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Higgs Boson Masses in Supersymmetric Models *

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ABSTRACT

Imposing supersymmetry on a Higgs potential constrains the parameters that define the potential. In supersymmetric extensions to the standard model containing only Higgs $SU(2)_L$ doublets there exist Higgs boson mass sum rules and bounds on the Higgs masses at tree level. The prescription for renormalizing these sum rules is derived. An explicit calculation is performed in the minimal supersymmetric extension to the standard model (MSSM). In this model at tree level the mass sum rule is $M_H^2 + M_h^2 = M_A^2 + M_Z^2$. The results indicate that large corrections to the sum rules may arise from heavy matter fields, e.g. a heavy top quark. Squarks significantly heavier than their fermionic partners contribute large contributions when mixing occurs in the squark sector. These large corrections result from squark-Higgs couplings that become large in this limit. Contributions to individual Higgs boson masses that are quadratic in the squark masses cancel in the sum rule. Thus the naturalness constraint on Higgs boson masses is hidden in the combination of Higgs boson masses that comprise the sum rule.

For my parents

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1. INTRODUCTION

One of the most important problems facing particle physicists today is our lack of knowledge about the mechanism of spontaneous electroweak symmetry breaking in the standard model. The neutral and charged current interactions of the standard model have been convincingly verified in many experiments. In the future it will be important to test the non-abelian nature of the theory and understand the mechanism that is responsible for the symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$. There is certainly new physics to be understood in the symmetry breaking sector because we know that the symmetry breaking takes place. Unfortunately the effects of electroweak symmetry breaking sector are notoriously difficult to detect. The elementary Higgs bosons or the bound states of a strongly interacting symmetry breaking sector might be too massive to observe directly, and there virtual effects are screened in electroweak radiative corrections.

Most of the models that have been proposed to explain the symmetry breaking have employed gauge theories, and with good reason as they have been so successful in their application to the standard model. Dynamical symmetry breaking is perhaps the most conservative solution to the symmetry breaking puzzle beyond the elementary scalar Higgs. This form of symmetry breaking has already been seen in the QCD sector of the standard model. A bit more daring is supersymmetry, in which the symmetry of spacetime transformations is extended to include transformations between fermions and bosons. No evidence for supersymmetry exists in nature, but physicists have for a long time been in the business of inventing new symmetries.

Faced with a lack of experimental information about the electroweak symmetry breaking sector of the standard model, theorists have invented there own constraints as a guide for further research and progress. Of these the hierarchy problem has probably received the most attention. Physicists hope to one day unify all of physics at some large energy scale. The hierarchy problem is just the question of why the electroweak scale and the proposed unification scale around the Planck mass are so divergent.

Closely related to the hierarchy problem is the problem of naturalness. Assuming that a hierarchy is generated at tree level, how is the hierarchy preserved once radiative corrections are introduced? Since the new physics is still unknown, the best we can do is take the view that the theories of today are effective theories below the scale of this new scale, and apply a cutoff Λ to divergent loop diagrams which embodies the unknown physics. However, the masses of fundamental scalar particles are subject to quadratic divergences. So if the cutoff parameter Λ is of the order of the Planck mass, then it is hard to understand why the Higgs bosons remain light.

In technicolor elementary scalar bosons are done away with entirely, and a confining gauge theory like QCD is employed. The fundamental states of technicolor are fermions and gauge bosons, and fermion-antifermion condensates lead to breaking of the electroweak symmetry. In supersymmetry scalar bosons are kept in the theory, but the new symmetries that exist ensure that the quadratic divergences cancel leaving only the milder and tolerable logarithmic divergences. The price to be paid for introducing supersymmetry is the introduction of many new states as each bosonic field must have a fermionic field that are connected by the supersymmetry transformations. It is the combined contribution of the bosons and their fermionic partners that give the vanishing quadratic divergence.

Supersymmetry must be broken. Exact supersymmetry would require that the supersymmetric partners have exactly the same mass. Since no such states have been observed, we must devise some means of breaking supersymmetry and boosting the masses of the supersymmetric particles to values above the range of present observation. The requirement of naturalness now presents itself as a limit on the amount of supersymmetry breaking that can be present. If the supersymmetric partners are sufficiently different in mass, then we have the naturalness problem all over again. The quadratic divergences may still cancel, but corrections to Higgs masses that are quadratic in the mass of the massive supersymmetric partner will remain. Thus the supersymmetric partners must be heavy enough to have escaped detection while not so heavy to reintroduce the problem of naturalness.

A good place to look for the radiative effects of the supersymmetric particles is in the Higgs masses themselves. Indeed the naturalness constraint is usually discussed in the context of the Higgs masses. Higgs bosons couple to all massive particles and is therefore sensitive to radiative effects from all sectors of the theory. In addition Higgs masses are particularly vulnerable to radiative corrections due to a heavy top quark (or a fourth generation) as the Higgs-quark-quark coupling is proportional to the quark mass.

At the moment supersymmetry is the only known way to reconcile the vast difference between the electroweak and GUT scales while still retaining scalars as fundamental fields. We shall refer to the two-Higgs model as the minimal supersymmetry extension to the standard model (MSSM). In this thesis we calculate radiative corrections from quark and squark loops to Higgs boson mass relations that arise in the MSSM. Radiative corrections to Higgs masses in the MSSM were first calculated in Reference [1] using the effective potential formalism. However a heavy top quark was not fashionable at that time. The radiative corrections arising from loops containing neutralinos and charginos to the Higgs boson mass sum rules have been considered in Reference [2]. No large corrections to the mass relations were found unless a dimensionless coupling constant becomes large. We find that large corrections can occur for quark and squark loops if the squark-squark-Higgs couplings are large. We also find that a large quark mass can yield large radiative corrections to the mass sum rules. In addition we develop a formalism for calculating radiative corrections to Higgs mass relations in a supersymmetric extension with an arbitrary number of Higgs doublets.

In the standard model, a single Higgs $SU(2)$ doublet suffices to break the electroweak symmetry. In supersymmetric extensions of the standard model, at least two doublets are required to cancel anomalies (the Higgs bosons have fermionic superpartners) and to give the up and down quarks a mass[3]. The empirical fact that $\rho = 1$ suggests a custodial symmetry in the Higgs sector. At tree-level there is the well-known result[4]:

$$\rho = \frac{\sum_i (4T_i(T_i + 1) - Y_i^2) v_i^2 c_i}{\sum_i 2Y_i^2 \, v_i^2}. \tag{1.1}$$

The index i runs over the Higgs representations. T is the weak isospin, Y is the hypercharge and $c = 1(\frac{1}{2})$ for complex (real) representations. Assuming $\rho = 1$ does not result from tuning the vacuum expectation values v_i, we obtain the requirement

$$(2T_i + 1)^2 - 3Y_i^2 = 1. \tag{1.2}$$

This custodial symmetry can be realized by taking a Higgs sector that contains weak $SU(2)$ doublets ($T = \frac{1}{2}, Y = \pm 1$) and singlets ($T = 0, Y = 0$). Other representations are possible, but these have large dimensionalities and appear rather ad hoc. The standard model contains just one complex Higgs doublet. Three of these four degrees of freedom are eaten by the W and Z gauge bosons, leaving a single physical Higgs boson. In this paper we are primarily concerned with extensions of the standard model that have two Higgs doublets only. The two-Higgs doublet model has eight degrees of freedom in the Higgs sector which become three neutral Higgs bosons(H, h, A), two charged Higgs bosons(H^+, H^-), and the usual three Goldstone bosons(G, G^+, G^-) that are eaten by the W and the Z. H and h are CP-even eigenstates while A is CP-odd. We follow the usual practice of calling these scalars and pseudoscalars respectively to indicate the form of their couplings to fermions. The general two-Higgs doublet extension of the standard model therefore has a much richer phenomenology than does the simple standard model. The general two doublet model (without supersymmetry) has quite a bit of arbitrariness in the masses and couplings of the physical Higgs bosons.

We will consider the supersymmetric version of the two-Higgs doublet extension to the standard model[3]. The restrictions imposed by supersymmetry constrain the couplings in the Higgs sector and lead to mass relations for the physical Higgs bosons. In addition, at tree level the lightest neutral Higgs h must be lighter than the Z, the heaviest neutral Higgs H must be heavier than the Z and the charged Higgs H^+ must be heavier than the W. In fact the first two inequalities remain true for supersymmetric extensions of the standard model containing an arbitrary number of Higgs doublets (containing no Higgs singlets or other representations)[5] though the charged Higgs does not have to be lighter than the W in these cases.

In this model, there exist the tree level mass sum rules

$$M_H^2 + M_h^2 = M_A^2 + M_Z^2 \tag{1.3}$$

and

$$M_{H^\pm}^2 = M_A^2 + M_W^2. \tag{1.4}$$

We explicitly calculate the $O(\alpha)$ corrections to the relation (1.3) arising from the quark and lepton sectors. The corrections to (1.3) and (1.4) will all be $O(\alpha)$ for the one-loop calculation since in supersymmetric models the cubic and quartic couplings in the Higgs potential are related to the gauge couplings g and g'. There is no arbitrary coupling in supersymmetric extensions of the standard model such as the quartic coupling λ in the standard model. The philosophy is therefore slightly different in the renormalization of the mass relation in (1.3) of the MSSM. The sum rule in (1.3) involves physically measurable masses, without any reference to couplings. So we can take these masses as the parameters that define the Higgs sector, and find radiative corrections to (1.3) in terms of these parameters. We find that large corrections to the mass relation in (1.3) can arise from matter loops but only if the significant mixing occurs between the squark fields, or if there is a heavy quark.

Large corrections ($O(\alpha \frac{m_q^4}{M_W^4})$ where m_q is a quark mass) to the Higgs boson masses arise as they do in the standard model. The squark $\tilde{q}$ corrections to Higgs masses that are $O(\alpha m_{\tilde{q}}^2)$ are quadratic in the supersymmetry breaking scale. If they become large, they destroy the stability of the electroweak scale to radiative corrections, necessitating large subtractions that require unnatural fine-tuning order by order in perturbation theory. We find that these contributions cancel exactly in the renormalization of the sum rule. Therefore the naturalness constraint is "hidden" in the sum rule. Mixing between left and right handed squarks occurs in general. If the off-diagonal entries in the left-right squark quark mass matrix are large, then large squark-Higgs couplings can arise and result in large corrections to the mass relation.

In Section II we review the aspects of the MSSM that are needed for this work. In Section III we explain in detail the formalism for renormalizing the Higgs sector of the MSSM. We discuss the results of an actual calculation we have performed in the MSSM in Section IV. Since the physical masses of the Higgs bosons(H, h, A) and the Z are measurable, the $O(\alpha)$ corrections to the mass relation in (1.3) is a physically measurable quantity. In Appendix A we display some Feynman vertices that are needed to calculate the Higgs self-energy diagrams in the MSSM. In Appendix B we display the full result for the correction to (1.3) arising from the up-type quark and up-type squark loops. This result is easily generalized to all contributions from other loops involving quarks, leptons and their supersymmetric partners. In Appendix C we show that the tadpole contributions cancel in the MSSM. Finally in Appendix D we discuss how the formalism developed in Section III can be generalized to models with more than two Higgs doublets.

Other work on radiative corrections to Higgs boson mass sum rules in the MSSM has also appeared[2,7,8]. The calculation in Reference [6] is a complete one-loop calculation of the radiative corrections from the fermion-sfermion sector. The propagating squark fields are the mass eigenstates, and the renormalized masses are the physical masses defined as the pole of the renormalized propagator. The only approximation is that flavor mixing is neglected. This is easily reincorporated into the result.

II. THE MINIMAL SUPERSYMMETRIC EXTENSION OF THE STANDARD MODEL

We shall follow the notation of Gunion and Haber[9] with the one exception that they refer to the neutral Higgs bosons H, h, A, and G as H_1^0, H_2^0, H_3^0, and G^0 respectively. Throughout this paper any mass without a subscript will be a *physical* mass(e.g. M_H, M_h, etc.). Any subscript on a mass parameter (e.g. $(M_H)_b$, $(M_H)_r$, etc.) indicates that this parameter is in general different from the physical mass. The definitions of these mass parameters will be given when they arise. Our review will be brief, and the interested reader is urged to consult References [3,5,9] for more details about the MSSM.

Supersymmetry requires that there be at least two Higgs doublets. The MSSM is minimal because it contains only these two Higgs doublets and the minimal particle content necessary to explain known phenomenology. Since it is the simplest viable supersymmetric model, it is the natural place to begin an investigation of radiative corrections in the Higgs sector. Call the two complex doublet scalar fields ϕ_1 and ϕ_2. The Higgs potential develops an asymmetric minimum, giving rise to spontaneous symmetry breaking. Then ϕ_1 gives mass to the d-type quarks and squarks, and ϕ_2 gives mass to the u-type quarks and squarks.

The MSSM can be obtained as the low-energy limit of a supergravity theory. The renormalization group equations are used to run the values of the parameters in the supergravity theory that obey certain boundary conditions at the unification scale. In this way constraints are placed on the parameters that define the MSSM. We shall ignore these constraints which can be imposed at any time. Imposing these constraints restricts ourselves to just this model, and weak-scale effective supersymmetry can arise in a more general way[10].

Supersymmetry constrains the otherwise independent quartic couplings in the MSSM to be combinations of the gauge couplings g and g'. This implies that the Higgs sector of the MSSM is weakly coupled as the coupling constants g and g' are certainly perturbative. We are allowed terms up to cubic order in the superfields in the superpotential by renormalizability, and it must of course be gauge invariant. The most general superpotential that conserves R parity contains the following pieces:

$$W = \epsilon_{ij}(\mu H_1^i H_2^j + f H_1^i \tilde{L}^j \tilde{R} + f_1 H_1^i \tilde{Q}^j \tilde{D} + f_2 H_2^j \tilde{Q}^i \tilde{U}) \tag{2.1}$$

where $\tilde{Q}$ and $\tilde{L}$ are the weak $SU(2)$ doublet quark and lepton superfields, $\tilde{U}$ and $\tilde{D}$ are the weak $SU(2)$ singlet quark superfields, and $\tilde{R}$ is the $SU(2)$ singlet lepton superfield. Only the first term in (2.1) contributes to the Higgs potential. The other terms contribute to the full scalar potential. f, f_1 and f_2 are the Yukawa couplings that yield the fermion masses and the masses of their supersymmetric partners. We can relax the constraint that the superpotential conserve R parity. An interesting discussion of some alternative models of low-energy supersymmetry can be found in Reference [11].

The scalar potential receives contributions from the so-called D terms and F terms. These are

$$V = \frac{1}{2}[D^a D^a + (D')^2] + F_i^* F_i \tag{2.2}$$

where

$$D^a = \frac{1}{2} g A_i^* \sigma_{ij}^a A_j, \tag{2.3a}$$

$$D' = \frac{1}{2} g' y_i A_i^* A_i + \xi, \tag{2.3b}$$

$$F_i = \frac{\partial W}{\partial A_i}. \tag{2.3c}$$

Here A_i denotes a generic scalar field appearing in the superpotential. ξ is the Fayet-Iliopoulos term[12] that may arise for U(1) gauge groups. The hypercharge assignments of the two Higgs doublets are $y_1 = -1$ and $y_2 = 1$, ensuring anomaly cancelation. Therefore, one Higgs doublet gives masses to the up-type quarks, while the other gives masses to the down-type quarks, so the MSSM by construction eliminates the unacceptable flavor-changing neutral currents.

In general we add all possible soft supersymmetry breaking terms[13] that can contribute to the scalar potential. These terms break supersymmetry but in such a way that no quadratic divergences appear. This allows the supersymmetry to be broken as is necessitated by phenomenology while preserving one of the major motivations for supersymmetry. The soft supersymmetry breaking terms must be of dimension three or less in the fields. The Higgs potential is then given by (we assume that the Fayet-Iliopoulos term associated with $U(1)_Y$ is small and neglect it)

$$V = \frac{1}{4} g^2 \sum_{a=1}^{3} |\phi_1^\dagger \sigma^a \phi_1 - \phi_2^\dagger \sigma^a \phi_2|^2 + \frac{g'^2}{8} (\phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2)^2 + |\mu|^2 (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2) + V_{soft} \tag{2.4a}$$

which can be rewritten

$$V = \frac{1}{8} g^2 \left[4|H_1^{i*} H_2^i|^2 - 2(H_1^{i*} H_1^i)(H_2^{i*} H_2^i) + (H_1^{i*} H_1^i)^2 + (H_2^{i*} H_2^i)^2 \right]$$

$$+\frac{1}{8}g'^2(H_2^{i*}H_2^i - H_1^{i*}H_1^i)^2 + |\mu|^2(H_1^{i*}H_1^i + H_2^{i*}H_2^i) + V_{soft} \tag{2.4b}$$

where

$$V_{soft} = m_1^2 H_1^{i*}H_1^i + m_2^2 H_2^{i*}H_2^i - (m_{12}^2\epsilon_{ij}H_1^iH_2^j + h.c.). \tag{2.4c}$$

the Higgs potential arises from three sources: (1) the terms proportional to g and g' that come form the D terms, (2) the term proportional to $|\mu|^2$ that comes from the F terms and (3) the soft supersymmetry breaking contributions in (2.4c). We are using the notation[9]

$$\phi_1^\dagger\phi_1 = H_1^{i*}H_1^i \tag{2.4d}$$

$$\phi_2^\dagger\phi_2 = H_2^{i*}H_2^i \tag{2.4e}$$

$$\phi_1^\dagger\phi_2 = \epsilon_{ij}H_1^iH_2^j. \tag{2.4f}$$

In this notation H_1^1 and H_2^2 are the neutral component of H_1 and H_2 respectively, while H_1^2 and H_2^1 are the charged components. The quantities m_1, m_2, and m_{12} are arbitrary mass parameters, and those terms in (2.4b) that depend on $|\mu|^2$ can be absorbed into the soft supersymmetry breaking terms of (2.4c). In low-energy supergravity models m_{12} is proportional to μ, but we will consider a more general MSSM and let m_{12} take any value that produces an acceptable vacuum (see below). Of course μ still has consequences on phenomenology; it appears in the squark mixing matrices for example. See Section IV below.

A troubling aspect of the MSSM is the very existence of the parameter μ. When the MSSM is viewed in the context of supergravity or grand-unified models, it is hard to understand why μ does not have a value of order the Planck or the GUT scale. This hierarchy problem can be cured by imposing an additional symmetry. It is necessary to remove μ as a fundamental scale in the theory. Two ways this can be accomplished are by going to a superstring model for which the Higgs mixing term is generated when a singlet is present or by expanding the R parity to be a continuous symmetry[14].

This Higgs potential has a minimum away from $H_1 = H_2 = 0$ so spontaneous symmetry breaking occurs. It is possible through a choice of phase to choose the vacuum expectation values to be real and non-negative. We are assuming no CP violation arising in the Higgs potential. We define v_1 and v_2 to be the vacuum expectation values of H_1 and H_2 respectively so that

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \tag{2.5}$$

To obtain the correct tree level mass $M_W^2 = \frac{1}{2} g^2 v^2$, we require $v_1^2 + v_2^2 = v^2$.

The Higgs masses arise from the quadratic parts of the Higgs potential. Define the scalar and pseudoscalar parts of the charge-neutral Higgs boson fields by

$$H_1^1 = v_1 + \frac{1}{\sqrt{2}}(S_1 + i\,P_1) \tag{2.6a}$$

$$H_2^2 = v_2 + \frac{1}{\sqrt{2}}(S_2 + i\,P_2). \tag{2.6b}$$

H and h are linear combinations of S_1 and S_2 while A and G are linear combinations of P_1 and P_2. The factor of $\sqrt{2}$ is included so the kinetic energy terms for the physical Higgs boson fields will have the canonical form. The soft supersymmetry breaking terms include

$$m_1^2 H_1^{i*} H_1^i + m_2^2 H_2^{i*} H_2^i - (m_{12}^2 \epsilon_{ij} H_1^i H_2^j + h.c.), \tag{2.7}$$

which contains the charge-neutral terms

$$\frac{1}{2}m_1^2(S_1^2+P_1^2)+\frac{1}{2}m_2^2(S_2^2+P_2^2)-m_{12}^2(S_1S_2-P_1P_2). \tag{2.8}$$

The F-terms contribute

$$|\mu|^2(H_1^{i*}H_1^i+H_2^{i*}H_2^i) \tag{2.9}$$

which we absorb into the soft supersymmetry breaking contribution. In order to break $SU(2)_L\times U(1)_Y$ the Higgs potential must have a minimum away from $H_1=H_2=0$, so that

$$m_1^2m_2^2<m_{12}^4. \tag{2.10}$$

Notice in Equation (2.4) that in the direction $\phi_1=\phi_2$ the quartic terms in the Higgs potential vanish. Therefore we require

$$m_1^2+m_2^2>2m_{12}^2 \tag{2.11}$$

to prevent the Higgs potential from being unbounded from below in this direction. Collecting the quadratic parts arising in the D-terms

$$\frac{1}{8}(g^2+g'^2)[H_1^{i*}H_1^i-H_2^{i*}H_2^i]^2, \tag{2.12}$$

$$\frac{1}{8}(g^2+g'^2)[(v_1^2-v_2^2)+\sqrt{2}(v_1S_1-v_2S_2)+\frac{1}{2}(S_1^2-S_2^2+P_1^2-P_2^2)]^2, \tag{2.13}$$

$$\frac{1}{8}(g^2+g'^2)[(v_1^2-v_2^2)(S_1^2-S_2^2+P_1^2-P_2^2)+2v_1^2S_1^2+2v_2^2S_2^2], \tag{2.14}$$

the mass matrix in the scalar sector is given by

$$M_S^2=\begin{pmatrix} m_1^2+\frac{1}{4}(g^2+g'^2)(3v_1^2-v_2^2) & -m_{12}+\frac{1}{4}(g^2+g'^2)v_1v_2 \\ -m_{12}+\frac{1}{4}(g^2+g'^2)v_1v_2 & m_2^2+\frac{1}{4}(g^2+g'^2)(3v_2^2-v_1^2)\end{pmatrix}, \tag{2.15}$$

while the mass matrix in the pseudoscalar sector is given by

$$M_P^2=\begin{pmatrix} m_1^2+\frac{1}{4}(g^2+g'^2)(v_1^2-v_2^2) & m_{12} \\ m_{12} & m_2^2+\frac{1}{4}(g^2+g'^2)(v_2^2-v_1^2)\end{pmatrix}. \tag{2.16}$$

Taking traces we obtain the sum rule in (1.3). The crucial point to notice is that the soft-supersymmetry breaking terms contribute equally to both sides of the sum rule. In other words, the sum rule is a result of the supersymmetric structure of the D-terms only since gauge invariance requires that contributions from both the F-terms and soft-supersymmetry breaking terms cancel.

Two parameters in the mass matrices above are determined by the others via the minimization condition. So we can solve for m_1 and m_2 in terms of m_{12}, v_1, and v_2:

$$m_1^2 = m_{12}^2\frac{v_2}{v_1} - \frac{1}{2}M_Z^2\cos 2\beta, \tag{2.17}$$

$$m_2^2 = m_{12}^2\frac{v_1}{v_2} + \frac{1}{2}M_Z^2\cos 2\beta, \tag{2.18}$$

where we have defined $\tan\beta = \frac{v_2}{v_1}$. Then the mass matrices can be written

$$M_S^2 = \begin{pmatrix} m_{12}^2\frac{v_2}{v_1} + M_Z^2\cos^2\beta & -m_{12} + \frac{1}{2}M_Z^2\sin\beta\cos\beta \\ -m_{12} + \frac{1}{2}M_Z^2\sin\beta\cos\beta & m_{12}^2\frac{v_1}{v_2} + M_Z^2\sin^2\beta \end{pmatrix}, \tag{2.19}$$

and

$$M_P^2 = \begin{pmatrix} m_{12}^2\frac{v_2}{v_1} & m_{12} \\ m_{12} & m_{12}^2\frac{v_1}{v_2} \end{pmatrix}. \tag{2.20}$$

The pseudoscalar mass matrix has a zero eigenvalue which corresponds to the neutral Goldstone boson. The eigenvalues of the mass matrices M_S^2 and M_P^2 are related by

$$M_{H,h} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2M_Z^2\cos^2 2\beta}\right]. \tag{2.21}$$

Therefore $M_h < M_Z$ and $M_H > M_Z$ at *tree level*. These results generalize to the case of 2N Higgs doublet models[5]. See also Appendix D.

In a non-supersymmetric two doublet model the Higgs masses M_H, M_h and M_A and the mixing angles are independent quantities. Supersymmetry, by constraining the quartic couplings, reduces the number of parameters needed to completely describe the Higgs sector at tree level to just two. Quantum corrections introduce dependence on the other masses and couplings in the theory.

When the MSSM is obtained from low-energy supergravity models, $\tan\beta > 1$ is preferred. In these models a heavy top quark is required to drive the renormalization group evolution and obtain the requisite electroweak symmetry breaking. Therefore v_2 larger than v_1 is favored.

The existence at tree level of a Higgs boson lighter than the Z boson has been of much interest recently as a Z factory has become available. If $M_h < M_Z$, then the decay $Z \to Z^* h$ is kinematically possible. This processes is suppressed by a mixing factor relative to the same process in the standard model. If the pseudoscalar Higgs A is also light (which is not a required condition in the MSSM), then the decay $Z \to Ah$ may also be possible. Experiments at LEP have used these processes to rule out regions of parameter space of the MSSM[15-16]. A discussion of the current status of these experiments from a theoretical perspective can be found in Reference [17,18].

Of course radiative corrections are important as well. Several recent calculations indicated that indeed at one-loop the lightest Higgs boson can be much heavier than the Z boson[19-22]. The necessary ingredient in these calculations is a large fermion mass (specifically the top quark mass). A heavy top quark mass is an important correction even for the sum rules[6].

If a singlet superfield N exists in the theory new terms can be included in the superpotential, an example of which is $\lambda \epsilon_{ij} H_1^i H_2^j N$. In E_6 superstring inspired models the two Higgs doublets are accompanied by a singlet[23]. The new terms in the superpotential can give rise to quartic terms in the Higgs potential. In addition there is no guarantee that λ is small, so strong coupling is a possibility in a supersymmetric model with an $SU(2)$ singlet.

The masses of the Higgs bosons can be obtained from (2.4) using the vacuum expectation values in (2.5). The mass matrices must be diagonalized to obtain M_H^2, M_h^2, and M_A^2. In the MSSM there is the tree level mass relation given in (1.3) where $M_h < M_Z$ and $M_H > M_Z$. Beyond tree level this relation is no longer exact but receives $O(\alpha)$ corrections. To implement the renormalization procedure, we fix M_H, M_A,and M_Z to be the physical masses which can in principle be measured by experiment. Then the physical mass of the other neutral Higgs boson h is given by a relation

$$M_h^2 = M_A^2 + M_Z^2 - M_H^2 + \Delta \tag{2.22}$$

where Δ is a correction that is $O(\alpha)$. There are two free parameters that characterize the tree-level masses in the Higgs sector if M_Z is fixed at its experimentally measured value. We shall take M_H and M_A to be the two parameters that define the theory. Then (2.22) provides a prediction for the light Higgs boson mass M_h. We can choose any two unknown masses we like and predict the mass of the third.

III. FORMALISM FOR RADIATIVE CORRECTIONS

We adopt a renormalization scheme is which external lines are evaluated with momenta on-shell. The physical mass is defined as the position of the pole in the propagator. The ultimate results of this section are the relations (3.41) and (3.49) below. These equations indicate that at the one-loop level the wave-function renormalization factors do not enter, and the corrections to the mass sum rules are given entirely by combinations of Higgs-boson and vector boson self-energies.

Before developing the formalism for calculating radiative corrections, we wish to discuss the applicability of the one-loop effective potential to determining physical Higgs masses. The effective potential cannot be used to calculate the poles of Higgs propagators exactly. It may be used to find an approximate result for the physical masses of the Higgs bosons in the MSSM. The calculation of the effective potential entails the summation of diagrams with external Higgs boson momenta set equal to zero. In the on-shell scheme, the external lines are put on-shell instead. The curvature of the scalar potential at its minimum is the physical mass of the Higgs only at tree level. The renormalized Higgs mass found using the renormalized one-loop effective potential is finite but is not necessarily equal to the physical Higgs mass (defined as the position of the pole in the Higgs propagator). There is no elementary method to relate these two quantities[24] without calculating the Higgs propagator to find the pole. However if the mass is sufficiently small, the difference between the Higgs self-energy with external momenta on-shell and with external momenta set to zero is small. Then the effective potential is a useful tool for calculating the physical Higgs mass. In fact, the Coleman-Weinberg mass[25] is the physical mass since setting external momenta to zero is the same to one-loop as setting them on-shell for this case. The calculation presented here goes beyond the effective potential in that the physical masses of the Higgs bosons are the quantities that enter into the formulae. In the MSSM we know that $M_H > M_Z$ at tree level, so setting the external legs to zero momenta is not necessarily a good approximation.

In this section, we denote all bare fields and parameters by the subscript b. Absence of this subscript indicates a renormalized field or a renormalized parameter. For example, H_b denotes the bare heavy-Higgs field, while H denotes the renormalized field.

In the multi-Higgs doublet models, renormalization is complicated by mixing of the physical Higgs bosons necessitating rediagonalization at each order. This is analogous to the mixing of the Z and the photon in the renormalization of the standard model[26]. Here we follow the method of Aoki et al.[27] for on-shell renormalization of fields when mixing is present.

Recall the definition of the scalar and pseudoscalar components of the charge-neutral Higgs boson fields:

$$H_1^1 = v_1 + \frac{1}{\sqrt{2}}(S_1 + i\,P_1) \tag{3.1a}$$

$$H_2^2 = v_2 + \frac{1}{\sqrt{2}}(S_2 + i\,P_2). \tag{3.1b}$$

H and h are linear combinations of S_1 and S_2 while A and G are linear combinations of P_1 and P_2. The factor of $\sqrt{2}$ is included so the kinetic energy terms for the physical Higgs boson fields will have the canonical form.

Renormalization proceeds in the standard way. Begin with a tree level Lagrangian $\mathcal{L}_t(f_1, f_2, ...; p_1, p_2, ...)$ which contains certain fields f_i and parameters p_j. To calculate at one-loop, renormalized fields and parameters are required. This is accomplished by breaking up the tree level Lagrangian into a piece containing renormalized fields and parameters and a counterterm piece. The fields and parameters in the tree level Lagrangian are now not physical quantities, contain infinities, and are called bare quantities. The counterterms Lagrangian is generated by shifting the parameters $p_{jb} \rightarrow p_{jr} + \delta p_j$ and introducing wave-function renormalizations Z_{f_i}. The wave-function renormalizations are of the form $Z_{f_i} = 1 + \delta Z_{f_i}$ where the δZ_{f_i} are in general divergent and of higher order in perturbation theory. $Z_{f_i} = I + \delta Z_{f_i}$ is a matrix equation if there is mixing. The renormalized Lagrangian has the same functional form as the tree Lagrangian but is expressed in terms of renormalized quantities.

$$\mathcal{L}_b(f_{1b}, f_{2b}, ...; p_{1b}, p_{2b}, ...)$$
$$= \mathcal{L}_r(f_{1r}, f_{2r}, ...; p_{1r}, p_{2r}, ...) + \mathcal{L}_{ct}(f_{1r}, f_{2r}, ...; p_{1r}, p_{2r}, ...; \delta p_1, ...; Z_{f_1}, ...). \tag{3.2}$$

Feynman rules are derived using the renormalized Lagrangian and the counterterm Lagrangian, and the infinities present in one-loop graphs are absorbed in the counterterm Lagrangian. The values of the renormalized parameters are fixed by experiment.

When tree-level mixing occurs, wave-function renormalization takes a matrix form. Define the matrices

$$Z_S^{1/2} = \begin{pmatrix} Z_{HH}^{1/2} & Z_{Hh}^{1/2} \\ Z_{hH}^{1/2} & Z_{hh}^{1/2} \end{pmatrix} \tag{3.3a}$$

and

$$Z_P^{1/2} = \begin{pmatrix} Z_{GG}^{1/2} & Z_{GA}^{1/2} \\ Z_{AG}^{1/2} & Z_{AA}^{1/2} \end{pmatrix}. \tag{3.3b}$$

In the bare Lagrangian we denote all parameters and fields with the subscript b. In particular the Higgs potential in (2.4) is rewritten in terms of bare fields and masses by attaching a subscript b to all quantities. Then the wave-function renormalization of the Higgs fields can be expressed as

$$\begin{pmatrix} H \\ h \end{pmatrix}_b = Z_S^{1/2} \begin{pmatrix} H \\ h \end{pmatrix} \tag{3.4a}$$

and

$$\begin{pmatrix} G \\ A \end{pmatrix}_b = Z_P^{1/2} \begin{pmatrix} G \\ A \end{pmatrix}. \tag{3.4b}$$

The matrices in (3.3) are not in general symmetric. There are four independent parameters for each matrix. We have that $Z_S^{1/2} = I + O(\alpha)$ so that $Z_{HH}^{1/2} = 1 + O(\alpha)$, $Z_{hh}^{1/2} = 1 + O(\alpha)$, $Z_{Hh}^{1/2} = O(\alpha)$, and $Z_{hH}^{1/2} = O(\alpha)$. The kinetic energy terms for the charge neutral pieces are

$$\frac{1}{2}\partial^\mu \begin{pmatrix} H & h \end{pmatrix} (Z_S^{1/2})^T Z_S^{1/2} \partial_\mu \begin{pmatrix} H \\ h \end{pmatrix} + \frac{1}{2}\partial^\mu \begin{pmatrix} G & A \end{pmatrix} (Z_P^{1/2})^T Z_P^{1/2} \partial_\mu \begin{pmatrix} G \\ A \end{pmatrix}. \tag{3.5}$$

Now we proceed to investigate the mass terms. In the usual way we shift the parameters that occur in the Higgs mass terms as follows

$$(m_1^2)_b = m_1^2 + \delta m_1^2 \tag{3.6a}$$

$$(m_2^2)_b = m_2^2 + \delta m_2^2 \tag{3.6b}$$

$$(m_{12}^2)_b = m_{12}^2 + \delta m_{12}^2 \tag{3.6c}$$

$$(M_Z^2)_b = M_Z^2 + \delta M_Z^2 \tag{3.6d}$$

$$(v_1)_b = v_1 + \delta v_1 \tag{3.6e}$$

$$(v_2)_b = v_2 + \delta v_2. \tag{3.6f}$$

The Higgs potential in (2.4) depends on five parameters, so we can choose five parameters in (3.6) to determine the potential. The parameters we use to define the theory are the physical masses M_H, M_h, M_A and M_Z as well as the coupling g (or α). The quantities in (3.6) are related to these five in a complicated way determined by the Higgs potential in (2.4) as was demonstrated in Section II. Other parameters such as $|\mu|^2$ and its associated counterterm are determined in terms of the five parameters and counterterms in (3.6). The dependence of μ on the other parameters is given in Equation (3.25) of Reference [9].

The shifts in v_1 and v_2 reflect the fact that the location of the minimum of the Higgs potential receives $O(\alpha)$ corrections. This is a generalization of the same statement in the standard model, where the tree level vacuum expectation value v receives $O(\alpha)$ corrections. In the tree level Lagrangian v_1 and v_2 are determined by finding the minimum of the Higgs potential. Therefore v_1 and v_2 are specified by the parameters in the Higgs potential, $m_1^2 + |\mu|^2$, $m_2^2 + |\mu|^2$, m_{12}, M_Z, etc. The constraints were given in Section II by (2.17) and (2.18). At one-loop these parameters are renormalized, and the same functional forms for v_1 and v_2 in terms of the renormalized parameters are no longer correct.

One approach is to define tadpole counterterms τ_H and τ_h so that they exactly cancel the one-loop tadpole diagrams. This would impose two constraints on the counterterms. We will show below and in Appendix C that it is in fact of no consequence how the tadpole divergences are handled as they are exactly cancelled in the radiative corrections to the sum rules. See also Reference [20].

Our goal then is to formulate renormalization conditions for the physical masses without any reference to the unmeasureable parameters that occur in (3.6). The basic idea is the following. The Higgs masses depend on M_Z and two mixing angles, usually called α and β (β was introduced in Section II). To obtain the one-loop corrected masses requires these angles to be renormalized. However in the sum rule we are interested only in the traces of the mass matrices, and if the rotation angles α and β that diagonalize the mass matrices are renormalized is of no consequence. We shall go through the detailed procedure of the renormalization procedure below. A more general argument valid for modes with an arbitrary number of Higgs doublets (including the two doublet case) is given in Appendix D.

The Higgs mass terms arise in the potential given by (2.4). The parameters m_1, m_2, and m_{12} are undetermined due to the arbitrariness of the soft supersymmetry breaking terms. The mass constraints arise because the quartic couplings in (2.4) are determined in terms of the gauge couplings by supersymmetry and gauge invariance. Then the mass terms that arise are of the form

$$\frac{1}{2}\begin{pmatrix} S_1 & S_2 \end{pmatrix}_b \begin{pmatrix} A & B \\ B & C \end{pmatrix}_b \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}_b \tag{3.7}$$

where

$$A_b = (m_1^2)_b + \frac{1}{2}(M_Z^2)_b \left(\frac{3v_1^2 - v_2^2}{v_1^2 + v_2^2}\right)_b \tag{3.8a}$$

$$B_b = -(m_{12}^2)_b + \frac{1}{2}(M_Z^2)_b \left(\frac{v_1 v_2}{v_1^2 + v_2^2}\right)_b \tag{3.8b}$$

$$C_b = (m_2^2)_b + \frac{1}{2}(M_Z^2)_b\left(\frac{3v_2^2 - v_1^2}{v_1^2 + v_2^2}\right)_b. \tag{3.8c}$$

This mass matrix is diagonalized by the real orthogonal matrix characterized by the angle α:

$$O_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \tag{3.9a}$$

where

$$\tan 2\alpha = \frac{2B_b}{(A-C)_b}. \tag{3.9b}$$

With a redefinition of fields given by

$$\begin{pmatrix} S_1 \\ S_2 \end{pmatrix}_b = O_\alpha \begin{pmatrix} H \\ h \end{pmatrix}_b \tag{3.10}$$

the mass matrix is diagonalized to give

$$O_{-\alpha}\begin{pmatrix} A & B \\ B & C \end{pmatrix}_b O_\alpha = \begin{pmatrix} M_H^2 & 0 \\ 0 & M_h^2 \end{pmatrix}_b \tag{3.11a}$$

where

$$(M_H^2)_b = \frac{1}{2}\left[(A_b + C_b) + \sqrt{(A_b - C_b)^2 + 4B_b^2}\right] \tag{3.11b}$$

$$(M_h^2)_b = \frac{1}{2}\left[(A_b + C_b) - \sqrt{(A_b - C_b)^2 + 4B_b^2}\right]. \tag{3.11c}$$

The shifts in the parameters introduced in (3.6) generate shifts in the parameters A_b, B_b, and C_b that appear in the unrenormalized mass matrix through the definitions in (3.8). We define the renormalized values of these parameters and the associated counterterms as $A_b = A+\delta A$, $B_b = B+\delta B$, and $C_b = C+\delta C$ where A, B, and C are defined just as the bare quantities are defined in (3.8) but in terms of the renormalized quantities. It is unnecessary to retain terms second order in the counterterms because these are higher order in perturbation theory.

The inverse propagator is a matrix due to the mixing of the Higgs bosons, and we denote it by:

$$i\Gamma_S(p^2) = \begin{pmatrix} i\Gamma_{HH}(p^2) & i\Gamma_{Hh}(p^2) \\ i\Gamma_{hH}(p^2) & i\Gamma_{hh}(p^2) \end{pmatrix}. \tag{3.12}$$

Then we have

$$i\Gamma_S(p^2) = (Z_S^{1/2})^T Z_S^{1/2} p^2 - (Z_S^{1/2})^T (M_S^2)_D Z_S^{1/2} - \delta M_S \tag{3.13a}$$

where

$$(M_S^2)_D \equiv \begin{pmatrix} (M_H^2)_r & 0 \\ 0 & (M_h^2)_r \end{pmatrix} \tag{3.13b}$$

and

$$\delta M_S \equiv \begin{pmatrix} \delta M_H^2 & \delta M_{Hh}^2 \\ \delta M_{hH}^2 & \delta M_h^2 \end{pmatrix} \tag{3.13c}$$

The subscript D in (3.13b) indicated that the renormalized mass matrix (with subscripts r) is diagonal. In obtaining (3.13) we have dropped terms that are second order in perturbation theory, used (3.11a), and defined

$$\delta M_H^2 = \delta A \cos^2\alpha + \delta B \sin 2\alpha + \delta C \sin^2\alpha \tag{3.14a}$$

$$\delta M_h^2 = \delta A \sin^2\alpha - \delta B \sin 2\alpha + \delta C \cos^2\alpha \tag{3.14b}$$

$$\delta M_{Hh}^2 = \delta M_{hH}^2 = (\delta C - \delta A)\sin\alpha\cos\alpha + \delta B \cos 2\alpha. \tag{3.14c}$$

We have neglected the pieces of the counterterms coming from $\delta\alpha$ (in $\alpha \to \alpha+\delta\alpha$) that in fact exactly cancel (3.14c). The off-diagonal terms are irrelevant in the renormalization of the sum rules. The inverse propagator matrix in (3.12) is symmetric as it should be. We have also defined the quantities

$$(M_H^2)_r = \frac{1}{2}\left[(A+C) + \sqrt{(A-C)^2 + 4B^2}\right] \tag{3.15a}$$

$$(M_h^2)_r = \frac{1}{2}\left[(A+C) - \sqrt{(A-C)^2 + 4B^2}\right] \tag{3.15b}$$

At this point the renormalized parameters $(M_H^2)_r$ and $(M_h^2)_r$ are not the physical masses M_H^2 and M_h^2. The connection between these quantities must be specified by renormalization conditions.

We have expressed the inverse propagator $i\Gamma_S(p^2)$ in terms of the wave-function renormalization parameters defined in (3.3) and the counterterms defined in (3.6). The expression is rather complicated, but fortunately we will only need to know the linear combination $\delta A + \delta C$ to calculate the radiative corrections to the mass relation (1.3). Notice that $\delta M_H^2 + \delta M_h^2 = \delta A + \delta C$, i.e. the trace of the mass matrix is invariant under the orthogonal transformation. We have that $\delta A + \delta C = \delta m_1^2 + \delta m_2^2 + \delta M_Z^2$ so that we arrive at the conclusion:

$$\delta M_H^2 + \delta M_h^2 = \delta m_1^2 + \delta m_2^2 + \delta M_Z^2. \tag{3.16}$$

We now repeat the analysis for the pseudoscalar sector in exactly the same way as we did the analysis for the scalar sector in Equations (3.7) to (3.16). Define $P_1 = \sqrt{2} Im(H_1^1)$ and $P_2 = \sqrt{2} Im(H_2^2)$. The mass terms are

$$\frac{1}{2}\begin{pmatrix} P_1 & P_2 \end{pmatrix}_b \begin{pmatrix} A' & B' \\ B' & C' \end{pmatrix}_b \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}_b \tag{3.17}$$

where

$$A_b' = (m_2^2)_b + \frac{1}{2}(M_Z^2)_b \left(\frac{v_1^2 - v_2^2}{v_1^2 + v_2^2}\right)_b \tag{3.18a}$$

$$B_b' = -(m_{12}^2)_b \tag{3.18b}$$

$$C_b' = (m_1^2)_b + \frac{1}{2}(M_Z^2)_b \left(\frac{v_2^2 - v_1^2}{v_1^2 + v_2^2}\right)_b. \tag{3.18c}$$

This mass matrix is diagonalized by an orthogonal transformation just as before. The real rotation matrix is characterized by the angle β:

$$O_\beta = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \tag{3.19a}$$

where

$$\tan 2\beta = \frac{2B'_b}{(A'-C')_b}. \tag{3.19b}$$

We obtain the mass eigenstates defined by

$$\begin{pmatrix} P_1 \\ P_2 \end{pmatrix}_b = O_\beta \begin{pmatrix} G \\ A \end{pmatrix}_b \tag{3.20}$$

The mass matrix in (3.17) is diagonalized to give

$$O_{-\beta} \begin{pmatrix} A' & B' \\ B' & C' \end{pmatrix}_b O_\beta = \begin{pmatrix} 0 & 0 \\ 0 & M_A^2 \end{pmatrix}_b \tag{3.21a}$$

where

$$(M_G^2)_b = \frac{1}{2}\left[(A'_b + C'_b) + \sqrt{(A'_b - C'_b)^2 + 4B'^2_b}\right] = 0 \tag{3.21b}$$

$$(M_A^2)_b = \frac{1}{2}\left[(A'_b + C'_b) - \sqrt{(A'_b - C'_b)^2 + 4B'^2_b}\right]. \tag{3.21c}$$

The Goldstone boson is exactly massless in the Landau gauge so the mass matrix has a zero eigenvalue.

The parameters defined by Equations (3.18) generate counterterms with the

$$\delta A' = \delta m_1^2 + \frac{1}{2}\delta M_Z^2 \left(\frac{v_1^2 - v_2^2}{v_1^2 + v_2^2}\right) + \frac{1}{2} M_Z^2 \delta \left(\frac{v_1^2 - v_2^2}{v_1^2 + v_2^2}\right) \tag{3.22a}$$

$$\delta B' = -\delta m_{12}^2 \tag{3.22b}$$

$$\delta C' = \delta m_2^2 + \frac{1}{2}\delta M_Z^2 \left(\frac{v_2^2 - v_1^2}{v_1^2 + v_2^2}\right) + \frac{1}{2} M_Z^2 \delta \left(\frac{v_2^2 - v_1^2}{v_1^2 + v_2^2}\right). \tag{3.22c}$$

We define the inverse propagator for the pseudoscalars in the same way as we did for the scalars:

$$i\Gamma_P(p^2) = \begin{pmatrix} i\Gamma_{GG}(p^2) & i\Gamma_{GA}(p^2) \\ i\Gamma_{AG}(p^2) & i\Gamma_{AA}(p^2) \end{pmatrix} \tag{3.23}$$

so that

$$i\Gamma_P(p^2) = (Z_P^{1/2})^T Z_P^{1/2} p^2 + (Z_P^{1/2})^T O_{-\beta} \begin{pmatrix} A' + \delta A' & B' + \delta B' \\ B' + \delta B' & C' + \delta C' \end{pmatrix} O_\beta Z_P^{1/2}. \tag{3.24}$$

The last term can be expanded again to obtain:

$$i\Gamma_P(p^2) = (Z_P^{1/2})^T Z_P^{1/2} p^2 + (Z_P^{1/2})^T (M_P^2)_D Z_P^{1/2} + \delta M_P \tag{3.25a}$$

where

$$(M_P^2)_D \equiv \begin{pmatrix} 0 & 0 \\ 0 & (M_A^2)_r \end{pmatrix} \tag{3.25b}$$

and

$$\delta M_P \equiv \begin{pmatrix} \delta M_G^2 & \delta M_{GA}^2 \\ \delta M_{AG}^2 & \delta M_A^2 \end{pmatrix} \tag{3.25c}$$

We have defined

$$\delta M_G^2 = \delta A' \cos^2\beta + \delta B' \sin 2\beta + \delta C' \sin^2\beta \tag{3.26a}$$

$$\delta M_A^2 = \delta A' \sin^2\beta - \delta B' \sin 2\beta + \delta C' \cos^2\beta \tag{3.26b}$$

$$\delta M_{GA}^2 = \delta M_{AG}^2 = (\delta C' - \delta A') \sin\beta \cos\beta + \delta B' \cos 2\beta \tag{3.26c}$$

and $(M_A^2)_r$ is defined just as $(M_A^2)_b$ is defined in (3.21) but in terms of the renormalized parameters (i.e. without the subscripts b on the parameters appearing on the RHS). As before we neglect $\delta\beta$ corrections to the off-diagonal terms. $(M_G^2)_r = 0$ since $(M_G^2)_b = 0$. The invariance of the trace gives

$\delta M_G^2 + \delta M_A^2 = \delta A' + \delta C'$. From (3.22) we have that $\delta A' + \delta C' = \delta m_1^2 + \delta m_2^2$, so we obtain

$$\delta M_G^2 + \delta M_A^2 = \delta m_1^2 + \delta m_2^2. \tag{3.27}$$

Making use of (3.16) we finally obtain the result:

$$\delta M_H^2 + \delta M_h^2 = \delta M_G^2 + \delta M_A^2 + \delta M_Z^2. \tag{3.28}$$

We define the self-energies of the scalars and the vector bosons as shown in Figure 1 with external legs amputated. The vacuum expectation values v_1 and v_2 are in general renormalized, and tadpole diagrams must be taken into account. We will argue below that the tadpole contributions to the final result Δ are zero with the renormalization conditions we choose. This will be shown explicitly in Appendix C. The renormalized inverse propagator

$$i\bar{\Gamma}_S(p^2) = \begin{pmatrix} i\bar{\Gamma}_{HH}(p^2) & i\bar{\Gamma}_{Hh}(p^2) \\ i\bar{\Gamma}_{hH}(p^2) & i\bar{\Gamma}_{hh}(p^2) \end{pmatrix} \tag{3.29}$$

includes the expression in (3.13) and the self-energy contributions shown in Figure 1. The inverse propagator matrix in (3.29) is symmetric. We have

$$i\bar{\Gamma}_{HH}(p^2) = (Z_{HH} + Z_{hH})p^2 - (M_H^2)_r Z_{HH} - (M_h^2)_r Z_{hH}$$
$$-\delta M_H^2 + \Pi_{HH}(p^2) \tag{3.30a}$$

$$i\bar{\Gamma}_{hh}(p^2) = (Z_{hh} + Z_{Hh})p^2 - (M_h^2)_r Z_{hh}^2 - (M_H^2)_r Z_{Hh}$$
$$-\delta M_h^2 + \Pi_{hh}(p^2) \tag{3.30b}$$

$$i\tilde{\Gamma}_{Hh}(p^2) = i\tilde{\Gamma}_{hH}(p^2) = (Z_{HH}^{1/2} Z_{Hh}^{1/2} + Z_{hh}^{1/2} Z_{hH}^{1/2})p^2 - (M_H^2)_r Z_{HH}^{1/2} Z_{Hh}^{1/2}$$

$$-(M_h^2)_r Z_{hh}^{1/2} Z_{hH}^{1/2} - \delta M_{Hh}^2 + \Pi_{Hh}(p^2). \qquad (3.30c)$$

In the on-shell scheme we adopt the renormalization conditions[27]:

$$i\tilde{\Gamma}_{HH}(M_H^2) = 0 \qquad (3.31a)$$

$$i\tilde{\Gamma}_{hh}(M_h^2) = 0 \qquad (3.31b)$$

$$i\tilde{\Gamma}_{Hh}(M_H^2) = i\tilde{\Gamma}_{Hh}(M_h^2) = 0 \qquad (3.31c)$$

$$i\tilde{\Gamma}'_{HH}(M_H^2) = 1 \qquad (3.31d)$$

$$i\tilde{\Gamma}'_{hh}(M_h^2) = 1 \qquad (3.31e)$$

where $i\tilde{\Gamma}'(p^2)$ is the derivative of $i\tilde{\Gamma}(p^2)$.

We choose as an additional renormalization condition that $(M_H)_r$ be set equal to the physical mass M_H of the H[28]. Then from (3.30) and (3.31) we conclude that

$$\delta M_H^2 = \Pi_{HH}(M_H^2). \qquad (3.32)$$

The pseudoscalar sector can be treated in the same way. The renormalized inverse propagator for the pseudoscalars is

$$i\tilde{\Gamma}_P(p^2) = \begin{pmatrix} i\tilde{\Gamma}_{GG}(p^2) & i\tilde{\Gamma}_{GA}(p^2) \\ i\tilde{\Gamma}_{AG}(p^2) & i\tilde{\Gamma}_{AA}(p^2) \end{pmatrix} \qquad (3.33)$$

which is

$$i\tilde{\Gamma}_{GG}(p^2) = (Z_{GG} + Z_{AG})p^2 - (M_A^2)_r Z_{AG}$$

$$-\delta M_G^2 + \Pi_{GG}(p^2) \tag{3.34a}$$

$$i\bar{\Gamma}_{AA}(p^2) = (Z_{AA} + Z_{GA})p^2 - (M_A^2)_r Z_{AA}^2$$
$$-\delta M_A^2 + \Pi_{AA}(p^2) \tag{3.34b}$$

$$i\bar{\Gamma}_{GA}(p^2) = i\bar{\Gamma}_{AG}(p^2) = (Z_{GG}^{1/2} Z_{GA}^{1/2} + Z_{AA}^{1/2} Z_{AG}^{1/2})p^2$$
$$-(M_A^2)_r Z_{AA}^{1/2} Z_{AG}^{1/2} - \delta M_{GA}^2 + \Pi_{GA}(p^2). \tag{3.34c}$$

The renormalization conditions are

$$i\bar{\Gamma}_{GG}(0) = 0 \tag{3.35a}$$

$$i\bar{\Gamma}_{AA}(M_A^2) = 0 \tag{3.35b}$$

$$i\bar{\Gamma}_{GA}(0) = i\bar{\Gamma}_{GA}(M_A^2) = 0 \tag{3.35c}$$

$$i\bar{\Gamma}'_{GG}(0) = 1 \tag{3.35d}$$

$$i\bar{\Gamma}'_{AA}(M_A^2) = 1 \tag{3.35e}$$

In this case we define $(M_A)_r$ to be the physical mass M_A of the A and require that the Goldstone boson G have zero mass at the one-loop level in the Landau gauge, i.e. $(M_G)_r = M_G = 0$. The masslessness of the Goldstone boson at one-loop follows from the Ward identities. Then we obtain

$$\delta M_A^2 = \Pi_{AA}(M_A^2) \tag{3.36}$$

$$\delta M_G^2 = \Pi_{GG}(0). \tag{3.37}$$

The remaining condition is obtained from (2.21) and (3.31b). This is

$$\delta M_h^2 = \Pi_{hh}(M_h^2) + \Delta. \tag{3.38}$$

where we have used the fact that $(M_H^2)_r + (M_h^2)_r = (M_A^2)_r + (M_Z^2)_r$. Similarly it can be shown that

$$\delta M_Z^2 = -A_{ZZ}(M_Z^2) \tag{3.39}$$

where $A_{ZZ}(p^2)$ is defined as the real part of the coefficient of $g^{\mu\nu}$ in the vacuum polarization tensor

$$\Pi_{ZZ}^{\mu\nu}(p^2) = \mathcal{A}_{ZZ}(p^2)g^{\mu\nu} + B_{ZZ}(p^2)p^\mu p^\nu \tag{3.40a}$$

$$A_{ZZ} = Re\ \mathcal{A}_{ZZ} \tag{3.40b}$$

defined as in Figure 1. Then using and Equations (3.28), (3.32) and (3.36)-(3.39) we find that

$$\Delta = -\Pi_{HH}(M_H^2) - \Pi_{hh}(M_h^2) + \Pi_{AA}(M_A^2) + \Pi_{GG}(0) - A_{ZZ}(M_Z^2). \tag{3.41}$$

So the calculation of Δ involves the determination of the Higgs and Z self-energies in (3.41). The final result for Δ must be finite even though the individual self-energies will not be. The expression in (3.41) depends only on self-energies. This is a somewhat unique result for a radiative correction to a physically measurable quantity. Usually one is required to calculate vertex corrections as well to do a precise comparison to experiment. Here supersymmetry and gauge invariance have conspired to produce a sum rule whose renormalization does not depend on the renormalization of the gauge couplings g and g' to one-loop. At two-loops and beyond the situation becomes more complicated, as we expect the gauge coupling renormalizations to enter as well as wave-function renormalizations.

The condition that the Goldstone boson mass be zero at one-loop ensures that the tadpole contributions will be zero. This is a consequence of a Ward identity. A discussion of this result in the context of the standard model is given in References [29-31]. The Goldstone self-energy at zero momentum is related to the tadpole diagrams of the H and h fields as

$$\Pi_{GG}(0) = -\frac{1}{\sqrt{2}v}\left[\cos(\beta-\alpha)T_H + \sin(\beta-\alpha)T_h\right]. \tag{3.42}$$

The counterterm Lagrangian contains the terms

$$-[\delta M_G^2 G^2 + \tau_H H + \tau_h h] \tag{3.43}$$

in which the coefficients satisfy

$$\delta M_G^2 = \frac{1}{\sqrt{2}v}\cos(\beta-\alpha)\tau_H + \sin(\beta-\alpha)\tau_h. \tag{3.44}$$

So we conclude that (3.37) is equivalent to taking $(T_H+\tau_H)\cos(\beta-\alpha)+(T_h+\tau_h)\sin(\beta-\alpha)=0$. The advantage in calculating $\Pi_{GG}(0)$ rather than the tadpole diagrams T_H and T_h is that the cancelation of divergences is more obvious in the former case. In terms of the Feynman rules, calculating the Goldstone boson self-energy is on an equal footing with calculating the Higgs boson self-energies in the Landau gauge. We have shown explicitly in Appendix C that in the context of the MSSM the tadpole contributions to Δ in (3.41) vanish identically. This result can be proved generally. In References [2,7] the tadpoles are evaluated instead of the Goldstone boson self-energy. This gives the same answer as (3.42) can be verified by direct calculation. As mentioned previously this is a generalization of a similar statement in the standard model. In the standard model the Goldstone boson self-energy is related to the Higgs tadpole (there is only one such tadpole in the standard model)

$$\Pi_{GG}(0) = -\frac{1}{\sqrt{2}v} T_H. \tag{3.45}$$

There is an elementary way to gain insight into the relationship between Goldstone boson counterterm and the tadpole counterterm. In any multi-Higgs model it is always possible to find a linear combination of Higgs fields whose vacuum expectation values is v, and all orthogonal components have zero vevs. In other words, in Higgs field space this linear combination is in the direction from the symmetry point to the asymmetric minimum. In the two doublet model we know that this direction is $\mathcal{H} = S_1 \cos\beta + S_2 \sin\beta$. Define the orthogonal combination $\mathcal{H}_\perp = -S_1 \sin\beta + S_2 \cos\beta$. We have

$$\begin{pmatrix} H \\ h \end{pmatrix} = O_{\beta-\alpha} \begin{pmatrix} \mathcal{H} \\ \mathcal{H}_\perp \end{pmatrix} \tag{3.46}$$

and the counterterms in (3.43) become

$$-\Big[\delta M_G^2 G^2 + \mathcal{H}[\cos(\beta-\alpha)\tau_H + \sin(\beta-\alpha)\tau_h] + \mathcal{H}_\perp[-\sin(\beta-\alpha)\tau_H + \cos(\beta-\alpha)\tau_h]\Big] \tag{3.47}$$

The Goldstone self-energy is related to the tadpole counterterms of the Higgs field combination $\mathcal{H}$ that lies in the direction of the asymmetric minimum. In the standard model this combination is just the physical Higgs field.

The tadpoles cancel in the Higgs mass sum rules and this requires the supersymmetric structure of the Higgs self-couplings. It is the constraints placed on these couplings by supersymmetry and gauge invariance that gives rise to the sum rules as well as the tadpole cancellation.

Another mass relation that holds at tree level in the MSSM was given in (1.4). It can be shown (in a method analogous to the preceding treatment of the mass relation in (1.3)) that the radiative corrections defined by

$$M_{H^\pm}^2 = M_A^2 + M_W^2 + \bar{\Delta} \tag{3.48}$$

are given by

$$\bar{\Delta} = -\Pi_{H^\pm H^\pm}(M_{H^\pm}^2) - \Pi_{G^\pm G^\pm}(0) + \Pi_{AA}(M_A^2) + \Pi_{GG}(0) - A_{WW}(M_W^2) \quad (3.49).$$

Again the tadpole contributions are exactly zero (see Appendix C).

We note that the result in (3.41) continues to hold when a Higgs singlet N is present in certain important cases. The criterion is that N not mix with the other Higgs bosons (H, h, A, G). Reference [9] discusses these cases. If the singlet mixes with the Higgs doublet then the mass relation (1.3) is destroyed even at tree level, and the tree-level constraints $M_h < M_Z$ and $M_H > M_Z$ also disappear. The mass relation (1.4) may be destroyed even if the singlet does not mix with the other fields.

IV. RADIATIVE CORRECTIONS

In this section we will discuss the contribution to Δ from quark and squark loops in the MSSM. It is necessary to know the Feynman rules for Higgs bosons in the MSSM to calculate the self-energy diagrams for the Higgs fields. Many of these have been derived previously in the literature[3,9,32-34]. We have derived some others in Reference [6] that appear in Appendix A.

The calculations involved are somewhat lengthy. Each individual diagram is divergent, and these divergences cancel only when loops involving the fermions and loops involving their superpartners are included. The divergent integrals are evaluated using dimensional regularization with the prescription for γ_5 given by Chanowitz et al.[35]. Since the γ_5's always occur in pairs in the amplitudes considered, this prescription guarantees the correct Ward identities. The calculation is straightforward, so we display only the final result in Appendix B. The diagrams evaluated are shown in Figure 3.

We have ignored the mixing between generations for simplicity, i.e. we approximate the CKM matrix and the super-CKM matrix as unit matrices. It is not difficult to adapt the answer to the general case. There is a contribution from each generation, and the contribution to Δ from the top quark is the same as that for the up quark with the appropriate mass substitutions. Of course the formulae are only relevant for quarks heavy compared to the hadronic scale. The calculation of the diagrams involving squark loops is complicated by the mixing in the squark sector.

We add soft supersymmetry breaking terms to the scalar potential. The terms in the scalar potential involving squarks are[9]

$$V = V_F + V_D + V_{soft}, \tag{4.1a}$$

where

$$V_F = (\mu^* H_1^{i*} + f_2\tilde{Q}^{i*}\tilde{U}^*)(\mu H_1^i + f_2\tilde{Q}^i\tilde{U}) + (\mu^* H_2^{i*} + f_1\tilde{Q}^{i*}\tilde{D}^*)(\mu H_2^i + f_1\tilde{Q}^i\tilde{D})$$

$$+f_1^2|\epsilon_{ij}H_1^i\tilde{Q}^j|^2 + f_2^2|\epsilon_{ij}H_2^i\tilde{Q}^j|^2 + (f_1 H_1^{i*}\tilde{D}^* - f_2 H_2^{i*}\tilde{U}^*)(f_1 H_1^i\tilde{D} - f_2 H_2^i\tilde{D}), \tag{4.1b}$$

$$V_D = \frac{1}{8}g^2\left[4|H_1^{i*}\tilde{Q}^i|^2 + 4|H_2^{i*}\tilde{Q}^i|^2 - 2(\tilde{Q}^{i*}\tilde{Q}^i)\left[H_1^{i*}H_1^i + H_2^{i*}H_2^i\right] + (\tilde{Q}^{i*}\tilde{Q}^i)^2\right]$$

$$+\frac{1}{8}g'^2\left[H_2^{i*}H_2^i - H_1^{i*}H_1^i + y_q\tilde{Q}^{i*}\tilde{Q}^i + y_u\tilde{U}^*\tilde{U} + y_d\tilde{D}^*\tilde{D}\right]^2, \tag{4.1c}$$

$$V_{soft} = \tilde{M}_Q^2\tilde{Q}^{i*}\tilde{Q}^i + \tilde{M}_u^2\tilde{U}^*\tilde{U} + \tilde{M}_d^2\tilde{D}^*\tilde{D}$$

$$+m_8\epsilon^{ij}(f_1 A_d H_1^i\tilde{Q}^j\tilde{D} - f_2 A_u H_2^i\tilde{Q}^j\tilde{U} + h.c.). \tag{4.1d}$$

y_u, y_d and y_q are the hypercharge quantum numbers of the corresponding fields. The conventional squark notation for the fields appearing in (2.1) and (4.1) is

$$\tilde{Q}^i = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \quad \tilde{U}^* = \tilde{u}_R, \quad \tilde{D}^* = \tilde{d}_R. \tag{4.2}$$

The mass terms for the up squarks, for example, are

$$-\begin{pmatrix} \tilde{u}_L^* & \tilde{u}_R^* \end{pmatrix}\begin{pmatrix} A_u & B_u \\ B_u & C_u \end{pmatrix}\begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix} \tag{4.3a}$$

where

$$A_u = \tilde{M}_Q^2 + M_Z^2\cos 2\beta(\frac{1}{2} - e_u\sin^2\theta_w) + m_u^2 \tag{4.3b}$$

$$B_u = m_u(A_u m_8 + \mu\cot\beta) \tag{4.3c}$$

$$C_u = \tilde{M}_U^2 + M_Z^2\cos 2\beta(e_u\sin^2\theta_w) + m_u^2. \tag{4.3d}$$

$A_u m_6$, $\tilde{M}_Q$, and $\tilde{M}_U$ are additional soft supersymmetry breaking parameters that enter into the part of the scalar potential that involves squarks. We assume $A_u m_6$ is real, which must be approximately the case to avoid unwanted CP violation. Notice that the left-right mixing term B_u is proportional to the fermion mass m_u. The mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$ can be defined as a mixture of these fields as

$$\begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} = O_{\theta_q} \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} \tag{4.4}$$

where O_{θ_q} are defined as in (3.8a). The mixing angles θ_q appear in the Feynman rules involving the squarks.

We note here that the soft supersymmetry breaking terms in (4.1d) do not include the so-called "mixed" trilinear contributions mentioned by Hall and Randall[10]. These terms are not present in the low-energy supergravity model but could be present in a more general model of weak-scale supersymmetry. These contributions are similar to those in (4.1d) in that they contribute to the off-diagonal elements of the squark mixing matrix and provide another source of coupling between the Higgs bosons and the squarks. In particular we have the terms

$$\mathcal{M}(f_1 A_d' H_2^{i*} \tilde{Q}^i \tilde{D} - f_2 A_u' H_1^{i*} \tilde{Q}^i \tilde{U} + h.c.). \tag{4.5}$$

This gives the additional contributions to the squark mass matrix off-diagonal entry B_u of $m_u A_u' \mathcal{M} \cot\beta$. Additional squark-squark-Higgs couplings arise. We expect these soft supersymmetry breaking terms to contribute to Δ in a similar way to the terms already in (4.1d).

The coupling of the squarks to the Higgs bosons come from three places in the scalar potential. First the D-terms contain contributions to the squark masses and to the squark-Higgs coupling that are of $O(gM_Z)$. The F-terms contain the Yukawa pieces that contribute a mass to the squarks equal to the quark mass (m_q), and terms of $O(gm_q)$ to the squark-Higgs couplings. The F-terms also contain the parameter μ which contributes to the off diagonal entries in the mass matrix (See Equation (4.3c)) as well as to the couplings. Finally the soft supersymmetry breaking terms contain the parameters $A_q m_6$ that contribute to the off diagonal terms in the mass matrix and in the couplings. The soft supersymmetry breaking parameters $\tilde{M}_Q^2$ and $\tilde{M}_U^2$ above in (4.3) do not contribute to the couplings.

The soft supersymmetry breaking parameters $\tilde{M}_Q^2$, $\tilde{M}_U^2$ and $A_u m_6$ are adjusted so that the squarks are sufficiently massive to have escaped detection while not so massive to destroy the stability of the electroweak scale to radiative corrections (i.e. the naturalness motivation for supersymmetry). The parameters $\tilde{M}_Q^2$ and $\tilde{M}_U^2$ show up in radiative corrections to Higgs masses in diagrams like that shown in Figure 4. In the renormalization of the mass sum rule, the combination of these diagrams that arises is shown in Figure 5. These diagrams sum exactly to zero. So while there are large corrections arising from $\tilde{M}_Q^2$ and $\tilde{M}_U^2$ to the mass of each Higgs boson, these contributions cancel in the sum rule. The sum rule is therefore insensitive to these parameters when they become large.

On the other hand, the supersymmetry breaking parameter $A_q m_6$ as well as the parameter μ contributes to the couplings of the squark to the Higgs bosons. If this parameter becomes large, substantial corrections can arise to the sum rule. It also generates mixing between the squark eigenstates. There are constraints on $A_q m_6$ from other considerations. When $A_q m_6$ becomes large, it usually produces large corrections to the rho parameter (although these contributions can be made to cancel against one another)[8]. In addition $A_q m_6$ is bounded by the requirement that the correct vacuum is obtained. Specifically if $A_q m_6$ is too large, the true vacuum breaks $SU(3)$ color which is of course ruled out phenomenologically.

The expression for Δ in Appendix B is composed of three parts, $\Delta = \Delta_4 + \Delta_2 + \Delta_0$. $\Delta_n \sim O(\alpha \frac{\tilde{m}^n}{M_W^n})$ where $\tilde{m}$ represents a mass parameter such as the up quark mass or a parameter involving the squark sector such as $A_u m_6$, μ, $m_{\tilde{u}_1}$ or $m_{\tilde{u}_2}$. We leave Δ in terms of the mixing angles α, β, and θ_q for convenience. The expressions for these angles in terms of physical masses are lengthy and not very illuminating. Expressions for α and β are given in Appendix A of Reference [36].

The terms in Δ_4 give the largest contribution to Δ for large quark and squark masses. The terms involving the off-diagonal entries in the squark mass matrix ($A_u m_6$ and μ) give large contributions provided the squark mixing angle θ_q is not small. Δ_2 contains terms that are $O(\alpha m_{\tilde{u}}^2)$, but these terms go to zero as the squark mass becomes large. This is a manifestation of the cancelation of the diagrams in Figure 5. The terms in Δ_2 of $O(\alpha m_{\tilde{u}}^2)$ come from the Z vacuum polarization only. Δ_0 is $O(\alpha M_Z^2)$ and is for our present purposes a negligible correction to the mass relation.

We will illustrate the result in Appendix B by considering the contribution from the top quark and the top squark. Four parameters characterize the squark mass matrix in (4.3). We can take these to be $m_{\tilde{t}_1}^2$, $m_{\tilde{t}_2}^2$, θ_t and μ. Then $A_t m_6$

is determined:

$$A_t m_6 = \frac{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)\sin 2\theta_t}{2m_t} - \mu \cot \beta. \tag{4.6}$$

First consider the case in which there is no squark mixing, i.e. $\theta_t = 0$. This is expected to be approximately the case for all squark species except possibly the top squark. When $\theta_t = 0^o$, the terms involving $A_t m_6$ and μ give only a small contribution to Δ. In this event the $A_t m_6$ and μ terms in the squark mass matrix are canceling one another. See (4.3). If the top quark and top squark are very massive ($m_t, m_{\tilde{t}} >> M_W, M_H, M_A, M_h$), we can neglect the other masses. Then we obtain

$$\Delta = \frac{g^2 m_t^4 N_c}{16\pi^2 M_W^2 \sin^2\beta} ln \left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} \right) \tag{4.7}$$

So we have large corrections to the mass relation just as there are large corrections ($O(m_t^4)$) in the Higgs sector of the standard model[37]. One factor of m_f^2 arises in the integration over the quark loop, while the Yukawa couplings at the vertices gives the other factor of m_f^2. We have plotted the correction Δ in Figure 6. We have chosen the parameters $m_t = 100 GeV$, $\alpha = -48^o$, $\beta = 30^o$ and $\mu = 0$. For these parameters the tree level Higgs boson masses are $M_H = 140\,GeV$, $M_h = 40\,GeV$ and $M_A = 110\,GeV$ and $M_H^2 + M_h^2 = 2 \times 10^4\,GeV^2$, so that each side of Equation (1.3) is equal to $2\times 10^4 GeV^2$ at tree level. So for $\Delta = 200 GeV^2$, *the correction is only one percent. We have plotted* Δ *for the case where* $\theta_t = 0^o$ in Figure 6a. The dependence on the squark masses is roughly logarithmic.

The expression in (4.7) diverges when $\sin^2\beta$ approaches zero. This reflects the fact that the Yukawa coupling giving the top quark a mass must diverge in this limit. The Yukawa coupling giving the bottom quark its mass diverges when $\cos^2\beta$ approaches zero. The non-decoupling of heavy quarks is just the standard evasion of the decoupling theorem[38] that arises when a coupling constant becomes large. When the supersymmetric limit is taken and the external momenta are set equal to zero rather than put on shell, the expression for Δ in Appendix B is zero. When the external legs are put on mass shell to obtain the physical masses, there are finite threshold effects that are in general non-zero even in the SUSY limit.

If there is significant mixing of the scalar quarks, large corrections can arise when there are large mass splittings between the squarks. In Figure 6b we have taken $\theta_t = 20^\circ$. Notice that the corrections are again small when $m_{\tilde{t}_1} \approx m_{\tilde{t}_2}$. If the squarks have significantly different masses, then there is a large negative Δ. These large corrections arise from large squark-Higgs couplings that arise because $A_t m_6$ is very large.

The results displayed in Figure 6 are typical. Other choices of the parameters m_t, α, β and μ give similar results. If $\theta_t \approx 0$, then corrections tend to be small (i.e. the same order as the contribution of a t quark with mass m_t in the standard model). If θ_t is significant, then large negative contributions arise when $|m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}|$ becomes significant. Negative values for Δ imply that the sum of the scalar Higgs boson masses squared $M^2_H + M^2_h$ is suppressed relative to the pseudoscalar boson mass squared M^2_A.

We note that large contributions to the mass sum rule are possible from a fourth generation as well, even when squark mixing is absent. As in the standard model the leading contribution for a heavy fermion ($m_f >> M_W$) goes like $\frac{\alpha m_f^4}{M_W^2}$[37]. So a priori if a heavy fermion exists, we can expect large corrections to the masses in the Higgs sector just as in the standard model. The results given here, however, are valid for any fermion mass, and it is only if $m_f >> M_W$ that Δ_4 can become very large. In Figure 7 we have held the squark masses fixed and plotted Δ as a function of the top quark mass. The values for α, β and μ are the same as in Figure 6. Notice that the correction Δ is positive as long as $m_t < m_{\tilde{t}_1}, m_{\tilde{t}_2}$. This is consistent with the radiative corrections to the light Higgs mass in Reference [21].

The contribution for a new top t' is given as in (4.7) while the new bottom b' will contribute (for $\theta_{b'} = 0$)

$$\Delta = \frac{g^2 m_{b'}^4 N_c}{16\pi^2 M_W^2 \cos^2\beta} ln\left(\frac{m_{\tilde{b}'_1}^2 m_{\tilde{b}'_2}^2}{m_{b'}^4}\right) \tag{4.8}$$

These contributions have the same sign. This differs from the renormalization of the ρ parameter in that the ρ parameter is protected by a custodial symmetry which is not broken by equal-mass fermion doublets. The effects of a mass-degenerate heavy doublet has been discussed before in the context of the standard model[39].

V. CONCLUSION

We have formulated the procedure for computing corrections to the Higgs mass relations in supersymmetric extensions to the standard model containing doublets. An explicit calculation in the case with just two doublets (the MSSM) was given. It was necessary to calculate self-energies of Higgs bosons and vacuum polarization tensors as shown in (3.41) and (3.49). Coupling constant and wave-function renormalizations are not necessary at one-loop. Tadpole contributions cancel exactly. The results in (3.41) and (3.49) are not destroyed in the presence *of other Higgs representations (singlets, triplets, etc.) provided that no mixing* between these fields and the Higgs doublets takes place. If mixing occurs, the tree-level mass relations (1.3) and (1.4) themselves will be destroyed as is easily understood in terms of the derivation of the mass sum rules in Section II. If a singlet or other state mixes with the Higgs fields, the relationship between the traces of the Higgs mass matrices will be destroyed. These results were generalized to the supersymmetric extensions to the standard model with more than two Higgs doublets (Appendix D).

We have performed an explicit computation of the radiative corrections to (1.3) from matter loops. We have found large corrections to the mass relation provided that the two complex squark fields mix. This results from large squark-Higgs couplings. The potentially large contributions of $O(\alpha m_{\tilde{q}}^2)$ or $O(\alpha m_{\tilde{l}}^2)$ to Higgs particle masses from a heavy squark and slepton sector in supersymmetric theories is hidden in the sum rule, i.e. cancels between the terms appearing in the sum rule. Provided that squark mixing is negligible, it is possible to imagine extremely large squark masses without inducing large radiative corrections to the sum rule.

APPENDIX A

Feynman Rules

In this appendix we display some Feynman rules that are needed in the calculation of Higgs boson self-energies in the MSSM. Other Feynman rules for the MSSM appear in References [3,9,32-34]. In Figure 8 we show the couplings of the Goldstone to the squarks. We have left the squarks in the weak interaction eigenstates for simplicity. In Figure 9 we show the trilinear couplings between the Goldstone bosons and the physical Higgs bosons. CP conservation demands that only an even number of pseudoscalars can emanate from a vertex.

APPENDIX B

The Correction to the Mass Relation

The $O(\alpha)$ corrections Δ can be divided into pieces

$$\Delta = \Delta_4 + \Delta_2 + \Delta_0 \tag{B.1}$$

where Δ_n is the part of Δ where the nth power of the up quark mass or parameters in the up squark mass matrix (such as $A_u m_6$, μ or the up squark masses themselves) occur. The results of the calculation are as follows:

$$\Delta_4 = \frac{g^2 m_u^4 N_c}{16\pi^2 M_W^2 \sin^2\beta}\Bigg[\sin^2\alpha(F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_H) + F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_H)$$

$$-3F(m_u, m_u, M_H))$$

$$+\cos^2\alpha(F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_h) + F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_h)$$

$$-3F(m_u, m_u, M_h))$$

$$+\cos^2\beta F(m_u, m_u, M_A) + \sin^2\beta F(m_u, m_u, 0)\Bigg]$$

$$+\frac{g^2 m_u^3 N_c[A_u m_6 \sin\alpha + \mu\cos\alpha]\sin 2\theta_u}{16\pi^2 M_W^2 \sin^2\beta}$$

$$\times\Big[\sin\alpha(F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_H) - F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_H))\Big]$$

$$+\frac{g^2 m_u^3 N_c[A_u m_6 \cos\alpha - \mu\sin\alpha]\sin 2\theta_u}{16\pi^2 M_W^2 \sin^2\beta}$$

$$\times\Big[\cos\alpha(F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_h) - F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_h))\Big]$$

$$-\frac{g^2 m_u^2 N_c}{64\pi^2 M_W^2 \sin^2\beta}\Bigg[\Big[2F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_A)[A_u m_6 \cos\beta - \mu\sin\beta]^2$$

$$+2F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, 0)[A_u m_6 \sin\beta + \mu\cos\beta]^2\Big]$$

$$-\sin^2 2\theta_u\Big[(F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_H) + F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_H))[A_u m_6 \sin\alpha + \mu\cos\alpha]^2$$

$$(F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_h) + F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_h))[A_u m_6 \cos\alpha - \mu\sin\alpha]^2\Big]$$

$$-\cos^2 2\theta_u\Big[2F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_H)[A_u m_6 \sin\alpha + \mu\cos\alpha]^2$$

$$+2F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_h)[A_u m_b \cos\alpha - \mu \sin\alpha]^2\Bigg]\Bigg], \qquad (B.2a)$$

$$\Delta_2 = \frac{g^2 m_u^2 N_c}{8\pi^2 \cos^2\theta_w \sin\beta}\Bigg[\big[\cos^2\theta_u(T_3 - e_u \sin^2\theta_w) + \sin^2\theta_u(e_u \sin^2\theta_w)\big]$$

$$\times\Big[\sin\alpha\cos(\alpha+\beta)F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_H) - \cos\alpha\sin(\alpha+\beta)F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_h)\Big]$$

$$+\big[\sin^2\theta_u(T_3 - e_u \sin^2\theta_w) + \cos^2\theta_u(e_u \sin^2\theta_w)\big]$$

$$\times\Big[\sin\alpha\cos(\alpha+\beta)F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_H) - \cos\alpha\sin(\alpha+\beta)F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_h)\Big]\Bigg]$$

$$+\frac{3g^2 m_u^2 N_c}{16\pi^2 M_W^2 \sin^2\beta}\Big[\sin^2\alpha M_H^2 G(m_u, m_u, M_H)$$

$$+\cos^2\alpha M_h^2 G(m_u, m_u, M_h) - \cos^2\beta M_A^2 G(m_u, m_u, M_A)\Big]$$

$$-\frac{g^2 m_u^2 N_c}{32\pi^2 \cos^2\theta_w} ln\ \mu_o^2 - \frac{g^2 m_u^2 N_c}{96\pi^2 \cos^2\theta_w}$$

$$+\frac{g^2 m_u N_c \cos(\alpha+\beta)}{16\pi^2 \cos^2\theta_w \sin\beta}\sin 2\theta_u[A_u m_6 \sin\alpha + \mu\cos\alpha]$$

$$\times\Bigg[\Big[\cos^2\theta_u(T_3 - e_u \sin^2\theta_w) + \sin^2\theta_u(e_u \sin^2\theta_w)\Big]F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_H)$$

$$-\Big[\sin^2\theta_u(T_3 - e_u \sin^2\theta_w) + \cos^2\theta_u(e_u \sin^2\theta_w)\Big]F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_H)$$

$$-\cos 2\theta_u(T_3 - 2e_u \sin^2\theta_w)F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_H)\Bigg]$$

$$-\frac{g^2 m_u N_c \sin(\alpha+\beta)}{16\pi^2 \cos^2\theta_w \sin\beta}\sin 2\theta_u[A_u m_6 \cos\alpha - \mu\sin\alpha]$$

$$\times\Bigg[\Big[\cos^2\theta_u(T_3 - e_u \sin^2\theta_w) + \sin^2\theta_u(e_u \sin^2\theta_w)\Big]F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_h)$$

$$-\left[\sin^2\theta_u(T_3 - e_u\sin^2\theta_w) + \cos^2\theta_u(e_u\sin^2\theta_w)\right]F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_h)$$

$$-\cos 2\theta_u(T_3 - 2e_u\sin^2\theta_w)F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_h)\Bigg]$$

$$-\frac{g^2 m_{\tilde{u}_1}^2 N_c}{8\pi^2\cos^2\theta_w}\Bigg[\left[\cos^2\theta_u(-T_3 + e_u\sin^2\theta_w)^2 + \sin^2\theta_u(e_u\sin^2\theta_w)^2\right]F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, 0)$$

$$-\left[\cos^2\theta_u(-T_3 + e_u\sin^2\theta_w) + \sin^2\theta_u(e_u\sin^2\theta_w)\right]^2 F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_Z)$$

$$-\frac{1}{2}\sin^2\theta_u\cos^2\theta_u H(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_Z)\Bigg]$$

$$-\frac{g^2 m_{\tilde{u}_2}^2 N_c}{8\pi^2\cos^2\theta_w}\Bigg[\left[\sin^2\theta_u(-T_3 + e_u\sin^2\theta_w)^2 + \cos^2\theta_u(e_u\sin^2\theta_w)^2\right]F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, 0)$$

$$-\left[\sin^2\theta_u(-T_3 + e_u\sin^2\theta_w) + \cos^2\theta_u(e_u\sin^2\theta_w)\right]^2 F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_Z)$$

$$-\frac{1}{2}\sin^2\theta_u\cos^2\theta_u H(m_{\tilde{u}_2}, m_{\tilde{u}_1}, M_Z)\Bigg], \qquad (B.2b)$$

$$\Delta_0 = \frac{g^2 M_Z^2 N_c}{16\pi^2 \cos^2\theta_\omega}\Bigg[\Big[\cos^2\theta_u(T_3 - e_u \sin^2\theta_\omega) + \sin^2\theta_u(e_u \sin^2\theta_\omega)\Big]^2$$

$$\times\Big[\cos^2(\alpha+\beta)F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_H) + \sin^2(\alpha+\beta)F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_h)\Big]$$

$$+\Big[\sin^2\theta_u(T_3 - e_u \sin^2\theta_\omega) + \cos^2\theta_u(e_u \sin^2\theta_\omega)\Big]^2$$

$$\times\Big[\cos^2(\alpha+\beta)F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_H) + \sin^2(\alpha+\beta)F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_h)\Big]$$

$$+\frac{1}{2}\sin^2 2\theta_u(T_3 - 2e_u \sin^2\theta_\omega)^2$$

$$\times\Big[\cos^2(\alpha+\beta)F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_H) + \sin^2(\alpha+\beta)F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_h)\Big]\Bigg]$$

$$-\frac{g^2 M_Z^2 N_c}{16\pi^2 \cos^2\theta_\omega}\Bigg[\Big[\cos^2\theta_u(-T_3 + e_u \sin^2\theta_\omega) + \sin^2\theta_u(e_u \sin^2\theta_\omega)\Big]^2 2G(m_{\tilde{u}_1}, m_{\tilde{u}_1}, M_Z)$$

$$+\Big[\sin^2\theta_u(-T_3 + e_u \sin^2\theta_\omega) + \cos^2\theta_u(e_u \sin^2\theta_\omega)\Big]^2 2G(m_{\tilde{u}_2}, m_{\tilde{u}_2}, M_Z)$$

$$+\sin^2\theta_u \cos^2\theta_u G(m_{\tilde{u}_1}, m_{\tilde{u}_2}, M_Z)$$

$$+4\Big[(-T_3 + e_u \sin^2\theta_\omega)^2 + (e_u \sin^2\theta_\omega)^2\Big]G(m_u, m_u, M_Z)\Bigg], \qquad (B.2c)$$

where $N_c = 3$ colors and

$$F(m_1, m_2, m_3) = \int_0^1 dx \, ln\Big[\frac{xm_1^2 + (1-x)m_2^2 - x(1-x)m_3^2}{\mu_o^2}\Big], \qquad (B.3)$$

$$G(m_1, m_2, m_3) = \int_0^1 dx \, x(1-x) ln\Big[\frac{xm_1^2 + (1-x)m_2^2 - x(1-x)m_3^2}{\mu_o^2}\Big], \qquad (B.4)$$

$$H(m_1, m_2, m_3) = \int_0^1 dx \, x ln\Big[\frac{xm_1^2 + (1-x)m_2^2 - x(1-x)m_3^2}{\mu_o^2}\Big]. \qquad (B.5)$$

T_3 is the weak isospin (which is $+\frac{1}{2}$ for left-handed up-type quarks).

The term $-\frac{g^2 m_u^2 N_c}{96\pi^2 \cos^2\theta_w}$ in Δ_2 arises in the self energy graphs with a quark in the loop. The contribution of each graph depends on the external momentum p^2 of the graph which is set equal to the physical Higgs mass when the renormalization conditions are applied:

$$-\frac{g^2 m_u^2 N_c}{96\pi^2 \cos^2\theta_w} = -\frac{g^2 m_u^2 N_c}{96\pi^2 M_W^2 \sin^2\beta}\left[\sin^2\alpha M_H^2 + \cos^2\alpha M_h^2 - \cos^2\beta M_A^2\right]. \quad (B.6)$$

This is an equality to this order in perturbation theory because there is the tree level relation

$$\sin^2\alpha M_H^2 + \cos^2\alpha M_h^2 = \cos^2\beta M_A^2 + \sin^2\beta M_Z^2. \quad (B.7)$$

If the external momentum of the graphs is set to zero rather that put on shell, then the term (B.6) vanishes.

The expression for Δ in (B.2) should be independent of the renormalization point μ_o. We have checked that this is indeed the case in both the analytic expression and in our computer program for calculating Δ, which provides a partial check of our answer (equivalent to the cancelation of divergences). We have also verified that (3.42) is satisfied which is a check on the value of the Goldstone self-energy that enters in (3.41).

The contribution for down quarks and down squarks is easily obtained from this result. The substitutions are shown below:

$$\theta_u \rightarrow \theta_d, \quad (B.8a)$$

$$m_u \rightarrow m_d, \quad (B.8b)$$

$$m_{\tilde{u}_{1,2}} \to m_{\tilde{d}_{1,2}}, \qquad (B.8c)$$

$$e_u \to e_d, \qquad (B.8d)$$

$$T_3 = \frac{1}{2} \to -\frac{1}{2}, \qquad (B.8e)$$

$$\sin\,\beta \to \cos\,\beta, \qquad (B.8f)$$

$$\cos\,\beta \to \sin\,\beta, \qquad (B.8g)$$

$$\cos\,\alpha \to \sin\,\alpha, \qquad (B.8h)$$

$$\sin\,\alpha \to \cos\,\alpha \qquad (B.8i)$$

The last four equations imply $\sin(\alpha+\beta) \to \sin(\alpha+\beta)$ and $\cos(\alpha+\beta) \to -\cos(\alpha+\beta)$. To obtain the proper result requires the *further* substitutions

$$\sin(\alpha+\beta) \to -\sin(\alpha+\beta), \qquad (B.8j)$$

$$\cos(\alpha+\beta) \to -\cos(\alpha+\beta). \qquad (B.8k)$$

For example, the first two terms in Δ_2 for the down quark and squarks should be

$$\frac{g^2 m_d^2 N_c}{8\pi^2 \cos^2\theta_w \cos\,\beta}[\cos^2\theta_d(T_3 - e_d \sin^2\theta_w) + \sin^2\theta_d(e_d \sin^2\theta_w)]$$
$$\times\left[\cos\,\alpha\cos(\alpha+\beta)F(m_{\tilde{d}_1}, m_{\tilde{d}_1}, M_H) + \sin\,\alpha\sin(\alpha+\beta)F(m_{\tilde{d}_1}, m_{\tilde{d}_1}, M_h)\right]$$
$$+[\sin^2\theta_d(T_3 - e_d \sin^2\theta_w) + \cos^2\theta_d(e_d \sin^2\theta_w)]$$
$$\times\left[\cos\,\alpha\cos(\alpha+\beta)F(m_{\tilde{d}_2}, m_{\tilde{d}_2}, M_H) + \sin\,\alpha\sin(\alpha+\beta)F(m_{\tilde{d}_2}, m_{\tilde{d}_2}, M_h)\right]. \qquad (B.9)$$

The contributions for the lepton and slepton loops are given in terms of the contributions for the up and down quark loops. The electron and selectron contribution is obtained from the expression for the down quarks with the appropriate mass and $SU(2) \times U(1)$ quantum number replacements. Similarly the contributions from the neutrino and the sneutrino are given by an expression similar to that for the up quark with the appropriate mass and $SU(2) \times U(1)$ quantum number substitutions.

APPENDIX C

Tadpole Contributions

In this appendix we demonstrate explicitly that the tadpole contribution to Δ in (3.41) and to $\tilde{\Delta}$ in (3.49) vanish in the MSSM. The result can be seen explicitly by examining the Feynman rules that are present in the MSSM. In the two doublet model there are two non-zero tadpoles shown in Figure 2. We display the vertices that are needed for the calculation of the tadpole diagrams in Figure 10. The contribution to the sum in (3.41) from the tadpole diagrams in Figure 11 are now easily seen to vanish using the couplings in Figure 10. We also display the vertices needed for the tadpole diagrams contributing to (3.49) in Figure 12. The combination of tadpole diagrams in Figure 13 vanishes.

These results generalize to the $2N$ Higgs doublet models discussed in Appendix D. The Π's in (D.16) and (D.18) therefore include all contributions to Higgs self-energies besides tadpole diagrams. Similarly, tadpoles are not to be included in the contributions from the vacuum polarization tensor either. In the $2N$ Higgs doublet model there are many more non-zero tadpole diagrams.

APPENDIX D

Generalization to 2*N* Higgs Doublets

Models with more than two Higgs doublets have mass relations analogous to (1.3) and (1.4). In an extension of the standard model with $2N$ Higgs doublets, there are $8N$ Higgs degrees of freedom. After spontaneous symmetry breaking three of these are Goldstone bosons, leaving $4N-2$ charged Higgs bosons $H_i^{\pm}$ and $4N-1$ neutral Higgs bosons. We shall denote the neutral scalar Higgs by H_i and the neutral pseudoscalar Higgs by A_i. In the supersymmetric version of the $2N$ doublet model, the couplings and masses in the Higgs sector are again constrained. The mass relations that arise are[5]

$$\sum_{i=1}^{2N} M_{H_i}^2 = \sum_{i=1}^{2N-1} M_{A_i}^2 + M_Z^2, \qquad (D.1)$$

$$\sum_{i=1}^{2N-1} M_{H_i^{\pm}}^2 = \sum_{i=1}^{2N-1} M_{A_i}^2 + M_W^2 \qquad (D.2)$$

which generalize (1.3) and (1.4).

The Higgs potential for the model in the extension with $2N$ doublets is[5]

$$V = \sum_{i=1}^{2N} m_i^2 \phi_i^\dagger \phi_i - \sum_{j<i}^{2N} m_{ij}^2 (\phi_i^\dagger \phi_j + \phi_j^\dagger \phi_i) + \frac{1}{8} g'^2 \sum_{i=1}^{2N} |(-1)^{i+1} \phi_i^\dagger \phi_i|^2$$

$$+\frac{1}{8} g^2 \sum_{a=1}^{3} \left| \sum_{i=1}^{2N} (-1)^{i+1} \phi_i^\dagger \sigma^a \phi_i \right|^2. \qquad (D.3)$$

This equation is the $2N$ doublet analog of (2.4) where arbitrary soft supersymmetry breaking terms have been included. There are possible terms that are $\mu_{ij} \phi_i^\dagger \phi_j$ that can be absorbed in the soft-supersymmetry breaking terms as in the two doublet case.

There are directions in Higgs field space where the quartic couplings vanish. For example, in the four doublet model the quartic couplings vanish when $\phi_1 = \phi_2$ and $\phi_3 = \phi_4$ as well as when $\phi_1 = \phi_4$ and $\phi_3 = \phi_2$.

There is now a vacuum expectation value v_i for each of the $2N$ doublets ϕ_i. We can eliminate the m_i in favor of the vevs v_i. The neutral scalar and neutral pseudoscalar mass matrices are $2N \times 2N$ matrices. The neutral scalar mass matrix M^2 is given by

$$M^2_{ii} = \sum_{j \neq i} m^2_{ij} \frac{v_j}{v_i} + \frac{1}{2}(g^2 + g'^2) v_i^2 \quad \text{(no sum on } i\text{)}, \qquad (D.4a)$$

$$M^2_{ij}(i \neq j) = -m^2_{ij} + (-1)^{j-i} \frac{1}{2}(g^2 + g'^2) v_i v_j \qquad (D.4b)$$

while the neutral pseudoscalar mass matrix M'^2 is given by

$$M'^2_{ii} = \sum_{j \neq i} m^2_{ij} \frac{v_j}{v_i}, \qquad (D.5a)$$

$$M'^2_{ij}(i \neq j) = m^2_{ij} \qquad (D.5b)$$

M'^2 has a zero eigenvalue corresponding to a neutral Goldstone boson. Since both M^2 and M'^2 are real and symmetric, they can be diagonalized by orthogonal transformations that preserve their traces, i.e. $\sum_i M^2_{H_i} = \sum_i M^2_{ii}$ and $\sum_i M^2_{A_i} = \sum_i M'^2_{ii}$. Using (D.4) and (D.5), one can obtain (D.1) and (D.2).

The renormalization of the mass relations in (D.1) and (D.2) is a generalization of the arguments in Section III. The wave-function renormalization matrices $Z_S^{1/2}$ and $Z_P^{1/2}$ become $2N \times 2N$ matrices. The [illegible]s matrices (D.4) and (D.5) are symmetric and are diagonalized by

$$(M_S^2)_D = O_S^{-1} M^2 O_S, \quad (M_P^2)_D = O_P^{-1} M'^2 O_P \qquad (D.6)$$

where O_S and O_P are orthogonal matrices. $(M_S^2)_D$ and $(M_P^2)_D$ are diagonal matrices whose nonzero entries are the masses $M_{H_i}^2$ and $M_{A_i}^2$ respectively. We shift parameters as in (3.6):

$$(m_{ij}^2)_b = m_{ij}^2 + \delta m_{ij}^2 \;\; (i \neq j), \tag{D.7a}$$

$$(v_i)_b = v_i + \delta v_i, \tag{D.7b}$$

$$(M_Z^2)_b = M_Z^2 + \delta M_Z^2. \tag{D.7c}$$

The unrenormalized propagators are given by formulas analogous to (3.12):

$$i\Gamma_S(p^2) = (Z_S^{1/2})^T Z_S^{1/2} p^2 - (Z_S^{1/2})^T (M_S^2)_D Z_S^{1/2} - \delta M_S^2, \tag{D.8}$$

$$i\Gamma_P(p^2) = (Z_P^{1/2})^T Z_P^{1/2} p^2 - (Z_P^{1/2})^T (M_P^2)_D Z_P^{1/2} - \delta M_P^2 \tag{D.9}$$

where $\delta M_S^2 = O_S^{-1} \delta M^2 O_S$ and $\delta M_P^2 = O_P^{-1} \delta M'^2 O_P$. δM^2 and $\delta M'^2$ are analogous to the matrices constructed in the two Higgs doublet case. Since the trace of the matrices is invariant under orthogonal transformations we have

$$Tr\, \delta M_S^2 = Tr\, \delta M^2, \tag{D.10}$$

$$Tr\, \delta M_P^2 = Tr\, \delta M'^2. \tag{D.11}$$

From the expressions for the mass relations in (D.4) and (D.5) we have

$$Tr\, \delta M^2 = Tr\, \delta M'^2 + \delta M_Z^2 \tag{D.12}$$

so that

$$Tr\, \delta M_S^2 = Tr\, \delta M_P^2 + \delta M_Z^2. \tag{D.13}$$

The renormalization conditions analogous to those in (3.20) are[27]

$$i\tilde{\Gamma}_{H_i H_i}(M_{H_i}^2) = 0 \;\; (no\, sum), \tag{D.14a}$$

$$i\tilde{\Gamma}_{H_iH_j}(M^2_{H_i}) = i\tilde{\Gamma}_{H_iH_j}(M^2_{H_j}) = 0 \ (no\,sum), \qquad (D.14b)$$

$$i\tilde{\Gamma}'_{H_iH_i}(M^2_{H_i}) = 1 \ \ (no\,sum). \qquad (D.14c)$$

If we define the radiative corrections to (D.1) as

$$\sum_{i=1}^{2N} M^2_{H_i} = \sum_{i=1}^{2N-1} M^2_{A_i} + M^2_Z + \Delta \qquad (D.15)$$

we obtain the result

$$\Delta = -\sum_{i=1}^{2N} \Pi_{H_iH_i}(M^2_{H_i}) + \sum_{j=1}^{2N} \Pi_{A_jA_j}(M^2_{A_j}) - A_{ZZ}(M^2_Z) \qquad (D.16)$$

where the sum over the pseudoscalar Higgs A_j self-energies includes the neutral Goldstone boson self-energy $\Pi_{GG}(0)$. It can be shown that the tadpoles cancel just as in the MSSM. Similarly it can be shown that the correction $\tilde{\Delta}$ to (D.2) defined as

$$\sum_{i=1}^{2N-1} M^2_{H_i^\pm} = \sum_{i=1}^{2N-1} M^2_{A_i} + M^2_W + \tilde{\Delta} \qquad (D.17)$$

is given by

$$\tilde{\Delta} = -\sum_{i=1}^{2N} \Pi_{H_i^\pm H_i^\pm}(M^2_{H_i^\pm}) + \sum_{j=1}^{2N} \Pi_{A_jA_j}(M^2_{A_j}) - A_{WW}(M^2_W) \qquad (D.18)$$

where the sum over the pseudoscalar Higgs A_j self-energies includes the neutral Goldstone boson self-energy $\Pi_{GG}(0)$, and the sum over the charged Higgs bosons $H_i^\pm$ self-energies includes the charged Goldstone boson self-energy $\Pi_{G^\pm G^\pm}(0)$.

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Figure Captions

Figure 1: Self-energy diagrams – The self-energy diagrams are defined as shown with the external legs amputated. $X, Y = H, h, A, G$. In the on-shell scheme the external legs are put on shell.

Figure 2: Tadpoles – The two kinds of tadpoles that exist in the MSSM.

Figure 3: One-loop Corrections – The diagrams calculated in the MSSM. There are the following number of nonvanishing diagrams of each type: (a) 4, (b) 12, (c) 8, (d) 1, (e) 3, (f) 2.

Figure 4: Quadratic SUSY Breaking Corrections – Contributions to Higgs boson masses that are quadratic in a scalar mass arise from diagrams of this topology.

Figure 5: Cancellation of Quadratic Corrections – The corrections to the mass sum rule that are quadratic in the squark mass cancel in the above diagrams. The restriction on naturalness from corrections to the Higgs boson masses is therefore hidden in the sum rule.

Figure 6: $\Delta(m_{\tilde{t}_1})$ – We have plotted the correction Δ using the full expression given in Appendix B. The parameters used are given in the text. The squark mixing angle is $\theta_t = 0^\circ$ and 20° in Figures 6a and 6b respectively. The curves

in the figures represent $m_{\tilde{t}_1}$ = (a)100 GeV, (b)400 GeV, (c)700 GeV, (d)1000 GeV, (e)1300 GeV. Large corrections occur when $\theta_t \neq 0$, and the squarks $\tilde{t}_1$ and $\tilde{t}_2$ have different masses. This occurs when the coupling parameter $A_t m_6$ becomes large.

Figure 7: $\Delta(m_t)$ – We have plotted Δ as a function of the top quark mass for five values of the squark masses: $m_{\tilde{t}_1} = m_{\tilde{t}_2}$ = (a)100 GeV, (b)400 GeV, (c)700 GeV, (d)1000 GeV, (e)1300 GeV. The radiative corrections behave like $\alpha m_t/M_Z^2$ for large m_t. The contribution can be of either sign depending on the relative sizes of the top quark mass and the top squark masses.

Figure 8: Feynman Rules – Feynman rules involving Goldstone bosons and squarks. We have written these in the $\tilde{u}_L - \tilde{u}_R$ basis for simplicity. These can be converted into Feynman rules in the mass eigenstates basis $\tilde{u}_1 - \tilde{u}_2$ by a rotation in the squark fields.

Figure 9: Trilinear Higgs Couplings – Trilinear Higgs couplings involving Goldstone bosons.

Figure 10: Trilinear Tadpole Couplings I – Trilinear couplings relevant to tadpole contributions to (3.41).

Figure 11: Tadpole Sum I – These diagrams contribute to the sum in (3.41). The couplings in Figure 10 show that this contribution is zero when the diagrams are summed with the appropriate signs.

Figure 12: Trilinear Tadpole Couplings II – Trilinear couplings relevant to tadpole contributions to (3.49).

Figure 13: Tadpole Sum II – These diagrams contribute to the sum in (3.49). The couplings in Figure 12 show that this contribution is zero when the diagrams are summed with the appropriate signs.

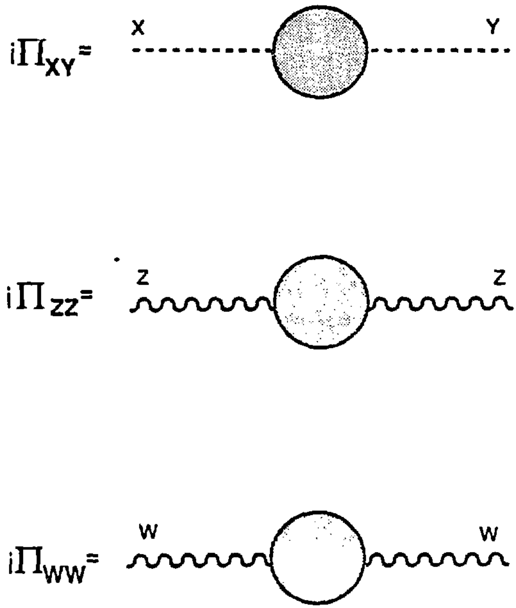


Figure 1

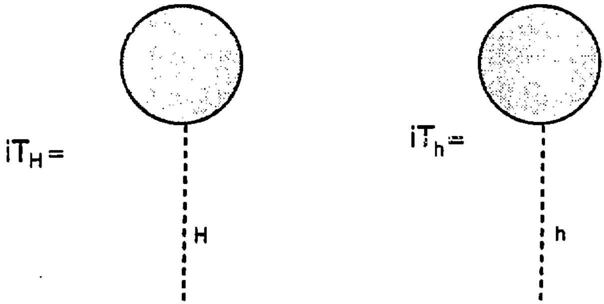


Figure 2

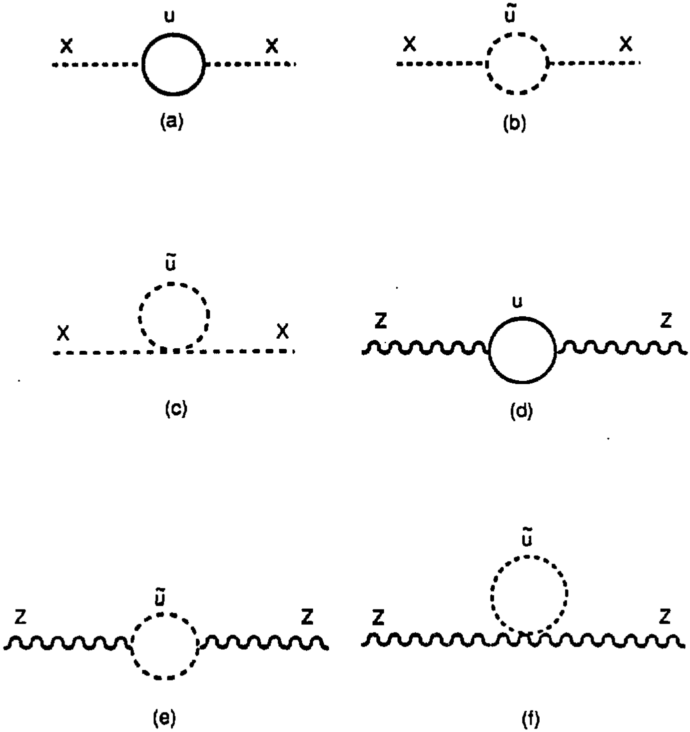


Figure 3

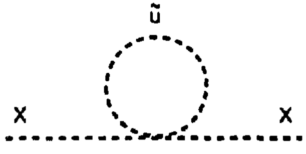


Figure 4

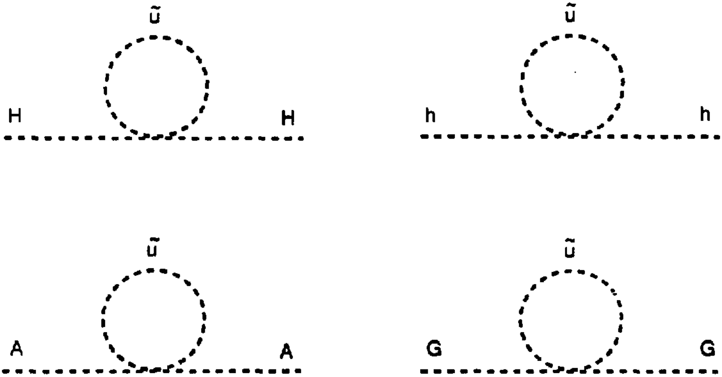


Figure 5

FIGURE 6a

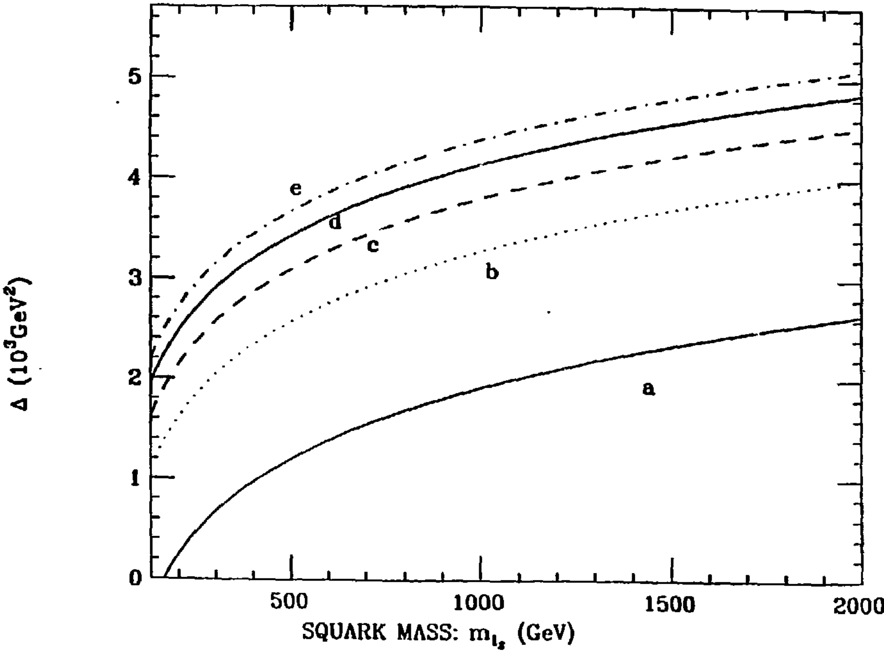

Δ (10^3GeV^2)
0
1
2
3
4
5
500
1000
1500
2000
SQUARK MASS: m_{l_s} (GeV)
a
b
c
d
e

FIGURE 6b

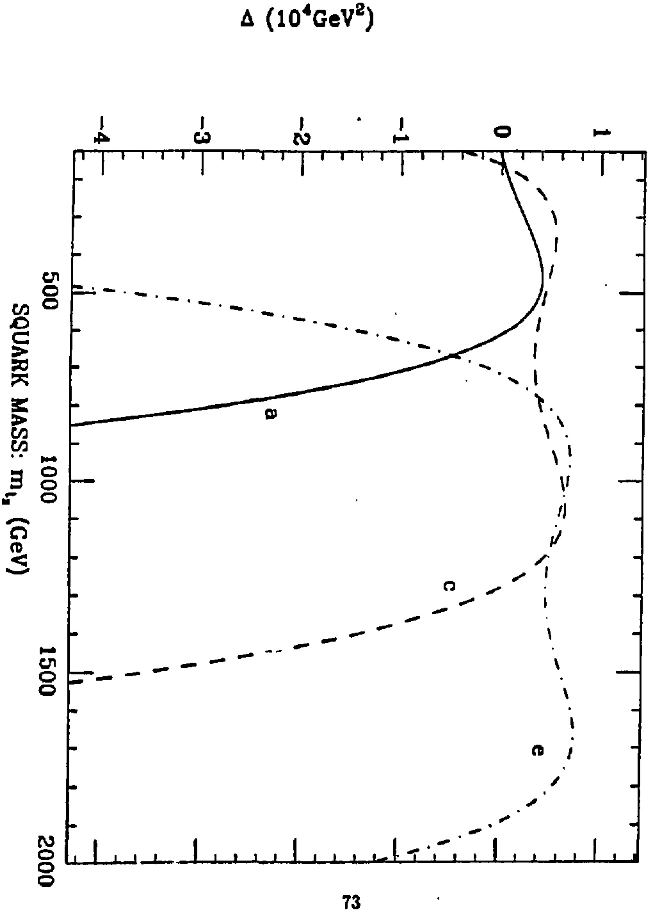
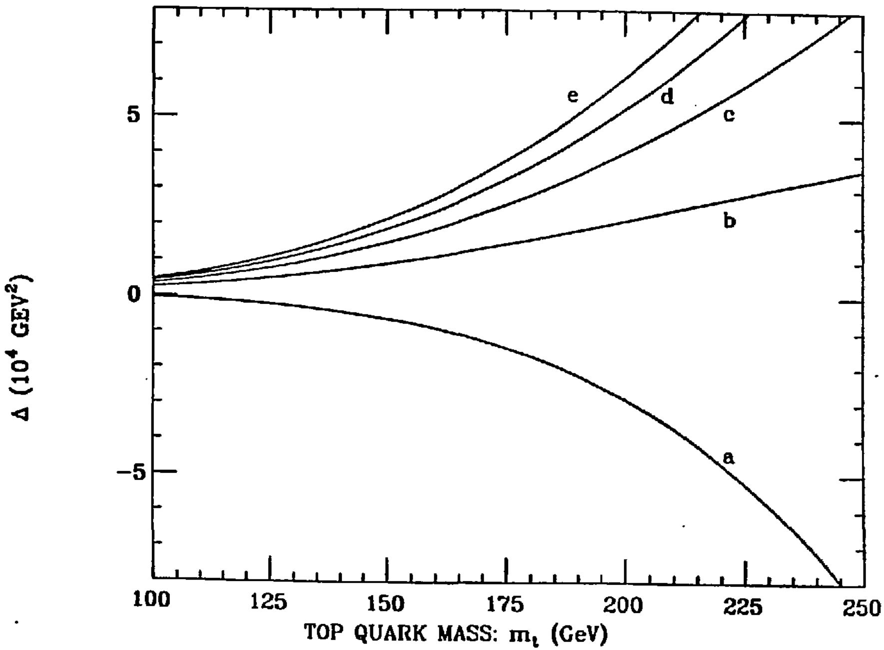
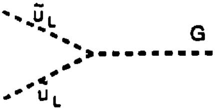


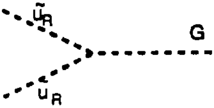
FIGURE 7



Δ (10^4 GEV^2)
5
0
−5
100
125
150
175
200
225
250
TOP QUARK MASS: m_t (GeV)
a
b
c
d
e



0



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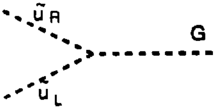
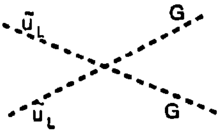
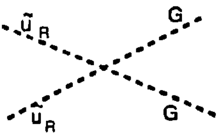


$\frac{g m_u}{2 M_W}(m_6 A_u + \mu \cot\ \beta)$

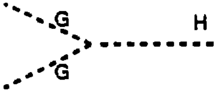


$-\frac{ig^2}{2}\left[\cos 2\beta\left(\frac{T_3 - e_u \sin^2\theta_w}{\cos^2\theta_w}\right) + \frac{m_u^2}{M_W^2}\right]$

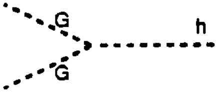


$-\frac{ig^2}{2}\left[\cos 2\beta\left(\frac{e_u \sin^2\theta_w}{\cos^2\theta_w}\right) + \frac{m_u^2}{M_W^2}\right]$

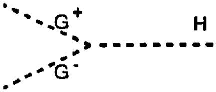
Figure 8



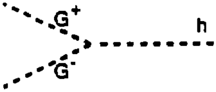
$$-\frac{igM_Z}{2\cos\theta_w}\cos 2\beta\cos(\beta+\alpha)$$



$$\frac{igM_Z}{2\cos\theta_w}\cos 2\beta\sin(\beta+\alpha)$$

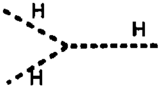


$$-\frac{igM_Z}{2\cos\theta_w}\cos 2\beta\cos(\beta+\alpha)$$

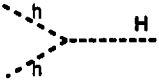


$$\frac{igM_Z}{2\cos\theta_w}\cos 2\beta\sin(\beta+\alpha)$$

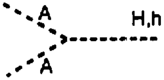
Figure 9



$-\frac{3igM_Z}{2\cos\theta_w}\cos 2\alpha\cos(\beta+\alpha)$

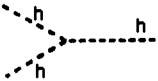


$-\frac{igM_Z}{2\cos\theta_w}[2\sin 2\alpha\sin(\beta+\alpha)-\cos 2\alpha\cos(\beta+\alpha)]$

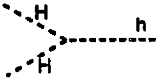


$\frac{igM_Z}{2\cos\theta_w}\cos 2\beta\,(\cos(\beta+\alpha),-\sin(\beta+\alpha))$

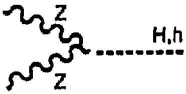
$-\frac{igM_Z}{2\cos\theta_w}\cos 2\beta\,(\cos(\beta+\alpha),-\sin(\beta+\alpha))$



$-\frac{3igM_Z}{2\cos\theta_w}\cos 2\alpha\sin(\beta+\alpha)$



$\frac{igM_Z}{2\cos\theta_w}[2\sin 2\alpha\cos(\beta+\alpha)+\cos 2\alpha\sin(\beta+\alpha)]$



$\frac{igM_Z}{\cos\theta_w}g^{\mu\nu}\,(\cos(\beta-\alpha),\sin(\beta-\alpha))$

Figure 10

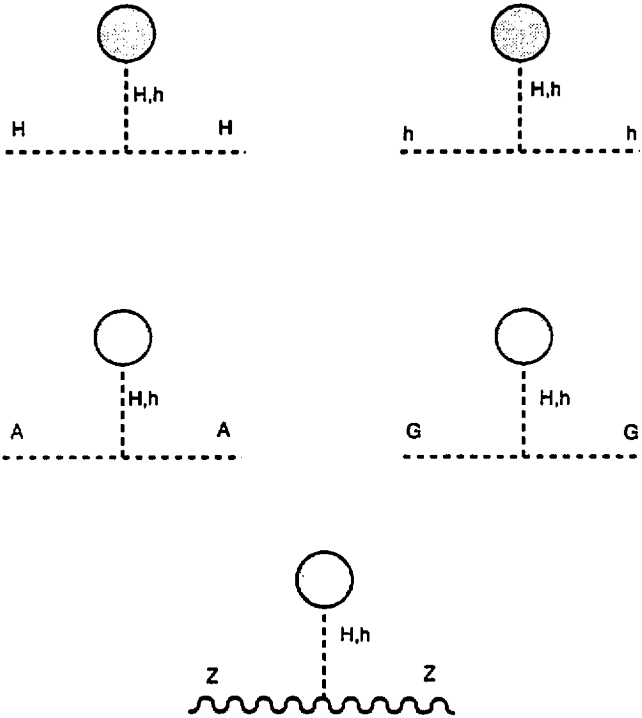
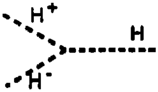
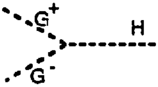


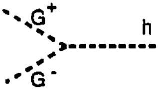
Figure 11



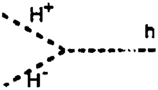
$$-ig\left[M_W\cos(\beta-\alpha)-\frac{M_Z}{2\cos\theta_w}\cos 2\beta\cos(\beta+\alpha)\right]$$



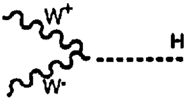
$$-\frac{igM_Z}{2\cos\theta_w}\cos 2\beta\cos(\beta+\alpha)$$



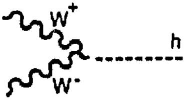
$$\frac{igM_Z}{2\cos\theta_w}\cos 2\beta\sin(\beta+\alpha)$$



$$-ig\left[M_W\sin(\beta-\alpha)+\frac{M_Z}{2\cos\theta_w}\cos 2\beta\sin(\beta+\alpha)\right]$$



$$igM_W\cos(\beta-\alpha)g^{\mu\nu}$$



$$igM_W\sin(\beta-\alpha)g^{\mu\nu}$$

Figure 12

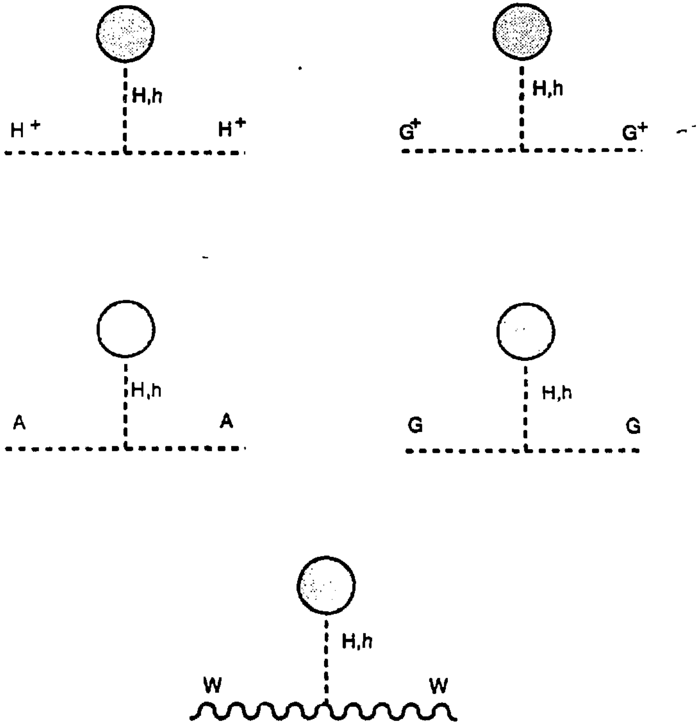


Figure 13