

Algorithms and Procedures in the Model Based Control of Accelerators

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## ALGORITHMS AND PROCEDURES IN THE MODEL BASED CONTROL OF ACCELERATORS

Eva Bozoki

### 1. INTRODUCTION

The overall design of a Model Based Control system was presented in [ref.1]. The system consists of PLUG-IN MODULES, governed by a SUPERVISORY PROGRAM and communicating via SHARED DATA FILES. Modules can be added or replaced without affecting the overall system. There can be more than one module (algorithm) to perform the same task. The user can choose the most appropriate algorithm or can compare the results using different algorithms.

Calculations, algorithms, file read and write, etc. which are used in more than one module, will be in a subroutine library. This feature will simplify the maintenance of the system.

The following is a **partial** list of modules, specifying the task they perform.

### 2. MODELING Modules

These modules calculate (i) **transport matrices** for each element and (ii) **machine functions** ( $\beta$ ,  $\alpha$ ,  $\phi$ ,  $\zeta$ ,  $\zeta'$ ) at each point element and before & after each finite-lengths elements.

**Elements** can be: drift, combined function dipole with arbitrary  $\epsilon_1$  and  $\epsilon_2$  beam entry and exit angles ( $B=0$  and  $\epsilon_1=\epsilon_2=0$  describes a pure wedge dipole) and edge focusing, quadrupole, sextupole, undulator, orbit correctors (no affect on the transport matrices, represent point kicks), orbit monitors (no affect on transport matrices, treated like a flag), flags (to specify locations).

Dipoles should be defined as consisting of  $N$  ( $N \geq 1$ ), sectors, with specified ratios of  $B_i$  (and  $B'_i$ ) in the sectors. This feature will keep the number of elements smaller and at the same time allows it to treat a single device with one controllable setpoint as a set of devices with different magnetic fields and gradients.

Elements can be specified as

- (i) belonging to the same family; they all have the same strengths and they have to be changed together in fitting procedures (one controllable setpoint in the on-line mode),
- (ii) coupled; their strength having a fixed ratio (one controllable setpoint in the on-line mode). This feature, for example, can be used to treat magnets fed by the same power supply but which have different number of coil-turns, as well as to treat sextupole content of dipoles, etc.

(iii) composite; their strengths is a sum of more components, (each component corresponding to a different controllable setpoint in the on-line mode).

The **input** to the modeling modules are the (1) lattice and element data, initial conditions, interrelation & availability & performance limit of the elements (DATA BASE), (2) device related data and (3) magnet strengths (see [refs.1,2])

The **output**, containing the name, type, length, transport matrix of the elements as well as the machine functions at the elements (see detailed description in [ref.2]) will serve as input to all other modules where that information is needed.

### 3. APPLICATION DRIVERS

In this section the terms calculate, simulate and match will be used in the following sense:

**calculate** - calculate the present status ("snapshot") of the system.

**simulate** - change magnet strengths ("knob") and calculate snapshots for each change. It is a repeated two-step procedure involving (1) change and (2) calculate.

**match** - perform optimization, fitting variables to satisfy desired conditions. It is a multistep procedure involving iterative use of (1) calculate and (2) optimizer modules

#### 3.1 MACHINE FUNCTION related

\*1 To **calculate** tune.

\*2 To **simulate** (via the use of modeling modules) the behavior of the tune and machine functions when changing ("knob"-ing) quadrupole strength (single quad or a set of quads with a specified ratio)

\*3 To **match** desired conditions, expressed in terms of machine functions - - by adjusting Q's.

The functions to be matched can be:

tune -  $\nu_{x,y}$

value of machine functions at any element -  $\beta_{x,y}, \alpha_{x,y}, \phi_{x,y}, \eta_{x,y}, \eta'_{x,y}$

phase advance between any 2 elements -  $\Delta\phi_{x,y}$

tune change for a given Q strengths change -  $\Delta\nu_{x,y}/\Delta Q$ ,

etc.

\*4 To **calculate** the

synchrotron integrals,

machine-related parameters: tune, momentum compaction, synchrotron integrals, dumping partitions and their rate of change, dumping times, etc.

beam-related parameters: phase space ellipses, uncoupled and coupled beam size, emittance, energy spread, Touchek life time, energy loss/turn, etc.

RF-related parameters: synchrotron phase angle, synchrotron tune, synchrotron frequency, bucket width, bunch length, quantum life-time, etc.

(See formulæ in [ref.3] and in Appendix 1).

- \*5 To **calculate chromaticity** and **simulate** its behavior when changing ("knob"-ing) sextupole strength (single sextupole or a set of them with a specified ratio).
- \*6 To **match** desired chromaticities -- by adjusting Sx's.
- \*7 To **calculate phase space ellipses** at any element in linear machines and transport lines and **simulate** their behavior at different quadrupole strengths.
- \*8 To **match** phase space ellipses (see ref. [4]) -- by adjusting Q's. The parameters of the ellipses (see Fig. 1) to be matched can be:

Twiss parameters:  $\beta_{x,y}$ ,  $\alpha_{x,y}$ ,  $\gamma_{x,y}$

maximum beam size:  $x_{\max}$ ,  $x'_{\max}$ ,  $y_{\max}$ ,  $y'_{\max}$

axes and rotation angle: A, B,  $\theta$   
(see Fig. 1).

- \*9 To **calculate and simulate beam envelope** in linear machines and transport lines.
- \*10 To **calculate, simulate and control positioning** of phase space ellipse (injection, septum,...) -- by adjusting steering magnets and septum strength (see ref. [4]).

### 3.2 ORBIT-related

#### Notation:

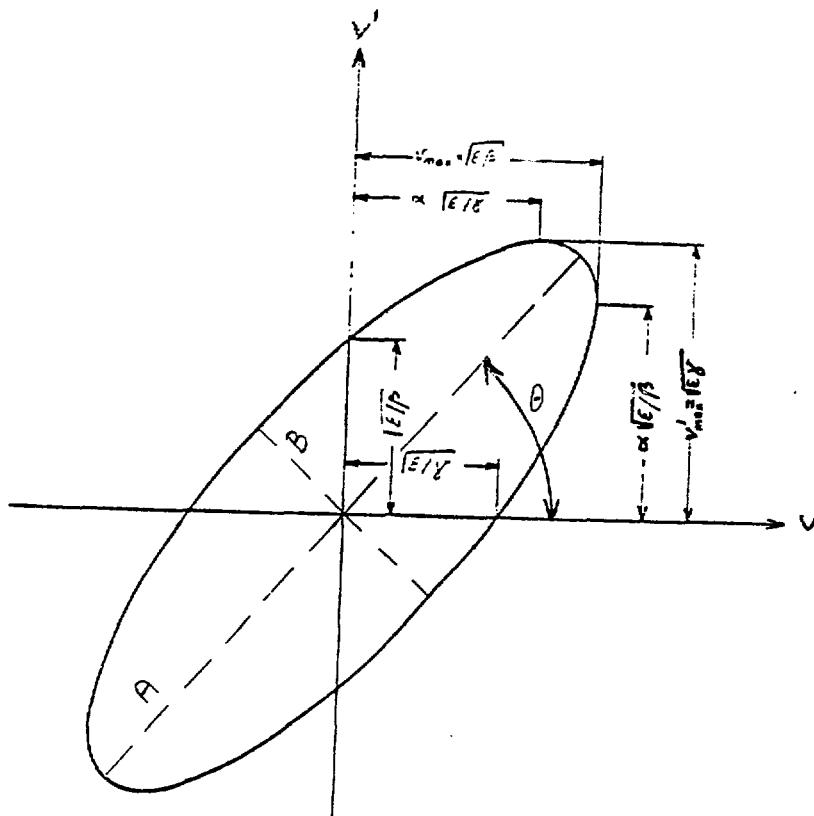
In this section v will be used to refer to either x or y.

The  $\vec{v}$  orbit consists of the  $(v_i, i=1, N_{PUE})$  orbit points. Similarly,  $\vec{\theta}$  refers to the set of  $(\theta_j, j=1, N_{COR})$  corrector strengths.

#### General Remarks:

There should be no difference in the format and in the handling of measured and calculated orbits.

Provide a way to assign weights to measured points to be able to mix measurements of different accuracy (e.g. PUE measurements on the e- beam and measurements on the photon beam an L distance away from the source point)



$$A = \frac{\sqrt{\varepsilon}}{2} \left( \sqrt{H+1} + \sqrt{H-1} \right)$$

$$B = \frac{\sqrt{\varepsilon}}{2} \left( \sqrt{H+1} - \sqrt{H-1} \right)$$

$$\tan(2\theta) = -\frac{2\alpha}{\beta-\gamma}$$

$$\therefore H = \frac{\beta+\gamma}{2}$$

Fig. 1

Provide a way of minimizing other then the  $v_i$  orbit points (e.g. minimize  $(v/\beta)_i$  or  $(v + Lv')_i$ ).

Provide a way to correct either the measured orbit or the bare orbit:

$$\vec{v}^b = \vec{v}^m - \vec{v}_c, \quad \text{where } \vec{v}_c = (A_{ij}) \vec{\theta}$$

Recognize the difference between correcting an  $\vec{x}$  orbit or a  $\vec{\Delta x}$  difference orbit when  $\eta \neq 0$  (in accelerators and storage rings, generally this applies to the horizontal plane). The orbit to be corrected is  $\vec{x} - \frac{\Delta p}{p} \vec{\eta}$  or  $\vec{\Delta x}$ , respectively.

- \*1 To calculate orbit changes due to specified kicks at specified locations ("knob"-ing of correctors).
- \*2 To calculate global orbit correction minimizing the RMS and/or the peak-to-peak orbit using different methods;
  - \*\*1 least square method minimizing orbit and corrector strengths for a given number of correctors to be used (MICADO algorithm, see [ref.5]).  
minimizing the  $\vec{r} = \vec{x} + (A)\vec{\theta}$  residual orbit or rather it's norm  $||\vec{r}||^2$ , yields for the corrector strengths

$$\vec{\theta} = (A^T A)^{-1} A^T \vec{x}. \quad \text{Equ.(1)}$$

MICADO solves this set of linear equation using pivoting and Hausholder transformation, choosing always the "most effective" correctors (needing the smallest strengths).

- \*\*2 PROCEDURE: repeated automatic execution of MICADO, increasing the number of correctors to be used starting from one, and using an algorithm which selects the solution with the "best" combination of residual orbit and necessary total corrector strengths (see [ref.6]).
- \*\*3 eigenvector method [ref. 7], which minimizes orbit only, regardless of the needed corrector strength.

This method solves Equ (1) by replacing  $(A^T A)$  by  $(U L U^T)$ , where  $U = (\vec{U}_1, \dots, \vec{U}_m)$ ,  $\vec{U}_m$  are the normalized eigenvectors of  $(A^T A)$  and  $L$  is the diagonal matrix of the  $\lambda_i$  eigenvalues. Since contribution to the closed orbit from the  $i$ -th eigenvector can be shown to be proportional to the  $i$ -th eigenvalue, those with small  $\lambda_i$  can be ignored reducing the overall strengths of correctors without reducing the quality of orbit correction.

- \*\*4 harmonic methods, where the harmonic content of an orbit is calculated and then the harmonics are corrected by similar harmonic distribution of the field errors [refs. 7,8].

The relationship between the  $A_b$ ,  $A_k$ ,  $B_k$  Fourier coefficients of the orbit and the  $a_o$ ,  $a_k$ ,  $b_k$  Fourier coefficients of the field errors is

$$a_k = \frac{\nu^2 - k^2}{\nu^2} A_k, \quad k = 0, K, \quad b_k = \frac{\nu^2 - k^2}{\nu^2} B_k, \quad k = 1, K$$

The harmonic content of the orbit can be calculated by different methods; by an FFT algorithm [ref. 9] or by a least-square fit of the orbit with a Fourier series as in [ref. 8].

Similarly, the "kicks" needed to correct selected harmonics can be calculated in different ways. To mention only two of the methods, they consist of

i) solving the set of linear equations (as in [ref. 8]):

$$\vec{V} = (A_{kj}) \vec{B},$$

where  $\vec{V}$  is a vector of the  $a_o$ ,  $a_k$ ,  $b_k$ ,  $k=1, K$  Fourier coefficients of the field errors,  $\phi$  is the azimuth angle and the  $A$  matrix is defined as

$$A_{kj} = \frac{\beta_j}{\pi\nu} * \begin{cases} \cos k\phi & \text{if } 0 \leq k \leq K \\ \sin k\phi & \text{if } K < k \leq 2k+1 \end{cases}$$

or ii) calculating the ratio of corrector strengths on all or on suitably positioned correctors, which would lead to a  $\cos k\phi$  or  $\sin k\phi$  orbit distortion. Then correcting the orbit, one harmonics at a time, working always on the most significant one.

Due to this variety of calculation techniques, some of the so called harmonic orbit corrections are in reality hybrid methods combining Fourier analysis and least square method.

\*\*5 A hybrid PROCEDURE can be developed using harmonic analysis and least-square correction of the orbit as follows:

A. Perform harmonic analysis (see 3.2.5) on the orbit.

B. Working on the most significant  $A_k$ ,  $B_k$  components (one at a time)

1. calculate the  $x_{A_k}^i = A_k \cos k\phi_i$  or  $x_{B_k}^i = B_k \sin k\phi_i$ :

harmonic orbit (see 3.2.6), which is the  $A_k$  or  $B_k$  "content" of the original orbit.

2. correct  $x_{A_k}^i$  or  $x_{B_k}^i$  with methods 3.2.1 or 3.2.2 or 3.2.3.

C. Implement the calculated corrections, remeasure the orbit and repeat A-C, if necessary.

\*\*6 "piece-wise" method [refs. 4, 10], where only the correctors, downstream of a monitor have effect on the orbit at that monitor.

It means solving the following equation for  $\vec{\theta}$ :

$$\vec{x} + (R) \vec{\theta} = 0$$

$$\text{where } R_{ji} = \begin{cases} \text{COR}_j \rightarrow \text{MON}_i & \text{if } \text{COR}_j \text{ is downstream of } \text{MON}_i \\ M_{12} & \\ 0 & \text{otherwise} \end{cases}$$

$M^{A \rightarrow B}$  is the transport matrix from A to B

\*3 To calculate & perform **local orbit changes** moving the beam at specified locations by specified x and/or x' (using 3 or 4 corrector algorithms) [refs. 4, 11]. Provide a way to either let the program choose the closest available correctors or be able to specify which correctors to use.

\*4 To **compare** two orbits,  $\vec{v}_1$  and  $\vec{v}_2$ , by calculating their difference and its RMS value

$$\vec{\Delta v} = \vec{v}_2 - \vec{v}_1 \rightarrow \begin{cases} \vec{\Delta x}, \vec{\Delta y} \\ \Delta x^{\text{RMS}}, \Delta y^{\text{RMS}} \end{cases}$$

Note, that  $\vec{v}_2 = \vec{0} \rightarrow \vec{\Delta v} = -\vec{v}_1$  (sign reversal)

$$\vec{v}_1 = \vec{0} \rightarrow \Delta v^{\text{RMS}} = v_2^{\text{RMS}}$$

\*5 To **perform harmonic analysis** on measured & calculated orbits either using an FFT algorithm (as in [ref. 9]) or using a least square fit (as in [ref. 8]).

\*6 To **calculate harmonic orbits** from specified  $A_0, A_k, B_k, k=1, K$  Fourier coefficients [ref. 12] as  $v_i^k = A_k \cos k\phi_i$  and  $v_i^k = B_k \sin k\phi_i$ .

#### 4. MEASUREMENT SUPPORT AND DIAGNOSTICS

\*1 To **measure orbit** with **validity checking** of raw data, offering automatic, default and manual disabling of PUE's (as in [ref. 13]).

\*2 **Consistency checking** of measured orbits to exclude "unphysical" orbit points or diagnose unexpected "kick". This can be done for example by first calculating  $v_i^m$  from the measured  $v_{i-1}$  and  $v_{i+1}$  as in [ref. 3a] and then comparing the calculated and measured orbit points for all PUE's ( $i=1, N_{PUE}$ ). If  $|v_i^m - v_i^c|$  is larger than the accuracy of orbit measurement, then either

- $v_i^m$  is wrong (PUE was not working correctly or orbit was outside the linear region, etc.), or
- there is a kick (misaligned magnet, gradient error, etc.) between the  $(i-1)$ th and  $(i+1)$ th PUE.

This method, when applied to different orbits, might prove to be a good diagnostic tool.

\*3 To **measure emittance** from beam-size observations (preferably digitized and under computer control) at different points along the beam (as e.g. in [ref. 14]).

\*\*1 The **projection method** uses the transport matrix between points A and B to project the measured beam size at B to point A (in the  $x$ - $x'$  and  $y$ - $y'$  phase space). The area defined by the intersection of these lines with the ones representing the measured beam size at A, represent an upper limit for the beam ellipse (Fig. 2). The more and at carefully selected points the beam-size is measured, the more accurate the estimate will be.

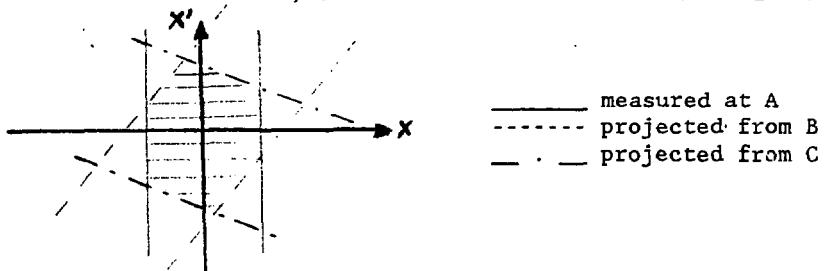


Fig. 2

\*\*2 The **transformation method** compares the  $\sigma_i^m$   $i=1, N$  measured and  $\sigma_i^c = \left\{ \left[ T_{li}(1,1)\beta_1 + T_{li}(1,2)\alpha_1 + T_{li}(1,3)\gamma_1 \right] \epsilon \right\}^{1/2}$  calculated beam sizes.  $\alpha_1, \beta_1, \gamma_1$  ( $\beta\gamma - \alpha^2 = 1$ ) are the Twiss parameters at point-1 and  $T_{li}$   $i=1, N$  are the transport matrices for the  $\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}$  vector. For  $N=3$ , the method yields the unique solution for  $\alpha_1, \beta_1, \gamma_1$  and  $\epsilon$  while for  $N > 3$ , a least-square estimate can be obtained by minimizing

$$\sum_{i=1}^N (\sigma_i^m - \sigma_i^c)^2$$

\*4 PROCEDURE for dispersion measurement

Since dispersion is measured by comparing orbits which were measured at different RF frequencies, a procedure can be developed to:

- A. Measure an  $\vec{x}_1$  orbit (Module 4.1),
- B. Change RF frequency (utility module),
- C. Measure the  $\vec{x}_2$  orbit (Module 4.1),
- D. Compare  $\vec{x}_1$  and  $\vec{x}_2$  (Module 3.2.4) calculating  $\vec{\Delta x}$  as well as  $\Delta x_{rms}$  and  $\Delta x_{aver}$
- E. Calculate  $\vec{\xi}$ ,  $\xi_{rms}$  and  $\xi_{aver}$ .

\*5 PROCEDURE for response matrix measurement (as e.g. in [ref. 15])

The  $(A_{ij})$   $i=1,N$ ,  $j=1,N$  response matrix can be measured as  $A_{ij} = \Delta v_{ij} / \Delta \theta_j$  where  $\Delta v_{ij}$  is the orbit change on the  $i$ -th monitor due to a  $\Delta \theta_j$  kick on the  $j$ -th orbit corrector. The procedure to measure  $(A_{ij})$  consists of

- A. Measuring a  $\vec{v}_{ref}$  reference orbit (Module 4.1),
- B. Changing the  $j$ -th corrector strength by  $\Delta \theta_j$  (utility Module),
- C. Measuring the  $\vec{v}_{\Delta \theta}$  changed orbit (Module 4.1),
- D. Restoring corrector strengths (utility Module),
- E. Calculating the  $\vec{\Delta v} = \vec{v}_{\Delta \theta} - \vec{v}_{ref}$  difference orbit and the  $A_{ij}$   $i=1,N$  matrix elements,
- F. Repeat A-E for all correctors.

\*6 PROCEDURE for chromaticity measurement

Chromaticity is measured by measuring tune at different RF frequencies and then evaluating the  $\Delta\nu/\Delta f_{RF}$  vs.  $f_{RF}$  measured values

\*7 PROCEDURE for measuring the  $\beta$ -function

$\beta$ -function can be measured at the location of the quadrupoles by evaluating the observed  $\Delta\nu$  tune change due to a  $\Delta Q$  change in the quadrupole strength.

- \*8 To calculate  $\nu$  as well as  $\beta$  and  $\phi$  at each corrector and monitor PUE from a measured response matrix. One can use an optimizer, like for example NPSOL [ref.16] to minimize the  $\|\vec{r}\|^2$  norm of the  $\vec{r} - \vec{x} - A(\nu, \beta_{PUE}, \beta_{COR}, \phi_{PUE}, \phi_{COR})^*$  residual vector with respect to all the shown variables in the  $A$  response matrix.
- \*9 To find **misalignment** of magnets and/or **gradient error** in magnets
  - \*\*1 by correcting bare orbit using least square method or eigenvector method with correctors "defined" at the location of the magnets. "Corrector" locations, picked by the 3.2.2.1 algorithm as most effective or corrector locations corresponding to the most significant required kicks as calculated by the 3.2.2.3 algorithm, indicate misalignments and/or gradient errors.

With this method one can even distinguish between dipole roll or edge focusing error by defining correctors at different points in the dipole. (This method was successfully used in some cases at NSLS and CERN PS)

- \*\*2 by measuring the orbit with two sets of quadrupole strengths, as in [ref. 17].

## 5. CALIBRATION [Ref. 18a]

- \*1 To **calibrate dipoles** with beam momentum measurements
- \*2 To **calibrate quadrupoles** comparing calculated and measured tunes, dispersions, etc. [ref. 18b].
- \*3 To **calibrate sextupoles** comparing calculated and measured chromaticities [ref. 18c].
- \*4 To **calibrate correctors** comparing calculated and measured orbit changes

Method 1: Measure the orbit charge at each orbit monitor when "kicking" the orbit by one corrector at a time. Then compare the measured and the model calculated orbit changes [ref. 18d].

Method 2: Make local bumps with 3 correctors at a time and use an iterative process until the bumps are truly local. Then compare the observed and model-calculated corrector ratios.

## 6. OPTIMIZERS

Optimizers are used by the application drivers

- (1) To make desired conditions by varying designated variables or
- (2) To extract information from measured data by optimizing an **objective function**, which might be subject to a set of constraints, to calculate (or obtain best estimates) on the variables. Both, the objective function and the constraints could be nonlinear. In most cases, however, one is faced with a constrained least-square problem.

7. UTILITY

- \*1 Choose data files as input/output  
Type data files:

Data Base (generic name: FDATA)  
Calculated Model Data (generic name: FMODEL)  
Orbit-files (generic name FMONVAL)  
Response Matrix-file (generic name FORBIT)  
Magnet strengths files (generic names: FRGTBQ, FRGTBD, FRGTBUN,  
FRGTBSX, FRGTBHC, FRGTBVC)

- \*2 Choose mode of execution as

ON-LINE: communicate with hardware; get/send setpoints  
OFF-LINE: get/send magnet strengths in design units from/to  
files  
EMULATE: get/send magnet strengths in setpoints from file

It is possible to mix modes; to get magnet strengths from files and send calculated setpoints (or increment/decrement of setpoints) to the device or visa versa. It is useful for example, to perform on-line orbit correction at a specified tune (avoiding the possible small day-to-day changes in the Q setpoints).

- \*3 Overwrite (for application drivers) the calculated tune with measured or operator specified value. (e.g. to use it in conjunction with measured response matrix).
- \*4 Choose if calculated or measured "Response Matrix" is to be used.
- \*5 Enable/disable monitors and correctors for calculation and control.
- \*6 Choose which magnets to be varied.
- \*7 Choose what constraints to be satisfied.
- \*8 "Knob" magnets (acts on the FRGTB... files).
- \*9 Implement calculated changes in magnet strengths.
  - (i) Send calculated changes to devices in one step (in the on-line mode overwrite the FRGTB... files)
  - (ii) Send calculated changes to devices in more steps.

8. GRAPHICS

To display lattice configuration as well as measured, calculated or expected behavior of the system.

9. SCREEN EDITOR

To create and/or modify input files to the Modeling program

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## APPENDIX 1

### Synchrotron integrals

The following notations will be used:

{ is the summation for all dipoles in the ring

$\langle \rangle_1$  is the average over the magnet

$\epsilon_1, \epsilon_2$  are the entrance and exit pole face rotation angles

$\alpha_o, \beta_o, \gamma_o, \eta_o, \eta'_o$ , are the values of the machine parameters before the magnet  
 $\eta_2$  is the dispersion after the magnet.  $\eta_2$  can also be calculated as

$$\eta_2 = \eta_0 CS + \eta'_1 \frac{SN}{k} \pm \frac{1}{\rho k^2} (1 - CS)$$

$$\eta'_1 = \eta'_o + \frac{1}{\rho} \xi_o \tan \epsilon_1$$

$$\alpha_1 = \alpha_o - \frac{1}{\rho} \beta_o \tan \epsilon_1 \quad \text{and} \quad \gamma_1 = \frac{1 + \alpha_1^2}{\beta_o}$$

$$CS = \begin{cases} \cos \alpha \\ \cosh \alpha \end{cases} \quad SN = \begin{cases} \sin \alpha \\ \sinh \alpha \end{cases} \quad TN = \begin{cases} \tan \frac{\alpha}{2} \\ \tanh \frac{\alpha}{2} \end{cases}$$

depending whether the magnet is focusing or defocusing in the horizontal plane.

$\alpha = K * L$  is the bend angle of the magnet, where  $L$  is the magnet length,

$$k = \frac{\sqrt{1-n}}{\rho} \quad \text{and} \quad n = -\frac{B_o}{B^2} B'$$

With the above notations, the synchrotron integrals are calculated as follows:

$$I1 = \oint \frac{n_x}{\rho} dS = \sum_i \frac{L_i}{\rho_i} \langle n_x \rangle_i = \sum_i \left[ \frac{n_{oi} + n_{2i}}{k_i * L_i} \tan \frac{1}{k_i^3 * L_i * \rho_i} (\alpha_i - 2 * \tan \epsilon_i) \right] \quad (1)$$

$$I2 = \oint \frac{1}{\rho^2} dS = \sum_i \frac{L_i}{\rho_i^2} \quad (2)$$

$$I3 = \oint \frac{1}{|\rho|^3} dS = \sum_i \frac{L_i}{|\rho_i|^3} \quad (3)$$

$$\begin{aligned} I4 = \oint \frac{(\delta - 2*n)}{\rho^3} \hat{x} \cdot dS &= \sum_i \left[ \frac{L_i}{\rho_i^3} \langle n_x \rangle_i - 2 * L_i * \langle \frac{n}{\rho_i^3} \rangle_i \right] = \\ &= \sum_i \left[ \frac{L_i}{\rho_i^3} (1 - 2 * n_i) \langle n_x \rangle_i - \frac{1}{2} (n_{oi} + n_{2i}) \tan \epsilon_i \right] \quad (4) \end{aligned}$$

where  $\delta = \begin{cases} 0 & \text{for parallel face magnet} \\ 1 & \text{for wedge magnet} \end{cases}$

$$I5 = \oint \frac{H}{|\rho|^3} dS \approx \sum_i L_i \frac{\langle H \rangle_i}{|\rho_i|^3} \quad \text{where} \quad H = \frac{1}{8} [n^2 + (\beta n' - \frac{1}{2} \beta' n)]$$

$$\begin{aligned} I5 &= \sum_i \left[ \frac{L_i}{|\rho_i|^3} [\gamma_1 n_o^2 + 2\alpha_1 n_o n'_1 + \beta_o (\eta'_1)^2] \right]_i^{\pm} \\ &\pm \sum_i \left[ \frac{2}{|\rho_i|^3} \frac{n_o}{k^3 \rho} [k\alpha_1 (1-CS) - \gamma_1 (\alpha-SN)] \right]_i^{\pm} \\ &\pm \sum_i \left[ \frac{2}{|\rho_i|^3} \frac{n'}{k^3 \rho} [k\beta_o (1-CS) - \alpha_1 (\alpha-SN)] \right]_i^{\pm} \quad (5) \\ &\pm \sum_i \left[ \frac{1}{|\rho_i|^3} \frac{\beta_o}{k^3 \rho} [\frac{1}{2} (\alpha-SN*CS) \mp \frac{2\alpha_1}{\beta_o k} (1-CS \mp \frac{1}{2} SN) \pm \frac{\gamma_1}{\beta_o k^2} (1.5\alpha - 2*SN + \frac{1}{2} SN*CS)] \right]_i^{\pm} \end{aligned}$$

Machine parameters

In addition to calculating the  $[\beta_x, \beta_y, \alpha_x, \alpha_y, \eta_x, \eta_y, \eta'_x, \eta'_y, \phi_x, \phi_y, S]$  lattice functions at injection point and after each element in the ring (which are printed as part of the detailed information), the following machine parameters are calculated.

Tune

$$\nu_{x,y} = \frac{N_{sp}}{2\pi} * \phi_{x,y}^S \quad \text{where } N_{sp} \text{ is the number of S.P. in the ring} \quad (6)$$

Circumference

$$L = N_{sp} * S^S \quad \text{where } S^S \text{ is the length until the symmetry point} \quad (7)$$

Momentum compaction factor

$$\alpha = \frac{dL/L}{dP/P} = \frac{I_1}{L} \quad \text{where the } I_1 \text{ synchrotron integral is defined in Eq.(1)} \quad (8)$$

Revolution time

$$t_0 = L/c \quad \text{where } c \text{ is the speed of light} \quad (9)$$

Dumping partitions

$$\begin{aligned} J_x &= 1 - I_4/I_2 & \text{where the } I_2, I_4 \text{ synchrotron integrals are} \\ J_y &= 1 & \text{defined in Eqs.(2) and (4)} \\ J_E &= I_4/I_2 + 2 \end{aligned} \quad (10)$$

Rate of change of dumping partitions

$$\begin{aligned} \frac{dJ_x}{dE/E} &= - \frac{2*N_{sp}}{I_2} \sum_{i=\text{quad}} L_i \left( \frac{B' * \eta_x}{B_p} \right)_i^2 \approx - \frac{2*N_{sp}}{I_2} \sum_i L_i \left( \frac{B' * \eta_x}{B_p} \right)_i^2 \\ \frac{dJ_y}{dE/E} &= - \text{ same as above with } \eta_y \\ \frac{dJ_E}{dE/E} &= - \frac{dJ_x}{dE/E} + \frac{dJ_y}{dE/E} \end{aligned} \quad (11)$$

Dumping times

$$\tau_x = \frac{2*E}{V_o} t_0 \frac{1}{J_x}, \quad \tau_y = \frac{2*E}{V_o} t_0 \frac{1}{J_y}, \quad \tau_E = \frac{2*E}{V_o} t_0 \frac{1}{J_E} \quad (12)$$

where  $V_o$  is the energy radiation/turn (see Eq.(13)) in Gev.

Beam parameters  
Energy radiation/turn

$$v_o = \frac{1}{2\pi} \gamma_c E^4 I_2 \text{ [Gev]}, \quad U_o = 10^3 * v_o \text{ [Mev]}, \quad \text{where } \gamma_c = \frac{4\pi}{3} \frac{R_e}{(mc^2)^3}, \quad (13)$$

$R_e$  is the electron radius and  $E$  is the beam energy

Energy spread:

$$\sigma_E = \left( \gamma_c' E^2 \frac{I_3}{I_2 J_E} \right)^{1/2}, \quad \text{where } \gamma_c' = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \frac{1}{(mc^2)^2} \quad (14)$$

$\sigma$ , due to betatron motion with no coupling:

$$\sigma_x^o = \left( \gamma_c' \frac{E^2}{J_x} * \frac{I_5}{I_2} \beta_x^o \right)^{1/2} \quad (15)$$

$$\sigma_y^o = 0$$

1) Coupled beam sizes and emittance

$$\sigma_x^c = \frac{1}{\sqrt{A^2+1}} \sigma_x^o, \quad \sigma_y^c = \sqrt{\frac{A^2}{A^2+1}} \sqrt{\frac{\beta_y^o}{\beta_x^o}} \sigma_x^o \quad (16)$$

$$\epsilon_x^c = (\sigma_x^c)^2 / \beta_x^o \quad \epsilon_y^c = (\sigma_y^c)^2 / \beta_y^o \quad (17)$$

where the  $A$  coupling constant is specified by the user via the operator screen.

2) Total beam sizes

$$\sigma_x^T = [(\sigma_x^c)^2 + (\eta_x \sigma_E)^2]^{1/2}, \quad \sigma_y^T = \sigma_y^c \quad (18)$$

3) Touschek lifetime (considering only the rf acceptance as in ref. [19]).

$$\tau_T = \frac{\gamma^3 \sigma' \left( \frac{dE}{E} \right)_{RF}^2 V}{r \pi r_e^2 c \frac{N}{h} C(\delta)} \quad (19)$$

$$\text{where } \gamma = \frac{E_o}{m_e c^2}$$

$$\sigma' = \sqrt{\frac{\epsilon_x}{\langle \beta_x \rangle} + \frac{\epsilon_y}{\langle \beta_y \rangle}} = \sqrt{\frac{2\pi}{L} (\epsilon_x \frac{v_x}{\beta_x} + \epsilon_y \frac{v_y}{\beta_y})}$$

RF-related parameters

Synchrotron phase angle

$$\phi_S = \pi - \arcsin(U/V_{RF}), \quad \text{and} \quad \phi_1 = \pi - \phi_S \quad (20)$$

Synchrotron tune

$$\nu_S = \left( - \frac{k*V_{RF}*\alpha_p*\cos\phi_S}{2*\pi*k*10} \right)^{1/2} \quad \text{where } V_{RF} \text{ is the RF voltage} \quad k \text{ is the harmonic-#} \quad (21)$$

Synchrotron frequency

$$f_S = \nu_S/t_0 \quad (22)$$

RF frequency

$$f_{RF} = k/t_0 \quad (23)$$

RF bucket width

$$\left( \frac{dE}{E} \right)_{RF} = \left( |\cos\phi_S - \sin\phi_S (\frac{\pi}{2} - \phi_S)| \right)^{1/2} \left( \frac{2*V_{RF}}{\pi*k*10^3*\alpha_p} \right)^{1/2} \quad (24)$$

Bunch length:

$$\sigma_z = 10^2 * \sigma_E * L \left( - \frac{\alpha_p * E}{2 * \pi * k * V_{RF} * \cos\phi_S} \right)^{1/2} = \frac{10^2 * L * \sigma_E}{2 * \pi * k} \left( - \frac{2 * \pi * k * \alpha_p * E}{V_{RF} * \cos\phi_S} \right)^{1/2} \quad (25)$$

Quantum life-time

$$\tau_Q = \frac{1}{60} \tau_E * \sigma_E^2 * \left( \frac{dE}{E} \right)_{RF} \exp \left[ \left( \frac{dE}{E} \right)_{RF}^2 / 2\sigma_E^2 \right] \quad (26)$$