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ALE Shock Calculations Using a Stabilized Serendipity Rezoning Scheme.*

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A rezone stencil for ALE shock calculations has been developed based on a stabilized variant of the serendipity element. This rezone stencil is compared to the Winslow rezone stencil. Unlike the Winslow stencil, which equalizes element volumes as well as node angles, the serendipity stencil equalizes node angles only. This may be advantageous for calculations involving strong density gradients such as those associated with shock compression.

1. Introduction

Our group is presently developing a new general-purpose shock code applicable to problems involving high compression, plastic flow, and vaporization. Thin regions of material, due either to initial geometry or high compression, will not be adequately represented by a fixed Eulerian mesh; on the other hand, no purely Lagrangian technique can represent highly distorted plastic or inviscid fluid flow adequately. We have chosen to implement arbitrary Lagrangian-Eulerian (ALE) techniques in the new shock code so that it will be able to handle the entire range of phenomena of interest.

1.1. Plastic Flow vs. Inviscid Fluid Flow

Plastic flow often results in the material behaving as if it was incompressible. Fully-integrated finite elements, in which the deformation field is approximated to the same order as the velocity gradient, cannot be used to solve an incompressible flow problem[1]; such elements are unable to represent the Stokes flow field resulting from certain deformation modes (the hourglass modes) with the result that these modes are filtered out of the velocity field. This is true no matter how fine the mesh is made. That is to say, the method is not convergent.

Generally, the problem is solved by reducing the order of approximation of the deformation field so that the hourglass modes are no longer coupled to the deformation. This renders the method convergent, but requires the introduction of hourglass control algorithms to prevent uncontrolled excitation of the hourglass modes.[2] A significant body of theoretical work now exists for hourglass control methods which are tied to the shear modulus of the material. In practice, *ad hoc* parametrization is used for calculational simplicity with the complete theory providing guidance on the values to use for the parameters.

If a calculation involves inviscid fluids, these hourglass control methods fail, since the shear modulus of the material is vanishingly small. This reflects the fact that a perfect fluid has no hourglass resistance. Since it is the hourglass distortion that is ultimately responsible for mesh tangling, this is a serious difficulty.

One is left with a number of alternatives for solving the problem. The first is to avoid incompressible materials and use a fully integrated element. This is not a satisfactory solution. Many materials of interest will have Poisson's ratio sufficiently close to 0.5 at some point in a calculation that a mesh of fully-integrated elements will be too stiff.

The second alternative is to use different element technologies for different portions of the mesh. This fails if a single material is both inviscid and nearly incompressible, which is not unknown.

The third alternative, which is explored in this paper, is to use an underintegrated element with standard hourglass control in an ALE setting. If a rezoning scheme can be found that reduces the hourglass component of the mesh, then application of this scheme may be adequate to control hourglass deformation.

1.2. ALE Rezoning Schemes

A finite-element based ALE scheme generally functions like a normal Lagrangian finite-element method until the mesh distortion exceeds some limit. The material is then permitted to flow through the mesh in such a way that mesh distortion is reduced to acceptable levels (which constitutes the semi-Eulerian mode of the ALE method).[3]

It is relatively easy to formulate a criterion for permitting material convection. For example, one can switch on

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convection when one of the angles formed by element sides at a node becomes too acute or when the elements connected to a node differ too greatly in volume. In our case, convection would be switched on when the hourglass component of the deformation field of an element becomes too great.

It is somewhat more challenging to come up with a good rezoning scheme. Benson[3] advocates the use of stencils that enforce equipotential relaxation, and this has become the *de facto* standard rezone method. However, as will be shown in this paper, there are classes of problems for which equipotential relaxation may be inferior to alternate methods if one is primarily interested in ALE hourglass control.

2. Rezoning stencils for ALE schemes

2.1. The Winslow stencil

This stencil is derived from a finite difference representation of the Laplace equation[4]

$$\nabla^2 f = 0 \qquad (1)$$

Hence it is referred to as an *equipotential relaxation* stencil.

The chief drawback of the Winslow stencil is that it tends to equalize element volumes as well as to reduce the vorticity of the mesh (that is, to equalize the angles formed by element sides at each node). This eliminates any initial mesh grading that may be introduced by the analyst. If the material contains strong density gradients, as is the case for many problems of interest to our group, such mesh grading is a highly desirable feature.

Figure 1 illustrates this behavior. Winslow's stencil has

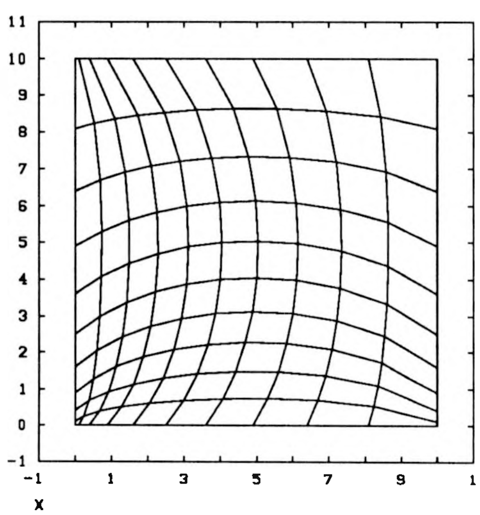

Figure 1. Winslow mesh

been applied to a mesh with unequal spacing of boundary nodes. One sees that elements away from the boundaries tend to be equal in volume despite the unequal boundary intervals. Although one could introduce a source term into the equipotential scheme so as to "attract" the mesh to regions of high density, Benson notes that this source term could lead to mesh overlap.

2.2. The stabilized serendipity stencil

This stencil gets its name from the fact that it was originally derived from the isoparametric 8-node serendipity element used in finite element methods. It may also be derived from the finite difference representation of the partial differential equation

$$\frac{\partial^4 f}{\partial^2 x \partial^2 y} = 0 \qquad (2)$$

with the general solution

$$f = y f_1(x) + x f_2(y). \qquad (3)$$

This stencil takes the form

$$x_1^{n+1} = 0.5\,[x_1^n + 0.5\,(x_2^n + x_4^n + x_6^n + x_8^n) - 0.25\,(x_3^n + x_5^n + x_7^n + x_9^n)] \qquad (4)$$

where the superscript denotes the iteration number and the subscript denotes the node number according to Benson's numbering scheme. (See Figure 2.) The x_1^n contribution on the right hand side is required for stability.

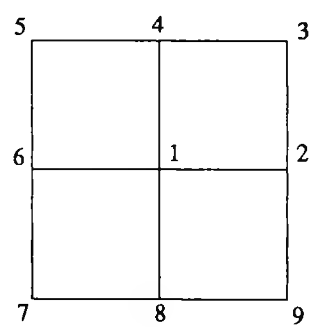

Figure 2. Node numbering schemel

Figure 3 illustrates the properties of this rezoning scheme. The boundary nodes are spaced identically with Figure 1. One sees that there is no tendency to equalize element volumes; only the vorticity of the mesh has been reduced.

Another striking feature of the serendipity stencil is that the resulting mesh is more sensitive to the boundaries than is the Winslow mesh. One therefore has a greater degree of control over the details of rezoning. Any suitable algorithm may be employed to redistribute the nodes along the bound-

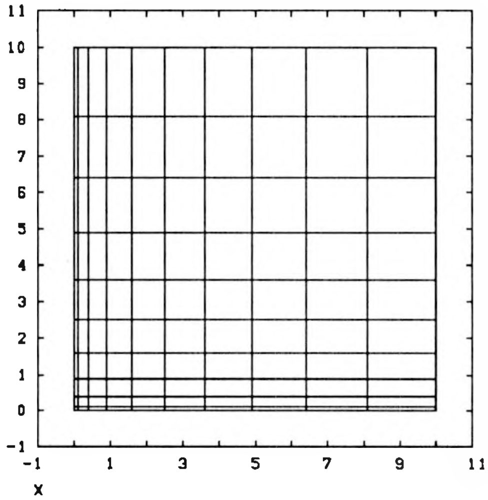

Figure 3. Stabilized serendipity mesh

aries of the mesh; the remainder of the rezoned mesh will then conform closely to the boundary.

2.3. Acceleration of convergence

The chief drawback of the stabilized serendipity stencil is that it converges very slowly. Figure 4 illustrates the prob-

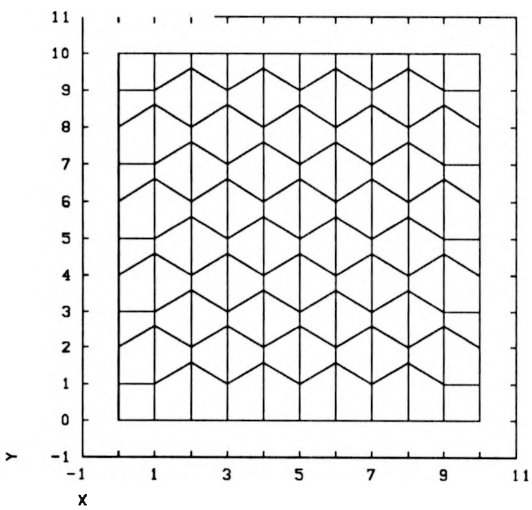

Figure 4a. Initial hourglassed mesh

lem. After five iterations, one sees that the initial, highly distorted mesh has begun to smooth near the boundaries; however, the center of the mesh remains highly hourglassed.

Ng acceleration[5] provides a means of increasing the rate of convergence. Figure 5 illustrates that the hourglass pattern

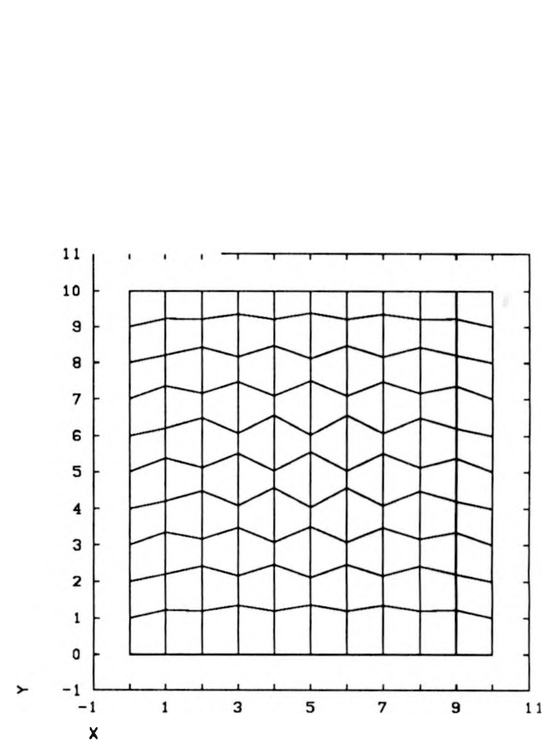

Figure 4b. Mesh after five iterations

is smoothed much more quickly by four Ng-accelerated iterations than by five iterations alone. (The computational cost is comparable). However, the global solution is still reached only very slowly; in the example, one sees an overall upwards distortion of the interior of the mesh that has not been completely removed by the Ng-acclerated iterations.

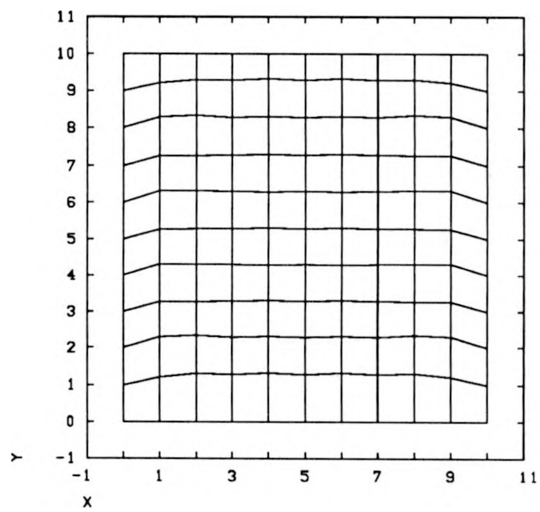

Figure 5. Mesh after four accelerated iterations

3. Conclusion

We are exploring the use of ALE techniques for the control of hourglassing in underintegrated inviscid fluid elements. We find that the stabilized serendipity stencil has great potential as a rezoning scheme because it reduces mesh vorticity without destroying mesh grading. Ng acceleration is useful for increasing the rather slow rate of convergence of the stabilized serendipity stencil.

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