932±0.119	+9.43	+9.02	-0.37
	H4	5.097	5.663±0.018	5.681±0.116	+11.10	+11.45	+0.32
	H11	5.855	6.436±0.020	6.294±0.120	+9.92	+7.49	-2.22
	J4	3.804	4.763±0.019	4.742±0.126	+25.20	+24.66	-0.43
	J14	4.361	5.372±0.024	5.353±0.117	+23.19	+22.75	-0.35
	J17	3.774	4.888±0.022	4.796±0.173	+29.52	+27.07	-1.89
	J18	3.626	4.683±0.020	4.672±0.117	+29.17	+28.87	-0.23
	K3	2.266	4.241±0.019	4.133±0.145	+87.19	+82.45	-2.53
	K8	3.077	4.998±0.023	4.838±0.116	+62.45	+57.24	-3.20
Totals		391.338	397.632	394.084	+1.61	+0.70	-0.89

TABLE VI-16. FISSILE LOADINGS AND UNCERTAINTIES FOR DESTRUCTIVELY ASSAYED RODS

Module	Rod Cell	Rod No.	Fissile Loading in Grams				Percent Differences		
			Calc. (PDO)	PIFAG Thermal	PIFAG Epithermal	Destructive (ANL)	ANL - PDO	ANL - Thermal	ANL - Epith.
Seed I-1	C10	0507057	25.570	25.598 ± .071	25.665 ± .045	25.637 ± .023	+0.26	+0.15	-0.11
	L29	0504042	23.185	22.949 ± .078	22.964 ± .050	22.956 ± .024	-0.99	+0.03	-0.03
	M49	0401744	17.608	17.449 ± .065	17.523 ± .043	17.456 ± .016	-0.86	+0.04	-0.38
	N63	0307602	15.638	15.635 ± .057	15.684 ± .039	15.665 ± .014	+0.17	+0.19	-0.12
	P39	0201562	14.143	13.910 ± .050	13.993 ± .037	13.913 ± .013	-1.63	+0.02	-0.57
Std. Blkt. I-3	A49	1200830	36.430	36.884 ± .097	36.796 ± .081	36.680 ± .026	+0.69	-0.55	-0.32
	C3	1400544	41.001	40.711 ± .091	40.756 ± .072	40.682 ± .030	-0.78	-0.07	-0.18
	D24	1302864	47.732	47.355 ± .129	47.384 ± .113	47.268 ± .034	-0.97	-0.18	-0.24
	E56	1605519	52.860	52.514 ± .154	52.471 ± .147	52.384 ± .037	-0.90	-0.25	-0.17
Pwr. Flat.	B62	2102187	29.357	29.367 ± .056	29.318 ± .049	29.296 ± .022	-0.21	-0.24	-0.08
Blkt. III-6	C13	2400408	34.042	33.914 ± .054	34.018 ± .053	33.936 ± .025	-0.31	+0.06	-0.24
	D29	2300711	45.753	45.364 ± .104	45.350 ± .077	45.309 ± .037	-0.97	-0.12	-0.09
	E31	2606481	52.120	51.575 ± .115	51.624 ± .082	51.564 ± .040	-1.07	-0.02	-0.12
	F73	2513854	53.436	53.670 ± .101	53.752 ± .076	53.779 ± .041	+0.64	+0.20	+0.05
	H1	2502102	57.460	57.295 ± .073	57.440 ± .070	57.419 ± .044	-0.07	+0.22	-0.04
Reflector IV-3	B1	3211456	35.659	34.845 ± .052	34.934 ± .111	34.805 ± .018	-2.39	-0.12	-0.37
	E3	3110505	23.692	23.874 ± .032	23.687 ± .111	23.746 ± .012	+0.23	-0.54	+0.25

TABLE VI-17. t-TEST FOR ANL DESTRUCTIVE ASSAY VERSUS
PIFAG NONDESTRUCTIVE ASSAY

Notation: n = sample size

$\bar{d}$ = average of percentage differences d between ANL and either
thermal or epithermal PIFAG result

s = standard deviation of the d values about $\bar{d}$

$$t_{n-1} = \sqrt{n} \bar{d}/s$$

	n	ANL/Thermal			ANL/Epithermal		
		$\bar{d}$	s	t_{n-1}	$\bar{d}$	s	t_{n-1}
Seed	5	+0.087	0.079	+2.47	-0.244	0.226	-2.42
Standard Blanket	4	-0.264	0.206	-2.56	-0.227	0.068	-6.66*
PF Blanket	6	+0.017	0.181	+0.23	-0.085	0.096	-2.16
Reflector	2	-0.327	0.299	-1.55	-0.061	0.438	-0.20
Overall	17	-0.069	0.231	-1.23	-0.162	0.187	-3.57*
Seed, SB, PFB	15	-0.035	0.210	-0.64	-0.176	0.157	-4.32*

* Significantly different from 0 at 5% two-sided significance level, where the critical value for t_{n-1} is a function of n and is 2.776 for $n=5$, 3.182 for $n=4$, 2.571 for $n=6$, 12.706 for $n=2$ and 2.120 for $n = 17$.

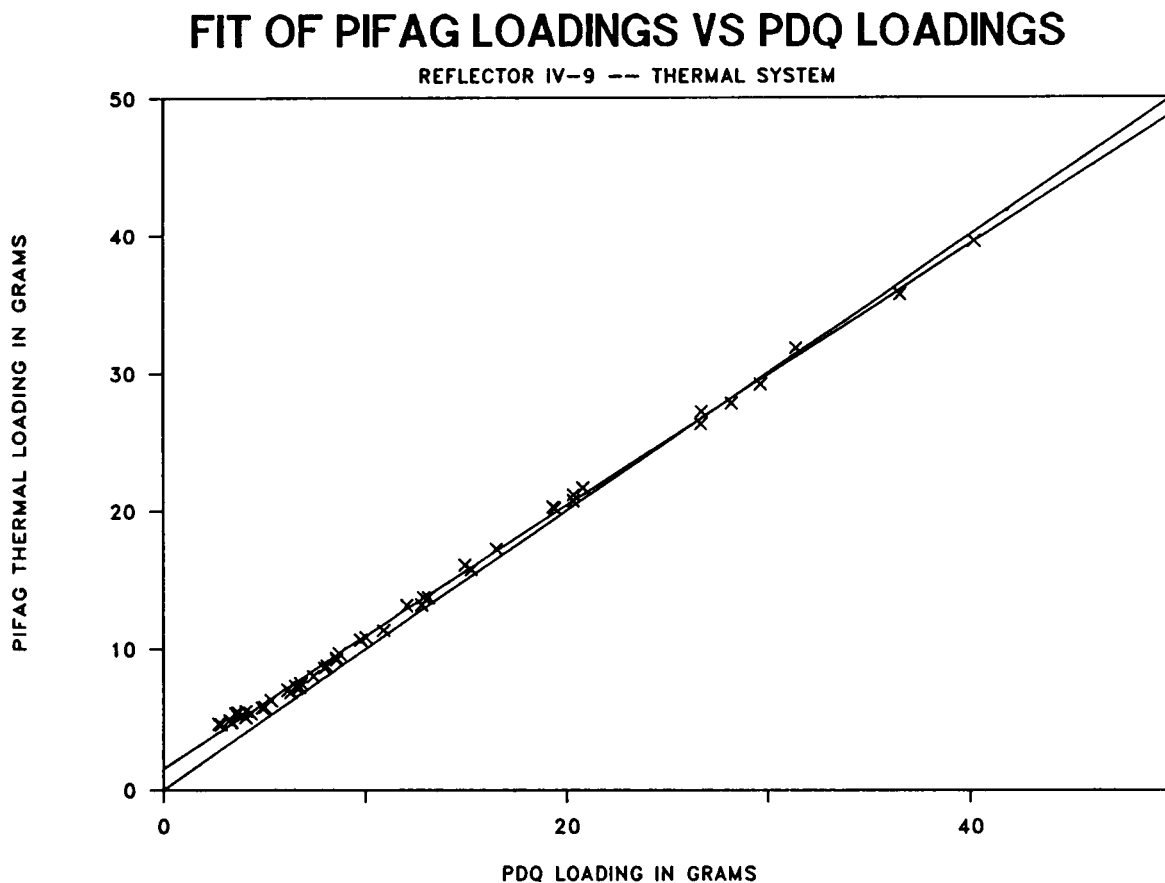


Figure VI-1. Fit of PIFAG Thermal Assay Loadings Versus PDQ Calculated Loadings for All Proof-of-Breeding Rods from Reflector Module IV-9. The Two Straight Lines are Discussed in the Text.

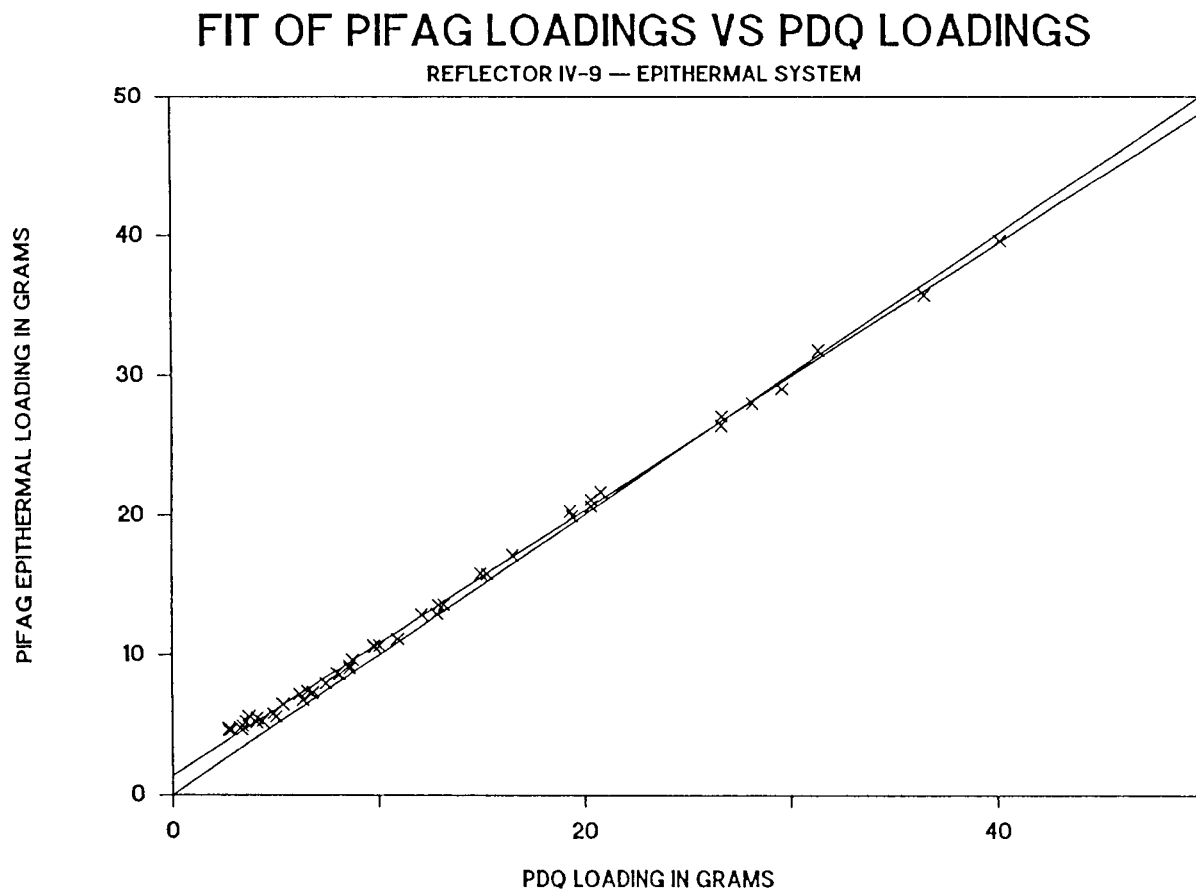


Figure VI-2. Fit of PIFAG Epithermal Assay Loadings Versus PDQ Calculated Loadings for All Proof-of-Breeding Rods from Reflector Module IV-9. The Two Straight Lines are Discussed in the Text.

VII. ROD STRATUM AND MODULE LOADINGS: ESTIMATION MODELS AND RESULTS

The objective of Sections VII and VIII is to establish the estimation procedure that was used to determine the fissile loading for the entire expended core from the nondestructive assay results for 524 POB rods given in the previous section, and to present the results. The POB rods came from 50 rod strata and nine module strata. It is convenient, however, to continue to treat the standard blanket and power flattening blanket rods separately, so we may speak instead of 11 effective module strata: Types I, II, and III seed; Types I, II, and III standard blanket; Types II and III power flattening blanket; and Types IVA, IVB, and V reflector. To arrive at the total expended core loading, four levels of estimation must be established: individual rod loadings, rod stratum loadings, module loadings, and, finally, the total core loading. The rod loadings were given in Section VI. This section presents the rod stratum and module loading models and the results for all POB modules. Section VIII will discuss the total expended core loading and the fissile inventory ratio calculations.

The first method of estimation considered was the stratum average method, in which the average measured loading of the sampled rods from a rod stratum was used as an estimate of the true mean loading of all the rods in the stratum. That is, the total loading of rod stratum i was estimated by multiplying the average loading from n_i sampled rods by the total number of rods, N_i , in the stratum. This estimation model will be discussed in Section VII.A below.

An improved model for stratum mean loading estimation was developed that utilized linear regression methodology and the predicted final loadings of all rods in the expended core based on PDQ calculations. This linear regression model is discussed in Section VII.B. Included in Section VII.B is a discussion of the special treatment given in the linear regression estimation model to rods adjacent to water channels.

Section VII.C discusses the estimation of module loadings and their variances. Section VII.D presents the results. Section VII.D also discusses the choice of linear regression as the preferred method of estimation based on the module loading standard deviations. Section VII.E discusses variation among modules and core symmetry.

A. Stratum Average Estimation

The first method of estimation to be considered is the stratum average method, in which the mean loading of all rods in a stratum is estimated by the average measured loading for the rods sampled from the stratum. Let the model for rod loading be

$$L_{tjiu} = \eta_{tji} + \epsilon_{tjiu}, \quad (\text{VII-1})$$

where L_{tjiu} is the observed loading, obtained by nondestructive assay, of the u^{th} rod ($u=1,2,\dots,n_{tji}$) from the i^{th} rod stratum of the j^{th} module of module stratum t ,

η_{tji} is the true mean loading of the rods in the i^{th} rod stratum of module j of module stratum t ,

and ϵ_{tjiu} is the error term, which is assumed to have a mean of 0 and variance of σ^2_{tji} . These errors are correlated within the stratum but independent between different strata, as discussed in Appendix A-1.

The error variance σ^2_{tji} has two components: the assay measurement error, σ^2_{Atji} , and the stratum population variance, σ^2_{Ptji} , which is the variance in true loading among the rods in the stratum. Since the POB rods were sampled randomly, σ^2_{Ptji} is also a measure of the sampling variance (i.e., the variance in true loading among the rods sampled.) For many of the rod strata — especially reflector strata and the 84 inch high zone seed and

power flattening blanket rod strata — the sampling variance dominated the measurement variance.

The estimate for the true mean rod loading based on this model is the average measured loading of the sampled rods:

$$\bar{L}_{tji} = \frac{\sum_{u=1}^{n_{tji}} L_{tj i u}}{n_{tji}} \quad (\text{VII-2})$$

The estimate for the variance in this quantity, as developed in Appendix A1, is

$$\hat{\text{Var}}(\bar{L}_{tji}) = \frac{N_{tji} - n_{tji}}{n_{tji} N_{tji}} s^2_{tji} + \frac{\hat{\sigma}^2_{A tji}}{N_{tji}}, \quad (\text{VII-3})$$

$$\text{where } s^2_{tji} = \frac{\sum_{u=1}^{n_{tji}} (L_{tj i u} - \bar{L}_{tji})^2}{(n_{tji} - 1)} \quad (\text{VII-4})$$

is the estimate of the combined sampling and assay measurement error variances,

N_{tji} is the total number of rods in the stratum,

$\hat{\sigma}^2_{A tji}$ is the estimate of the assay measurement error variance,

and $\frac{N_{tji} - n_{tji}}{n_{tji} N_{tji}}$ is called the finite population correction factor, which appears because the sampling is being performed from a population with restricted size.

The stratum average estimate for the total loading of all the rods in the stratum is obtained by multiplying $\bar{L}_{tji}$ from Equation (VII-2) by the number of rods in the stratum, N_{tji} :

$$\hat{L}_{tji}(\text{SA}) = N_{tji} \bar{L}_{tji} . \quad (\text{VII-5})$$

The stratum average estimate of the whole module loading is then obtained by summing the stratum loadings over all strata in the module, as discussed in Section VII.C.

B. Linear Regression Estimation

1. The Basic Model

In Section V.D.2 it was stated that if the stratum average method is used to estimate the loading of all rod strata, then the theoretically optimum sampling allocation scheme would require over 37% of the POB rods to be taken from the reflector modules, even though these modules were predicted to contain less than 8% of the fuel in the expended core. This is a consequence of the large stratum population variances for the reflector rod strata, which are caused by the large range of reflector rod loadings (predicted to be from less than three to more than 40 grams). It was suspected, and readily confirmed through simulation studies, that an improved module loading estimate could be achieved for reflector modules by utilizing the predicted rod loadings, calculated using the PDQ computer code as described in Section IV, as an input variable in a regression model. Thus, a linear regression estimation method was developed, first for reflector modules and finally for all modules.

The linear regression model for an individual rod can be written as

$$L_{tjiu} = \eta_{tji} + \beta_{tj} (q_{tjiu} - \bar{q}_{tj}) + \epsilon_{tjiu} \quad (\text{VII-6})$$

where L_{tjiu} is, as before, the measured assay loading for the u^{th} rod from rod stratum i of the j^{th} module from module stratum t ,

η_{tji} is the true mean loading of rods in this stratum,

β_{tj} is a parameter to be estimated,

q_{tju} is the PDQ-predicted loading for this rod,

$\bar{q}_{ti} = \frac{\sum_{u=1}^{n_{tji}} q_{tju}}{n_{tji}}$ is the average of the PDQ-predicted loadings for the n_i sampled rods from the i^{th} stratum,

and ϵ_{tjju} is the random error term (different from the random error term in Equation VII-1).

Note that the quantities q_{tju} and $\bar{q}_{ti}$ need no subscript j (to represent the j^{th} module) because the PDQ model considered all modules in a given module stratum to be identical due to the assumption of sixth-core symmetry.

As noted previously in Section V.D.2, the linear regression model assumes that there is a linear relationship between the predicted rod loadings and the measured loadings. Equation (VII-6) defines a straight line relationship for each stratum in a module, but with a common slope, β_{tj} . It was originally expected that a different value of the slope parameter might be needed for each rod stratum. However, when the nondestructive assay work was performed, it was found that the measured loadings for all the POB rods from a reflector module fitted the predicted loadings very well using a common slope. (The fit was even better when the water channel adjustment was made for a few rods, as described in the next section.) This meant that a single value of β_{tj} could be used for all rods in the module, instead of a different value for each rod stratum. This approach reduced the number of fitted parameters, thereby increasing the number of degrees of freedom and consequently improving the quality of the module loading estimate. The uncertainties in the module loading estimates obtained using the linear regression method were substantially lower than the stratum average loading estimate uncertainties, as will be discussed further in section VII.D.

It was further expected, when the model was originally devised, that the predictions would be sufficiently good that the value of β_{tj} would be close to one. However, the linear regression method does not require that this be true; it is necessary only that the relationship between calculated and true loadings be capable of representation by a straight line, regardless of the slope of that line. (If the straight-line assumption is not valid — that is, if the errors in the calculated loadings are strongly and nonlinearly correlated with loading — then the linear regression method may give a biased estimate of the loadings.) In fact, the assay results provided excellent confirmation of the assumed linear relationship. It was shown in Section VI.E that the measured loadings for the nondestructively assayed reflector rods fit very well on a straight line versus predicted loading, and it will be shown that a similar relationship holds for rods from other module types as well.

The regression variable, q , is an a priori estimate of a rod loading. By definition it follows that

$$\sum_{u=1}^{n_{tji}} (q_{tju} - \bar{q}_{ti}) = 0, \text{ for all } i, j, \text{ and } t. \quad (\text{VII-7})$$

The error term ϵ_{tjiu} in Equation (VII-6) is assumed to be independently and identically distributed as a normal population with mean 0 and a common variance, σ_{tji}^2 , for all rods in the stratum. The estimation model and the development of the variance function and its estimation are discussed in more detail in Appendix A2.

The stratum loading is estimated by replacing the parameters η_{tji} and β_{tj} in Equation (VII-6) with their least-squares estimates, which are obtained by minimizing the sum of the squares of the random error term, ϵ_{tjiu} . These least-squares estimates are "best" in that they provide minimum variance among all linear, unbiased estimates. The least-squares estimate for η_{tji} is, as before, the average measured loading of the sampled rods, given by Equation (VII-2). The least-squares estimate for β_{tj} will be designated b_{tj} . The expression for b_{tj} is

$$b_{tj} = \frac{\sum_{j=1}^{r_t} \sum_{u=1}^{n_{tji}} L_{tj i u} (q_{t i u} - \bar{q}_{t i})}{\sum_{j=1}^{r_t} \sum_{u=1}^{n_{tji}} (q_{t i u} - \bar{q}_{t i})^2} \quad (\text{VII-8})$$

where r_t is the number of rod strata in the module.

The individual rod loadings obtained using these best estimates are then summed over all N_{tji} rods in the stratum to give the linear regression estimate of the total loading of all the rods in the stratum:

$$\begin{aligned} \hat{L}_{tji}(\text{LR}) &= \sum_{u=1}^{N_{tji}} [\bar{L}_{tji} + b_{tj} (q_{t i u} - \bar{q}_{t i})] \\ &= N_{tji} [\bar{L}_{tji} + b_{tj} (\delta_{t i} - \bar{q}_{t i})] \end{aligned} \quad (\text{VII-9})$$

$$\text{where } \delta_{t i} = \sum_{u=1}^{N_{tji}} q_{t i u} / N_{tji} \quad (\text{VII-10})$$

is the average loading of all rods in the stratum, as predicted by the PDQ code.

A comparison of Equations (VII-9) and (VII-5) shows that the linear regression estimate of the stratum loading differs from the stratum average estimate due to the presence of the additional term, $b_{tj}(\delta_{t i} - \bar{q}_{t i})$, which is a correction for the difference between the average loading of the sampled rods and the average loading of all the rods in the stratum; in other words, this term represents a sampling error correction to the estimate of the stratum loading. To illustrate the significance of this term, consider a situation where the sampled rods happened to contain significantly more fuel on the average than the rods in the stratum as a whole. In this case the estimate of the stratum loading obtained using the stratum average method would probably be too large. But if the PDQ calculations are reasonably accurate, the term $b_{tji}(\delta_{t i} - \bar{q}_{t i})$ would be negative in this case, so the

linear regression estimate would reduce the stratum average estimate to a value closer to the true mean stratum loading. Similarly, if the sampled rods had less fuel on the average than the population PDQ mean, then the correction term would make the linear regression estimate larger than the stratum average estimate.

Since the linear regression method proved successful for reflector rod strata, it was decided to try the method for seed and blanket rod strata as well. A single value of b_{tj} was used for all strata in a seed module, and separate values of b_{tj} were used for all standard blanket strata and all power flattening blanket strata in a blanket module. (It would not have been possible to use a different value of b_{tj} for each stratum in seed or blanket modules because many of the strata in these modules had too few POB rods, or had too small a range of loading values, for the linear regression model to be successfully applied to a single stratum by itself.) It was found that the measured rod loadings for all seed, standard blanket, and power flattening blanket rods from a module fitted the common slope model very well, especially when the water channel correction was applied as indicated in the following section. As was the case for reflector modules, the linear regression method provided seed and blanket module loading estimates with significantly smaller uncertainties than the stratum average method.

2. Additional Terms in the Linear Regression Model: Water Channel Effect

It was observed in Section VI that rods located in the outermost rod row in seed modules and those from both the innermost and outermost rod rows in blanket modules generally had significantly more fuel than predicted by PDQ. These rods were located next to the water channel between modules. Most of the other seed and blanket rods, on the other hand, had slightly less fuel than predicted. In the reflector modules, measured loadings for rods adjacent to support posts and for rods located in the last two rows (see Figure VII-1) appear to be greater than expected from the straight line fit between prediction and measurement discussed in the previous section. These effects

may be caused by difficulties in handling fuel/water interfaces and attenuation across a reflector module using the modified diffusion theory method of PDQ.

It was decided to modify the linear regression model to account for these effects, which will be denoted collectively as "water channel" effects. (It should be noted here that the water channel effects apply only to the linear regression model, since these effects relate to the accuracy of the PDQ calculations, which are not used in the stratum average model.) Accordingly, one or two additional terms, depending on the module type, were added to the linear regression model. These terms applied only to those rods adjacent to a water channel (or, in the case of reflector rods, a support post), effectively splitting the rod stratum next to the water channel (or support post) into two substrata. Only one term was added for seed modules because only the outermost rod row in seed modules was adjacent to a water channel. In blanket modules, both the innermost and outermost rod rows were adjacent to water channels. However, only one additional term was needed in the model for standard blanket strata since the row of standard blanket rods next to the outer water channel constituted an entire rod stratum, hence all rods in the stratum were water channel rods. Two terms were added for power flattening blanket strata since both strata adjacent to a water channel consisted of more than one rod row, so it was necessary to separate the water channel rods from the other rods in the stratum. Two terms were also added for reflector modules to permit the rods next to the front support posts to be treated separately from those at the back of the module. The modified linear regression model for an individual rod, as derived in Appendix A2, can be written as:

$$\begin{aligned}
 L_{tj i u} = & \eta_{t j i} + \beta_{t j} (q_{t i u} - \bar{q}_{t i}) \\
 & + \alpha_{1 t j} (w_{1 t j i u} - \bar{w}_{1 t j i}) \\
 & + \alpha_{2 t j} (w_{2 t j i u} - \bar{w}_{2 t j i}) \\
 & + \epsilon_{t j i u}
 \end{aligned}
 \tag{VII-11}$$

where, in addition to the terms already defined for equation (VII-6) above,

$$w_{vtj i u} = \begin{cases} 1, & \text{if the } u^{\text{th}} \text{ rod belongs to the } v^{\text{th}} \text{ water channel} \\ & \text{effect region;} \\ 0, & \text{otherwise,} \end{cases}$$

for $v = 1 \text{ or } 2$;

$\bar{w}_{vtj i}$ is the average value of $w_{vtj i u}$ over the $n_{tj i}$ rods sampled from stratum i ;

and α_{vtj} , for $v = 1 \text{ or } 2$, is a parameter to be estimated by least-squares fitting the measured POB rod loadings to the predicted loadings.

Note that the water channel indicators, $w_{vtj i u}$, like the PDQ values, $q_{t i u}$, satisfy the relation

$$\sum_{u=1}^{n_{tj i}} (w_{vtj i u} - \bar{w}_{vtj i}) = 0, \text{ for all } v, t, j, \text{ and } i. \quad (\text{VII-12})$$

The loading of an individual rod is estimated using the modified linear regression model of Equation (VII-11) by replacing the Greek-letter symbols with their best estimate values to obtain

$$\hat{L}_{tj i u}(\text{LR}) = \bar{L}_{tj i} + b_{tj} (q_{t i u} - \bar{q}_{t i}) + \sum_{v=1}^2 a_{vtj} (w_{vtj i u} - \bar{w}_{vtj i}) \quad (\text{VII-13})$$

where $\hat{L}_{tj i u}(\text{LR})$ is the estimated rod loading,

a_{vtj} is the best estimate for the parameter α_{vtj} , calculated as indicated in Appendix A2,

and $\bar{w}_{vtj i} = \frac{\sum_{u=1}^{n_{tj i}} w_{vtj i u}}{n_{tj i}}$ is the ratio of the number of sampled water channel rods to the total number of sampled rods in the stratum.

The stratum loading is estimated by adding the estimated loadings of all N_{tji} rods in the stratum (that is, by summing Equation (VII-13) over the index u from 1 to N_{tji}) to obtain

$$\hat{L}_{tji}(\text{LR}) = N_{tji} [\bar{L}_{tji} + b_{tj} (\delta_{ti} - \bar{q}_{ti}) + \sum_{v=1}^2 a_{vtj} (\Omega_{vtji} - \bar{w}_{vtji})] \quad (\text{VII-14})$$

where $\Omega_{vtji} = \frac{\sum_{u=1}^{N_{tji}} w_{vtjiu}}{N_{tji}}$ is the ratio of the number of water channel rods in the stratum to the total number of rods in the stratum.

A summary of the estimated model parameters b_{tj} , a_{1tj} , and a_{2tj} , and the residual standard deviations for all the POB modules is given in Table VII-1. The residual standard deviation is a measure of the variation of the error term ϵ_{tjiu} in the regression model, Equation (VII-6). More specifically, it measures the variation of the observed loadings, L_{tjiu} , about the fitted model values, $\hat{L}_{tji}$, after all the other parameters in the model have been evaluated. It is given by

$$s_{Rtj}^2 = \sum_{u,i} (L_{tjiu} - \hat{L}_{tjiu})^2 / f_{tj} \quad (\text{VII-15})$$

where s_{Rtj}^2 is the residual standard deviation for the j th POB module from module stratum t ,

and f_{tj} is the number of degrees of freedom, equal to the total number of assayed rods in the module minus the number of fitted parameters b_{tj} , a_{1tj} , and a_{2tj} .

The estimate of variance in the stratum loading obtained using the linear regression method is discussed in Appendix A2.

3. Detection of Water Channel Effects

Figure VII-2 shows the PIFAG thermal assay results for the POB rods from seed module I-1 plotted against the loadings calculated with the basic linear regression model without the water channel effect, Equation (VII-6), using the least-squares estimates for the parameters. Ideally, all the data points should fit on a straight line with a slope of 1. Note that the five values at the lower left of the plot generally do not fit the line as well as the other points. These values are for the 42 inch low-zone seed rods. Two of these rods were located in the outermost rod row of the module, next to a water channel, while the remaining three rods were not adjacent to a water channel. When the modified linear regression model of Equation (VII-11) is used, the five 42 inch low-zone rod values in question are much closer to the theoretical line, as can be seen in Figure VII-3. Similar results were obtained with the thermal and epithermal PIFAG data from all POB seed modules. The standard blanket, power flattening blanket, and reflector data also were fitted better when the water channel terms were included in the linear regression model.

Table VII-2 gives a summary of the analyses of the statistical significance of the water channel effects for all modules assayed. The residual sum of squares (abbreviated Res. SS in the table) is the sum of squares of the differences between the measured and estimated rod loadings; in other words, the quantity that is minimized in the least squares fitting procedure. The column labeled "Std. Dev." is the residual standard deviation defined in Equation (VII-15). The extra sum of squares (see Reference 21) due to the presence of the water channel term(s) in the model is given by the difference in the residual sum of squares obtained with and without these terms. An F-test for significance is obtained by dividing the extra sum of squares by the number of water channel parameters being estimated (1 or 2) and comparing it to the residual variance estimate obtained when the water channel effects are included in the model. For seed module I-1 the F-test value for

the water channel effect is 7.95, which is significant at the 1% level — $F(1,27,.01) = 5.49$. This means that there is less than 1% probability that the reduction in residual sum of squares achieved by adding the extra term occurred merely by chance. The table shows that with few exceptions the water channel effects are found to be significant at least at the 5% level and those that are not found to be significant are nearly so. Therefore, the water channel effects were included in every module analysis. One such term was applied to the seed and standard blanket modules and two were applied to the power flattening blanket and reflector modules.

C. Module Loading Estimation and Variance

The loading for a module is estimated by summing the estimated stratum loadings, obtained either by the stratum average method using Equation (VII-5) or by the modified linear regression method using Equation (VII-14), over all the strata in the module:

$$\hat{L}_{tj} = \sum_{i=1}^{r_t} \hat{L}_{tji} \quad (\text{VII-16})$$

where $\hat{L}_{tj}$ is the loading of the j^{th} POB module from module stratum t ,

and r_t is the number of rod strata in a module from module stratum t .

The variance in the loading estimate for a POB module has two components: the PIFAG measurement error and the sampling error. The PIFAG measurement error is discussed in Reference VII-2, and the uncertainties (standard deviations) in the thermal and epithermal measurements for all POB rods are given in Tables VI-1 through VI-15 of this report. For most seed and blanket rods, the PIFAG thermal and epithermal relative standard deviations were between 0.2% and 0.5%. For reflector rods, the thermal measurement standard deviations were generally in the same range, but the epithermal standard deviations were larger for many rods. Most of the reflector epithermal

uncertainties were between 0.3% and 2.0%, although larger values were obtained for some lightly loaded rods.

The sampling error is the uncertainty inherent in estimating the total loading of all rods from measurements made on a random sample of those rods. The estimate of the variance in the module loading, including both the sampling error and the measurement error contributions, is discussed in Appendix A1 for the stratum average method and in Appendix A2 for the linear regression method. It was found that for most strata the sampling error contribution to the stratum loading variance was larger than the measurement error contribution. Those strata for which the two error terms were comparable (primarily the 56 inch and 70 inch seed and blanket strata) made a rather small contribution to the module loading variance. Therefore, the sampling error part of the module loading variance tended to dominate the measurement error part. Because of this, a simplifying assumption was made in calculating the module loading variances: The measurement uncertainty (both thermal and epithermal) was taken to be 0.5% for all seed and blanket rods and 2.0% for all reflector rods. In light of the information in the previous paragraph, these assumptions are conservative.

D. Results for Proof-Of-Breeding Modules

Table VII-3 lists the estimation results for the assayed modules. Included in the table are the module loadings and the estimated standard deviations in the module loadings for both the stratum average and linear regression methods and for both the thermal and epithermal PIFAG data. The difference between the linear regression and stratum average estimates for the loading of the same module is always well within twice the square root of the sum of the squared standard deviations. This indicates that the module loading estimates by the two methods are not significantly different. However, for both the thermal and epithermal cases it can be seen that the stratum average standard deviation is about three times larger than the linear regression standard deviation. This illustrates the improvement in estimation

that was achieved by using the linear regression model and makes linear regression the clear choice as the preferred method of estimation. All module loadings quoted in the remainder of this report are linear regression estimates.

Table VII-4 presents a comparison of the thermal and epithermal module loadings with each other and with the predicted loadings calculated by the method described previously in Section IV. As the epithermal loadings for the individual POB seed and blanket rods were slightly larger on the average than the thermal loadings, so the epithermal module loadings are generally slightly larger. However, the difference is smaller than twice the standard deviation in the difference for all seed and blanket modules except Seed II-3 and Power Flattening Blanket III-2, for which the difference is slightly outside two standard deviations. The total epithermal loading for all seed and blanket POB modules is larger than the total thermal loading by 0.19%. For the reflector modules the epithermal loadings are smaller than the thermal loadings, as was generally the case for the individual rod loadings. The reflector thermal and epithermal module loadings agree within two times the standard deviation in the difference in all cases. The total epithermal loading for all reflector modules is smaller than the total thermal loading by 0.81%. When the seed, blanket, and reflector results are combined, the total epithermal loading for all POB modules is larger than the total thermal loading by 0.12%.

The thermal loadings are less than calculation for all seed and blanket modules. The difference is 0.32% when the loadings of all modules are combined. For reflector modules the thermal loadings are several percent greater than calculation, with the difference in total reflector loading being 4.61%. However, when the results from all POB modules are combined, the reflector difference almost exactly cancels the seed and blanket difference so that the total thermal loading is almost identical to the calculated value. The epithermal seed and blanket loadings tend to be smaller than calculation, but not by as much as the thermal loadings. The epithermal reflector loadings

are larger than predicted, but again not by as much as the thermal loadings. Finally, the total epithermal loading for all POB modules is greater than predicted by only 0.12%.

In summary, both the thermal and epithermal measurements indicate that there was slightly less fuel than predicted in the seed and blanket regions of the expended core, while the reflector had a few percent more fuel than predicted. But the differences tended to cancel so that on balance the total fuel in all POB modules was very close to prediction.

E. Module-to-Module Variation and Core Symmetry

If the LWBR core were perfectly symmetric, then all modules in a given stratum would have had identical loadings both initially and at end of life. In fact, however, there were several factors that gave rise to small asymmetries in the core. Some modules had a flux well at a location where other modules in the same stratum had a fuel rod. There were small deviations from symmetry in the as-built core, as shown in Table III-1. The neutron flux distribution in the core was found to have a small asymmetry of about 1% or less, as determined from physics testing during core lifetime (Reference 17). It is to be expected that these and other differences would give rise to small deviations from symmetry in the expended core loading. To allow these deviations to be investigated and to permit the module-to-module variation in loading to be evaluated, two different POB modules were chosen from each of three strata. These modules are Type III seeds 1 and 2, Type III blankets 2 and 6, and Type IVA reflectors 4 and 9. It is important to determine the degree of variability in the loading among modules of a given stratum, since this affects the module sampling variance — that is, the uncertainty in the core loading estimate due to the fact that the sampled rods came from only 12 out of the 39 modules in the core. It will be shown below that the expended core was in fact highly symmetric, and the deviations from module loading symmetry were small.

Table VII-5 gives the relevant data for the symmetrically located modules. The expended module loading estimates in this table were obtained using the linear regression method, in keeping with the observations of the previous section. For each pair of modules the difference is defined to be the loading of the first module minus that of the second. It may be seen that seed modules III-1 and III-2 differed in final loading by $0.20\% \pm 0.17\%$ based on the thermal PIFAG data and by $0.32\% \pm 0.17\%$ based on the epithermal data, where the uncertainties are one standard deviation in the difference in module loadings. Neither of these differences is significantly different from zero at the 5% significance level for a two-sided test. However, if the thermal and epithermal results are assumed to be independent and are combined to give an average difference of $0.26\% \pm 0.12\%$, then the difference is significant. That is, there is greater than 95% chance that the true final loading of seed module III-2 was indeed greater than that of seed module III-1. Part of this difference almost certainly arises from the fact that a difference of 0.24% was already present initially, as module III-1 had the smallest as-built fissile loading of all seed modules while module III-2 had a larger-than-average loading (see Table III-1). It is to be expected that the difference in module loading at end of life should be smaller than the initial difference because the module with higher loading would have higher neutron flux during life, which would lead to greater depletion and therefore greater net loss of fuel. Nevertheless, in view of the uncertainty in the measured final loading difference, it appears that most or all of this difference can be accounted for by the initial difference.

For the standard blanket rods from symmetrically located blanket modules III-2 and III-6, Table VII-5 shows an insignificant difference in final loading using either the thermal or the epithermal PIFAG data. The initial differences were negligible as well. To compare the loadings of the power flattening blanket rods from these two modules, it is first necessary to correct for the presence of a flux well at one rod location in module III-2. The rod at the corresponding location in module III-6 was deliberately included in the POB rod sample to facilitate this comparison. After the

measured loading of the extra rod in module III-6 is subtracted from the estimated total loading, there is an insignificant difference from the estimated power flattening blanket loading for module III-2. When the standard blanket and power flattening blanket results are added to give the total loadings for the two blanket modules, the differences remain insignificant.

For symmetrically located reflector modules IV-4 and IV-9, the estimated final loadings differ by about 0.5% using either the thermal or epithermal PIFAG data. Although these differences are not significant at the 5% significance level, they are consistent with the existence of a small asymmetry, or "tilt," detected in the core neutron flux distribution (Reference 17). The reflector module loadings were more strongly influenced by the presence of the flux tilt than the seed or blanket module loadings. This is a consequence of the fact that reflector modules had no fissile fuel initially. Therefore, the final fuel content of a reflector module was approximately proportional to the integral of the neutron flux in the module over core life. If, for example, the flux in one reflector module had been 1% greater than the flux in another module throughout life, the final loading of the first module would have been nearly 1% greater than that of the second. For seed and blanket modules, on the other hand, the close competition between growth of new fuel and depletion of old fuel meant that a 1% difference in flux would not give rise to a 1% difference in final loading. In fact, a seed module with 1% greater flux than another, otherwise identical, module would have slightly less fuel at end of life than the other module, and for blanket modules a 1% difference in flux would have very little effect on final loading. Therefore, the observed difference in the reflector POB module loadings may be an effect of the flux tilt that was not seen in the seed or blanket module loadings.

The observed flux tilt is illustrated in Figure VII-4, which shows the difference in power (which is proportional to neutron flux) among the different seed-blanket module combinations, as determined from end-of-life

physics testing. The number at each seed module location gives the percent deviation in the power for that seed-blanket combination from the average for all modules of the same type, as deduced from reactivity worth measurements. These numbers were obtained by averaging results from a series of reactivity worth measurements tabulated in Reference 17. (The assumption is made here that power generated in symmetric modules is approximately proportional to the square root of the differential reactivity worths, as discussed in Reference 22.) It may be noted that the figures for one half of the core are generally positive and those in the other half are negative. This indicates that the power (and hence the flux) is higher towards one side of the core. While we have no direct measure of the power in the reflector modules themselves, it is reasonable to take the relative power in the adjacent seed-blanket combinations as an approximate indication of the difference in power between reflector modules IV-4 and IV-9. On this basis it is estimated that the power in module IV-4 was about 0.3% lower than average and in module IV-9 about 0.5% above average, for a net difference in power of 0.8% between the two modules. Based on the discussion above, one would therefore expect to find the final loading of module IV-4 a little less than 0.8% lower than that of module IV-9. This is in excellent agreement with the measured loadings.

A bit of caution should be exercised here in view of the statistical uncertainties in the module loadings. The uncertainty in the difference between the loadings of the two modules is approximately as great as the difference itself, using either the thermal or the epithermal PIFAG results. Thus, the possibility cannot be ruled out that the agreement with the flux tilt measurement is a statistical accident.

The degree of symmetry exhibited in the measured module loadings is better than was assumed when the proof of breeding program was developed. The simulation studies that were performed to define various aspects of the program, as described in Section V.B, generally assumed a random module-to-module variation in loading with 0.5% standard deviation, superimposed on the effect of a 1% flux tilt across the core. The measured differences given in

Table VII-4 are smaller than this. The measured differences were used in calculating the sampling variance in the total core loading, as will be discussed in Section VIII.C. The estimation of the module-to-module variance is presented in Appendix A.3.

TABLE VII-1. SUMMARY OF LINEAR REGRESSION ESTIMATION PARAMETERS

$$\text{Model: } \hat{L}_{iu}(\text{LR}) = \bar{L}_i + b (q_{iu} - \bar{q}_i) + \sum_{v=1}^2 a_v (w_{v i u} - \bar{w}_{vi})$$

Module	Thermal				Epithermal			
	b	a ₁ (grams)	a ₂ (grams)	Residual Std.Dev. (grams)	b	a ₁ (grams)	a ₂ (grams)	Residual Std.Dev. (grams)
Seed								
I-1	1.06440	0.40243	-	0.155	1.04970	0.31389	-	0.165
II-3	1.14071	0.41053	-	0.119	1.15812	0.38222	-	0.148
III-1	1.05276	0.22347	-	0.147	1.07588	0.26470	-	0.147
III-2	1.05582	0.28994	-	0.121	1.10145	0.26470	-	0.126
Standard Blanket								
I-3	0.93654	0.89272	-	0.174	0.94876	0.75698	-	0.162
II-2	1.17383	0.67557	-	0.178	1.19236	0.54564	-	0.178
III-2	1.32281	0.90598	-	0.151	1.28195	0.87638	-	0.180
III-6	1.07448	0.76899	-	0.118	0.98146	0.85189	-	0.122
Power Flattening Blanket								
II-2	0.99222	0.59604	1.39440	0.237	1.04436	0.49815	1.40289	0.244
III-2	1.00132	0.29372	1.30208	0.241	1.00283	0.26689	1.07658	0.252
III-6	1.00279	0.39396	1.66301	0.264	0.97613	0.45811	1.29159	0.269
Reflector								
IV-3	0.96460	0.69253	0.59394	0.308	0.97768	0.80270	0.64518	0.270
IV-4	0.94184	0.95370	0.81626	0.333	0.93395	0.85351	0.84332	0.373
IV-9	0.94240	0.71228	0.63434	0.324	0.94162	0.73566	0.50406	0.263
V-4	0.85284	0.81581	1.07688	0.147	0.87213	0.77531	1.13294	0.251

Note: a₁ represents the effect of the water channel next to the rod stratum with a 42" binary length for seed and blanket modules and the effect of the outside (back) of the reflector modules.

a₂ represents the effect of the water channel next to the rod stratum with an 84" binary length for power flattening blanket rods and the effect on the rods immediately surrounding posts on the front side of the reflector modules (i.e., the side adjacent to the blanket modules).

TABLE VII-2. SIGNIFICANCE OF WATER CHANNEL EFFECT

Module	Without Water Channel(s)	With Water Channel(s)			Extra SS due to Water Channels	Significance Test F
	Res.SS	Res. SS	Std. Dev.	degr.of freedom#		
Seed-Thermal						
I-1	0.8374	0.6470	0.155	1,27	0.1904	7.95**
II-3	0.5909	0.3936	0.119	1,28	0.1973	14.03**
III-1	0.6637	0.6055	0.147	1,28	0.0582	2.69
III-2	0.4754	0.3925	0.121	1,27	0.0829	5.70*
Seed-Epithermal						
I-1	0.8516	0.7357	0.165	1,27	0.1159	4.25*
II-3	0.7881	0.6170	0.148	1,28	0.1711	7.77**
III-1	0.7011	0.6030	0.147	1,28	0.0981	4.56*
III-2	0.4976	0.4285	0.126	1,27	0.0691	4.35
Standard Blanket-Thermal						
I-3	2.8177	0.8774	0.174	1,29	1.9403	64.1**
II-2	1.2802	0.5389	0.178	1,17	0.7413	24.5**
III-2	1.2979	0.2497	0.151	1,11	1.0482	46.0**
III-6	0.9496	0.1542	0.118	1,11	0.7954	57.1**
Standard Blanket-Epithermal						
I-3	2.1556	0.7605	0.162	1,29	1.3951	53.2**
II-2	1.0217	0.5382	0.118	1,17	0.4835	18.4**
III-2	1.1609	0.1801	0.128	1,11	0.9808	59.9**
III-6	1.1391	0.1629	0.122	1,11	0.9762	65.6**
Power Flattening Blanket-Thermal						
II-2	6.5986	1.8501	0.234	2,33	4.7845	43.7**
III-2	11.8113	2.8493	0.241	2,49	8.9620	77.2**
III-6	13.5744	3.4040	0.264	2,49	10.1704	73.2**
Power Flattening Blanket-Epithermal						
II-2	6.6481	1.9663	0.244	2,33	4.6818	39.3**
III-2	9.2729	3.1236	0.252	2,49	6.1493	48.4**
III-6	9.8716	3.5387	0.269	2,49	6.3329	43.8**
Reflector-Thermal						
IV-3	2.9692	1.9030	0.308	2,20	1.0662	5.62*
IV-4	4.5108	2.3250	0.333	2,21	2.1858	9.86**
IV-9	4.9456	2.0127	0.224	2,40	2.9329	29.2**
V-4	4.1407	0.6039	0.147	2,28	3.5368	81.8**
Reflector-Epithermal						
IV-3	2.8008	1.4528	0.270	2,20	1.3480	9.25**
IV-4	4.9318	1.7694	0.373	2,21	2.0113	7.23**
IV-9	5.5933	2.7632	0.263	2,40	2.8301	20.5**
V-4	5.4329	1.7694	0.251	2,28	3.6635	29.1**

Degrees of freedom for F-test of water channel significance: number of water channel effects in model, number of rods minus the total number of parameters estimated

* Statistically significant at the 5% level

** Statistically significant at the 1% level

TABLE VII-3. ESTIMATION RESULTS FOR MODULE LOADINGS AND STANDARD DEVIATIONS

Thermal Results

		<u>Linear Regression Estimation</u>		<u>Stratum Average Estimation</u>	
<u>Module</u>		<u>Module Loading (Grams)</u>	<u>Standard Deviation (Grams)</u>	<u>Module Loading (Grams)</u>	<u>Standard Deviation (Grams)</u>
Seed	I-1	12490.5	18.1	12470.2	55.4
	II-3	12891.1	13.9	12920.3	58.5
	III-1	13229.9	17.1	13228.2	61.0
	III-2	13256.8	14.6	13276.4	63.1
Std.	I-3	19591.1	13.6	19551.7	31.9
Blkt.	II-2	11273.2	9.5	11282.9	33.4
	III-2	7955.0	6.8	7942.3	26.3
	III-6	7951.3	5.3	7960.1	22.9
Pwr.	II-2	14424.7	11.5	14426.7*	36.2
Flat.	III-2	21516.5	15.0	21531.2	61.5
Blkt.	III-6	21608.7	16.3	21594.3	65.5
Ref1.	IV-3	3050.9	14.5	3080.7*	65.4
	IV-4	3323.8	14.8	3332.8*	72.4
	IV-9	3341.9	7.9	3342.3	78.9
	V-4	1721.1	4.5	1723.6	44.4

Epithermal Results

		<u>Linear Regression Estimation</u>		<u>Stratum Average Estimation</u>	
<u>Module</u>		<u>Module Loading (Grams)</u>	<u>Standard Deviation (Grams)</u>	<u>Module Loading (Grams)</u>	<u>Standard Deviation (Grams)</u>
Seed	I-1	12514.9	19.3	12495.9	55.0
	II-3	12936.0	17.3	12966.1	60.0
	III-1	13256.1	17.1	13253.6	62.6
	III-2	13298.8	15.3	13319.2	66.9
Std.	I-3	19618.0	12.7	19578.5	29.7
Blkt.	II-2	11297.7	9.5	11307.4	32.9
	III-2	7954.9	5.8	7942.6	25.2
	III-6	7965.6	5.4	7973.7	22.2
Pwr.	II-2	14440.2	11.9	14442.6*	38.0
Flat.	III-2	21570.7	15.7	21586.5	61.7
Blkt.	III-6	21632.4	16.6	21622.0	65.2
Ref1.	IV-3	3016.9	12.7	3048.4*	67.2
	IV-4	3303.2	16.6	3311.8*	71.0
	IV-9	3321.3	9.3	3321.5	78.6
	V-4	1704.8	7.6	1708.0	45.7

* Stratum average loading estimated without use of PIFAG results for core examination rods (see Section V.E)

TABLE VII-4. COMPARISON OF CALCULATED AND MEASURED MODULE LOADING ESTIMATES

Module	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)*	PIFAG Epithermal Loading and Std. Dev. (Grams)*	---- % Differences ----		
				Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
Seed I-1	12571.8	12490.5±18.1	12514.9±19.3	-0.65	-0.45	+0.20
II-3	12942.0	12891.1±13.9	12936.0±17.3	-0.39	-0.05	+0.35
III-1	13276.8	13229.9±17.1	13256.1±17.1	-0.35	-0.16	+0.20
III-2	13276.8	13256.8±14.6	13298.8±15.3	-0.15	+0.17	+0.32
Std. I-3	19662.8	19591.1±13.6	19618.0±12.7	-0.36	-0.23	+0.14
Blkt. II-2	11341.1	11273.2± 9.5	11297.7± 9.5	-0.60	-0.38	+0.22
III-2	7994.2	7955.0± 6.8	7954.9± 5.8	-0.49	-0.49	-0.00
III-6	7994.2	7951.3± 5.3	7965.6± 5.4	-0.54	-0.36	+0.18
Pwr. II-2	14445.4	14424.7±11.5	14440.2±11.9	-0.14	-0.04	+0.11
Flat. III-2	21565.1	21516.5±15.0	21570.7±15.7	-0.23	+0.03	+0.25
Blkt. III-6	21622.6	21608.7±16.3	21632.4±16.6	-0.06	+0.05	+0.11
Totals, seed and blanket modules	156692.8	156188.8	156485.3	-0.32	-0.13	+0.19
Ref1. IV-3	2911.3	3050.9±14.5	3016.9±12.7	+4.80	+3.63	-1.11
IV-4	3179.7	3323.8±14.8	3303.2±16.6	+4.53	+3.88	-0.62
IV-9	3179.7	3341.9± 7.9	3321.3± 9.3	+5.10	+4.45	-0.62
V-4	1662.8	1721.1± 4.5	1704.8± 7.6	+3.51	+2.53	-0.95
Totals, reflector modules	10933.5	11437.7	11346.2	+4.61	+3.77	-0.81
Totals, all POB modules	167626.3	167626.5	167831.5	+0.00	+0.12	+0.12

* Linear regression estimation

TABLE VII-5. COMPARISON OF MODULE LOADINGS FOR SYMMETRICALLY LOCATED MODULES

Module	Thermal Loading and Std. Dev. (Grams)*	Epithermal Loading and Std. Dev. (Grams)*
Seed III-1	13229.9 ± 17.1	13256.1 ± 17.1
Seed III-2	13256.8 ± 14.6	13298.8 ± 15.3
Difference	-26.9 ± 22.4	-42.7 ± 22.9
% Difference	-0.20± 0.17	-0.32± 0.17
Std.Blkt. III-2	7955.0 ± 6.8	7954.9 ± 5.8
Std.Blkt. III-6	7951.3 ± 5.3	7965.6 ± 5.4
Difference	+3.7 ± 8.6	-10.7 ± 7.9
% Difference	+0.05± 0.11	-0.13± 0.10
Pwr.Flat.Blkt.III-2	21516.5 ± 15.0	21570.7 ± 15.7
Pwr.Plat.Blkt.III-6#	21551.4 ± 16.3	21575.0 ± 16.6
Difference	-34.9 ± 22.1	-4.3 ± 22.8
% Difference	-0.16± 0.10	-0.02± 0.11
Reflector IV-4	3323.8 ± 14.8	3303.2 ± 16.6
Reflector IV-9	3341.9 ± 7.9	3321.3 ± 9.3
Difference	-18.1 ± 16.8	-18.1 ± 19.0
% Difference	-0.54± 0.50	-0.54± 0.57

* Linear regression estimation

Corrected for flux well rod loading

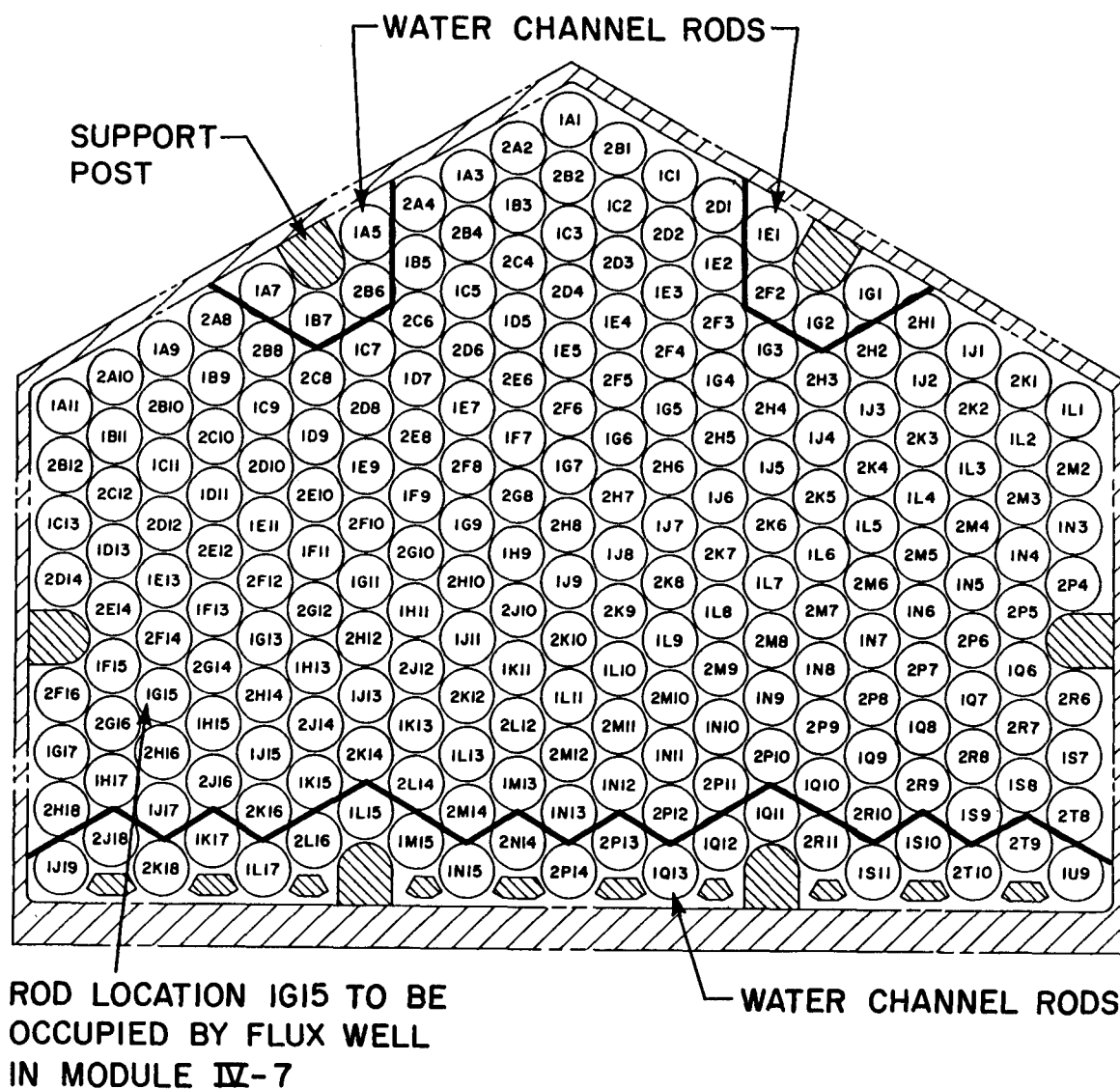


Figure VII-1. Rods Included in Water Channel Effect for Reflector
Type IV Module

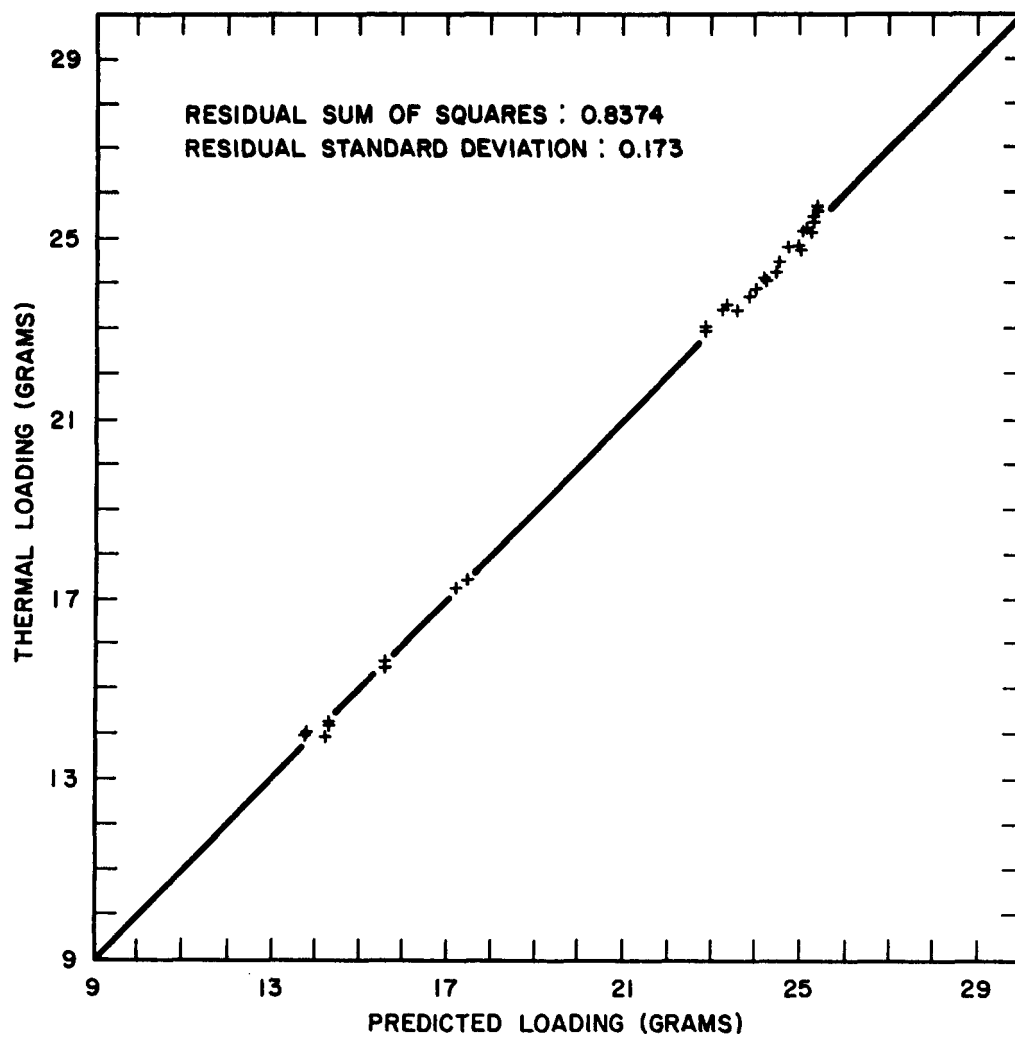


Figure VII-2. Seed I-1 Thermal Data Without Water Channel Effect

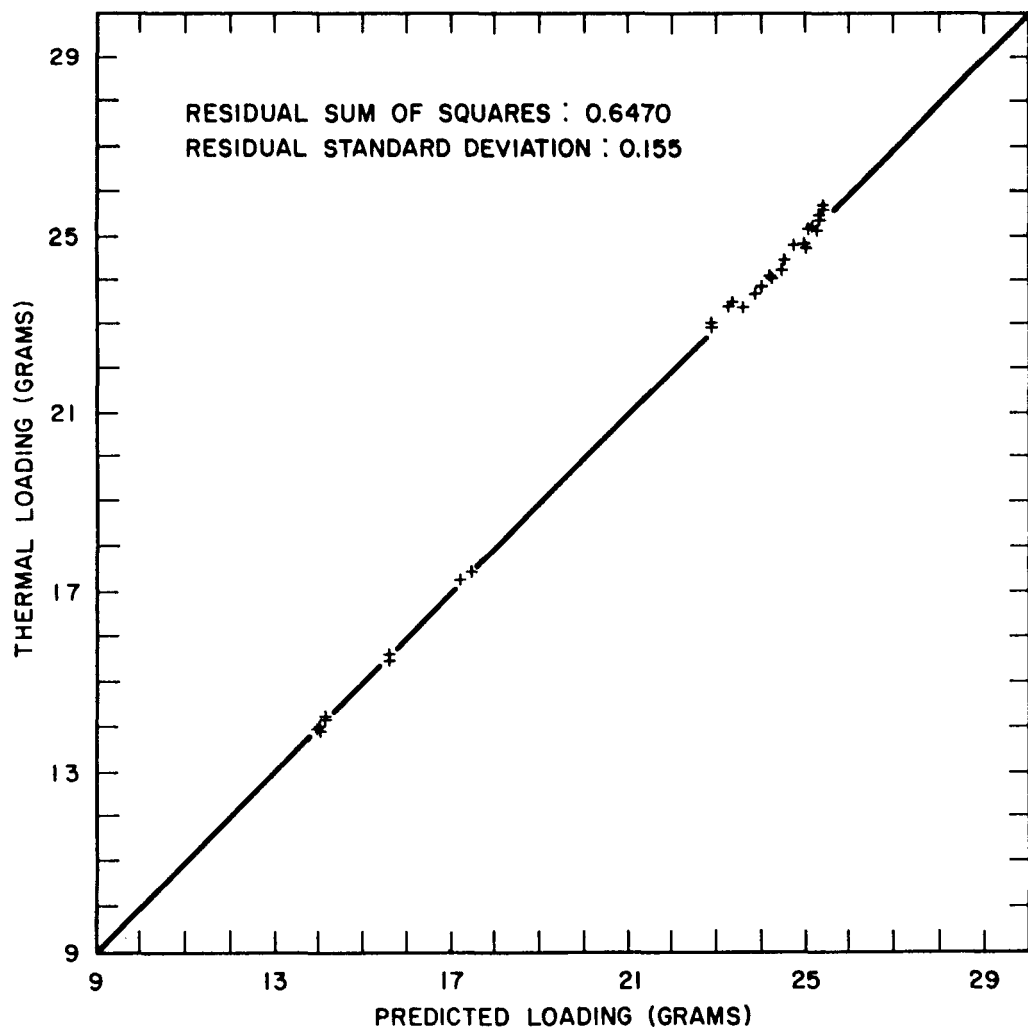


Figure VII-3. Seed I-1 Thermal Data With Water Channel Effect

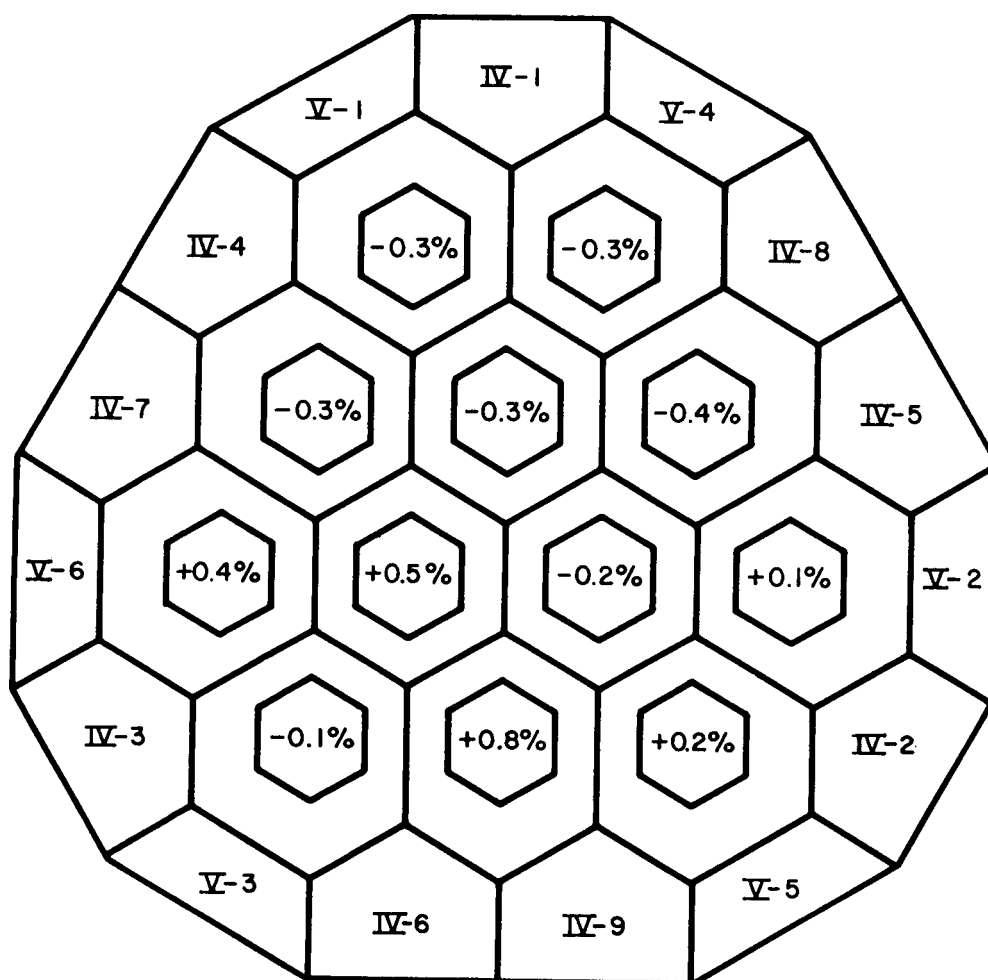


Figure VII-4. Deviation of Power from Average for Module Type,
As Deduced from End-Of-Life Reactivity Worth Measurements

VIII. ESTIMATION OF TOTAL EXPENDED CORE LOADING AND FISSILE INVENTORY RATIO

A. Total Expended Core Loading Estimation

If all modules within each module stratum had the same number of fuel rods, then the total expended core fissile loading, T , would be obtained by multiplying the estimate of the mean module loading, $\bar{L}_t$, for each stratum by the number of modules in that stratum and then summing over all strata; i.e.,

$$T = \sum_{t=1}^{11} M_t \bar{L}_t, \quad (\text{VIII-1})$$

where M_t is the total number of modules in module stratum t ,

and the estimate of the mean module loading is given by

$$\bar{L}_t = \frac{\sum_{j=1}^{m_t} \hat{L}_{tj}}{m_t} \quad (\text{VIII-2})$$

where $\hat{L}_{tj}$ is the linear regression estimate of the final loading of the j^{th} assayed module as given by Equation (VII-16), and

m_t is the number of assayed (POB) modules from module stratum t .

In fact, however, some modules contained one fewer rod than their counterparts from the same module stratum due to the replacement of a rod by a flux well. To account for this difference, Equation (VIII-1) is replaced by

$$T = \sum_{t=1}^{11} M_t \left[\sum_{j=1}^{m_t} \tilde{L}_{tj} \right], \quad (\text{VIII-3})$$

where $\tilde{L}_{tj}$ is an estimate of the j^{th} module loading which is

- (1) equal to the linear regression module loading estimate, $\hat{L}_{tj}$, if the j^{th} module was assayed, or

- (2) equal to $\bar{L}_j$ given by Equation (VIII-2) if the j^{th} module was not assayed but did contain the same number of rods as all the assayed modules in the stratum, or
- (3) equal to a modified loading, if the j^{th} module was not assayed and did not contain the same number of rods as the assayed modules.

The modules which required calculation of a modified loading were power flattening blanket modules II-1, II-3, III-1, III-3, III-4, and III-5, and reflector module IV-7. The modified loading calculations are described below.

1. Loading Estimate for Unassayed Power Flattening Blanket Modules II-1 and II-3

Blanket modules II-1 and II-3, which were not part of the POB sample, each had a flux well in the power flattening blanket region at rod location H1, where POB module II-2 had a fuel rod. Therefore, the linear regression estimate of the loading for rod H1, as obtained from Equation (VII-13), was subtracted from the linear regression estimate of the total loading for module II-2 (as given in Table VII-3) to give the modified loading estimate for power flattening blanket modules II-1 and II-3:

$$\bar{L}_{II-j} = \hat{L}_{II-2} - \hat{l}_{II-2(H1)} \quad (\text{VIII-4})$$

where $j = 1$ and 3 , and

$\hat{l}_{II-2(H1)}$ is the estimated loading for rod H1 in POB module II-2.

2. Loading Estimate for Unassayed Power Flattening Blanket Modules III-1, III-3, III-4, and III-5

Blanket modules III-1 and III-2 each had a flux well in the power flattening blanket region at rod location H18 whereas modules III-3 through III-6 had a fuel rod at that location. Thus, assayed module III-2 had a flux

well but assayed module III-6 did not. Some care must be exercised in comparing Type III blanket modules because the symmetry of the core was such that modules 1 through 3 were "mirror images" of modules 4 through 6 (see Figure V-1). Therefore, the rod in assayed module III-6 that was in the location symmetric to the flux well in unassayed module III-1 was rod H1, not rod H18. This fact was used to provide the modified loading estimate for power flattening blanket module III-1 as the average of the measured loading of power flattening blanket module III-2 and the measured loading of power flattening blanket module III-6 corrected for the flux well rod:

$$\tilde{L}_{III-1} = [\hat{L}_{III-2} + \hat{L}_{III-6} - \hat{l}_{III-6(H1)}] / 2 \quad (VIII-5)$$

where $\hat{l}_{III-6(H1)}$ is the linear regression estimate of the loading for rod H1 in module III-6.

The estimated loading, $\tilde{L}_{III-j}$, for the unassayed power flattening blanket modules III-3, III-4, and III-5 is given by

$$\tilde{L}_{III-j} = [\hat{L}_{III-2} + \hat{l}_{III-2(H18)} + \hat{L}_{III-6}] / 2 \quad (VIII-6)$$

where $j = 3, 4, \text{ and } 5$, and

$\hat{l}_{III-2(H18)}$ is the linear regression estimate of the loading for the rod that would be in location H18 of module III-2 if there were not a flux well at that location. This loading is calculated using Equation (VII-13).

3. Loading Estimate for Unassayed Reflector Module IV-7

Unassayed Type IVA module IV-7 contained a flux well at rod location G15. This module was the only reflector module with a flux well and hence was the only reflector module that needed special treatment. The estimated loading of reflector module IV-7 was obtained from the equation

$$\tilde{L}_{IV-7} = [\hat{L}_{IV-4} - \hat{l}_{IV-4(G7)} + \hat{L}_{IV-9} - \hat{l}_{IV-9(G15)}] / 2 \quad (\text{VIII-7})$$

where $\hat{l}_{IV-4(G7)}$ is the linear regression estimate of the loading for the rod in POB module IV-4 that was in the mirror image location corresponding to the flux well in module IV-7, and

$\hat{l}_{IV-9(G15)}$ is the linear regression estimate of the loading for the rod in POB module IV-9 that was in the location corresponding to the flux well in module IV-7.

B. Variance in Total Expended Core Loading Calculation

The variance in the total expended core loading estimate has two parts: random variance and systematic variance. These will be discussed separately.

1. Random Variance

There are two contributions to the random variance in the total expended core loading estimate. The first contribution comes from the variances in the individual module loadings. For the core loading estimate of Equations (VIII-1) and (VIII-2), this "within module" (WM) contribution to the random variance is estimated by

$$\hat{\text{Var}}(T)_{WM} = \sum_{t=1}^{11} \sum_{j=1}^{m_t} \left[\frac{M_t}{m_t} \right]^2 \sigma^2(\hat{L}_{tj}) \quad (\text{VIII-8})$$

where $\sigma(\hat{L}_{tj})$ is the estimate of the standard deviation in the linear regression module loading, as tabulated in Table VII-3.

Properly, Equation (VIII-8) should be modified to account for the flux wells in some modules. The modification involves the addition of a small correction to account for the variance in the estimated loading of the missing rods. This correction was included in the calculation of the numerical values

given below for the core loading variance. However, the correction has an insignificant effect on the total variance and therefore will not be discussed further.

The second contribution to the random variance in the core loading estimate is the "module-to-module" (MM) variance, or module sampling variance, which is a measure of the uncertainty caused by the fact that the core loading estimate is based on assay results from a sample of the modules in the core. It is shown in Appendix A-3 that an unbiased estimate of the module-to-module variance is given by

$$\hat{\text{Var}}(T)_{\text{MM}} = \frac{11}{\sum_{t=1}^T} \frac{(M_t - m_t)}{m_t} M_t \hat{S}_t^2, \quad (\text{VIII-9})$$

where

$$\hat{S}_t^2 = \frac{m_t}{\sum_{j=1}^{m_t}} (\hat{L}_{tj} - \bar{L}_t)^2 / (m_t - 1) - \frac{m_t}{\sum_{j=1}^{m_t}} \sigma^2(\hat{L}_{tj}) / m_t. \quad (\text{VIII-10})$$

Clearly, Equation (VIII-10) can be used only for those module strata having more than one assayed module; namely, Type III seed, Type III standard blanket and power flattening blanket, and Type IVA reflector. The relative module-to-module standard deviation (i.e., the estimated standard deviation divided by the average module loading, or $\hat{S}_t / \bar{L}_t$) for each of these module strata was used for the similar module strata having only one assayed module. That is, the relative module-to-module standard deviation for the Type I and Type II seeds was taken to be equal to that for the Type III seed modules. Similarly, the value determined for the Type III standard blanket module stratum was applied to the Types I and II standard blanket strata, the value for the Type III power flattening blanket stratum was applied to the Type II power flattening blanket stratum, and the value for the Type IVA reflector stratum was applied to the Types IVB and V reflector strata.

It may be noted that $\hat{S}_t^2$ defined by Equation (VIII-10) would be negative if the square of the difference in loading between the two assayed modules in a stratum happened to be smaller than the sum of the variances in

the individual module loadings. Since the true module-to-module variance cannot be negative, a value of zero was used in that case.

The within module and module-to-module variance terms defined by Equations (VIII-9) and (VIII-10) respectively are added to give the total random variance in the core loading, designated as $\text{Var}(T)_{\text{Random}}$. The values obtained for the random standard deviation (square root of the total random variance) in total loading for each of the four kinds of fuel (seed, standard blanket, power flattening blanket, and reflector) and for the whole core are given in Table VIII-1. The table entries are relative standard deviations in percent.

2. Systematic Variance

There are two avenues of approach to the determination of the systematic uncertainty in the expended core loading estimate. One approach is to identify the possible sources of systematic error and to quantify their potential effect. The second approach is to make use of the results from the destructive assay of 17 POB rods at Argonne National Laboratory (ANL), discussed in Section VI.F, to estimate the systematic uncertainty in the PIFAG measurements. Both of these approaches were used, and the results from the two methods were added to give the total systematic variance.

It is conservative to add the estimates of systematic variance obtained from these two approaches, since they are in effect two different estimates of the same quantity. However, combining the two estimates helps to account for any additional sources of error that may not have been identified in the first approach and for the relatively small size of the sample of destructively assayed rods in the second approach.

a. Sources of Systematic Error

The uncertainties in the PIFAG measurements were discussed in

detail in Reference 5. That discussion will not be repeated here, but a brief description of the primary sources of systematic error will be presented along with an estimate of the relative importance of each. The primary sources of systematic error in the nondestructive assay results are as follows:

- (1) calibration curves
- (2) ^{232}Th correction
- (3) ^{234}U correction
- (4) ^{235}U correction
- (5) absorption cross sections
- (6) atomic densities for fission products and ^{234}U in the assayed rod

These sources will be discussed individually.

Two different PIFAG calibration curves, one for thermal assays and one for epithermal assays, were obtained for each of the four kinds of fuel. Each curve was recalibrated several times during the assay campaign using newly acquired data from the calibration rods. (See Reference 5 for a chronology of PIFAG operations.) Since the results from each recalibration were used in the assay of a number of POB rods, any error in the calibration curve would have had a systematic effect on the determination of the core rod loadings. The relative standard deviations in the calibration curves for the four kinds of fuel are given in Table VIII-2. These values represent the combination of the variances due to differences among the different data sets for each kind of fuel, uncertainties in the calibration rod loadings, and the residual variances in the calibration curve fits.

The ^{232}Th correction was the largest single correction applied to the PIFAG data. It arose from the fact that a part of the PIFAG signal came from fast fission in the large amount of ^{232}Th present in every fuel rod. The calculation of the correction was described in detail in Appendix D of Reference 5. The correction involved the measurement of the response of the

PIFAG to ^{232}Th , which was done by assaying fuel rods containing pellets of pure thorium oxide. The measurements were repeated each time a new evaluation of the calibration curve was performed. The ^{232}Th response was different for each kind of fuel, and the effect of the correction was considerably greater for epithermal assays than for thermal. Based on the accuracy with which the correction was calculated, it is estimated that the relative standard deviation in the core loading determination due to uncertainty in the ^{232}Th correction is 0.016% for the thermal measurement and 0.089% for the epithermal measurement.

The ^{234}U correction arose from the fact that fast fission of ^{234}U , like ^{232}Th , contributed to the PIFAG signal. The correction was very small for the thermal assays but significantly larger for the epithermal assays. Although there was much less ^{234}U than ^{232}Th in the assayed rods, there was additional uncertainty in the ^{234}U correction due to the fact that the response of the gauge to this isotope could not be measured directly, since no ^{234}U calibration rods were available. Instead, the response was calculated as described in Appendix C of Reference 20. The standard deviation in the core loading determination resulting from the uncertainty in the ^{234}U correction is estimated to be 0.005% for the thermal measurement and 0.042% for the epithermal measurement.

The ^{235}U correction arose from the fact that the LWBR core rods contained an average of about 1% as much fissile ^{235}U as ^{233}U at end of life. The response of the gauge was different for the two isotopes. The ^{235}U response was calculated as described in Appendix B of Reference 20. The ^{235}U correction was larger for thermal assays because the difference in gauge response for the two isotopes was greater for the thermal PIFAG configuration than for the epithermal. Based on the uncertainty in the calculated response of the gauge to ^{235}U , the standard deviation in core loading due to this correction is estimated as 0.046% thermal and 0.002% epithermal.

The response of the PIFAG to fissile fuel was affected by neutron absorption in fission products and ^{234}U that grew into the fuel during core operation. The correction for this absorption made use of calculated atomic densities and absorption cross sections for fission products and ^{234}U . The atomic densities were calculated as described in Sections IV.C and IV.D. The cross sections were determined as described in Appendix C of Reference 5. The uncertainties in these quantities gave rise to standard deviations in the thermal core loading determination of 0.054% from the cross sections and 0.084% from the atomic densities, and in the epithermal determination, 0.029% from the cross sections and 0.043% from the number densities.

A summary of the effect of the PIFAG correction factor systematic errors on the core loading is given in Table VIII-3.

b. PIFAG-ANL Difference as a Measure of Systematic Error

The differences between the nondestructive PIFAG assay and the ANL destructive assay were summarized previously in Tables VI-16 and VI-17. Since the uncertainties in the destructive assay measurements are substantially smaller than those for the nondestructive assay, the differences between the two measurements provide an indication of the error in the nondestructive results. The standard deviations in the average differences between the destructive and nondestructive results can be considered to estimate the systematic uncertainty in the PIFAG loadings. As discussed above, it was found that a significant difference between the epithermal PIFAG results and ANL results did exist for seed and blanket rods. Thus, the variance in the ANL-PIFAG differences for seed and blanket rods was applied separately from the variance for the reflector rods. This contribution to the total core loading variance was

$$\begin{aligned} \text{Var}(T)_{\text{ANL-PIFAG}} = & \left[\sum_k M_k \bar{L}_k / 100 \right]^2 (\sigma\%)^2_{\text{S/B,ANL-PIFAG}} & (\text{VIII-11}) \\ & + \left[M_{\text{Ref}} \bar{L}_{\text{Ref}} / 100 \right]^2 (\sigma\%)^2_{\text{Ref,ANL-PIFAG}} , \end{aligned}$$

where the subscript k represents seed, standard blanket, and power flattening blanket rods in one group and Ref refers to the reflector rods, and $(\sigma\%)$ is the estimated percent standard deviation in the difference between the ANL and PIFAG results for the respective kinds of fuel (equal to the quantity "s" in Table VI-17).

The variance defined by Equation (VIII-11) is based on the standard deviation in the differences between the PIFAG and ANL measurements, but not on the differences themselves. As has been noted previously, the average difference between the thermal PIFAG and ANL measurements is not statistically significant, regardless of whether the test for significance is applied to the four kinds of fuel rods separately or to the whole sample of destructively assayed rods. On the other hand, the average difference of 0.176% between the epithermal PIFAG and ANL measurements for the seed and blanket rods is significant. This bias between the two types of measurement is probably caused by systematic error in the epithermal results. Accordingly, an additional term has been added to the ANL-PIFAG variance for the epithermal measurement only that is equal to the square of the product of the percent difference between the PIFAG and ANL results for seed and blanket rods times the total seed and blanket loading.

The ANL-PIFAG variance was added to the various contributions to the systematic variance described in the previous section to give the total systematic variance; this in turn was added to the random variance (evaluated as described in section VIII.B.1) to give the total variance in expended core loading.

C. Expended Core Loading - Results

Table VIII-4 gives the total loading for each of the four kinds of fuel and for the whole expended core. These loadings were obtained from Equation (VIII-3). The thermal and epithermal loadings are compared to each other and to the predictions from the LWBR calculational model. These figures show the

same general trends as those observed in Section VII for the POB modules and in Section VI for the POB rods. Specifically, the epithermal loadings are slightly greater than the thermal for seed and blanket fuel but smaller for reflector fuel. Both thermal and epithermal loadings are slightly smaller than predicted for seed and blanket fuel but about 4% larger than predicted for reflector fuel. However, when the differences are expressed in absolute rather than relative units (i.e., in grams rather than percent), the excess fuel in the reflector almost exactly balances the deficiency in the central core, so that the total measured core loading is very close to prediction.

The thermal and epithermal values are in very good agreement with each other, differing by only 0.11%. Both values are within one standard deviation of the predicted expended core loading.

D. Estimation of Fissile Inventory Ratio

As noted earlier, the fissile inventory ratio (FIR) is the ratio of the total expended core fissile loading to the initial core loading, and a value greater than 1.0 for this quantity means that breeding occurred. The measured values of fissile inventory ratio obtained from the PIFAG thermal and epithermal data, respectively, are 1.0139 ± 0.0014 and 1.0150 ± 0.0021 , as given in Table VIII-4. The uncertainties are one standard deviation, obtained as discussed above.

Up to this point the PIFAG thermal and epithermal data have been treated independently, yielding two different values of fissile inventory ratio as a result. Of course the two values are not completely independent because the same assay gauge was used for both, and some parts of the analysis (e.g., the flux dip data, the gamma scan data, and the PDQ/CINDER atomic densities) were common to both. On the other hand, independent thermal and epithermal passes were made on each POB rod, separate calibration curves were obtained, different cross sections were calculated, and independent corrections to the data (which in most cases were substantially different in magnitude for the

two systems) were made for delayed neutrons from ^{232}Th , ^{234}U , and ^{235}U , shielding from ^{234}U and fission products, and deadtime. Thus, there is a substantial degree of independence between the two measurements of fissile inventory ratio. The fact that they are in good agreement gives added confidence in the demonstration of breeding. However, it has been pointed out that there is evidence of a small but significant bias in the epithermal assay measurements, based on the comparison between the epithermal and destructive assay results. As observed previously, it is more likely that the disagreement is caused by error in the PIFAG epithermal measurements than in the destructive assay, especially since there is no statistically significant difference between the PIFAG thermal and destructive assay results. Further, as was noted in Section VI, there is a statistically significant difference between the PIFAG thermal and epithermal results for some rod types. Because of these facts, it is not appropriate to average the thermal and epithermal values of fissile inventory ratio together to get a "best value." It is possible to apply a bias correction to the epithermal value of fissile inventory ratio based on the average difference between the epithermal and destructive assay values for the loadings of the 17 rods that were assayed at ANL. If such a correction were applied, the epithermal value of fissile inventory ratio would be reduced to 1.0134, which is very close to the thermal value of 1.0139. However, to apply such a correction adds to the uncertainty in the value of fissile inventory ratio due to the implicit assumption that the epithermal bias for the whole POB sample of rods is the same as for the 17 destructively assayed rods. Therefore, the best measured value of fissile inventory ratio is considered to be the PIFAG thermal value of 1.0139 ± 0.0014 . This value is consistent with the epithermal result regardless of whether a bias correction is applied or not. The close agreement between the PIFAG thermal, PIFAG epithermal, and destructive assay results gives strong support to this best value for fissile inventory ratio.

E. Determination of the Lower 95% Confidence Bound

To provide a measure of confidence in the estimated value of fissile

inventory ratio, we wish to establish a lower 95% confidence bound. That is, there should be 95% probability that the true value of fissile inventory ratio is greater than this lower bound, based on the measured value and its standard deviation. The calculation will be done for both the thermal and epithermal values. Note that this is a one-sided test in that we are concerned about the lower limit on fissile inventory ratio but not the upper limit. The lower 95% confidence bound, CB, is obtained from

$$CB = FIR (1.0 - t_{f,0.05} (\sigma\%)_{FIR}) , \quad (VIII-12)$$

where FIR is the thermal or epithermal value of fissile inventory ratio,

$t_{f,0.05}$ is the Student's t value that allows 5% of the distribution to the right of the value,

and $(\sigma\%)_{FIR}$ is the estimated relative (percent) standard deviation in fissile inventory ratio, obtained from

$$(\sigma\%)^2_{FIR} = (\sigma\%)^2_T + (\sigma\%)^2_B , \quad (VIII-13)$$

where $(\sigma\%)^2_T$ and $(\sigma\%)^2_B$ are the estimated percent variances in the expended core and initial core loadings, respectively. The uncertainty in the initial loading was discussed in Section III.

The t test depends on the number of degrees of freedom, f, in the estimate of fissile inventory ratio. This has been evaluated using Satterthwaite's approximation (Reference 23):

$$f = \frac{[\sum_i \sigma_i^2]^2}{[\sum_i (\sigma_i^2)^2 / f_i]} \quad (VIII-14)$$

where σ_i^2 and f_i are the estimated variances and degrees of freedom,

respectively, for each of the variance terms involved. The values of f for the expended core loading estimates are 49 for the thermal estimate and 82 for the epithermal estimate.

The values obtained for the lower 95% confidence bound are 1.0115 and 1.0114, respectively, for the PIFAG thermal and epithermal data. As before, the thermal value is the preferred value, but the difference between the two is very small. These values indicate with high confidence that LWBR was a breeder.

TABLE VIII-1. RANDOM STANDARD DEVIATION IN TOTAL LOADING FOR EACH KIND OF FUEL AND WHOLE CORE

Kind of Fuel	Thermal Std.Dev. (%)	Epithermal Std.Dev. (%)
Seed	0.069	0.106
Std. Blkt.	0.039	0.047
Pwr. Flat. Blkt.	0.044	0.045
Reflector	0.190	0.211
Total	0.032	0.041

TABLE VIII-2. CALIBRATION CURVE STANDARD DEVIATIONS

Kind of Fuel	Thermal Std.Dev. (%)	Epithermal Std.Dev. (%)
Seed	0.199	0.152
Std. Blkt.	0.103	0.152
Pwr. Flat. Blkt.	0.253	0.206
Reflector	0.063	0.125

TABLE VIII-3. PIFAG CORRECTION FACTOR STANDARD DEVIATIONS

Correction Factor	Thermal Std.Dev. (%)	Epithermal Std.Dev. (%)
^{232}Th	0.016	0.089
^{234}U	0.005	0.042
^{235}U	0.046	0.002
Cross Sections	0.054	0.029
Atomic Densities	0.084	0.043

TABLE VIII-4. RESULTS FOR EXPENDED CORE LOADING AND FISSILE INVENTORY RATIO

Kind of Fuel	Calc. Final Loading (Grams)	PIFAG Thermal Loading and Std. Dev. (Grams)	PIFAG Epithermal Loading and Std. Dev. (Grams)	% Differences		
				Thermal - Calc.	Epith. - Calc.	Epith. - Ther.
Seed	156202.4	155604.8 $\pm$ 398.7	156017.4 $\pm$ 351.8	-0.38	-0.12	+0.27
Std.Blkt.	140977.2	140311.7 $\pm$ 284.2	140508.5 $\pm$ 300.4	-0.47	-0.33	+0.14
Pwr.Fl.Blkt.	172844.2	172594.4 $\pm$ 511.5	172874.6 $\pm$ 426.1	-0.14	+0.02	+0.16
Reflector	37780.3	39467.4 $\pm$ 123.5	39143.8 $\pm$ 160.5	+4.47	+3.61	-0.82
Core Total	507804.1	507978.4 $\pm$ 711.1	508544.3 $\pm$ 1050.6*	+0.03	+0.15	+0.11
Fissile Inventory Ratio	1.0135	1.0139 $\pm$ 0.0014	1.0150 $\pm$ 0.0021			
Lower 95% Confidence Bound		1.0115	1.0114			

* Standard deviation adjusted to include effect of ANL-epithermal bias.

IX. SUMMARY AND DISCUSSION OF SIGNIFICANCE

The work described in this report, and in the references cited herein, represents the most comprehensive and accurate demonstration of breeding ever undertaken. Unique aspects of the program include the following:

1. The high precision achieved (standard deviation of 0.034%) in determining the amount of fuel in the as-built core, which was based on both destructive and nondestructive measurements made during core manufacture;
2. The large number (524) of fuel rods nondestructively assayed to determine the fissile fuel content of the spent core;
3. The use of a stratified random sampling technique to select the fuel rods to be assayed, thereby guaranteeing that a representative sample of rods of all types present in the core and from all parts of the core were assayed;
4. The high precision achieved in the nondestructive assay of the spent fuel rods (standard deviation of 0.2% to 0.5% for most rods);
5. The use of two different types of nondestructive assay measurements (thermal and epithermal) to provide a check on the assay results for each fuel rod;
6. The amount of destructive assay work (17 whole fuel rods) and the precision achieved (standard deviation of better than 0.11%), which provided a further check on the validity of the nondestructive assay results; and
7. The overall precision (standard deviation of 0.14%) in the determination of the amount of fissile fuel in the spent core.

The agreement among the nondestructive thermal, nondestructive epithermal, and destructive assay results was excellent, even though they indicated that

the epithermal measurements had a very small bias of about 0.16%. The agreement of the measured fissile inventory ratio (1.0139 ± 0.0014) with prediction (1.0135) was also excellent.

The results demonstrate conclusively that LWBR was a breeder. They show that breeding can be achieved in a light-water reactor using ^{233}U as fissile fuel and the naturally occurring, relatively abundant ^{232}Th as fertile material. Thus, the Light Water Breeder Program which the Department of Energy pursued for more than twenty years has demonstrated and proven unequivocally that ^{233}U - ^{232}Th breeders can be built, operated in light water reactor plants to produce electrical energy, and breed more fissile fuel than they consume. This means that the plentiful domestic supply of low and moderate cost thorium represents a potential resource for providing about fifty times the amount of energy which could be produced using current light water reactors and the domestic supply of low and moderate cost uranium. This light water breeder system could supply the entire electrical energy need of the United States for centuries.

To accomplish this, however, it would be necessary to build shielded, remotely operated, and automated fuel factories, light water reactor plants with somewhat larger reactor vessels than those in current commercial use (because of the approximately 30% lower power density in the LWBR core — see Reference 24 for example), prebreeder cores similar to the LWBR to convert thorium to ^{233}U using available fissile fuel such as ^{235}U (see Reference 25 for example), and fuel reprocessing capability to recover and recycle the bred uranium and the residual thorium in expended cores. The technology to do all of these things is either immediately available or is known to be practical and achievable. The acceptability of associated environmental impacts and estimated costs for such a system of breeders were reported in Reference 26.

There are other points of significance which are by no means as important as proving breeding. The knowledge of the thorium fuel cycle was initially rudimentary, and much work was done as part of LWBR development to determine

accurately the physical properties of this fuel system. A book describing the results of this work has been published (Reference 27). The close agreement between predicted and measured breeding confirms the high accuracy of the nuclear properties of fuel nuclides developed, published, and used in predicting LWBR performance. The neutron regeneration ratio η of ^{233}U which was used for LWBR appears to be accurate to much less than 0.5%.

Similarly, the analytical methods developed and used for LWBR have been shown to be accurate. However, neutron leakage from the seed and blanket modules into the thorium reflector has not been predicted as accurately as it might have been, as indicated by the fissile content of the reflector being about 4% higher than predicted. This result attests to the wisdom of the design approach for the Shippingport demonstration of LWBR to simulate the small overall leakage of a large (greater than 500 MW(e)) LWBR in the small 60 MW(e) Shippingport core. In a large LWBR the inaccuracy of neutron leakage predictions would be less important, but it would nevertheless be desirable to improve the predictions of neutron leakage. Also, the LWBR destructive assay work at Argonne National Laboratory demonstrated that the buildup of radioactive ^{232}U in LWBR was less than predicted, and this would reduce the predicted radiation levels of recycled fuel.

The primary significance of proving breeding in LWBR is the demonstrated potential for greatly increasing our nation's electrical energy generation capability for many years to come.

X. ACKNOWLEDGMENTS

A number of people have made technical contributions to the LWBR Proof-of-Breeding program beyond those cited in the references in the following section. Specific acknowledgment is given to the work of A. C. Kahler, H. C. Hecker, and C. J. Simon.

The supervision of S. Milani, I. Goldberg, and W. J. Babyak is gratefully acknowledged. The nondestructive assay operations were carried out at the Naval Reactors Expended Core Facility by a staff of engineers and technicians under the supervision of D. S. Duncan and W. F. Irvine.

At several stages of the nondestructive assay work, an independent review committee of experimental physicists investigated the program and provided many helpful suggestions. Committee members included S. S. Glickstein, C. J. Emert, and (in the early stages of the work) J. J. Volpe, all of Bettis Atomic Power Laboratory; R. E. Slovacek of Knolls Atomic Power Laboratory; and (in the latter stages of the work) S. Milani, formerly of Bettis.

Finally, the overall direction of H. F. Raab of the Division of Naval Reactors, United State Department of Energy, throughout the program is gratefully acknowledged.

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APPENDIX A1The Stratum Average Module Loading Model

The objective of this appendix is to derive the stratum average estimation model, including the estimate of the module loading and its variance.

The stratum average method of estimating a module loading begins with the rod loading model as follows: let

$$L_{tjiu} = \eta_{tjiu} + \epsilon_{Atjiu} \quad (A1-1)$$

where L_{tjiu} is the observed loading in grams fissile for the u^{th} rod of rod stratum i from module j in module stratum t ,

ϵ_{Atjiu} is the random assay measurement error for the u^{th} rod, where these errors are assumed to be independently distributed with $E(\epsilon_{Atjiu})=0$, and $\text{Var}(\epsilon_{Atjiu})=\sigma^2_{Atji}$, which is constant over all rods within a stratum,

and η_{tjiu} is the true mean loading of the u^{th} rod of rod stratum i from module j of module stratum t .

However, the rods in the i^{th} stratum form a rod population that can be described by

$$\eta_{tjiu} = \eta_{tji} + \epsilon_{Ptjiu}, \quad (A1-2)$$

where η_{tji} is the true mean stratum loading,

and ϵ_{Ptjiu} is the random population error where $E(\epsilon_{Ptjiu})=0$, and whose variance-covariance function is

$$\text{Var}(\epsilon_{ptjiu}, \epsilon_{ptjiu'}) = \begin{cases} \sigma^2_{ptji}, & \text{for } u=u' \\ \frac{-1}{N_{tji}-1} \sigma^2_{ptji}, & \text{for } u \neq u' \end{cases} \quad (\text{A1-3})$$

indicating that these errors are correlated within stratum.

The stratum average loading is the average loading, $\bar{L}_{tji}$, of the sampled rods in the stratum, n_{tji} ; that is,

$$\bar{L}_{tji} = \frac{1}{n_{tji}} \sum_{u=1}^{n_{tji}} L_{tjiu}. \quad (\text{A1-4})$$

The variance of the stratum average can be derived by substituting Equations (A1-2) and (A1-1) into Equation (A1-4) as follows:

$$\begin{aligned} \text{Var}(\bar{L}_{tji}) &= \text{Var}\left[\frac{1}{n_{tji}} \sum_{u=1}^{n_{tji}} (\epsilon_{Atjiu} + \epsilon_{ptjiu}) \right] \\ &= \frac{\sigma^2_{Atji}}{n_{tji}} + \frac{1}{n_{tji}^2} \left[n_{tji} \sigma^2_{ptji} + n_{tji}(n_{tji}-1) \left(\frac{-1}{N_{tji}-1} \sigma^2_{ptji} \right) \right] \\ &= \frac{1}{n_{tji}} \left[\sigma^2_{Atji} + \frac{N_{tji} - n_{tji}}{N_{tji} - 1} \sigma^2_{ptji} \right]. \end{aligned} \quad (\text{A1-5})$$

Redefining the rod sampling variance σ^2_p according to a convention discussed by Cochran (Reference 18), Equation (A1-5) becomes

$$\text{Var}(\bar{L}_{tji}) = \frac{1}{n_{tji}} \left[\sigma^2_{Atji} + \frac{N_{tji} - n_{tji}}{N_{tji}} S^2_{ptji} \right], \quad (\text{A1-6})$$

$$\text{where } S^2_{ptji}/N_{tji} = \sigma^2_{ptji}/(N_{tji}-1), \quad (\text{A1-7})$$

and $\frac{N_{tji} - n_{tji}}{N_{tji}}$ is called the finite population factor.

The variance of the module loading is obtained by noting that the total loading for a stratum is simply N_{tji} times $\bar{L}_{tji}$ and then summing over the strata in the module. Thus, the module loading variance for the stratum average method of estimation is

$$\begin{aligned} \text{Var}(\sum_i N_{tji} \bar{L}_{tji}) &= \sum_i \frac{N_{tji}^2}{n_{tji}} \left[\sigma_{Atji}^2 + \frac{N_{tji} - n_{tji}}{N_{tji}} S_{Ptji}^2 \right] \\ &= \sum_i \frac{N_{tji}^2}{n_{tji}} \sigma_{Atji}^2 + \sum_i N_{tji} \frac{N_{tji} - n_{tji}}{n_{tji}} S_{Ptji}^2 \quad (\text{A1-8}) \end{aligned}$$

Estimation of Variance

An unbiased estimate of the variance of $\bar{L}_{tji}$ can be obtained from

$$s_{tji}^2 = \frac{n_{tji}}{\sum_{u=1}^{n_{tji}} (L_{tjiu} - \bar{L}_{tji})^2} / (n_{tji} - 1) . \quad (\text{A1-9})$$

To see this, consider the expected value of the sum of squares:

$$\begin{aligned} E\{ \sum_u [L_{tjiu} - \bar{L}_{tji}]^2 \} &= E\{ \sum_u [(\epsilon_{Atjiu} + \epsilon_{Ptjiu}) - (\bar{\epsilon}_{Atji} + \bar{\epsilon}_{Ptji})]^2 \} \\ &= E\{ \sum_u [(\epsilon_{Atjiu} - \bar{\epsilon}_{Atji}) + (\epsilon_{Ptjiu} - \bar{\epsilon}_{Ptji})]^2 \} \quad (\text{A1-10}) \end{aligned}$$

Since the ϵ_{Atjiu} are independent of the ϵ_{Ptjiu} and of each other (i.e., $E(\epsilon_{Atjiu}, \epsilon_{Atjiu'}) = E(\epsilon_{Atjiu}, \epsilon_{Ptjiu}) = 0$), this component can be derived separately as follows:

$$\begin{aligned} E\{ \sum_u [\epsilon_{Atjiu} - \bar{\epsilon}_{Atji}]^2 \} &= E\{ \sum_u [\epsilon_{Atjiu} (1 - \frac{1}{n_{tji}}) - \frac{1}{n_{tji}} \sum_{u' \neq u} \epsilon_{Atjiu'}]^2 \} \\ &= \{ \sum_u \frac{(n_{tji} - 1)^2}{n_{tji}^2} E(\epsilon_{Atji}^2) + \sum_u \frac{1}{n_{tji}^2} \sum_{u' \neq u} E(\epsilon_{Atjiu'}^2) \} \end{aligned}$$

$$\begin{aligned}
&= \frac{(n_{tji}-1)^2}{n_{tji}} \sigma^2_{Atji} + \frac{(n_{tji}-1)}{n_{tji}} \sigma^2_{Atji} \\
&= \frac{(n_{tji}-1)}{n_{tji}} (n_{tji}-1 + 1) \sigma^2_{Atji} \\
&= (n_{tji}-1) \sigma^2_{Atji} \tag{A1-11}
\end{aligned}$$

For the second term of Equation (A1-10) consider the following:

$$\begin{aligned}
E\left\{ \sum_u (\epsilon_{ptjiu} - \bar{\epsilon}_{ptji})^2 \right\} &= E\left\{ \sum_u [\epsilon^2_{ptjiu} - 2 \epsilon_{ptjiu} \bar{\epsilon}_{ptji} + \bar{\epsilon}^2_{ptji}] \right\} \\
&= E\left\{ \sum_u \epsilon^2_{ptjiu} - \frac{2}{n_{tji}} \sum_u \epsilon_{ptjiu} \sum_{u'} \epsilon_{ptjiu'} \right. \\
&\quad \left. + n_{tji} \left[\sum_{u'} \frac{\epsilon_{ptjiu'}}{n_{tji}} \right]^2 \right\} \\
&= \sum_u E[\epsilon^2_{ptjiu}] - \frac{1}{n_{tji}} E\left[\sum_u \epsilon_{ptjiu} \right]^2 \\
&= \sum_u E[\epsilon^2_{ptjiu}] - \frac{1}{n_{tji}} \{ E\left[\sum_u \epsilon^2_{ptjiu} \right] + E\left[\sum_u \sum_{u'} \epsilon_{ptjiu} \epsilon_{ptjiu'} \right] \} \\
&= n_{tji} \sigma^2_{ptji} - \frac{1}{n_{tji}} \{ n_{tji} \sigma^2_{ptjiu} + n_{tji} (n_{tji}-1) \left[-\frac{\sigma^2_{ptji}}{n_{tji}-1} \right]^2 \} \\
&= (n_{tji}-1) \left[1 + \frac{1}{n_{tji}-1} \right] \sigma^2_{ptji} \\
&= (n_{tji}-1) \frac{n_{tji} \sigma^2_{ptji}}{n_{tji}-1} \\
&= (n_{tji}-1) S^2_{tji} \tag{A1-12}
\end{aligned}$$

where Equation (A1-7) has been used in the last step of Equation (A1-12).

Thus, combining Equations (A1-11) and (A1-12), we have

$$E[s^2_{tji}] = \sigma^2_{Atji} + S^2_{Ptji}.$$

If an independent unbiased estimate, $\hat{\sigma}^2_{Atji}$, of the measurement error variance is obtained from replicated assays or other considerations, then an unbiased of the sampling error variance can be obtained from

$$\hat{S}^2_{Ptji} = s^2_{tji} - \hat{\sigma}^2_{Atji}. \quad (A1-13)$$

Substituting σ^2_{Atji} and S^2_{Ptji} into Equation (A1-8) yields an estimator of the variance of module loading:

$$\hat{\text{Var}}(\sum_i N_{tji} \bar{L}_{tji}) = \sum_i \frac{N_{tji} (N_{tji} - n_{tji})}{n_{tji}} s^2_{tji} + N_{tji} \hat{\sigma}^2_{Atji}.$$

It was found for the LWBR modules that the stratum population variance, S^2_p , dominated the assay measurement variance, σ^2_A , for most strata.

APPENDIX A2The Linear Regression Module Loading Model

The objective of this appendix is to derive the linear regression estimation model, including the estimate of the module loading and its variance.

The basic rod loading model is

$$L_{tj i u} = \eta_{tj i u} + \epsilon_{tj i u} \quad (A2-1)$$

where $L_{tj i u}$ is the observed PIFAG loading in grams fissile for the u^{th} rod of rod stratum i from module j in module stratum t ;

$\epsilon_{tj i u}$ is the random measurement error for the u^{th} rod, where

$$E(\epsilon_{tj i u})=0, \text{ and } \text{Var}(\epsilon_{tj i u})=\sigma^2_{\Lambda t j i}$$

$\sigma^2_{\Lambda t j i}$ is the variance for rod type t , stratum i , module j , constant over all rods in the stratum;

and $\eta_{tj i u}$ is the true mean loading of u^{th} rod of rod stratum i from module j of module stratum t .

The true mean loading can be further modeled by

$$\eta_{tj i u} = \theta_{t j i} + \beta_{t j} q_{t i u} + \sum_{v=1}^2 \alpha_{v t j i} w_{v j i u} + \epsilon_{p t j i u} \quad (A2-2)$$

where θ_{tji} is the intercept for the i^{th} rod stratum in module j for module stratum t ;

q_{tju} is the predicted PDQ loading for the u^{th} rod of rod stratum i for module stratum t , where PDQ assumed a symmetric core;

β_{tj} is the slope parameter for the true loading to PDQ loading relationship over all strata in the module;

w_{vtjiu} is a variable that identifies whether the u^{th} rod of rod stratum i in module j of module stratum t belongs to sub-stratum v (where $v=1$ for seed and standard blanket modules, and $v=1,2$ for power flattening blanket and reflector modules), which is generally referred to as a water channel effect,

$$w_{vtjiu} = \begin{cases} 1, & \text{if rod } tjiu \text{ belongs to sub-stratum } v, \\ 0, & \text{if not;} \end{cases} \quad (\text{A2-3})$$

α_{vj} is the parameter measuring the effect of water channel effect v ;

ϵ_{ptjiu} is the random sampling error from the population of rods in the i^{th} rod stratum in module j of module stratum t , where

$$E(\epsilon_{ptjiu})=0, \text{ and } \text{Var}(\epsilon_{ptjiu})= \sigma^2_{ptji} .$$

For the stratum average model (Appendix A1), it was stated that the errors were correlated within stratum and that the stratum population variances generally dominated the error measurement variances. Two simplifications are made here for the linear regression estimation. First, since the finite population correction factor can generally be ignored when the sampling ratio, n/N , is about 10% or less, which is true generally true in the sampling plan except for the 84 inch high and medium zone power flattening

blanket strata and some reflector strata, the covariance term was ignored for the regression estimate. Thus, the errors were assumed to be independent for all rods in the module and the population correction factor was not applied. The effect of this is at worst to overestimate the actual variance, an occurrence that is not a concern for this problem. Second, the linear regression estimation procedure greatly reduced the differences between the error measurement variances and stratum population variances, and among the stratum population variances themselves. Thus, the assumptions of independence and homogeneity for the error terms in the linear regression model (Equation A2-2) are reasonable.

By reparameterizing the model by subtracting out the PDQ stratum averages and the water channel averages, the model can be written in its most useful form as

$$L_{tj i u} = \eta_{tj i} + \beta_{tj} (q_{t i u} - \bar{q}_{t i}) + \sum_{v=1}^2 \alpha_{vtj i} (w_{vtj i u} - \bar{w}_{vtj i}) + \epsilon_{\eta t j i u} \quad (A2-4)$$

where $\eta_{tj i}$ is the mean stratum loading ($\eta_{tj i} = \theta_{tj i} + \beta_{tj} \bar{q}_{t i} + \sum_v \alpha_{vtj i} \bar{w}_{vtj i}$),

$\bar{q}_{t i}$ is the PDQ average for the sampled rods,

$\bar{w}_{vtj i}$ is the ratio of the rods in stratum i that belong to substratum v to the total number rods in stratum i ,

$$\text{and } \epsilon_{\eta t j i u} = \epsilon_{ptj i u} + \epsilon_{Atj i u}, \quad (A2-5)$$

$$\text{where } \sigma_{L_{tj i u}}^2 = \sigma_{ptj i}^2 + \sigma_{Atj i}^2.$$

In the linear regression analysis to follow, it was found that the variance terms could be reasonably assumed to be constant. Thus, in the remainder of this derivation,

$$\sigma^2_{\text{L}_{tjiu}} = \sigma^2_{tj} \quad \text{for all values of } i \text{ and } u.$$

To derive the linear regression estimation equations for module loading, matrix notation will be used. Since the discussion will dwell on a single module loading, the subscripts specifying the specific module will be dropped in this derivation.

Let

$$\underline{L} = \underline{X} \underline{\beta} + \underline{\epsilon}, \quad (\text{A2-6})$$

where $\underline{L}$ is the $n \times 1$ column vector of assay results,

where $n = \sum_i n_i$ is the total number of rods sampled,

$\underline{X}$ is the $n \times p$ matrix of input values and will be described fully below,

$\underline{\beta}$ is the $p \times 1$ column vector of parameters to be estimated

$$= (\eta_1 \quad \eta_2 \quad \dots \quad \eta_r \quad \beta \quad \alpha_1 \quad \alpha_2)^T$$

where r is the number of strata in the module,

and $p = r + 3$ is the total number of parameters estimated,

$\underline{\epsilon}$ is the $n \times 1$ column vector of random errors which will be assumed to be independently and identically distributed with

$$E[\underline{\epsilon}] = \underline{0}, \quad \text{and} \quad \text{Var}(\underline{\epsilon}) = \underline{I} \sigma^2,$$

with σ^2 assumed equal for all u ($u=1, \dots, n$).

The input or design matrix $\underline{X}$ can be partitioned as follows:

$$\underline{X} = [\underline{x}_1 \ \underline{x}_2 \ \dots \ \underline{x}_p \ \tilde{\underline{q}} \ \tilde{\underline{w}}_1 \ \tilde{\underline{w}}_2]$$

where $\underline{x}_i = \{ x_{i1} \ \dots \ x_{iu} \ \dots \ x_{in} \}^T$

$$\text{and } x_{iu} = \begin{cases} 1 & , \text{ if rod } u \text{ belongs to the } i^{\text{th}} \text{ stratum,} \\ 0 & , \text{ otherwise,} \end{cases}$$

$$\tilde{\underline{q}} = \{ \tilde{q}_{11} \ \tilde{q}_{12} \ \dots \ \tilde{q}_{1n_1} \mid \tilde{q}_{21} \ \dots \ \tilde{q}_{2n_2} \mid \dots \mid \tilde{q}_{s1} \ \dots \ \tilde{q}_{sn_s} \}^T$$

where $\tilde{q}_{iu} = q_{iu} - \bar{q}_i$ with q_{iu} being the loading predicted by the PDQ physics model, and $\bar{q}_i$ being the average for the i^{th} stratum,

$$\tilde{\underline{w}}_v = \{ \tilde{w}_{v11} \ \dots \ \tilde{w}_{vin} \mid \tilde{w}_{v21} \ \dots \ \tilde{w}_{v2n} \mid \dots \mid \tilde{w}_{vs1} \ \dots \ \tilde{w}_{vsn} \}^T$$

where $\tilde{w}_{viu} = w_{viu} - \bar{w}_{vi}$

$$\text{with } w_{viu} = \begin{cases} 1 & , \text{ if rod } u \text{ of stratum } i \text{ is located} \\ & \text{in a row next to the } v^{\text{th}} \text{ water} \\ & \text{channel for } v=1,2, \\ 0 & , \text{ otherwise} \end{cases}$$

and $\bar{w}_{vi}$ is the average value for the i^{th} stratum.

Following the usual least squares procedures gives the parameter estimates, $\underline{b}$, and the variance-covariance matrix of $\underline{b}$ as

$$\underline{b} = (\underline{X}^T \underline{X})^{-1} \underline{X}^T \underline{L} \quad , \quad (\text{A2-7})$$

$$\text{Var}(\underline{b}) = (\underline{X}^T \underline{X})^{-1} \sigma^2 \quad , \quad (\text{A2-8})$$

where

$$\underline{X}^T \underline{X} = \begin{bmatrix} n_1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & n_2 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots & & & \\ 0 & 0 & \dots & n_s & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & \tilde{q}^T \tilde{q} & \tilde{q}^T \tilde{w}_1 & \tilde{q}^T \tilde{w}_2 \\ 0 & 0 & \dots & 0 & \tilde{q}^T \tilde{w}_1 & \tilde{w}_1^T \tilde{w}_1 & \tilde{w}_1^T \tilde{w}_2 \\ 0 & 0 & \dots & 0 & \tilde{q}^T \tilde{w}_2 & \tilde{w}_1^T \tilde{w}_2 & \tilde{w}_2^T \tilde{w}_2 \end{bmatrix}$$

The inverse of the $\underline{X}^T \underline{X}$ matrix can be written

$$(\underline{X}^T \underline{X})^{-1} = \begin{bmatrix} 1/n_1 & 0 & \dots & 0 & | & \\ 0 & 1/n_2 & \dots & 0 & | & \\ \dots & & & & | & \underline{0} \\ 0 & 0 & \dots & 1/n_s & | & \\ \hline & & & \underline{0} & | & (\underline{Z}^T \underline{Z})^{-1} \end{bmatrix}$$

where $\underline{Z}^T \underline{Z}$ is the lower right-hand corner of $\underline{X}^T \underline{X}$.

Let $\tilde{\underline{X}}$ be the $N \times p$ matrix of input values, x_{i_u} , that accounts for all the rods in the module, $N = \sum_i N_i$, where $u=1, \dots, N$. Then, the estimated loading of all of the rods in the module can be expressed by

$$\hat{\underline{L}} = \tilde{\underline{X}} \underline{b},$$

and the total module loading can be estimated by summing all of these rod loadings as follows:

$$\begin{aligned}
\sum_{u=1}^N \hat{L}_u &= \underline{1}^T \hat{\underline{L}} = \underline{1}^T \tilde{\underline{X}} \underline{b} , \\
&= \sum_i N_i [\bar{L}_i + b(\delta_i - \bar{q}_i) + \sum_v a_v (\Omega_{vi} - \bar{w}_{vi})] . \quad (A2-9)
\end{aligned}$$

The variance of the total module loading becomes

$$\begin{aligned}
\text{Var} \left(\sum_{u=1}^N \hat{L}_u \right) &= \text{Var} \left(\underline{1}^T \hat{\underline{L}} \right) \\
&= \underline{1}^T \tilde{\underline{X}} (\tilde{\underline{X}}^T \tilde{\underline{X}})^{-1} \tilde{\underline{X}}^T \underline{1} \sigma^2 \quad (A2-10) \\
&= \left[\sum_i N_i^2 / n_i + \underline{V}^T (\underline{Z}^T \underline{Z})^{-1} \underline{V} \right] \sigma^2
\end{aligned}$$

where

$$\underline{V} = \begin{bmatrix} \sum_i N_i (\delta_i - \bar{q}_i) \\ \sum_i N_i (\Omega_{1i} - \bar{w}_{2i}) \\ \sum_i N_i (\Omega_{2i} - \bar{w}_{2i}) \end{bmatrix} ,$$

and

$$(\underline{Z}^T \underline{Z})^{-1} = \begin{bmatrix} \text{Var}(b) & \text{Cov}(b, a_1) & \text{Cov}(b, a_2) \\ \text{Cov}(b, a_1) & \text{Var}(a_1) & \text{Cov}(a_1, a_2) \\ \text{Cov}(b, a_2) & \text{Cov}(a_1, a_2) & \text{Var}(a_2) \end{bmatrix} \frac{1}{\sigma^2} .$$

Estimation of Variance Term

The variance terms above can be estimated by replacing the residual variance, σ^2 , by its unbiased estimate s^2 , where

$$\begin{aligned}
 s^2 &= \frac{1}{(n-p)} \sum_u [L_u - \hat{L}_u]^2 \quad . \\
 &= \frac{1}{(n-p)} \left\{ \sum_i \sum_{u=1}^{n_i} [L_{iu} - \bar{L}_i - b(q_{iu} - \bar{q}_i) \right. \\
 &\quad \left. - \sum_v a_v (w_{v i u} - \bar{w}_{vi})]^2 \right\} \quad . \quad (A2-11)
 \end{aligned}$$

Appendix A3The Module-to-Module Variance

The objective of this appendix is to derive the variance and the estimated variance functions for the module-to-module variation.

If the LWBR core could be perfectly loaded and operated in a symmetric manner, then all module loadings for a given type would be exactly the same. This is not possible in the real world, so the variation that does occur will be represented by a random variable, μ_{tj} , in the simplified model in which measurement errors are ignored:

$$\eta_{tj} = \bar{\eta}_t + \mu_{tj} \quad , \quad (A3-1)$$

where η_{tj} is the true loading of the j^{th} module of module stratum t ,

$\bar{\eta}_t$ is the true mean module loading for module stratum t ,

and μ_{tj} is the module-to-module error for which

$$E(\mu_{tj}) = 0 \quad ,$$

$$\text{Var}(\mu_{tj}) = \sigma_{\mu t}^2$$

$$\text{and} \quad \text{Cov}(\mu_{tj}, \mu_{tj'}) = \frac{1}{K_t - 1} \sigma_{\mu t}^2 \quad ,$$

with $j=1,2,\dots, M_t$.

That is, the module loadings are correlated within module-type.

The total loading for the t^{th} module stratum is estimated from the sample module loadings by

$$L_t = (1/m_t) \sum_{j=1}^{m_t} M_t \hat{L}_{tj} , \quad (\text{A3-2})$$

where M_t is the number of modules in the core for module stratum t . The variance of the module stratum total loading estimate, ignoring measurement and rod sampling errors, is

$$\text{Var}(L_t) = (M_t/m_t)^2 \text{Var}\left(\sum_{j=1}^{m_t} \hat{L}_{tj}\right) . \quad (\text{A3-3})$$

Let the measured module loading model be

$$L_{tj} = \bar{\eta}_t + \mu_{tj} + \epsilon_{\eta tj} , \quad (\text{A3-4})$$

where $\epsilon_{\eta tj}$ is the error made in estimating the module loading, and since we are ignoring these errors in this section, it can be seen that the module-to-module variance for Equation (A3-3) can be determined from consideration of the model in Equation (A3-1) only. That is,

$$\begin{aligned} \text{Var}\left(\sum_{j=1}^{m_t} \eta_{tj}\right) &= m_t \sigma_{\eta t}^2 + m_t(m_t-1) \left[-\frac{1}{M_t-1} \sigma_{\eta t}^2 \right] \\ &= m_t \sigma_{\eta t}^2 \left[\frac{M_t - 1 - (m_t-1)}{M_t - 1} \right] \\ &= [m_t (M_t - m_t) / (M_t - 1)] \sigma_{\eta t}^2 \\ &= [m_t (M_t - m_t) / M_t] S_{\eta t}^2 \\ &= [m_t (1 - m_t/M_t)] S_{\eta t}^2 \end{aligned} \quad (\text{A3-5})$$

where $S^2_{Mt} = [M_t/(M_t-1)] \sigma^2_{Mt}$ is the same kind of variance redefinition for a finite population that was applied to the rod strata in Appendix A1. The term $(1 - m_t/M_t)$ is a finite population correction factor for modules.

The variance in the total loading for all modules from stratum t is, from Equation (A3-3), $(M_t/m_t)^2$ times the variance given in Equation (A3-5). That is,

$$\text{Var}(L_t) = [M_t (M_t - m_t) / m_t] S^2_{Mt} . \quad (\text{A3-6})$$

Estimation of Module-to-Module Variance

The natural estimate of S^2_{Mt} is the usual sum of squared deviations from the average,

$$s^2_{Mt} = \frac{1}{m_t - 1} \sum_{j=1}^{m_t} (\hat{L}_{tj} - \bar{L}_t)^2 , \quad (\text{A3-7})$$

or equivalently, from equation (A3-4),

$$= \frac{1}{m_t - 1} \sum_{j=1}^{m_t} [(\mu_{tj} - \bar{\mu}_t) + (\epsilon_{\eta tj} - \bar{\epsilon}_{\eta t})]^2$$

where for this calculation any differences in the number of rods in a module is accounted for.

Since the module loading estimation errors, $\epsilon_{\eta tj}$, are independent of the module-to-module errors, μ_{tj} , the expected value of s^2_{Mt} can be separated into two parts:

$$E[s^2_{mt}] = \frac{1}{m_t - 1} \{ E[\sum_{j=1}^{m_t} (\mu_{tj} - \bar{\mu}_t)^2] + E[\sum_{j=1}^{m_t} (\epsilon_{\eta t j} - \bar{\epsilon}_{\eta t})^2] \} \quad (A3-8)$$

By applying the variance conditions of Equation (A3-1) and following the same procedure as for the evaluation of the rod population variance in the expected value of the rod stratum variance estimate of Appendix A1, the first term in the bracket above can be shown to be

$$E[\sum_{j=1}^{m_t} (\mu_{tj} - \bar{\mu}_t)^2] = (m_t - 1) S^2_{mt} ,$$

and by performing a procedure similar to that done for the assay measurement errors in Appendix A1, the second term in the bracket of Equation (A3-8) can be shown to be

$$E[\sum_{j=1}^{m_t} (\epsilon_{\eta t j} - \bar{\epsilon}_{\eta t})^2] = [(m_t - 1)/m_t] \sum_{j=1}^{m_t} \sigma^2_{\eta t j} .$$

Thus,

$$E[s^2_{mt}] = S^2_{mt} + (1/m_t) \sum_{j=1}^{m_t} \sigma^2_{\eta t j} .$$

Therefore, an unbiased estimate of the module population variance S^2_{mt} is

$$\hat{S}^2_{mt} = s^2_{mt} - (1/m_t) \sum_{j=1}^{m_t} \hat{\sigma}^2_{\eta t j} , \quad (A3-9)$$

where $\hat{\sigma}^2_{\eta t j}$ is the estimated variance for the loading of the j^{th} module of module stratum t .

APPENDIX B

Comparison of Predicted Isotopic Distributions to Destructive Analysis

The destructive analysis of 17 fuel rods performed by the Argonne National Laboratory (ANL) provided isotopic distributions of fuel and fission products, as well as the fissile loadings discussed in Section VI.F. This appendix compares data for whole rods and for segments of rods to predictions made using the LWBR nuclear analysis model, which is based on the PDQ diffusion theory-depletion program and the CINDER fission product program (Section IV).

Whole Rod Data

Table B-1 compares the predicted to measured uranium isotopic contents for the 17 fuel rods destructively analyzed. The measured data in this table and the succeeding tables of this appendix are labeled "ANL". The comparison to the calculations are in the form of experiment-to-calculation ratios and are labeled "E/C".

The beginning-of-life nuclide concentrations input to PDQ and CINDER for the depletion calculations were uniform over a fuel rod type or composition. For example, all high zone seed rods (including rods C10 and L29 in Table B-1) were assigned the same beginning-of-life nuclide concentrations in the calculations. In reality, each rod in the core had its own set of as-built nuclide concentrations. The approximation of using composition average concentrations for all rods of a given type makes only a small difference in predicted individual rod ^{233}U loadings, but can lead to substantial errors in the predicted loadings of higher mass isotopes in individual rods. This is due to the fact that the controls on ^{233}U content of the fuel rods during manufacture were tighter than the controls on the other isotopes. For this reason, a correction for the deviation of the as-built individual rod isotopic

inventories from the composition average has been applied to the uranium isotope predictions before forming the experiment-to-calculation ratios in this appendix.

The approximate correction for as-built inventories was made as follows: For ^{234}U , ^{235}U , and ^{236}U the lifetime process is primarily a buildup; so the end-of-life calculated isotopic loading was incremented by the difference between beginning-of-life individual rod and beginning-of-life composition average loadings. This assumes that the increment does not deplete. For ^{238}U the lifetime effect is essentially depletion, so the ratio of beginning-of-life individual rod to beginning-of-life composition average loadings multiplied the end-of-life calculated loading. For ^{233}U in binary fuel zones (Section II), the ratio correction like ^{238}U was used in most cases; the increment correction like ^{234}U was used for low zone standard blanket rod A49.

From Table B-1 it is seen that the discrepancies in ^{233}U , the most abundant isotope, are between -1.6% and +0.3% except for one reflector rod where a -2.5% discrepancy occurs. The agreement is excellent, in view of the extensive amounts of depletion and conversion that occurred at different rates in different regions of the core. This attests to the overall accuracy of predicting the important reactions. Experimental errors for ^{233}U (and for ^{234}U) are a few hundredths of one percent. The pattern of experiment falling slightly below calculation is the same as discussed in Section VI.

The ^{234}U content, the second most abundant of the U isotopes, has discrepancies ranging from -4% to +3%, except for rod A49 which has a +9% difference. Rod A49 is the only water channel rod in the seed or blanket which was destructively assayed. As pointed out in Section VI, rods next to the water channels showed a pattern of having more fissile fuel than predicted. This same pattern may contribute to the discrepancies in the higher mass isotopes.

^{235}U and ^{236}U appear in the smallest quantities among the isotopes formed by neutron capture in the initial ^{233}U . The ^{235}U discrepancies range from -5% to +8% except for water channel rod A49 which is +18% discrepant. The ^{236}U discrepancies range from -5% to +9% except for this rod which is +23% discrepant. For both of these isotopes as well as ^{234}U , the calculated quantities are somewhat low for most rods.

The ^{238}U is essentially all residue from the initial amount and is accurately predicted. Experimental errors for ^{235}U , ^{236}U , and ^{238}U are several tenths of one percent.

The ^{232}U arises from an (n,2n) reaction on ^{232}Th , with ^{231}Pa being an intermediate isotope in the chain. Experimental errors for ^{232}U are a few percent. The large discrepancy between prediction and measurement, with E/C averaging about 0.7, indicates that the cross sections used in the calculations were probably not correct. In fact, recent investigations have uncovered an error in the processing of data for the (n,2n) cross section which would account for about half the discrepancy. This isotope chain, from ^{232}Th to ^{232}U , has only a small effect on core performance; its main significance lies in the handling and reprocessing of fissile fuel, because of gamma activity in the ^{232}U decay chain.

Segment Data

During the destructive assay, the fuel rods were sheared into axial segments, with the larger diameter rods divided into more segments than the smaller diameter rods. Calculational results were integrated over the same segments. Tables B-2 through B-7 compare segment data to predictions for two seed rods, two standard blanket rods, one power flattening blanket rod, and one reflector rod. The data compared are ^{233}U content and ^{137}Cs content. The ^{137}Cs was calculated with the CINDER fission product code, with fluxes input from PDQ calculations (Section IV). Then a correction was applied to the CINDER data to improve its accuracy compared to the PDQ calculation. The

correction was the ratio of the depletion (fissions per cm^3) as calculated in PDQ to the depletion as calculated in CINDER. The production of ^{137}Cs closely follows depletion.

Consider first the ^{233}U comparison. Experimental errors are several hundredths of one percent. For the five seed and blanket rods, the range of discrepancies in all the segments except the end segments is -2.8% to +1.8%; this includes thorium and binary segments, and indicates a very good description of the axial distributions of ^{233}U depletion and Th conversion in the main part of the core. The end reflector discrepancies are as large as 23.6% in rod A49, the water channel rod, and 9.6% in the other seed and blanket rods. These segments have much less ^{233}U than the others. The end reflector comparison shows that too little leakage is predicted; this is similar to the trend of predicting too little radial leakage to be discussed below. The top end segment of rod F73 is an exception to this trend. One other trend may be noted: starting at the top of each rod, the E/C values are high, gradually reducing to a minimum in the middle of the rod, and then increasing again toward the bottom.

In the reflector rod, many segments have several percent less fuel than predicted, though four segments at or near the rod ends have more fuel than predicted, as with the other rods. Overall agreement is not as good as with the other rods, with the predicted loading being 2.5% high. This reflector rod was near the apex of the module, close to the power flattening blanket region of the adjacent module (Figures V-12 and II-1). In Section VI.E, it is pointed out that there is a trend in reflector rod loadings such that rods near the power flattening blanket have less fuel than predicted, but rods far from the power flattening blanket have more fuel than predicted. Rod B1 is consistent with this observation. For the reflector as a whole, however, the calculation predicts too little radial leakage and has too small a fissile loading (Tables VI-12 to VI-15). The axial distribution trends in the reflector rod are similar to those in the seed and blanket rods.

The ^{137}Cs isotope has a 30.7 year half-life and is a good indicator of depletion. For the five seed and blanket rods, the range of discrepancies is -7% to +12% except for end segments. The range varies considerably depending on the rod location; for rod F73, the range is -7% to -2%; for rod A49, the range is +4% to +12%. The axial distribution is thus predicted within an 8% range. The end blankets have very small amounts of ^{137}Cs and are not predicted very well. The experiment-to-calculation ratios are smaller for the reflector rod than for the seed and blanket rods, consistent with the ^{233}U behavior. The axial trends from top to bottom observed for the ^{233}U distributions occur also in the ^{137}Cs distributions in most cases. The experimental errors are a few tenths of one percent.

Conclusions

The uranium isotopic distributions were predicted very well with two exceptions: ^{232}U , for which an error in the cross sections used in the calculational model was discovered, and the rod next to the water channel, which indicates a more general problem with the calculational model.

The ^{233}U axial distributions were also predicted very well except in the thorium end segments where very little fissile fuel was generated. Too little leakage is predicted axially, as well as radially. The ^{137}Cs shows distribution patterns which are similar to the ^{233}U distributions but more variable. This is because ^{137}Cs reflects burnup of ^{233}U while the ^{233}U itself also reflects the creation of new material.

The comparisons in this appendix served to check the accuracy of the PDQ-CINDER calculational model, but were not used in the determination of fissile loading from the PIFAG measurements, with one exception. The exception is the very small correction for ^{232}U described in section VI.A. The calculated amount of ^{232}U present at end of life was reduced by the E/C ratio of 0.7 discussed previously in this appendix.

TABLE B-1. ANL MEASURED URANIUM ISOTOPIC CONTENTS VERSUS PREDICTIONS
(Loadings in Grams)

Module	Cell	^{232}U		^{233}U		^{234}U	
		ANL	E/C	ANL	E/C	ANL	E/C
Seed	C10	0.023	0.69	25.22	0.999	2.51	1.02
	L29	0.024	0.69	22.48	0.988	2.74	1.01
	M49	0.023	0.71	17.07	0.998	2.12	1.01
	N63	0.022	0.72	15.31	0.998	1.88	0.99
	P39	0.022	0.75	13.60	0.987	1.69	1.02
Std Blkt	E56	0.072	0.66	51.45	0.991	5.23	1.01
	D24	0.061	0.66	46.50	0.990	4.35	1.01
	C3	0.061	0.69	40.06	0.984	3.59	1.03
	A49	0.075	0.72	35.94	1.003	3.87	1.09
PF Blkt	E31	0.049	0.70	50.75	0.987	4.94	1.01
	F73	0.038	0.67	53.14	1.002	4.27	0.98
	H1	0.011	0.62	57.18	1.000	2.51	1.00
	D29	0.047	0.71	44.57	0.989	4.40	1.01
	C13	0.024	0.67	33.64	0.997	2.21	0.96
	B62	0.036	0.74	28.96	0.995	2.27	1.02
Reflector	E3	0.014	0.72	23.68	1.001	0.68	1.03
	B1	0.036	0.66	34.64	0.975	1.49	1.03

TABLE B-1 (Cont'd)

Module	Cell	^{235}U		^{236}U		^{238}U	
		ANL	E/C	ANL	E/C	ANL	E/C
Seed	C10	0.40	1.01	0.030	1.05	0.083	1.00
	L29	0.46	1.04	0.040	1.07	0.082	1.00
	M49	0.37	1.02	0.032	1.03	0.073	1.01
	N63	0.34	1.00	0.036	1.08	0.051	1.00
	P39	0.29	1.04	0.028	1.07	0.044	1.00
Std Blkt	E56	0.88	1.04	0.075	1.02	0.12	1.00
	D24	0.73	1.01	0.070	1.03	0.10	0.98
	C3	0.59	1.03	0.044	1.06	0.073	1.00
	A49	0.70	1.18	0.066	1.23	0.036	1.01
PF Blkt	E31	0.78	1.06	0.087	1.09	0.15	1.00
	F73	0.62	1.01	0.069	1.04	0.15	1.00
	H1	0.23	1.01	0.014	0.99	0.46	1.00
	D29	0.71	1.05	0.076	1.08	0.12	1.01
	C13	0.28	0.95	0.023	0.95	0.078	1.01
	B62	0.32	1.03	0.025	1.07	0.047	1.02
Reflector	E3	0.045	1.05	0.0010	1.08	0.0016	NA
	B1	0.14	1.08	0.0050	0.99	0.0024	NA

TABLE B-2. ANL MEASURED AXIAL DISTRIBUTIONS VERSUS PREDICTIONS
FOR HIGH ZONE SEED ROD IN CELL C10, MODULE I-1

ANL Segment	Segment Height (inches)	^{233}U		^{137}Cs	
		ANL(gm)	E/C	ANL(10^{20} atoms)	E/C
1, Thoria (Top)	10.1	0.32	1.048	0.093	1.12
2, Binary	21.4	6.58	1.006	5.92	0.99
3, Binary	21.2	5.68	1.001	8.92	0.97
4, Binary	21.2	5.61	0.986	9.13	0.99
5, Binary	21.2	6.84	0.997	5.16	0.99
6, Thoria	9.6	0.19	1.050	0.037	1.14
Total		25.22	0.999	29.25	0.98

TABLE B-3. ANL MEASURED AXIAL DISTRIBUTIONS VERSUS PREDICTIONS
FOR LOW ZONE SEED ROD IN CELL P39, MODULE I-1

ANL Segment	Segment Height (inches)	^{233}U		^{137}Cs	
		ANL(gm)	E/C	ANL(10^{20} atoms)	E/C
1, Thoria (Top)	17.9	0.89	1.048	0.53	1.09
2, Thoria	17.6	1.71	1.005	2.31	1.03
3, Thoria	17.4	1.83	0.972	3.04	1.01
4, Binary	21.3	3.92	0.979	10.19	0.99
5, Binary	21.2	5.06	0.989	5.80	0.99
6, Thoria	8.9	0.20	1.077	0.043	1.21
Total		13.60	0.987	21.91	1.03

TABLE B-4. ANL MEASURED AXIAL DISTRIBUTIONS VERSUS PREDICTIONS
FOR HIGH ZONE STANDARD BLANKET ROD IN CELL E56, MODULE I-3

ANL Segment	Segment Height (inches)	^{233}U		^{137}Cs	
		ANL(gm)	E/C	ANL(10^{20} atoms)	E/C
1, Thoria (Top)	11.2	0.42	1.030	0.047	1.05
2, Binary	14.3	8.74	1.000	4.82	1.01
3, Binary	14.1	8.46	0.986	10.35	0.98
4, Binary	14.1	8.39	0.983	13.88	0.98
5, Binary	14.1	8.32	0.987	14.45	0.97
6, Binary	14.1	8.17	0.988	12.55	0.97
7, Binary	14.2	8.18	0.992	7.26	0.99
8, Thoria	9.4	0.77	1.042	0.16	1.12
Total		51.45	0.991	63.52	0.98

TABLE B-5. ANL MEASURED AXIAL DISTRIBUTIONS VERSUS PREDICTIONS
FOR LOW ZONE STANDARD BLANKET ROD IN CELL A49, MODULE I-3

ANL Segment	Segment Height (inches)	^{233}U		^{137}Cs	
		ANL(gm)	E/C	ANL(10^{20} atoms)	E/C
1, Thoria (Top)	11.4	0.55	1.236	0.080	1.45
2, Binary	14.2	6.12	1.018	4.79	1.12
3, Binary	14.0	6.55	0.991	10.57	1.06
4, Binary	14.1	6.89	0.984	14.33	1.04
5, Thoria	13.9	5.88	0.990	9.50	1.08
6, Thoria	10.5	4.14	0.993	5.73	1.04
7, Thoria	14.0	4.25	1.014	3.92	1.07
8, Thoria	13.0	1.56	1.064	0.52	1.13
Total		35.94	1.003	49.45	1.06

TABLE B-6. ANL MEASURED AXIAL DISTRIBUTIONS VERSUS PREDICTIONS
FOR HIGH ZONE POWER FLATTENING BLANKET ROD IN CELL F73, MODULE III-6

ANL Segment	Segment Height (inches)	^{233}U		^{137}Cs	
		ANL (gm)	E/C	ANL (10^{20} atoms)	E/C
1, Thoria (Top)	8.0	0.22	0.962	0.019	0.87
2, Binary	18.0	12.07	1.015	5.37	0.93
3, Binary	17.5	10.52	1.001	11.59	0.95
4, Binary	17.5	10.14	0.999	13.84	0.95
5, Binary	17.5	10.36	0.994	11.95	0.97
6, Binary	14.5	9.39	0.996	5.06	0.98
7, Thoria	11.4	0.45	1.096	0.058	1.21
Total		53.14	1.002	47.88	0.96

TABLE B-7. ANL MEASURED AXIAL DISTRIBUTIONS VERSUS PREDICTIONS
FOR REFLECTOR ROD IN CELL B1, MODULE IV-3

ANL Segment	Segment Height (inches)	^{233}U		^{137}Cs	
		ANL (gm)	E/C	ANL (10^{20} atoms)	E/C
1, Thoria (Top)	6.4	0.17	1.062	0.007	1.05
2, Thoria	7.0	0.72	0.993	0.086	0.94
3, Thoria	7.0	1.54	0.968	0.36	0.88
4, Thoria	7.0	2.29	0.934	0.80	0.81
5, Thoria	7.0	3.01	0.960	1.48	0.90
6, Thoria	7.0	3.39	0.938	1.96	0.88
7, Thoria	7.0	3.66	0.933	2.35	0.89
8, Thoria	7.0	3.72	0.930	2.48	0.91
9, Thoria	7.0	3.66	0.932	2.36	0.89
10, Thoria	6.9	3.56	0.961	2.27	0.96
11, Thoria	7.0	3.24	0.972	1.79	0.95
12, Thoria	7.0	2.61	0.981	1.11	0.94
13, Thoria	7.0	1.85	1.021	0.53	0.99
14, Thoria	7.0	0.96	1.055	0.15	1.07
15, Thoria	6.0	0.26	1.099	0.014	1.13
Total		34.64	0.975	17.74	0.91