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REGGEON FIELD THEORY AND THE PHASES OF QCD *

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Abstract

We propose a Reggeon Field Theory phase diagram involving Sub-critical and Super-critical Pomeron behavior and the Expanding Disc. We describe the derivation of Reggeon Field Theory from QCD using infra-red analysis of the reggeon diagrams of the spontaneously broken theory. Matching the Reggeon Field Theory phase-diagram to that of lattice QCD with many fermions has significant implications for the chiral properties of continuum QCD when the number of flavors is less than the maximum allowed by asymptotic freedom.

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1. Introduction

Historically Reggeon Field Theory produced the first application[1,2] to High Energy Physics of the renormalization group treatment of Critical Phenomena. It was developed before QCD as describing the appropriate degrees of freedom for hadron interactions at *high energy* and *low momentum transfer*. Nowadays we think of hadron interactions almost exclusively in terms of QCD. We do, however, identify two kinematic regimes, that is the ultra-violet or short-distance region where perturbation theory and the parton model is believed to be valid and the infra-red or long-distance region where confinement and chiral symmetry breaking are believed to appear and lattice gauge theory provides perhaps the most promising formulation of the theory.

Since the Regge region is a mixed infra-red and ultra-violet regime in which the dynamics clearly couples infra-red and ultra-violet effects, Reggeon Field Theory could be a powerful formalism for studying the dynamical interrelation and self-consistency of any description of the two regimes in QCD. In particular the phase-diagram for lattice gauge theories as a function of the number of fermions seems to display interesting structure[3,4] which has received little attention as yet, but which could potentially have great significance for the continuum limit defining QCD. Since this particular degree of freedom should be especially relevant in the Regge region it is perhaps a-priori plausible that there might be a direct relationship between the phase-diagram of Reggeon Field Theory and that of lattice QCD involving the number of flavors. This is in fact what we shall propose in this talk.

There have been both theoretical[3] and numerical analyses[4] suggesting that as the number of flavors is increased in lattice QCD a new first-order phase-transition appears in which both the chiral and confinement properties of the theory undergo significant change. A major conclusion from the arguments we shall present here is that this phase-transition is essentially the Critical Pomeron phase-transition[1,2] of Reggeon Field Theory. This enables us to be more specific about the nature of the transition as well as its flavor dependence. Indeed it was argued in Ref. 3 that the new transition while being bad news for strong-coupling expansions in lattice QCD does not destroy the chiral symmetry breaking properties of the weak-coupling theory (and therefore—it was hoped—the same properties of the continuum theory). In fact we shall suggest that the transition involves chiral symmetry restoration via parity doubling of the physical (fermion) states. As a consequence the weak-coupling theory will actually have parity-doubling chiral symmetry restoration for all numbers of flavors *except* that giving Critical Pomeron behavior in the continuum theory. We have previously suggested[5] that Critical Pomeron behavior is necessary for the parton-model to be realized in hadron scattering at short distances in QCD. If the above connection of the Critical Pomeron to a chiral symmetry restoring transition in QCD is established then we similarly see its importance from the infra-red standpoint.

Our argument proceeds in several stages. We begin in Section 2 by describing the phase-diagram of Reggeon Field Theory for a single Pomeron Regge pole. We identify the Critical Pomeron,[1,2] the Super-Critical Pomeron[6] and the Expanding Disc.[7] The Super-Critical Pomeron is identified as containing a Pomeron condensate and an odd-signature vector Regge trajectory exchange-degenerate with the Pomeron. We also describe how the parity-doubling of fermion states is removed as the Pomeron becomes Super-Critical.

In Section 3 we describe the derivation of Reggeon Field Theory from QCD beginning with the reggeon diagrams of spontaneously broken QCD. We show how the Super-Critical Pomeron emerges from infra-red divergences as the gauge-symmetry is restored to SU(2). The Pomeron condensate is identified with a *winding-number condensate* whose origin can, we suggest, also be understood as arising from the regularization of the infinite momentum quark sea in the presence of instantons. The Critical Pomeron occurs as the gauge symmetry is restored to SU(3). The cut-off dependence of this last step relates directly to the flavor dependence of asymptotic freedom. From this we can deduce the flavor dependence of the Reggeon Field Theory phase diagram.

We briefly review what is known about the flavor dependence of lattice QCD in Section 4 focussing on the "chiral" and "deconfinement" transitions which are argued to occur respectively from the corresponding β-functions of the theory.

Finally, in Section 5, we bring all the lattice and Reggeon Field Theory arguments together into what we argue is a single consistent picture. Effectively the "chiral" transition on the lattice is identified with the Critical Pomeron transition and the "deconfinement" transition is identified with the transition to the expanding disc of Reggeon Field Theory. We discuss the general implications of this identification—the most striking of which is that QCD with less than the maximum number of flavors allowed by asymptotic freedom, if it exists as a continuum theory, does not have the chiral symmetry breaking properties of the strong-coupling lattice theory.

2. The Phase-diagram of Reggeon Field Theory

The RFT lagrangian for a Pomeron Regge pole with trajectory $\alpha_{\mathbb{P}}(t)$ is, near the critical point $\Delta = 1 - \alpha_{\mathbb{P}}(0) = 0$,

$$\begin{aligned} \mathcal{L}(\bar{\psi}, \psi) = & \bar{\psi} \frac{\overleftrightarrow{\partial}}{\partial y} \psi + \alpha' \nabla \psi \nabla \psi + \Delta \bar{\psi} \psi + ir \left[\bar{\psi}^2 \psi + \bar{\psi} \psi^2 \right] \\ & + \lambda \left[\bar{\psi}^3 \psi + \bar{\psi} \psi^3 \right] + 2\mu \left[\bar{\psi}^2 \psi^2 \right] + \ldots . \end{aligned} \tag{2.1}$$

$\bar{\psi}(y, \underline{x})$ and $\psi(y, \underline{x})$ are respectively creation and destruction operators with y rapidity and $\underline{x}$ impact parameter. The Critical Pomeron was discovered[1,2] over a decade ago as a strong-coupling solution of the theory described by (2.1) in which Δ(renormalized) $= 0$ and λ and μ (together with all higher-order couplings) are driven to zero compared to r. The resulting set of scaling laws for diffractive cross-sections and multiplicity distributions is still the only known completely unitary asymptotic description of rising cross-sections.

If $\Delta < 0$ ($\alpha_{\mathbb{P}}(0) > 1$) then the theory has a negative "mass-gap". The potential near the critical point

$$V(\bar{\psi}, \psi) = \Delta \bar{\psi} \psi + ir \left[\bar{\psi}^2 \psi + \bar{\psi} \psi^2 \right] \tag{2.2}$$

has two stationary points as potential new vacua, that is

$$(\bar{\psi}, \psi) = \left(\frac{2i\Delta}{r}, 0 \right), \quad \left(0, \frac{2i\Delta}{r} \right) \tag{2.3}$$

The Super-Critical Pomeron[6] is obtained by shifting ψ on the negative rapidity-axis and shifting $\bar{\psi}$ on the positive rapidity-axis. This generates a Pomeron with intercept

$$\alpha_{\mathbb{P}}(0) = 1 + \Delta \tag{2.4}$$

and *Pomeron source terms*—giving a perturbation expansion as illustrated in Fig. 2.1. By studying the "energy" and momentum singularities of the reggeon graphs we find threshold singularities which imply[6] a vector particle has entered the theory on a Regge trajectory degenerate with that of the Pomeron.

The expanding disc solution for $\Delta < 0$ is obtained[7] by not shifting $\bar{\psi}$ or ψ. Instead the configurations (2.3) contribute as a zero-energy state inside a disc in impact-parameter space that expands with increasing rapidity. For $\Delta \to -\infty$ (with $\lambda, \mu = 0$ *and* a transverse momentum cut-off) the expanding disc solution can be established[8] via a spin model. We shall assume that it is then the unique solution of the problem. If we include λ and μ in the discussion the Super-Critical Pomeron solution is modified to

$$\alpha_{\mathbb{P}}(0) = 1 + \tilde{\Delta}, \tag{2.5}$$

where

$$\tilde{\Delta} = \frac{(-ir + x)x}{2\lambda} \qquad x = \left(-r^2 - 4\lambda\Delta\right)^{1/2} \tag{2.6}$$

The two-Pomeron source term is also modified to

$$s = \tilde{\Delta} + 2(\lambda - \mu)\left[\frac{-ir + x}{2\lambda}\right]^2 . \tag{2.7}$$

Note that there is now a second zero (or critical point) of $\tilde{\Delta}$ at $x = 0$. Also if $\lambda = \mu$ the two Pomeron source term, which represents the physical effect of the Pomeron condensate, vanishes at this point. Consequently the condensate effectively disappears again at this second critical point and it is consistent that the expanding disc now appears. Note that

$$x = 0 \Rightarrow \Delta = \Delta_M \equiv -\frac{r^2}{4\lambda} \equiv -\frac{r^2}{4\mu}, \tag{2.8}$$

and it is known[8] that for $\lambda = 0$ the expanding disc is certainly present at the "magic value" of Δ given by $\Delta = 2\Delta_M$. Consequently if we draw a phase diagram in terms of Δ and another parameter, say λ/r, then we expect something like that shown in Fig. 2.2(a) where the distinct transitions shown for λ small could meet, presumably, if μ (and other higher-order couplings) behave appropriately as λ increases.

We have so far ignored the renormalization effects which will actually be an important part of our discussion—in particular the dependence on the transverse momentum cut-off $\Lambda_\perp$. The one-loop renormalization of the Pomeron intercept gives the only divergent contribution as the cut-off is removed (that is if λ, μ and other higher-order couplings are zero) and moves the critical point $\Delta = 0$ to

$$\Delta_c = \frac{r^2}{\alpha'} \ln \frac{r^2}{\alpha' \Lambda_\perp} + O(r^2). \tag{2.9}$$

We expect such effects to make only the minor modification of Fig. 2.2(a) shown in Fig. 2.2(b).

The physical significance of the Pomeron condensate will become apparent after we relate the Super-Critical Pomeron to QCD. There is, however, an important chiral property of the Critical and Super-Critical Pomeron which we can elaborate on at this stage. It is shown in Ref. 9 that the Critical Pomeron solves the old strong-interaction parity doublet problem. That is for positive t one parity part of a fermion Regge trajectory $j = \alpha_F\left(\sqrt{t}\right)$ is absent from the physical sheet of the j-plane—being hidden by the accumulation of the fermion-Pomeron Regge cuts. For negative t *both parity branches* of the trajectory are physical.

A straightforward model of a sub-critical Pomeron to be expected in QCD would be a string-like theory in which all amplitudes have asymptotic Regge pole behavior with *linear* Regge trajectories and all Regge cuts, including those generated by the Pomeron, are non-leading (in the neighborhood of $t = 0$). In these circumstances it follows from the basic analyticity properties of amplitudes that fermion trajectories must be parity-doubled (McDowell symmetry). In this case fermion states will be parity-doubled until the Pomeron becomes critical and is able to shield one parity. The Super-Critical Pomeron should maintain this shielding through the additional t-channel singularities produced by the new vector particle entering the theory. Consequently we anticipate that the Critical Pomeron phase-transition could be coupled to a chiral phase-transition via the parity-doubling properties of fermion states.

3. Reggeon field Theory from QCD

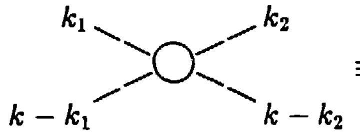
Our starting point here is the perturbative results on the reggeization of massive gluons (and quarks) in spontaneously broken gauge theories.[10] To be specific we consider QCD—that is SU(3) gauge theory—with the gauge symmetry "spontaneously broken" by the Higgs mechanism utilizing two color triplet Higgs fields ϕ_1 and ϕ_2 (with vacuum expectation values initially chosen to give all gluons a common mass M).

In the Regge region the leading power behavior at all orders of perturbation theory (and all logarithms) is given[11] by transverse momentum diagrams which can be organized[12] into reggeon diagrams for massive reggeized gluons—with propagator

$$ \text{---}\bigcirc\text{---} = \Gamma_{1,1}(E = 1 - j, k^2) = \left[E - \Delta(k^2)\right]^{-1} \left[k^2 - M^2\right]^{-1} \tag{3.1} $$

$$ \Delta(k^2) = \frac{g^2}{16\pi^2}(k^2 - M^2) \int \frac{d^2k}{[q^2 - M^2]\,[(k - q)^2 - M^2]} + O(g^4), \tag{3.2} $$

and interaction vertices, for example



$$ \equiv \Gamma_{2,2}\,(k, k_1, k_2) = \frac{g^2\,(k_1^2 - M^2)\,(k_2^2 - M^2)}{(k^2 - M^2)} + \cdots. \tag{3.3} $$

If massive quarks are included they also will lie on Regge trajectories.

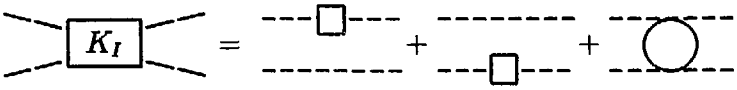
To utilize the reggeon diagrams to discuss unbroken QCD we must remove the gluon mass M by decoupling the Higgs fields ϕ_1 and ϕ_2. We do this, within the reggeon diagrams, by first imposing a transverse momentum cut-off $\Lambda_\perp$. Qualitatively this should be equivalent to introducing a transverse lattice. We also expect it to be possible to define the complete theory—with a cut-off—by the (Borel) sum of the perturbation expansion (which should also be the sum of all reggeon diagrams). Most importantly we assume that the *principle of complementarity*[13]—derived from lattice gauge theory—can be applied directly in that *fundamental* representation Higgs fields can be decoupled smoothly from the theory. Equivalently there is no confinement phase-transition to be encountered in the decoupling.

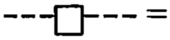
We shall first decouple one Higgs field ϕ_1 so that an SU(2) subgroup of gluons (with mass which we now denote by M_1) becomes massless. The vital relationship to Pomeron reggeon field theory will come at this stage since we shall claim that the Super-Critical Pomeron is produced from the gluon reggeon diagrams as $M_1^2 \to 0$. After this it will be straightforward to relate the decoupling of the second Higgs field ϕ_2 (or the remaining gluon mass $M_2 \to 0$) to the Critical Pomeron.

The starting point in analyzing the infra-red divergences of reggeon diagrams as $M_1^2 \to 0$ is to note that the exponentiation of reggeization is also an exponentiation of infra-red divergences,[14] that is

$$ S^{\Delta(k^2)} \underset{M^2 \to 0}{\sim} \exp\left[\frac{-g^2}{16\pi^2} \ln s \ln\left(\frac{k^2}{M^2}\right) + O(g^4)\right]. \tag{3.4} $$

At first sight this exponentiation sends *all* reggeon diagrams to zero! However, the reggeon interactions also produce exponentiating divergences which can potentially cancel those due to reggeization. The full divergence structure can be analyzed by defining interaction "kernels" for fixed numbers of (elementary) gluons which have definite SU(2) color and which combine the reggeization and interactions e.g.

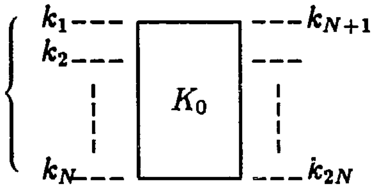
$I = 0, 1, 2$ 

 $= \dfrac{\Delta\,(k^2)}{k^2 - M^2}$ (3.5)

It is found that reggeization dominates *all* color non-zero kernels, that is

$$K_I \underset{M^2 \to 0}{\longrightarrow} -\infty \qquad \text{all } I \neq 0. \tag{3.6}$$

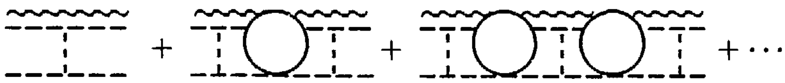
Also any color-zero kernel

$I = 0$ 

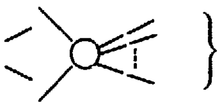
(3.7)

diverges if any subset of all the transverse momenta go to zero. K_0 is finite (and scale-invariant) *only* if *all momenta* are taken uniformly to *zero*.

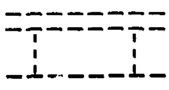
Further exponentiation can result from interaction with the remaining massive reggeized gluons (and quarks) of the theory. For example, if ~~~ denotes the SU(2) singlet gluon which remains massive, exponentiation is produced by

 (3.8)

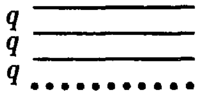
This removes all zero transverse momentum gluon configurations except for those without the appropriate local reggeon interaction. Since such interactions can be computed from a (multiparticle) dispersion relation it can be shown that there will be an interaction provided only that the gluon configuration involved can couple to on-shell physical states—that is

on-shell states  zero transverse momentum gluons $\neq 0$. (3.9)

This is the case *except* for the "unnatural" color charge parity configuration

$\tilde{K}^0 \equiv$ ····· $\equiv$  $\Big\}$ color zero, *axial* vector configuration with *all* transverse momenta zero (3.10)

The infra-red divergence associated with the $\tilde{K}^0$ configuration of SU(2) gluons dominates the $M_1^2 \to 0$ limit. To analyze its effect systematically it is necessary to go to the multiquark S-Matrix and study (exchanged) reggeon scattering amplitudes. We find the surviving reggeons are picked out by the ***infra-red divergence*** $\tilde{K}^0$ ***coupling through (anomalous) fermion loops.*** The resulting "infinite momentum hadrons" have as lowest approximation (in terms of quarks q and antiquark $\bar{q}$)

 $\leftrightarrow$ "meson",  $\leftrightarrow$ "baryon" (3.11)

and they scatter by exchange of the "Pomeron"

 $\leftrightarrow$ IP (3.12)

The infra-red divergence $\tilde{K}^0$ and its coupling through anomalous fermion loops has a straightforward physical interpretation. The simplest contribution to $\tilde{K}^0$ comes from the three-gluon configuration

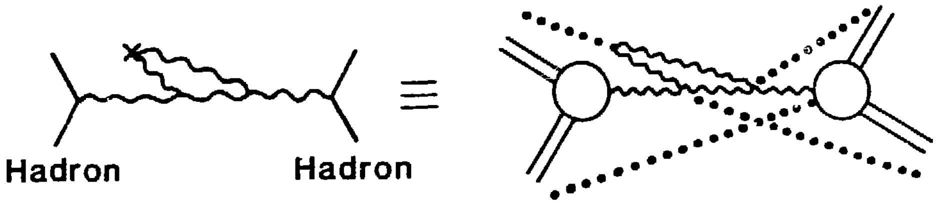
$$K^- = \epsilon_{+\alpha\beta\gamma}\epsilon^{ijk}\left(A^i_\alpha A^j_\beta A^k_\gamma\right)_{P_-=\infty,\ P_+=0,\ P_\perp=0}, \tag{3.13}$$

which is the pure gauge light-cone component of the familiar *winding number operator* defined as a three dimensional integral of the current

$$K^\mu(x) = \frac{g^2}{8\pi^2}\epsilon_{\mu\alpha\beta\gamma}Tr\left[A^i_\alpha\partial_\beta A^i_\gamma - \frac{2ig}{3}\epsilon^{ijk}A^i_\alpha A^j_\beta A^k_\gamma\right]. \tag{3.14}$$

Our infra-red analysis thus produces hadrons effectively defined as color-zero fermion states in an (infinite momentum) *winding-number condensate* with the Pomeron similarly defined as a gluon in the same background field. By close analogy with the Schwinger Model Manton[15] has argued that such a winding-number condensate could indeed arise in QCD as a consequence of the regularization of the *massless* quark sea in the presence of instantons and the anomaly. Remarkably this seems to be just the effect that our infra-red analysis of reggeon diagrams has discovered. Infinite momentum quarks are effectively massless and since the anomalous fermion loop reggeon interactions can be regarded as simulating instanton interactions, the dynamical origin of the winding-number condensate is just as envisaged by Manton.

The real significance of the winding number condensate for us, however, is that it can be identified with the Pomeron condensate of the previous Section. Also the Pomeron trajectory is exchange degenerate with that on which the SU(2) singlet massive gluon lies. Finally, if we develop the higher-order Pomeron interactions we find the Pomeron source diagrams of the Super-Critical theory arise from the presence of the winding-number condensate in the hadron states e.g.



(3.15)

We conclude that QCD with a transverse momentum cut-off and the gauge symmetry broken from SU(3) to SU(2) gives Super-Critical Pomeron high-energy behavior.

That the SU(2) singlet gluon lies on the Pomeron trajectory implies that

$$\alpha_{\mathbf{P}}(M_2^2) = 1, \tag{3.16}$$

and so the limit $M_2^2 \rightarrow 0$ given by the decoupling of the second Higgs multiplet ϕ_2 necessarily gives the Critical Pomeron limit $\alpha_{\mathbf{P}}(0) = 1$. Consequently restoring the full SU(3) gauge symmetry, with a transverse momentum cut-off, can give the Critical Pomeron. However, from (2.9) we see that subsequent variation of the cut-off $\Lambda_\perp$ will immediately move the theory away from the critical surface. In particular if we suppose that Δ, r and α' are dominated by infra-red transverse momenta and are relatively insensitive to $\Lambda_\perp$ then if (2.9) is satisfied for a particular $\Lambda_\perp$ an increase will imply that $\Delta < \Delta_c$ and hence the theory is sub-critical. Consequently we expect to obtain the sub-critical Pomeron by the limit $M_2^2 \rightarrow 0$, with a sufficiently large $\Lambda_\perp$, *unless* we can take the limit $\Lambda_\perp \rightarrow \infty$ first.

To take the limit $\Lambda_\perp \rightarrow \infty$ before $M_2^2 \rightarrow 0$ we require asymptotic freedom of the theory with the Higgs sector ϕ_2 still present. (Otherwise sums of reggeon diagrams will be undefined because of large ultraviolet contributions.) For color triplet quarks this is posible[16] only for

$$N_F \text{ (number of flavors)} = 16. \tag{3.17}$$

We conclude therefore that the Regge limit in QCD, defined by removing the transverse momentum cut-off in reggeon diagrams *after* removing all infra-red (mass) cut-offs, gives the sub-critical Pomeron for $N_F < 16$ and the Critical Pomeron for $N_F = 16$. For $N_F \geq 17$ we enter a new phase of QCD because of the loss of asymptotic freedom. Consequently if we take N_F as one parameter and the gauge coupling g (which we assume decreases via renormalization as $\Lambda_\perp$ increases) as a second parameter then we obtain the phase-diagram of Fig. 3.1. If we assume that Fig. 3.1 maps relatively straightforwardly onto Fig. 2.2(b) then we conclude that, as shown, the expanding disc phase of Reggeon Field Theory is associated with the non-asymptotically free phase of QCD with a large number of quarks.

We have exploited the Higgs mechanism only as a technical device. However, we conclude from Fig. 3.1 that the dynamical effects of the Higgs mechanism can be induced spontaneously, simply by increasing N_F at finite values of g (or equivalently at finite values of the transverse momentum cut-off). In particular the odd-signature trajectory which is an SU(2) singlet gluon trajectory when the Higgs mechanism is used must emerge as an SU(3) singlet. [From our analysis it is clear that the lowest-order approximation to this "odd-signature Pomeron" will be the odd-signature component of four gluons in an odd color charge parity configuration.] Effectively a color singlet massless vector state enters the theory at the critical point, develops a mass as we go into the Super-critical phase and then becomes massless again at the second critical point which is now identified with a phase-transition to the non-asymptotically free phase of QCD. As we note in the next Section this phase is indeed expected to be "Coulomb-like" in lattice gauge theory.

Finally we note from our discussion in the last Section that we expect chiral symmetry to be spontaneously broken as we cross into the Super-Critical phase—if it is represented by massive parity doublets in the sub-critical phase.

4. The Flavor Number Dependence of Lattice QCD

The first study of the phase-diagram to be expected for the N_F-dependence of QCD was by Banks and Zaks.[3] Their arguments combined a study of lattice β-functions, that is the usual β-function β_T (defined essentially from the string tension) and the chiral β-function β_B (defined from the spontaneously-induced baryon mass), with the existence of an infra-red fixed-point in the perturbative β-function for $N_F \lesssim 16\frac{1}{2}$. They concluded that as a function of N_F, β_T develops as shown in Fig. 4.1. This implies the existence of a line of phase-transitions, as shown in Fig. 4.2, which is expected to be a deconfining transition to a coulomb-like phase. For small N_F the chiral β-function β_B is positive for large g^2 and negative or *zero* for small g^2. Consequently a chiral transition is anticipated at some value of g as illustrated in Fig. 4.3. For large N_F, β_B is positive or *zero* for all values of g and there is in fact no reason for the chiral transition to be present, although Banks and Zaks speculated[3] that it persists as illustrated by the dotted line in Fig. 4.3.

The first numerical results for a large number of flavors were published by Kogut *et al.*[4] These results were obtained on a finite temperature (that is asymmetric) lattice of 4×8^3. With eight flavors the plot of Fig. 4.4 is obtained—showing a dramatic first-order transition. Although this was at finite temperature it seems likely[17] (on the based of further unpublished results) that the phase-transition persists at zero temperature. Kogut *et al.* speculated that this "new first-order transition" might be the chiral transition anticipated by Banks and Zaks.

From Fig. 4.4 it is clear that the transition involves a drop in the value of the chiral-condensate $\langle\bar{\psi}\psi\rangle$ and so could be a chiral restoring transition. It was argued by Banks and Zaks that the transition is present for all (low) values of N_F. However, by arguing that the theory can be analytically continued to non-integer N_F, and to $N_F = 0$ in particular, they concluded that chiral symmetry is not restored and so is still present at $g = 0$ for small N_F. This is, of course, essential if continuum QCD can be defined from the lattice and if chiral symmetry breaking is indeed present in the theory for small N_F. However, the simplest interpretation of Figs. 4.3 and 4.4 is surely that chiral symmetry is in fact restored in continuum QCD with eight flavors, apparently producing a major conflict with our understanding of the physical applicability of QCD. We shall elaborate on this issue after bringing the Pomeron phase-transition into the discussion in the next Section.

The chiral transition has been seen numerically for $N_F = 8$ only and appears to be absent for smaller N_F. It could, of course, simply be much weaker for small N_F. If it is absent we would still expect the physical effects of the transition—namely the fermion parity doubling that we shall argue for in the next Section—to develop smoothly as we approach the continuum limit.

5. Reggeon Field Theory and Lattice QCD

As we described in Section 3 our development of Reggeon Field theory from QCD depended on the introduction of a transverse momentum cut-off in gluon reggeon diagrams. Qualitatively we expect this to be analogous to defining lattice QCD on a transverse lattice. If we also assume that the qualitative structure of the QCD phase-diagram, close to the continuum, is independent of the nature of the lattice utilized then we can expect to directly relate the small g^2 Reggeon Field Theory diagram of Fig. 3.1 to that of lattice QCD described in the last Section.

The simplest way to blend Fig. 3.1 with Fig. 4.4 is clearly the phase diagram of Fig. 5.1. Banks and Zaks acknowledged that they had no motivation for the chiral and confinement lines to cross as they do in Fig. 4.4 and so Fig. 5.1 is in fact also consistent with their analysis. Note that while the deconfinement transition is expected in any gauge theory, the chiral (or Critical Pomeron) transition will be present in SU(3) gauge theory but not SU(2) gauge theory. From the Critical Pomeron viewpoint it is clear from Section 3 that the full structure of the SU(3) gauge group is necessary while the Banks and Zaks argument relies on the existence of a spontaneously induced baryon fermion mass as a chiral parameter. (In SU(2) there are no fermion baryons!)

Consider the implications of Fig. 5.1 if we accept it as a valid phase diagram for QCD. It implies

i) the expanding-disc solution of Reggeon Field Theory is identified with a Coulomb-like phase of QCD—this is plausible both from QED based calculations[19] leading to an expanding disc and from the cut-off dependence of the expanding disc[8].

ii) the parity-doubling of fermion trajectories we expect to be associated with the sub-critical Pomeron suggests the chiral transition in lattice QCD involves a restoration of chiral symmetry via (massive) parity-doubled states. Although forbidden in the continuum this is possible on the lattice because the anomaly is not properly represented. [The crucial role of the anomaly in the critical theory is actually apparent[5] when the Critical Pomeron is approached via the Super-Critical Pomeron as in Section 3.]

iii) The Super-Critical Pomeron analysis shows that there is a color singlet gluon state which becomes massless at the critical point—this would explain the jump of the Wilson line value illustrated in Fig. 4.3. It is part of the Pomeron phase-transition that the gluon Regge pole decouples from physical states at the critical point. However, it need not decouple in lattice Green's functions!

iv) if the chiral transition extends down to all numbers of flavors (or its physical effects) then the strong coupling lattice results of confinement and chiral symmetry breaking persist into the weak-coupling asymptotically-free theory only at the Critical Pomeron value $N_F = 16$.

This last point is the most striking since it implies that for $N_F < 16$ continuum QCD, even if it can be defined from the asymptotically-free fixed-point will not have the desired chiral symmetry breaking properties! We have previously argued[5] that $N_F = 16$ or "QCD saturated with quarks" is necessary for a parton-model description of the *soft* high-energy collisions producing Regge behavior and the Critical Pomeron. This has been a prime motivation for arguing that electroweak dynamical symmetry breaking should be due to a doublet of color sextet quarks.[18] We have also argued[18] that the anomaly together with topological properties of the theory can play a special role in the existence of the chiral limit when $N_F = 16$.

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Figure Captions

Fig. 2.1 The Super-Critical Pomeron perturbation expansion.

Fig. 2.2 Reggeon Field Theory phase-diagram (a) without renormalization effects (b) with renormalization.

Fig. 4.1 The variation of β_T as N_F increases.

Fig. 4.2 the line of phase-transitions implied by the development of β_T shown in Fig. 4.1.

Fig. 4.3 The chiral transition required by the zero of β_B.

Fig. 4.4 $N_F = 8$, SU(3) data for $\langle\bar{\psi}\psi\rangle$, the Wilson line WL and the fermion internal energy on a 4×8^3 lattice with bare fermion mass 0.10—Kogut et al.[4]

Fig. 5.1 Phase-diagram obtained by matching Reggeon Field Theory and lattice gauge theory analyses.

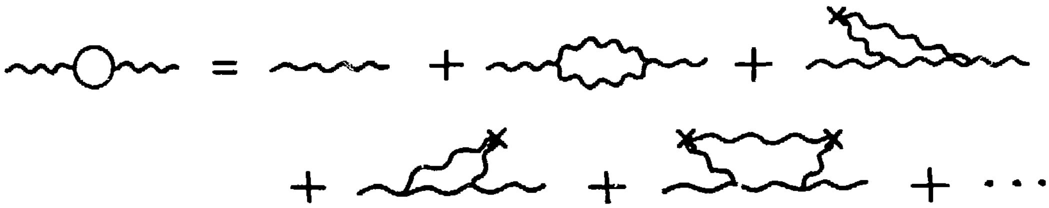


Fig. 2.1

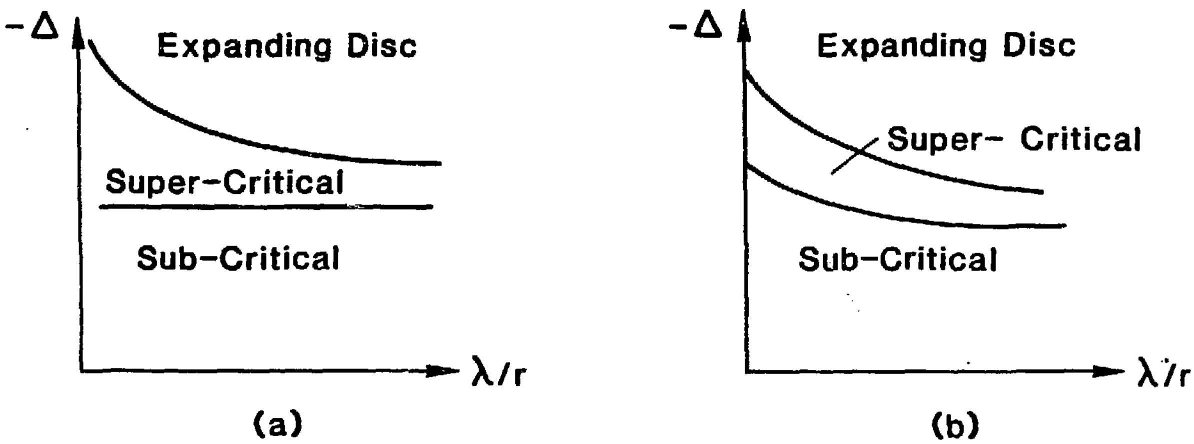

-Δ
Expanding Disc
Super-Critical
Sub-Critical
λ/r
(a)
-Δ
Expanding Disc
Super- Critical
Sub-Critical
λ/r
(b)

Fig. 2.2

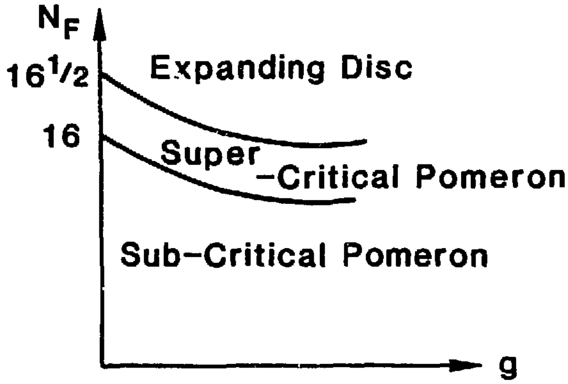
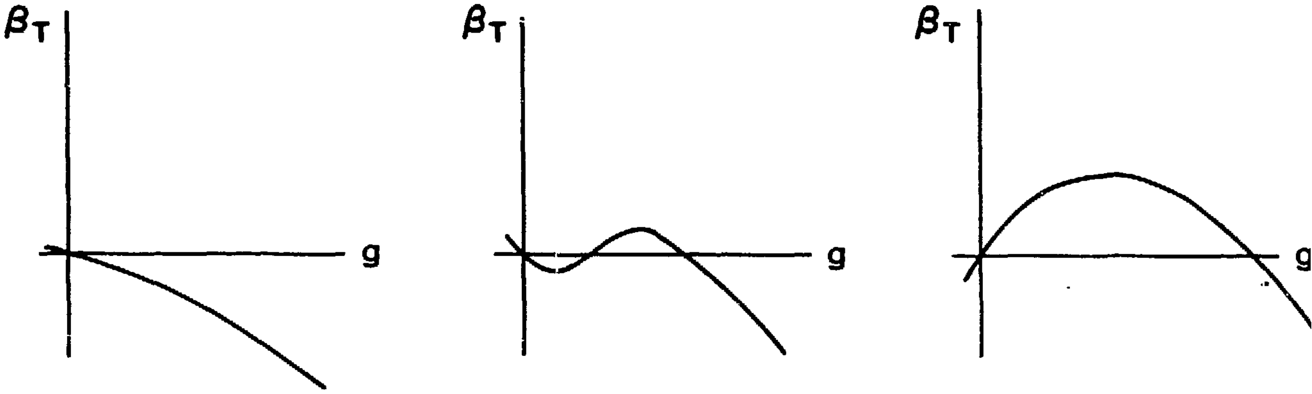


Fig. 3.1



(a) $N_F \ll 16^{1/2}$ (b) $N_F \lesssim 16^{1/2}$ (c) $N_F \gg 16^{1/2}$

Fig. 4.1

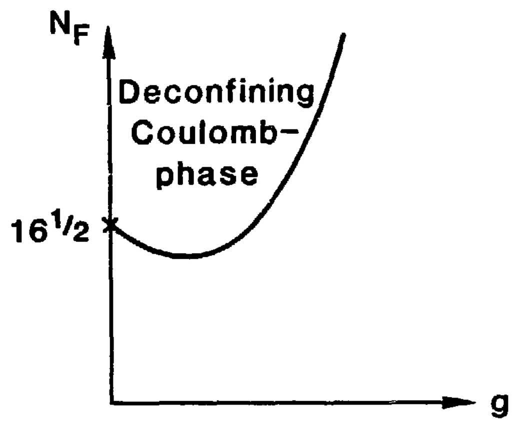


Fig. 4.2

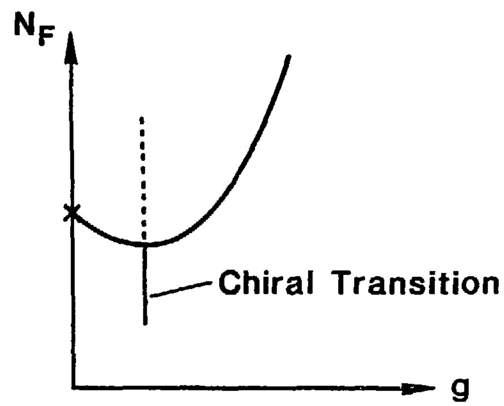
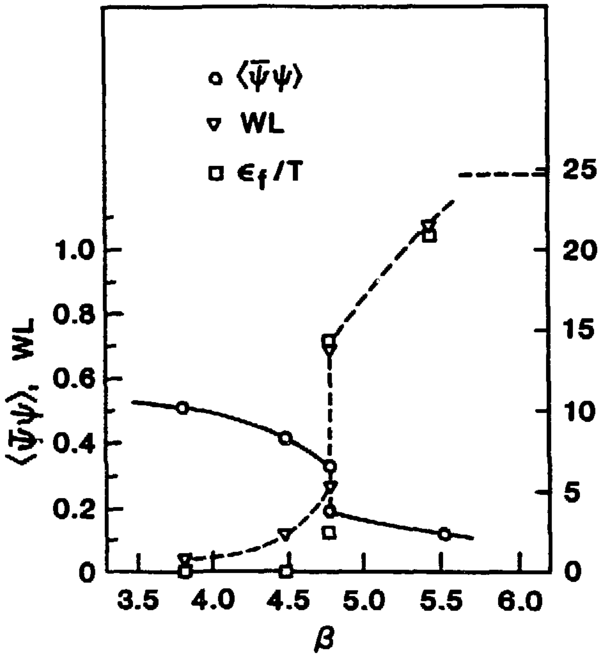
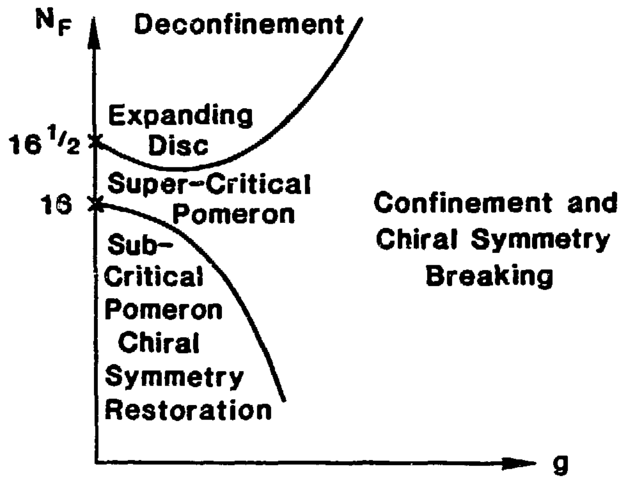


Fig. 4.3



$\langle\bar{\psi}\psi\rangle$
WL
ϵ_f/T
1.0
0.8
0.6
0.4
0.2
0
$\langle\bar{\psi}\psi\rangle$, WL
25
20
15
10
5
0
ϵ_f/T
3.5
4.0
4.5
5.0
5.5
6.0
β

Fig. 4.4



N_F
Deconfinement
Expanding Disc
$16\,^1/_2$
Super-Critical Pomeron
16
Confinement and Chiral Symmetry Breaking
Sub-Critical Pomeron
Chiral Symmetry Restoration
g

Fig. 5.1