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PROJECTILE OSCILLATIONS IN AUGMENTED RAIL GUNS*

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ABSTRACT

The projectile in an inductive store-powered rail gun, augmented by an external magnetic field, will oscillate under certain conditions. This behavior is easily understood when there is no resistance in the circuit comprising the storage coil, rails and armature. In this case, the flux in the complete circuit is conserved. However, as the projectile moves down the rails, more flux from the augmenting field is picked up. This must be accompanied by a decrease in current in the system to conserve the total flux. At a certain distance down the rails, the current must reverse to conserve the flux, and thus the force on the projectile reverses. This mechanism leads to oscillation of the projectile. An analytic solution is given for the case in which the resistance is zero.

INTRODUCTION

Simple conducting railguns use the force from the self-magnetic field inside the rails to accelerate projectiles. Augmentation and superconducting augmentation of the magnetic field have been proposed as ways to increase the efficiency of the transfer from magnetic to mechanical energy. The energy and efficiency of augmented systems has been analyzed.[1] In this paper and in a following paper,[2] we shall examine the behavior of the projectile.

In the system that we will consider, an external inductor provides the initial current that flows through the rails of the gun and through a conducting path located inside the projectile. As shown in Fig. 1, another external coil provides the augmenting magnetic field. We will assume that, in an x-y-z coordinate system, the rails lie in planes parallel to the x-axis and that the projectile is induced to move in the

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x-direction. Although the projectile is a cube of height h, we will denote its position at time t by the location, x(t), of the conductor embedded in it. Thus, the current flows from the inductor through one rail to the point x(t) where it is shunted to the other rail by the conductor in the projectile. This current gives rise to the self-magnetic field which acts on the projectile through the Lorentz force. Since the inductance of the rail gun can be written as $L'x$, where L' is the inductance per unit length, the Lorentz force due to the current I is $L'I^2/2$.

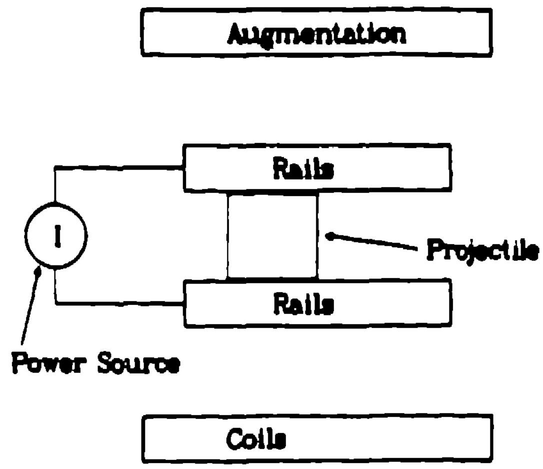

Fig. 1. Generalized augmented rail gun layout.

The projectile is subject to an additional force due to the augmenting field B_o. In this system, B_o is independent of both time and position and lies in the same direction as the self field. Thus, the component of Lorentz force due to the augmenting field is $B_o hI$. The total force on the projectile is the sum of these two forces:

$$F = m\ddot{x} = \frac{1}{2} L'I^2 + B_o hI \quad , \tag{1}$$

where m is the mass of the projectile and $\ddot{x}$ is the projectile acceleration.

The current is determined by the circuit elements L_o, $L'x$, and resistance R, and by Faraday's law for the change in flux due to the projectile moving in the augmenting field:

$$\frac{d}{dt}\left[(L_o + L'x)I + B_o hx\right] + RI = 0 \quad . \tag{2}$$

For this analysis, we will assume that there is no coupling between the augmenting coil, the power source, and the gun. We will also neglect distortions of the rails and the projectile so that the projectile height h remains constant.

We will solve (1) and (2) for the projectile position and velocity, x and $\dot{x}$, and the current, I. Under certain circumstances, we will find that the projectile oscillates; that is, it moves first in the positive x direction, stops at some distance which we denote by $x(\tau)$, then reverses directions and moves back toward its original position.

These projectile oscillations are caused by sign changes in the current. To see how these changes arise, let us assume that at time $t = 0$, the projectile position is zero, $x(0) = 0$, the resistance R is zero and the current I(0) has the strictly positive value I_0.

By (2), the current at time t is given by

$$I(t) = \frac{L_o I_o}{L_o + L'x(t)} - \frac{B_o hx(t)}{L_o + L'x(t)} .$$

This current consists of two terms. The first is due to the inductive circuit. The second is of opposite sign and is the result of the change in flux in the rails from the augmenting field. As x increases, the second component increases and the total current decreases. When x is such that $B_o hx$ equals $L_o I_o$, the current and hence the force on the projectile are zero. Since the kinetic energy of the projectile is not zero, it continues to move, x increases, and the current becomes negative. At some point, $x(\tau)$, the kinetic energy is zero and projectile stops. The force in the opposite direction now acts on the projectile, driving it back to its original position when the pattern repeats. With small values of resistance, this oscillatory motion is slightly damped and the projectile ultimately comes to rest near $x(\tau)/2$. At greater values of R, the oscillations vanish.[2]

These highly undesirable motions have a great effect on augmented rail gun design. For example, they determine that there is a length that gives maximum launch velocity. This is in contrast to simple rail guns in which the launch velocity is in proportion to the length.

EXACT SOLUTION

Closed-form solutions of (1) and (2) can be obtained for the case in which the resistance is zero. As such solutions show the limiting behavior for cases in which the resistance is small and as they are useful in determining the validity of approximate solutions, we will obtain the closed-form solutions first.

Let us define the current J_o as the ratio of $B_o h$ to L':

$$J_o = \frac{B_o h}{L'} . \tag{3}$$

With the initial conditions

$$x(0) = 0, \ \dot{x} = 0, \text{ and } I(0) = I_o , \tag{4}$$

and with R = 0, the total current in the circuit at time t is

$$I(t) = \frac{L_o I_o - J_o L'x(t)}{L_o + L'x(t)} . \tag{5}$$

By (2), (3) and (5), the acceleration can be written

$$\ddot{x} = \frac{1}{2} L' \left[\left(\frac{I_o + J_o}{1 + \frac{L'x}{L_o}} \right)^2 - J_o^2 \right] . \tag{6}$$

In terms of a new variable

$$y = 1 + \frac{L'x}{L_o} \tag{7}$$

and the parameter

$$\beta = \frac{J_o}{I_o + J_o} \quad , \tag{8}$$

equation (6) becomes

$$\ddot{y} = \frac{(L')^2}{2mL_o} (I_o + J_o)^2 \left[\frac{1}{y^2} - \beta^2\right] \quad . \tag{9}$$

Multiplication of (9) by $\dot{y}$ yields the following expression, which shows that the quantity below enclosed in brackets is conserved:

$$\frac{d}{dt}\left[\frac{\dot{y}^2}{2} + \frac{(L')^2}{2mL_o} (I_o + J_o)^2 \left(\frac{1}{y} + \beta^2 y\right)\right] = 0 \quad . \tag{10}$$

By (4a) and (4b), the initial conditions on y are

$$y(0) = 1 \text{ and } \dot{y}(0) = 0 \quad . \tag{11}$$

Thus, by (10) and (11), we obtain the following first integral of (9):

$$(\dot{y})^2 = \frac{(L')^2(I_o + J_o)^2}{mL_o} \quad \frac{\beta^2(y - 1)\,(\beta^{-2} - y)}{y} \quad . \tag{12}$$

The normalized velocity $\dot{y}$ may have positive and negative values and, in addition to its zero initial value,

$$\dot{y} = 0 \text{ when } y = \beta^{-2} \quad . \tag{13}$$

Since by (8), β is strictly less than 1, and by (11a), $y(0) = 1$, the normalized acceleration $\ddot{y}$ is initially positive; at the point $y = \beta^{-2}$, the acceleration is negative:

$$\ddot{y} = \frac{(L')^2}{2mL_o} (I_o + J_o)^2 (1-\beta^2) \text{ when } y = 0$$

$$\ddot{y} = -\beta^2\ddot{y}(0) \text{ when } y = \beta^{-2} \quad . \tag{14}$$

When $y = \beta^{-1}$, $\ddot{y}$ is zero and the magnitude of the normalized velocity achieves its maximum value of $L'I_o/\sqrt{mL_o}$. Thus, we find that the velocity itself is initially positive, reaches a maximum, decreases to zero and then becomes negative:

$$x = \begin{cases} 0, \text{ when } x = 0 \text{ and } x = \dfrac{L_oI_o}{B_oh}\left[\dfrac{I'I_o}{B_oh} + 2\right] \quad , \\ \pm \quad I_o\sqrt{\dfrac{L_o}{m}}, \text{ when } x = \dfrac{L_oI_o}{B_oh} \quad . \end{cases} \tag{15}$$

Let us write (12) as

$$\frac{\beta L'}{\sqrt{mL_o}} (I_o + J_o)\, dt = \sqrt{\frac{y}{(y - 1)\,(\beta^{-2} - y)}}\, dy \quad ,$$

from which we may obtain the following function of y:

$$\frac{\beta L'(I_o + J_o)}{\sqrt{mL_o}} t = \int_1^y \left(\frac{\sigma}{(\sigma - 1)\,(\beta^{-2} - \sigma)}\right)^{1/2} d\sigma \quad . \tag{16}$$

The integral on the right may be expressed in terms of elliptic integrals[3,4]:

$$\int_1^y \left(\frac{\sigma}{(\sigma - 1)\,(\beta^{-2} - \sigma)}\right)^{1/2} d\sigma = \frac{2}{\beta} \left[E\left(\frac{\pi}{2};\ \sqrt{1 - \beta^2}\right) - E\left(\theta;\ \sqrt{1 - \beta^2}\right)\right] \tag{17}$$

where

$$\sin\theta = \sqrt{\frac{1 - \beta^2 y}{1 - \beta^2}} \quad ,$$

and E is the elliptic integral of the second kind:

$$E(\theta,k) = \int_0^\theta \sqrt{1 - k^2 \sin\phi}\ d\phi \quad .$$

Substitution of (17) into (16) yields the following expression from which y(t) and the solution of (1), that is, the projectile position x(t), may be determined:

$$E\left(\arcsin\sqrt{\frac{1-\beta^2 y}{1-\beta^2}}\ ;\ \ 1 - \beta^2\right) = E\left(\frac{\pi}{2}\ ;\ \sqrt{1 - \beta^2}\right) - \frac{B_o{}^2 h^2}{2\sqrt{mL_o}\,\left(I_o L'_o + B_o h\right)}\ t \quad . \tag{18}$$

We will define τ as the time at which $y = \beta^{-2}$ and, by (13), $\dot{y} = 0$. By (18), τ has the value

$$\tau = \frac{2\sqrt{mL_o}\,\left(I_o L' + B_o h\right)}{B_o^2 h^2} E(\pi/2;\sqrt{1 - \beta^2}) \tag{19}$$

For the case $B_o = 100$ T, $I_o = 90$ kA, $h = 9.53$ mm (3/8 in), $L' = 0.35$ µH/m, $L_o = 5$ µH and $m = 0.005$ kg, Eqs. (7), (15) and (19) give the following values for τ and $x(\tau)$:

$\tau = 0.529$ ms

$x(\tau) = 0.96$ m .

The value of the magnetic field in this example is much greater than the maximum value of 20 T for a typical augmenting field.[1] In our analysis, the augmenting field appears only in the product $B_o h$, the augmenting flux per unit length. Thus, the above values for τ and $x(\tau)$ hold for any system in which $B_o h$ is approximately 0.953, e.g., $B_o = 20$ Tesla and $h_o = 4.765$ cm. Variations of τ and $x(\tau)$ with $B_o h$ are presented in the following paper[3] in which the effect of nonzero resistance on projectile oscillations is discussed.

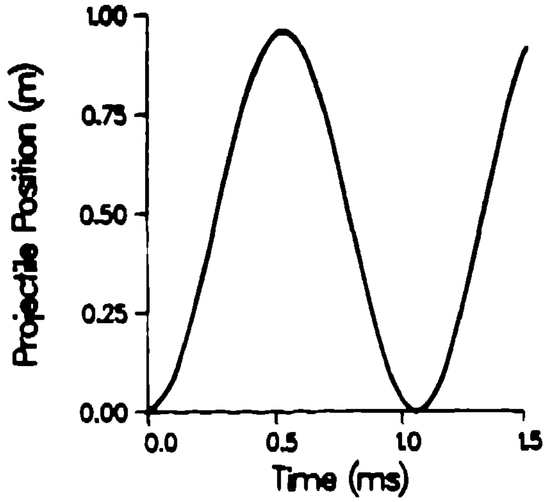

Fig. 2. In the augmented rail gun discussed in the text, the projectile moves 0.96 m toward the muzzle of the gun, then stops, reverses direction and moves back toward the breech. As this is the lossless case, these oscillations continue indefinitely. Maximum velocity occurs near 0.47 m.

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