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# *Statistical Near-Real-Time Accountancy Procedures Applied to AGNS Minirun Data Using PROSA*

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# STATISTICAL NEAR-REAL-TIME ACCOUNTANCY PROCEDURES APPLIED TO AGNS MINIRUN DATA USING PROSA

by

Rainer Beedgen

**ABSTRACT**

The computer program PROSA (PROgram for Statistical Analysis of near-real-time accountancy data) was developed as a tool to apply statistical test procedures to a sequence of materials balance results for detecting losses of material. First applications of PROSA to model facility data and real plant data showed that PROSA is also usable as a tool for process or measurement control. To deepen the experience for the application of PROSA to real data of bulk-handling facilities, we applied it to uranium data of the Allied General Nuclear Services miniruns, where accountancy data were collected on a near-real-time basis. Minirun 6 especially was considered, and the pulsed columns were chosen as materials balance area. The structure of the measurement models for flow sheet data and actual operation data are compared, and methods are studied to reduce the error for inventory measurements of the columns.

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## I. INTRODUCTION

The computer program PROSA [PROgram for Statistical Analysis of near-real-time accountancy (NRTA) data[1]] was developed as a tool to apply statistical test procedures to a sequence of materials balance results for detecting losses of material under consideration especially nuclear material. First applications of PROSA to model facility data[1,2] and real plant data[3] showed that PROSA is also usable as a tool for process or measurement control. To get more experience for the application of PROSA to real data of bulk-handling facilities, we applied it to uranium data from the Allied General Nuclear Services (AGNS) miniruns,[4] where accountancy data were collected on a near-real-time basis. In our case, the four pulsed columns 2A, 2B, 3A, and 3B are considered as one materials balance area. The pulsed columns are interesting from the measurement uncertainty and measurement model point of view. There is not much information available about the sizes of these uncertainties under routine operating conditions.

If we are to get reasonable results out of such an analysis, it is important to have a realistic measurement model for the process data because it is an essential input for PROSA and the application of statistical test procedures. The measurement model for a steady-state operation based on the flow sheet data is compared with the actual model derived from the actual facility data. The measurement models (dispersion matrices) allow an estimate of the performance of the NRTA test procedures and a determination of those loss patterns that are the most difficult to detect.

We studied some experiments for reducing the error of the inventory measurements and determined what the changes of the data mean for the structure of the measurement model. The analysis of the Minirun 6 data may serve as an example of how to evaluate other real plant data with PROSA. The study should be valuable for data analysts as well as plant operators. The analysis was carried out under the bilateral U.S. DOE/BMFT (Bundesministerium fuer Forschung und Technologie) Cooperation in Reprocessing Safeguards R&D.

## II. SHORT DESCRIPTION OF PROSA

PROSA has been developed as a tool to apply truncated sequential statistical tests to a sequence of materials balance results, the origin of which is a model facility or an existing plant. PROSA is used to decide, on the basis of statistical considerations, whether in a given sequence of materials balance periods a loss of material might have occurred. The evaluation of the materials balance data is based on statistical test procedures.

In the present version of PROSA 1.0, three statistical tests,

(1) Truncated Sequential CUMUF Test,

(2) CUSUM Test with Power One thresholds, and

(3) CUSUM Test with Page's thresholds,

are selected. These three test procedures are the result of several years of statistical research in the international community and, at the moment, are promising ones, as far as the detection probability for a loss of material and the timeliness of detection of a loss is concerned.

PROSA has been developed for evaluating accountancy data from reprocessing facilities. However, it is also able to evaluate accountancy data from all kinds of facilities as long as they possess a particular, but fairly general, structure.

The evaluation of a given data set can be performed with a desired false alarm probability $\alpha$. This enables sensitivity studies for given data sets.

To use PROSA, it is not necessary to understand all of the statistical details, but it is important that the user is aware of the measurement model of the plant under consideration. The measurement model is the basis for the statistical tests performed on a given sequence of materials balance results.

## A. Multiple Balances Model

We assume a discrete number of balance periods $k = 1,2,\ldots,n$ for a well-defined class of material. For each period k we establish the materials balance equation

$$MUF_k = I_{k-1} + T_k - I_k \quad . \tag{1}$$

Here $I_k$ is the inventory at the end of period k ($I_0$ is the beginning inventory), and the net transfer is $T_k = R_k - S_k$ with $R_k$ equal to receipts and $S_k$ equal to shipments.

The concept of multiple balances is used for detection of possible nuclear materials losses in a bulk-handling facility. The detection has to be timely and have sufficiently high probability. The true $MUF_k$ values are zero in the ideal situation of no losses and no measurement errors. In actual practice, however, nonzero $MUF_k$'s may occur for a number of reasons, for example, measurement errors or loss of material.

Measurement errors in our model are represented as random variables in determining the materials balance.

We assume that $I_k$, $R_k$, and $S_k$ are random variables that can be written as

$$I_k = E(I_k) + ZI_k + SI_k \quad ,$$

where $E(I_k)$ is the true value of inventory, $ZI_k$ is the random error of measurement, and $SI_k$ is the systematic measurement error. Furthermore, we define

$$T_k = R_k - S_k = E(T_k) + ZT_k + ST_k$$

for all k, where $E(T_k)$ are the true values, $ZT_k$ the random measurement errors, and $ST_k$ the systematic measurement errors.

A further assumption is that all measurement errors are stochastically independent.

The variances for period k are defined as

$$\text{var}\ (I_k) = \text{var}\ (ZI_k) + \text{var}\ (SI_k) \quad \text{and}$$

$$\text{var}(T_k) = \text{var}\ (ZT_k) + \text{var}\ (ST_k) \quad .$$

For two periods i and j, we define the covariance of $MUF_i$ and $MUF_j$ as

$$\sigma_{ij} = \text{cov}\ (MUF_i, MUF_j) \quad .$$

All the variance and covariance calculations may be summarized in the variance-covariance matrix $\Sigma$, also called the dispersion matrix, of the sequence $MUF_1, MUF_2, \ldots, MUF_n$:

$$\Sigma = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{pmatrix} . \quad (2)$$

The matrix $\Sigma$ is the condensed form of the measurement model of the facility under consideration. It is an essential component of the statistical analysis of the MUF sequence.

Given a sequence of nonzero MUF values, we have to decide whether the values are caused by measurement errors or loss. In our case, we use the theory of statistical hypothesis testing to decide on the basis of a given sequence of MUF values $(MUF_1, \ldots, MUF_n)$ whether the situation of no loss or loss of nuclear material pertains. Loss of material may occur in a variety of patterns, and we have to take into account that the actual loss pattern is unknown.

We assume two hypotheses for the mean values of the random variables $MUF_k$. If there is no loss of material, all materials balances have zero mean. This situation is described by the null hypothesis

$$H_0: E(MUF_k) = 0 \text{ for all periods } k = 1,2,\ldots,n \quad . \quad (3)$$

A loss of material can take place in one or more balance periods.

Taking this into account, we formulate the alternative hypothesis:

$$H_1: E(MUF_k) = m_k \neq 0 \quad \text{with } \Sigma m_k > 0 \quad . \quad (4)$$

Hypothesis $H_1$ means that a loss of material occurred in at least one balance period k. In our considerations, we are not restricted to a fixed number of inventory periods.

## B. NRTA Test Procedures

The sequential tests in PROSA are truncated versions; that is, they give a decision at the end of the $n^{th}$ balance period or earlier. We use three sequential test procedures in PROSA, all of which are evaluated with the same selected false alarm probability $\alpha$.

### 1. Test Based on MUFs.

a. Truncated Sequential CUMUF Test. CUMUF is defined as the cumulative sum of the materials balance results $MUF_i$:

$$CUMUF_i = MUF_1 + \dots + MUF_i \quad , \quad i = 1,2,\dots,n \quad . \tag{5}$$

The test is performed as follows:

for $i = 1,2,\dots,n-1$,

$$CUMUF_i \begin{cases} > s_i, \text{ reject } H_0 \\ \le s_i, \text{ no decision and go to the next period} \end{cases} ;$$

for $i = n$,

$$CUMUF_n \begin{cases} \le s_n, \text{ reject } H_1 \\ > s_n, \text{ reject } H_0 \end{cases} .$$

The significance thresholds $s_1,s_2,\dots,s_n$ are determined by a Monte Carlo simulation to give a given false alarm probability $\alpha$. In our case, we select

$$s_i = \text{var}\,(CUMUF_i)^{1/2}\, U_{1-\alpha'} \quad ,$$

where U is the inverse standard normal distribution function. The value $\alpha'$ corresponds to the total false alarm probability $\alpha$.

<u>b. The GeMUF Test</u>. The application of PROSA 1.0 to various data sets revealed that the application of the Power One Test does not provide a substantial increase in detection capability of anomalies among the data compared with the Page's Test. This is not very surprising because the statistics of both tests are very similar. There are cases, however, where the CUMUF Test as well as Page's Test do not perform very well, so at Kernforschungszentrum Karlsruhe (KfK) we were looking for a test that is based on the idea of the Neyman-Pearson Test, which should close the gap. The idea is to replace the Power One Test by this newly developed test. We know that there exists exactly one best test to test $H_0$ against $H_1$ when in case of loss the loss pattern is known exactly. This is the Neyman-Pearson Test that may be formulated as

$$Z \begin{cases} > k, \text{ reject } H_0 \\ < k, \text{ reject } H_1 \end{cases} , \qquad (6)$$

where

$$Z = (m_1, m_2, \ldots, m_n)\Sigma^{-1}(MUF_1, MUF_2, \ldots, MUF_n)^t \quad .$$

Because in the case of loss the exact pattern will normally not be known, this test cannot be applied. The idea of the new test, which is called the GeMUF Test, is to estimate the loss $m_i$ of period i, i=1,2,...,n, by $MUF_i$, which is an unbiased estimate. The statistics of this test may be written as

$$GeMUF_i = (MUF_1, MUF_2, \ldots, MUF_i)\Sigma_i^{-1}(MUF_1, MUF_2, \ldots, MUF_i)^t \quad ,$$

where $\Sigma_i^{-1}$ is the inverse of the dispersion matrix for the random vector $(MUF_1 MUF_2,\ldots,MUF_i)$. The test may now be formulated as follows:

for $i = 1,2,\ldots,n-1$,

$$GeMUF_i \begin{cases} \leq t_i, & \text{no decision and go to the next period} \\ > t_i, & \text{reject } H_0 \end{cases} ;$$

for $i = n$,

$$GeMUF_n \begin{cases} \leq t_n, & \text{reject } H_1 \\ > t_n, & \text{reject } H_0 \end{cases} .$$

The thresholds $t_i$ have to be calculated by a Monte Carlo simulation to allow a total overall false alarm probability $\alpha$.

2. Tests Based on the MUF Residuals. The materials balance equations $MUF_i$ are stochastically dependent random variables. With a linear transformation, it is possible to transform the sequence $MUF_1,\ldots,MUF_n$ to a sequence of stochastically independent random variables $MUFR_1,\ldots,MUFR_n$. There are numerous possibilities for this transformation. We selected the transformation given by

$$MUFR_i = MUF_i - E(MUF_i | MUF_1,\ldots,MUF_{i-1}) \tag{7}$$

for $i = 2,\ldots,n$ with $MUFR_1 = MUF_1$. The values for $MUFR_i$ are called MUF residuals because they describe the difference between the estimate for the mean of $MUF_i$ based on the last $i - 1$ results and the realization of $MUF_i$.

The transformation can be described as an $n \bullet n$ matrix L with

$$(MUFR_1,\ldots,MUFR_n) = (MUF_1,\ldots,MUF_n) \bullet L \quad . \tag{8}$$

The diagonal matrix $L \cdot \Sigma \cdot L^t$ is the dispersion matrix of the MUFR vector.

For the hypotheses we get

$$(H_0): E(MUFR_i) = 0 \quad \text{for } i = 1,2,\ldots,n \quad . \tag{9}$$

Under the alternative hypothesis $H_1$, positive or negative values for the sum of the means of $MUFR_i$ are possible, and this is an important difference to $MUF_i$'s. Therefore, we have a two-sided hypothesis:

$$(H_1): E(MUFR_i) \neq 0 \quad \text{for at least one } i \quad . \tag{10}$$

a. Power One Test. The Power One Test was proposed by Robbins as a procedure that accepts $H_1$ with probability one when it is true and testing can continue indefinitely. For this test, we use the cumulative sum of the standardized $MUFR_i$ variables:

$$T_i = \sum_{j=1}^{i} \frac{MUFR_j}{\text{var }(MUF_j)^{1/2}} \quad . \tag{11}$$

The test procedure is defined as follows:

for $i = 1,2,\ldots,n-1$,

$$|T_i| \begin{cases} > b_i, \text{ reject } H_0 \\ \leq b_i, \text{ no decision and go to the next period} \end{cases} ;$$

for i=n,

$$|T_n| \begin{cases} > b_n, \text{ reject } H_0 \\ \le b_n, \text{ reject } H_1 \end{cases} .$$

The parameters $b_i$ are calculated as

$$b_i = \{(i + m) [-2\ln (a) + \ln (1 + \frac{i}{m})]\}^{1/2} ,$$

where a is determined by simulation to obtain a specific false alarm probability $\alpha$ and m is a parameter that influences the distribution of the false alarms.

b. CUSUM Test with Page Thresholds. The test was proposed by Page and uses the following statistics:

$$S_o = 0 ,$$

$$T_o = 0 ,$$

$$S_i = \max \{0, S_{i-1} + MUFR_i - k\} , \text{ and}$$

$$T_i = \min\{0, T_{i-1} + MUFR_i + k\} ,$$

for i = 1,2,...,n, where k is a fixed real number. The test procedure called Page's Test is defined as follows, where h is a real number:

for i = 1,2,...,n-1,

1. $S_i > h$ or $T_i < -h$ , reject $H_0$

2. $S_i \le h$ and $T_i \ge -h$, no decision and go to the next period ;

for $i = n$,

1. $S_n > h$ or $T_n < -h$, reject $H_0$

2. $S_n \leq h$ and $T_n \geq -h$, reject $H_1$

The parameters h and k are determined by simulation to guarantee a false alarm probability $\alpha$ for the n balance periods. In our case, we selected $k = 0$.

## III. DESCRIPTION OF AGNS MINIRUN DATA

The data that are used for a demonstration of PROSA were collected during a demonstration of near-real-time nuclear materials accountancy at the AGNS Barnwell Nuclear Fuels Plant.[4] The demonstration was structured in several experiments for processing uranium solutions. The experiments lasted 1 week each and were called miniruns. The measurement system for collecting the data consisted primarily of process-monitoring measurements. The accounting data were collected in 4-minute intervals, but only the hourly readings are used for the following analysis and only a part of the facility is used for the accountancy analysis. The materials balance area under consideration consists of the four pulsed columns 2A,2B,3A and 3B (see also Fig. 1). The demonstration of PROSA is restricted to Minirun 6, which was conducted during the week 13-19 May 1981. Miniruns 1-5 were devoted primarily to the testing of equipment, measurement system, and materials accountancy software. Full-scale tests of the complete material and data collection systems were conducted during Miniruns 6 and 7.

The hourly flow and concentration measurements for the input and output are shown in Figs. 2-5. For the determination of the hourly input and output, we assumed that they are constant during the 1-hour time period. The hourly inventories of the four pulsed columns are illustrated in Figs. 6-9. For a further description of the measurement system, we refer to Ref. 4.

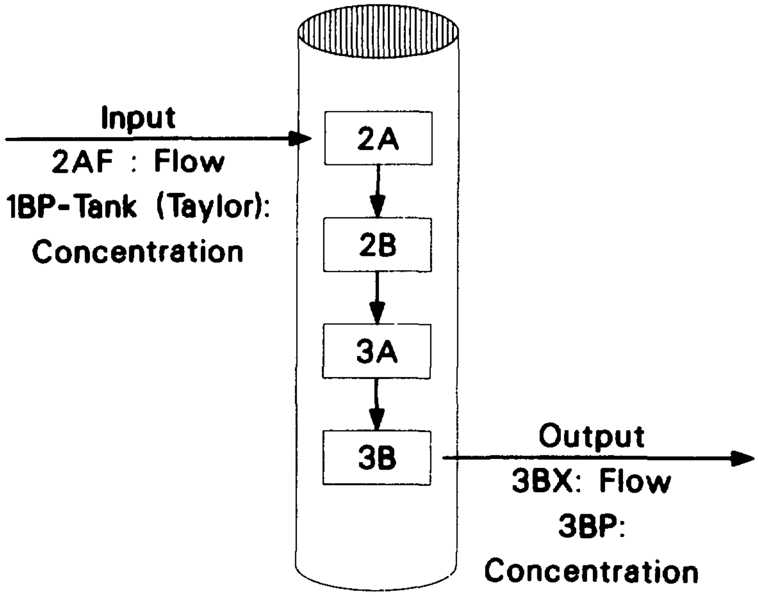


**Fig. 1. Selected materials balance area of AGNS Minirun 6 for demonstrations of NRTA with PROSA.**

## IV. DEVELOPMENT OF A MEASUREMENT MODEL

An essential part of the the NRTA analysis of a sequence of materials accountancy data is the calculation of the dispersion matrix $\Sigma$ [see Eq. (2)]. This matrix has to be calculated based on the real inventories, inputs, and outputs for a series of materials balances under consideration. For the inventory measurements of the pulsed columns, we assume a random error of 20% (measurement-to-measurement variation) and a systematic error of 20%, which is a random error that is constant for the whole sequence of accounting periods. The operator information (Table A-IV in Ref. 4) ranges from 20% to 76% standard deviation. We take the lowest value for all pulsed columns. The high uncertainties mainly originate from the influence of pulsing the columns.

The following data in Table I, taken from Table A-IV in Ref. 4, are used as relative standard deviations for data of the transfer measurements. Based on the information from the flow sheet data, we calculate the relative standard deviations for the input R and output S where $F_R$ and $C_R$ are the flow and concentration data, respectively, for the input and $F_S$ and $C_S$ are the flow and concentration data, respectively, for the output.

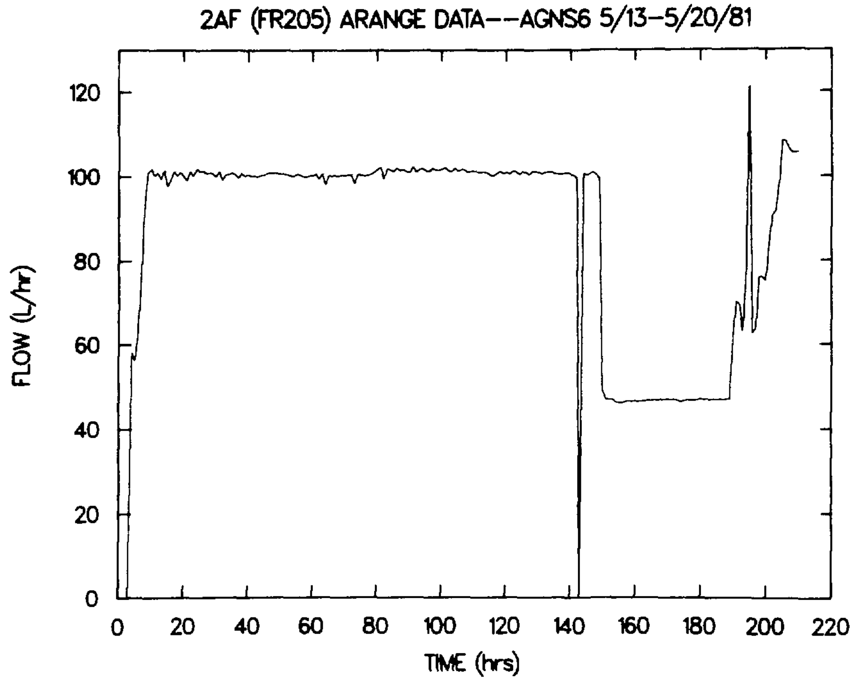


**Fig. 2. Flow data for the input (2AF stream), measured hourly.**

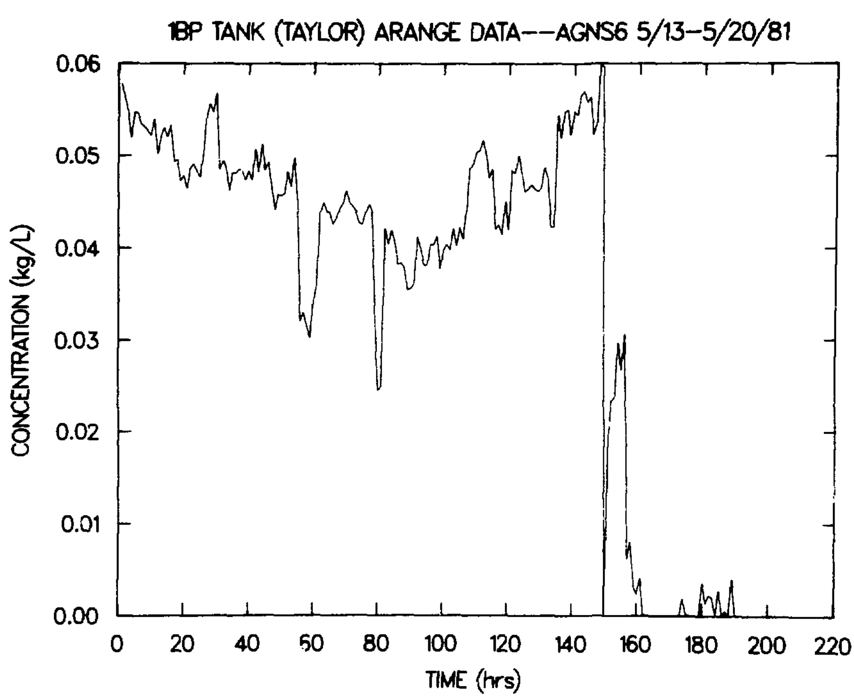


**Fig. 3. Concentration data for the input (1BP tank, Taylor), measured hourly.**

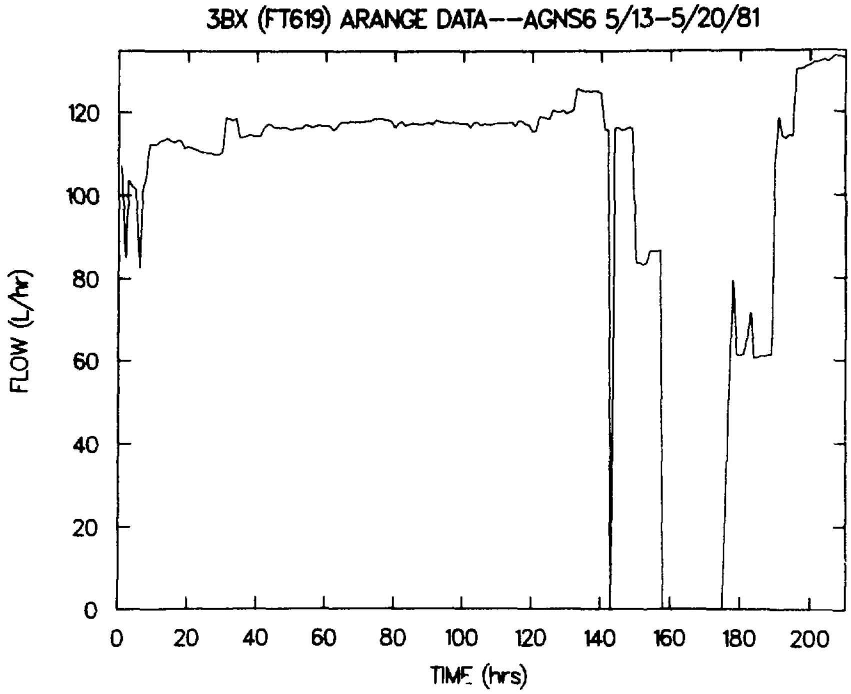


**Fig. 4. Flow data for the output (3BX stream), measured hourly.**

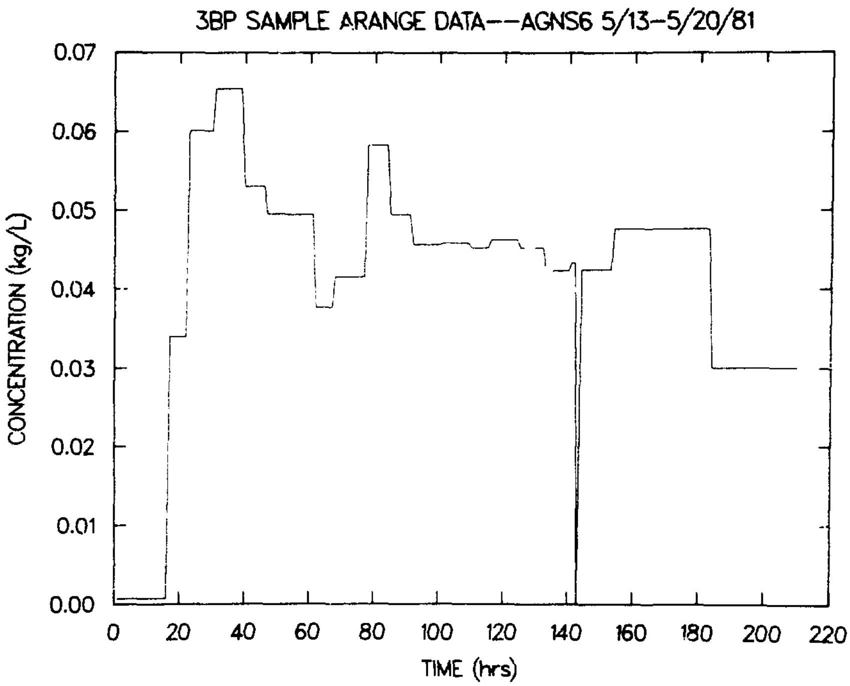


**Fig. 5. Concentration data for the output (3BP sample), measured hourly.**

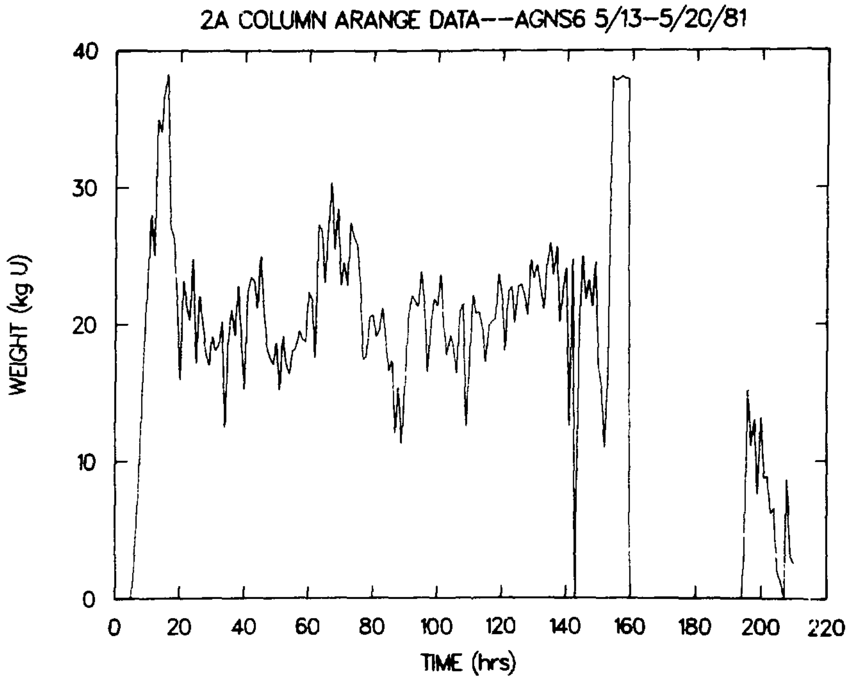


**Fig. 6. Uranium inventory of 2A column, measured hourly.**

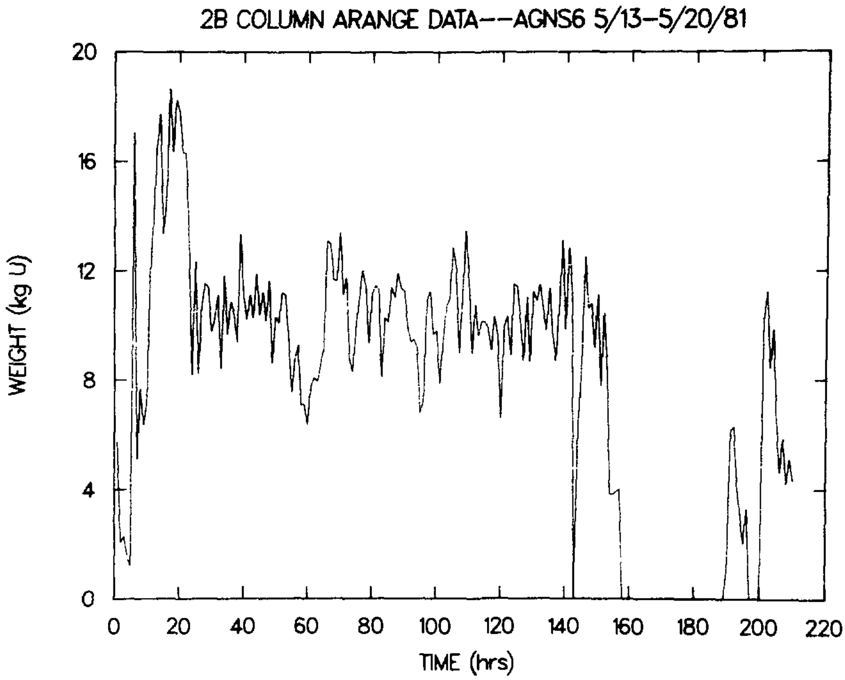


**Fig. 7. Uranium inventory of 2B column, measured hourly.**

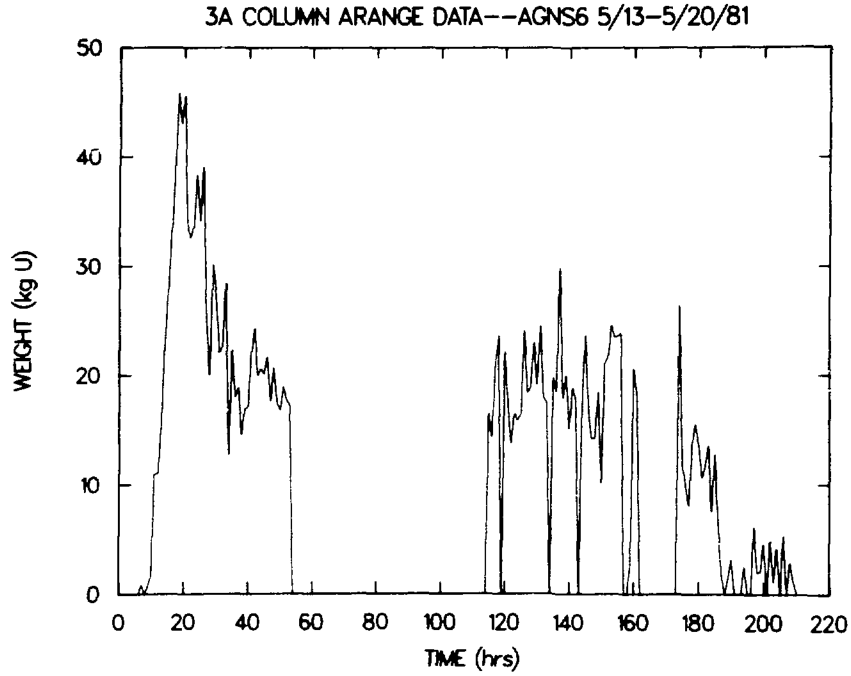


**Fig. 8. Uranium inventory of 3A column, measured hourly.**

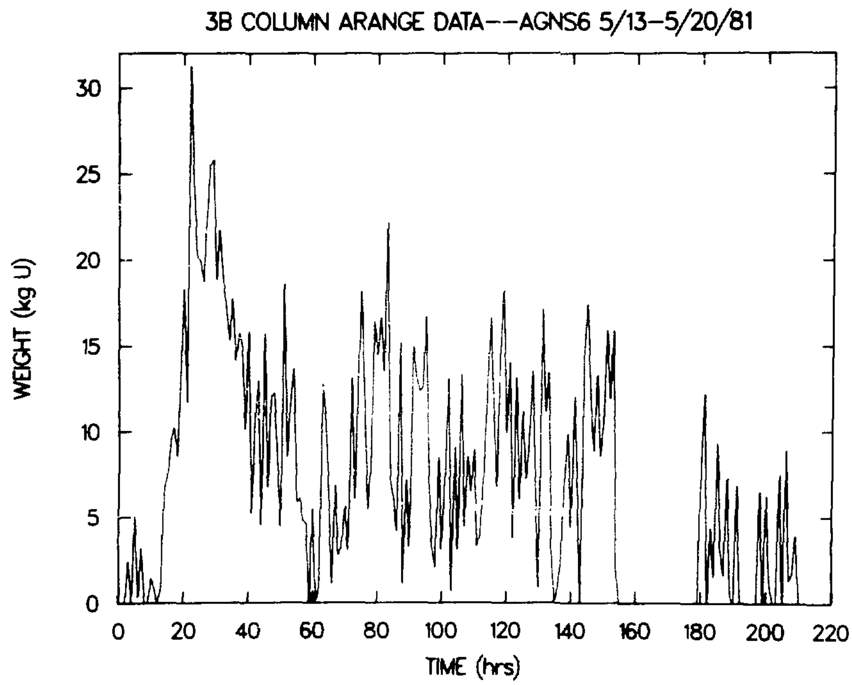


**Fig. 9. Uranium inventory of 3B column, measured hourly.**

Table I. Relative Standard Deviations for Input and Output Streams

| | | Random | Systematic |
|---|---|---|---|
| Input: | Concentration - 1 BP Tank (Taylor) | 0.064 | 0.142 |
| | Flow - 2 AF | 0.02 | 0.02 |
| Output: | Concentration - 3 BP Sample | 0.06 | 0.01 |
| | Flow - 3 BX | 0.02 | 0.02 |

We get

$$R = C_R \cdot F_R \quad \text{and} \quad S = C_S \cdot F_S , \tag{12}$$

and using error propagation based on the first two terms of the Taylor series, we get

$$\text{var } (R) = E(C_R)^2 \cdot \text{var } (F_R) + E(F_R)^2 \cdot \text{var}(C_R) \tag{13}$$

$$= (60 \cdot 100 \cdot 0.02)^2 + (100 \cdot 60 \cdot 0.064)^2 \quad \text{(random)}$$

$$+ (60 \cdot 100 \cdot 0.02) + (100 \cdot 60 \cdot 0.142) \quad \text{(systematic)}$$

$$= 161\ 856\ (\text{g U/h})^2 \quad \text{(random)}$$

$$+ 740\ 304\ (\text{g U/h})^2 \quad \text{(systematic)}$$

and

$$\text{var } (S) = E(C_R)^2 \cdot \text{var } (F_S) + E(F_S)^2 \cdot \text{var } (C_S) \tag{14}$$

$$= (50 \cdot 120 \cdot 0.02)^2 + (120 \cdot 50 \cdot 0.06)^2 \quad \text{(random)}$$

$$+ (50 \cdot 120 \cdot 0.02)^2 + (120 \cdot 50 \cdot 0.01)^2 \quad \text{(systematic)}$$

$$= 14\,400\ (\text{g U/h})^2 \qquad \text{(random)}$$

$$+\ 18\,000\ (\text{g U/h})^2\ . \qquad \text{(systematic)}$$

Based on the flow sheet information, we have an input and output of 6000 g of uranium/hour. This allows us to calculate the relative standard deviations for input and output (Table II).

**Table II. Relative Standard Deviations of Input and Output**

| | Random | Systematic |
|---|---|---|
| Input | 0.07 | 0.14 |
| Output | 0.06 | 0.02 |

## A. Steady-State Model for Pulsed Columns

If we assume an ideal operation of the facility according to the flow sheet data, we can set up a steady-state operation and measurement model for the pulsed columns that can serve as a guideline for the uranium materials accountancy considerations. The steady-state operation model is summarized in the Table III. Based on Table III, we get for the variance of a 5-hour materials balance 65 kg of uranium, which is a standard deviation of 8.065 kg of uranium. The standardized 19 • 19 dispersion matrix for the materials balance model in Table III has the following structure:

$$\Gamma = \begin{bmatrix} a & b & c & c & c & c & \cdots\cdots & c \\ b & a & b & c & c & c & \cdots\cdots & c \\ c & b & a & b & c & c & \cdots\cdots & c \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & c & b & a & b \\ c & \cdots & \cdots & \cdots & \cdots & c & b & a \end{bmatrix}, \qquad (15)$$

where a = 1, b = -0.073, and c = 0.277.

**Table III. Parameter of Steady-State Operation Measurement Model for Balancing Uranium in the Pulsed Columns of AGNS Minirun 6**

| | |
|---|---|
| Number of working hours | 95 |
| Balance interval in hours | 5 |
| Number of balances | 19 |

| | U - Inventory (kg) | Relative Standard Deviation Random | Systematic |
|---|---|---|---|
| 2A Column | 19.2 | $0.20 = \sigma_{I,r}$ | $0.20 = \sigma_{I,s}$ |
| 2B Column | 5.4 | 0.20 | 0.20 |
| 3A Column | 11.4 | 0.20 | 0.20 |
| 3B Column | 5.6 | 0.20 | 0.20 |
| | U/hour (kg) | | |
| Input (2AF,1BP) | 6.0 | $0.07 = \sigma_{R,r}$ | $0.14 = \sigma_{R,s}$ |
| Output (3BX,3BP) | 6.0 | $0.06 = \sigma_{S,r}$ | $0.02 = \sigma_{S,s}$ |

The structure of $\Gamma$ means that the systematic errors of the transfers are quite influential[5] and allows a first estimate about the performance of NRTA measures to detect losses of material. The most difficult to detect loss pattern in such a case is a uniform loss over all 19 balance periods. A loss of about 40 materials balance standard deviations is detectable by the Truncated Sequential CUMUF Test with 95% probability if the amount is distributed uniformly over the 19 balance periods. The standard deviation for the materials balance over 95 hours is 81.071 kg of uranium.

## B. Measurement Model for AGNS Minirun 6 Data

A steady-state model that is considered in Table I is only an approximation of a real process when this process is running according to the flow sheet data. But a running facility is often not in a flow sheet state. For an exact NRTA analysis of real process data, one has to use a measurement model that is based on the facility data that led to the series of materials balances. The establishment of a measurement model for a sequence of materials balance periods means that PROSA must use the plant data for inventories, input, and output to calculate, based on the relative standard deviations, the dispersion matrix $\sum = (\sigma_{ij}{}^2)$. In our special case, we have the same standard deviation for all inventory components 2A, 2B, 3A, and 3B, so we use $I_i$ as total inventory measurement at the end of balance period i. Here $I_0$ is the beginning inventory, and $R_i$ and $S_i$ are the receipts and shipments, respectively, during balance period i.

The symmetric dispersion matrix $\sum$ for n balances is calculated according to the following equations:

for $i = 1,2,\ldots,n$,

$$\sigma_{ii}^2 = I_{i-1}^2 \bullet (\sigma_{I,r}^2 + \sigma_{I,s}^2) + I_i^2 \bullet (\sigma_{I,r}^2 + \sigma_{I,s}^2) - 2 \bullet I_{i-1} \bullet I_i \bullet \sigma_{I,s}^2$$
$$+ R_i^2 \bullet (\sigma_{R,r}^2 + \sigma_{R,s}^2) + S_i^2 \bullet (\sigma_{S,r}^2 + \sigma_{S,s}^2) \quad ; \qquad (16)$$

for $j = i + 1$,

$$\sigma_{ij}^2 = -I_i^2 \bullet \sigma_{I,r}^2 + (I_{i-1} \bullet I_i - I_i^2 - I_{i-1} \bullet I_j + I_i \bullet I_j) \bullet \sigma_{I,s}^2$$
$$+ R_i \bullet R_j \bullet \sigma_{R,s}^2 + S_i \bullet S_j \bullet \sigma_{S,s}^2 \quad ; \qquad (17)$$

and for $j > i + 1$,

$$\sigma_{ij}^2 = (I_{i-1} \bullet I_{j-1} - I_i \bullet I_{j-1} - I_{i-1} \bullet I_j + I_i \bullet I_j)^2 \bullet \sigma_{I,s}$$
$$+ R_i \bullet R_j \bullet \sigma_{R,s}^2 + S_i \bullet S_j \bullet \sigma_{S,s}^2 \quad . \qquad (18)$$

## V. APPLICATION OF PROSA TO THE MINIRUN 6 DATA

In our case PROSA is going to be applied to the data of Minirun 6 as explained in Sec. III. As a first step, we use for our NRTA analysis a time period when we had a controlled, nearly steady-state input according to the flow sheet. Based on Fig. 2, we select the time period beginning after operation hour 55 to hour 150 of Minirun 6 divided into 19 balance periods of 5 hours.

### A. Application to Data Without any Changes

In a first approach, we use the data without any adjustments. In Figs. 10-12, the input, output, and inventories are illustrated as they have been derived from Figs. 2-9. The values of the materials balances are shown in Fig. 13. Based on Eqs. (16)-(18) and using the data in Figs. 10-12, we calculate the dispersion matrix, which is shown in Fig. 14 in lower triangular form. PROSA calculates the standardized dispersion matrix because it generally uses standardized values. The structure of the standardized dispersion matrix shown in Fig. 15 allows a first analysis of the materials balance data. We see that the two following materials balances are negatively correlated and that otherwise materials balances are not strongly correlated. That means, the measurement uncertainties of the transfers are dominated by the measurement uncertainties of the inventories. Furthermore, the general structure of the standardized dispersion matrix in Fig. 15 differs considerably from the standardized dispersion matrix $\Gamma$ of the flow sheet model in Sec. IV.A. One of the reasons is that for the real data the systematic measurement uncertainties have an influence on the dispersion matrix. The readings in Fig. 12 show a considerable fluctuation in the inventory readings.

Now PROSA is going to be used to decide if the MUF series shown in Fig. 13 may be explained by the assumed measurement model--the result of which is the dispersion matrix in Fig. 14. We assume for the determination of the alarm thresholds a false alarm probability of 5% for the total period of 19 balance periods.

The results coming out of PROSA are shown in Figs. 16-19. We do not generally get an alarm by PROSA. Only the Power One Test shows a slight anomaly, which is caused by the negative tendency of the CUMUFs. In such a situation, the CUMUF Test is not useful because it is one-sided and only designed to detect material losses, that is, positive CUMUFs. A further analysis of the data shows that a false alarm level of 1% does not lead to an alarm. The fluctuations of the inventories lead to the hypothesis that

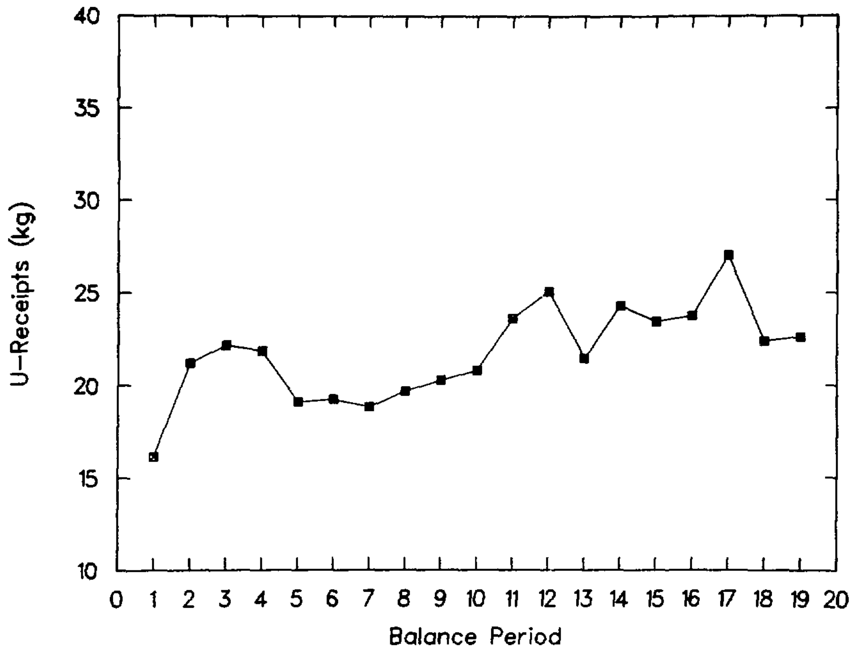


**Fig. 10. Uranium receipts for 19 balance periods beginning with operation hour 56.**

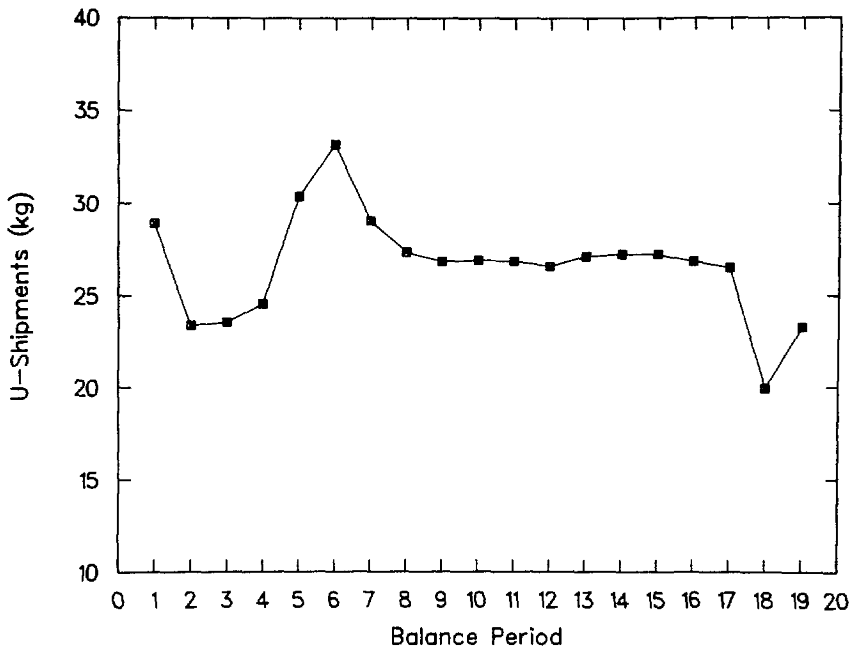


**Fig. 11. Uranium shipments for 19 balance periods beginning with operation hour 56.**

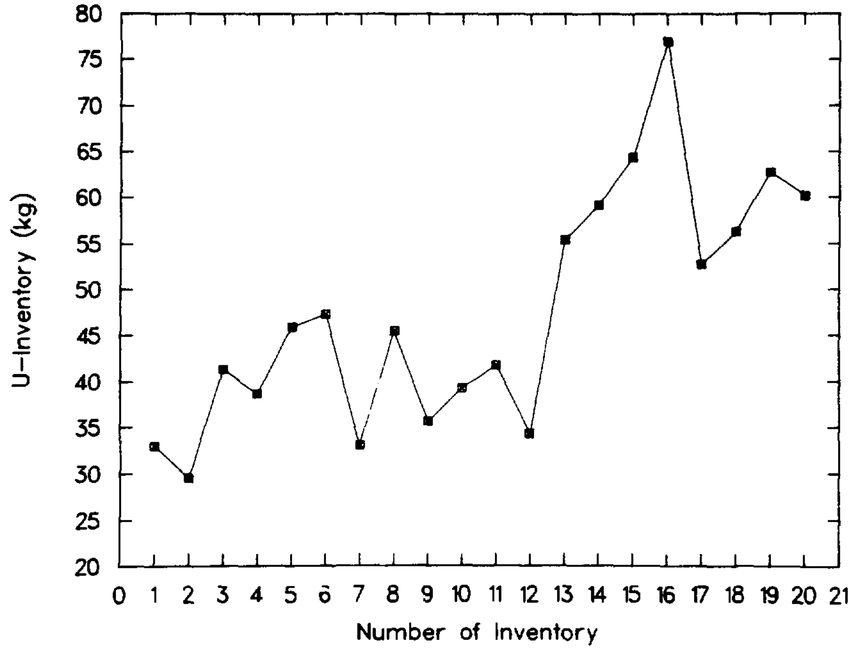


**Fig. 12. Uranium inventories of pulsed columns 2A, 2B, 3A, and 3B for 19 balance periods.**

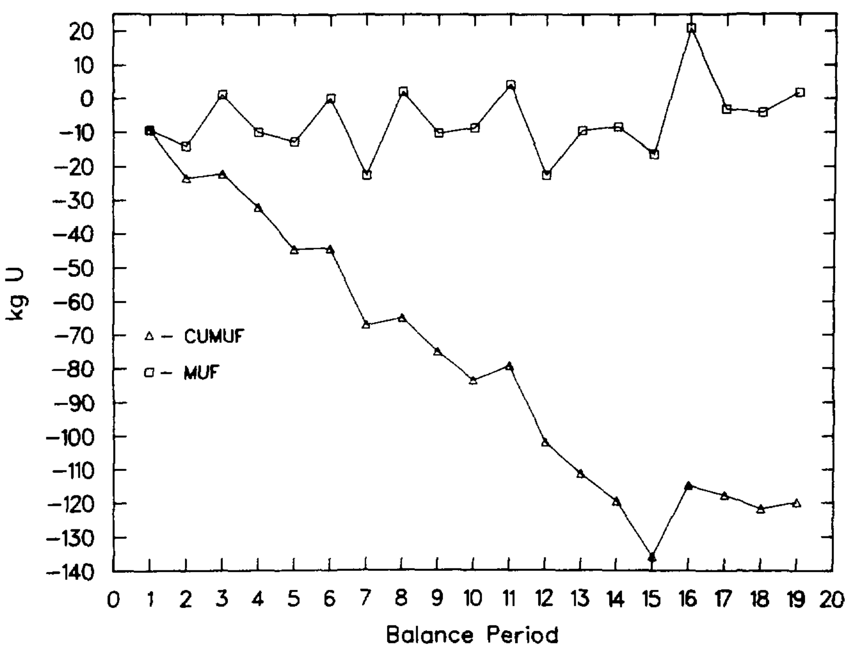


**Fig. 13. Results for the materials balances (MUF) and cumulative materials balances (CUMUF).**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 88.738 | | | | | | | | | |
| -29.589 | 122.150 | | | | | | | | |
| 7.649 | -60.236 | 142.908 | | | | | | | |
| 6.202 | 12.731 | -50.964 | 160.482 | | | | | | |
| 6.209 | 8.877 | 8.452 | -75.434 | 186.434 | | | | | |
| 8.445 | 1.604 | 10.188 | 4.482 | -82.531 | 154.663 | | | | |
| 4.574 | 13.965 | 7.149 | 11.940 | 8.089 | -43.257 | 144.574 | | | |
| 7.898 | 3.819 | 9.850 | 5.891 | 7.180 | 13.360 | -79.905 | 149.841 | | |
| 6.225 | 10.405 | 8.689 | 10.013 | 8.131 | 5.956 | 9.608 | -44.176 | 126.140 | |
| 6.541 | 10.095 | 9.029 | 9.914 | 8.263 | 6.780 | 9.249 | 7.338 | -52.822 | 145.441 |
| 8.813 | 6.524 | 11.304 | 8.225 | 8.766 | 13.529 | 5.312 | 12.337 | 8.591 | -60.766 |
| 135.805 | | | | | | | | | |
| 5.311 | 20.629 | 8.913 | 17.086 | 10.864 | -2.164 | 20.021 | 1.716 | 13.318 | 12.636 |
| -41.532 | 205.790 | | | | | | | | |
| 6.578 | 10.946 | 9.183 | 10.548 | 8.576 | 6.330 | 10.099 | 7.110 | 9.372 | 9.422 |
| 9.095 | -108.711 | 277.349 | | | | | | | |
| 7.262 | 12.842 | 10.251 | 12.200 | 9.719 | 6.532 | 11.900 | 7.606 | 10.718 | 10.729 |
| 9.947 | 16.666 | -128.565 | 324.291 | | | | | | |
| 5.986 | 15.932 | 9.114 | 13.938 | 9.799 | 2.075 | 15.200 | 4.431 | 11.440 | 11.120 |
| 7.374 | 22.389 | 12.036 | -151.801 | 425.497 | | | | | |
| 11.173 | -1.309 | 13.133 | 3.471 | 7.911 | 23.078 | -2.921 | 18.929 | 6.222 | 7.533 |
| 18.528 | -8.446 | 6.649 | 6.491 | -237.810 | 388.288 | | | | |
| 8.367 | 13.185 | 11.631 | 12.892 | 10.658 | 8.540 | 12.072 | 9.336 | 11.568 | 11.682 |
| 11.734 | 16.586 | 12.203 | 13.921 | 14.513 | -101.884 | 259.432 | | | |
| 6.412 | 12.530 | 9.229 | 11.645 | 8.978 | 5.054 | 11.687 | 6.334 | 10.049 | 9.989 |
| 8.637 | 16.630 | 10.591 | 12.224 | 13.728 | 4.399 | -113.865 | 299.873 | | |
| 7.770 | 8.393 | 10.310 | 9.173 | 8.610 | 10.305 | 7.335 | 9.983 | 8.863 | 9.205 |
| 11.476 | 9.174 | 9.367 | 10.463 | 9.339 | 13.262 | 11.861 | -148.067 | 317.239 | |

**Fig. 14. Lower triangular form of the dispersion matrix for the series of MUFs in Fig. 10.**

the standard deviation of 20% of the random and systematic error for measuring the column inventories may be too optimistic. Indeed, if we go closer to the operator information and go up to 60% relative standard deviations, we do not get an alarm anymore. In this case the measurement model is dominated even more by the measurement uncertainties of the inventories. These may originate from the random influence of pulsing the columns on a single reading for the weight measurement. Furthermore, it is not yet clear what a systematic error in such an environment really means and how it has to be propagated.

## B. Data Adjustments to Reduce the Random Error for Pulsed-Column Inventory Measurements

The first adjustment of data we are going to make is for the inventory measurements of the 3A column, where we have from reading 54 to 114 a zero for the column inventory, which was caused by an error in the measurement system. We change these data by assuming a constant inventory of 17.263 kg of uranium, which was the last reading before the failure. This measure

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.0000 | -0.2842 | 0.0679 | 0.0520 | 0.0483 | 0.0721 | 0.0404 | 0.0685 | 0.0588 | 0.0576 |
| 0.0803 | 0.0393 | 0.0419 | 0.0428 | 0.0308 | 0.0602 | 0.0551 | 0.0393 | 0.0463 | |
| -0.2842 | 1.0000 | -0.4559 | 0.0909 | 0.0588 | 0.0117 | 0.1051 | 0.0282 | 0.0838 | 0.0757 |
| 0.0507 | 0.1301 | 0.0595 | 0.0645 | 0.0699 | -0.0060 | 0.0741 | 0.0655 | 0.0426 | |
| 0.0679 | -0.4559 | 1.0000 | -0.3365 | 0.0518 | 0.0685 | 0.0497 | 0.0673 | 0.0647 | 0.0626 |
| 0.0811 | 0.0520 | 0.0461 | 0.0476 | 0.0370 | 0.0558 | 0.0604 | 0.0446 | 0.0484 | |
| 0.0520 | 0.0909 | -0.3365 | 1.0000 | -0.4361 | 0.0285 | 0.0784 | 0.0380 | 0.0704 | 0.0649 |
| 0.0557 | 0.0940 | 0.0500 | 0.0535 | 0.0533 | 0.0139 | 0.0632 | 0.0531 | 0.0407 | |
| 0.0483 | 0.0588 | 0.0518 | -0.4361 | 1.0000 | -0.4860 | 0.0493 | 0.0430 | 0.0530 | 0.0502 |
| 0.0551 | 0.0555 | 0.0377 | 0.0395 | 0.0348 | 0.0294 | 0.0485 | 0.0380 | 0.0354 | |
| 0.0721 | 0.0117 | 0.0685 | 0.0285 | -0.4860 | 1.0000 | -0.2893 | 0.0878 | 0.0426 | 0.0452 |
| 0.0933 | -0.0121 | 0.0306 | 0.0292 | 0.0081 | 0.0942 | 0.0426 | 0.0235 | 0.0465 | |
| 0.0404 | 0.1051 | 0.0497 | 0.0784 | 0.0493 | -0.2893 | 1.0000 | -0.5429 | 0.0711 | 0.0638 |
| 0.0379 | 0.1161 | 0.0504 | 0.0550 | 0.0613 | -0.0123 | 0.0623 | 0.0561 | 0.0342 | |
| 0.0685 | 0.0282 | 0.0673 | 0.0380 | 0.0430 | 0.0878 | -0.5429 | 1.0000 | -0.3213 | 0.0497 |
| 0.0865 | 0.0098 | 0.0349 | 0.0345 | 0.0175 | 0.0785 | 0.0474 | 0.0299 | 0.0458 | |
| 0.0588 | 0.0838 | 0.0647 | 0.0704 | 0.0530 | 0.0426 | 0.0711 | -0.3213 | 1.0000 | -0.3900 |
| 0.0656 | 0.0827 | 0.0501 | 0.0530 | 0.0494 | 0.0281 | 0.0639 | 0.0517 | 0.0443 | |
| 0.0576 | 0.0757 | 0.0626 | 0.0649 | 0.0502 | 0.0452 | 0.0638 | 0.0497 | -0.3900 | 1.0000 |
| -0.4324 | 0.0730 | 0.0469 | 0.0494 | 0.0447 | 0.0317 | 0.0601 | 0.0478 | 0.0429 | |
| 0.0803 | 0.0507 | 0.0811 | 0.0557 | 0.0551 | 0.0933 | 0.0379 | 0.0865 | 0.0656 | -0.4324 |
| 1.0000 | -0.2484 | 0.0469 | 0.0474 | 0.0307 | 0.0807 | 0.0625 | 0.0428 | 0.0553 | |
| 0.0393 | 0.1301 | 0.0520 | 0.0940 | 0.0555 | -0.0121 | 0.1161 | 0.0098 | 0.0827 | 0.0730 |
| -0.2484 | 1.0000 | -0.4550 | 0.0645 | 0.0757 | -0.0299 | 0.0718 | 0.0669 | 0.0359 | |
| 0.0419 | 0.0595 | 0.0461 | 0.0500 | 0.0377 | 0.0306 | 0.0504 | 0.0349 | 0.0501 | 0.0469 |
| 0.0469 | -0.4550 | 1.0000 | -0.4287 | 0.0350 | 0.0203 | 0.0455 | 0.0367 | 0.0316 | |
| 0.0428 | 0.0645 | 0.0476 | 0.0535 | 0.0395 | 0.0292 | 0.0550 | 0.0345 | 0.0530 | 0.0494 |
| 0.0474 | 0.0645 | -0.4287 | 1.0000 | -0.4087 | 0.0183 | 0.0480 | 0.0392 | 0.0326 | |
| 0.0308 | 0.0699 | 0.0370 | 0.0533 | 0.0348 | 0.0081 | 0.0613 | 0.0175 | 0.0494 | 0.0447 |
| 0.0307 | 0.0757 | 0.0350 | -0.4087 | 1.0000 | -0.5851 | 0.0437 | 0.0384 | 0.0254 | |
| 0.0602 | -0.0060 | 0.0558 | 0.0139 | 0.0294 | 0.0942 | -0.0123 | 0.0785 | 0.0281 | 0.0317 |
| 0.0807 | -0.0299 | 0.0203 | 0.0183 | -0.5851 | 1.0000 | -0.3210 | 0.0129 | 0.0378 | |
| 0.0551 | 0.0741 | 0.0604 | 0.0632 | 0.0485 | 0.0426 | 0.0623 | 0.0474 | 0.0639 | 0.0601 |
| 0.0625 | 0.0718 | 0.0455 | 0.0480 | 0.0437 | -0.3210 | 1.0000 | -0.4082 | 0.0413 | |
| 0.0393 | 0.0655 | 0.0446 | 0.0531 | 0.0380 | 0.0235 | 0.0561 | 0.0299 | 0.0517 | 0.0478 |
| 0.0428 | 0.0669 | 0.0367 | 0.0392 | 0.0384 | 0.0129 | -0.4082 | 1.0000 | -0.4801 | |
| 0.0463 | 0.0426 | 0.0484 | 0.0407 | 0.0354 | 0.0465 | 0.0342 | 0.0458 | 0.0443 | 0.0429 |
| 0.0553 | 0.0359 | 0.0316 | 0.0326 | 0.0254 | 0.0378 | 0.0413 | -0.4801 | 1.0000 | |

**Fig. 15. Standardized dispersion matrix of matrix shown in Fig. 14.**

will only influence two materials balance results, namely, one at the beginning and one at the end of the failure period. Furthermore, it may reduce the fluctuations in the total inventory readings. Another measure is to take for an inventory at the end of 5 hours the average of the last 5 readings, which should under the assumption of nearly constant real inventory reduce the random error from 60% to $1/\sqrt{5} \cdot 60\%$, which is about 25%. The inventory measurements that result from these measures are shown in Fig. 20, where the readings are averaged over 5 hourly readings. The adjusted inventory readings are used to calculate a new sequence of materials balance results and a new dispersion matrix, which are illustrated in Figs. 21-23, where a 25% measurement uncertainty of random and systematic error for the inventory measurements is assumed. Also, in this case the CUMUF series shows a negative trend. It is explainable by the input

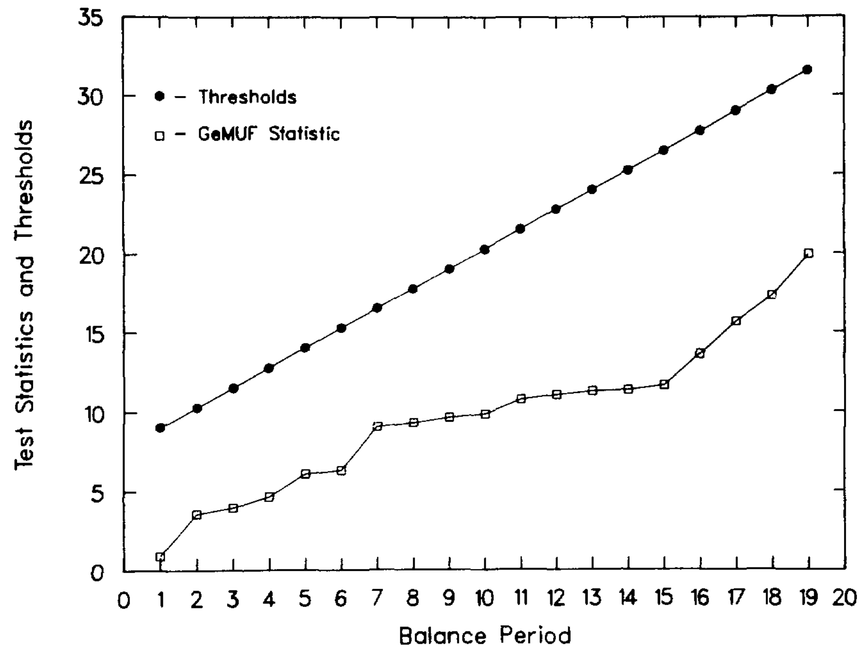


**Fig. 16. PROSA results for the Truncated Sequential CUMUF Test using the data in Fig. 13 and measurement model in Fig. 14; no alarm is initiated.**

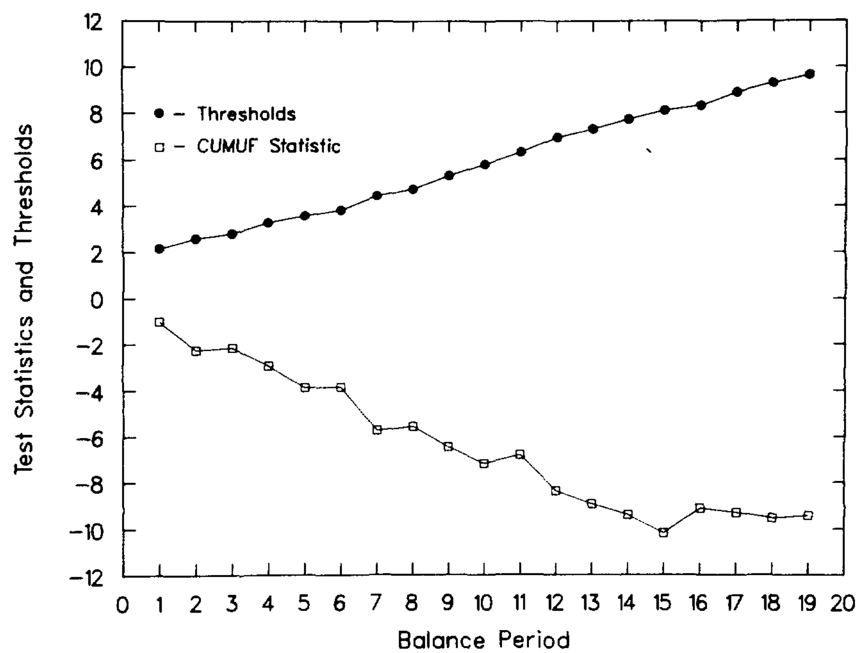


**Fig. 17. PROSA results for the GeMUF Test using the same data as in Fig. 16; no alarm is initiated.**

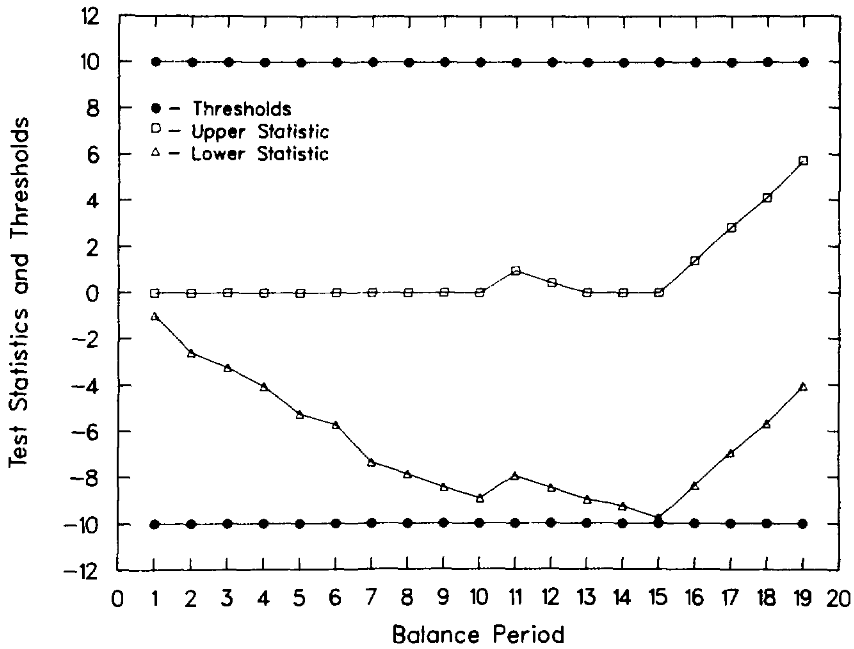


**Fig. 18. PROSA results for Page's Test using the same data as in Fig. 16; no alarm is initiated.**

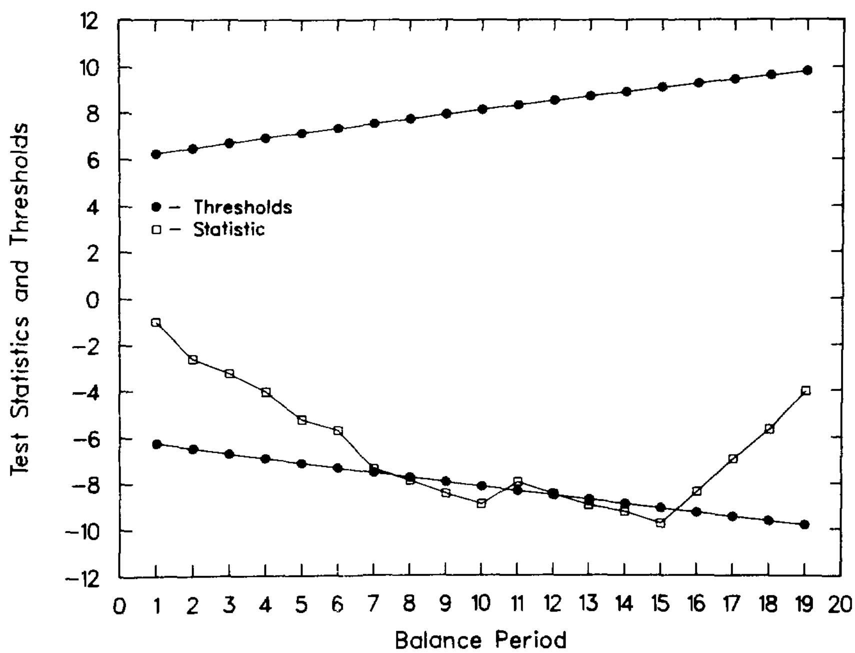


**Fig. 19. PROSA results for the Power One Test using the same data as in Fig. 16; alarm in balance period 8 is initiated.**

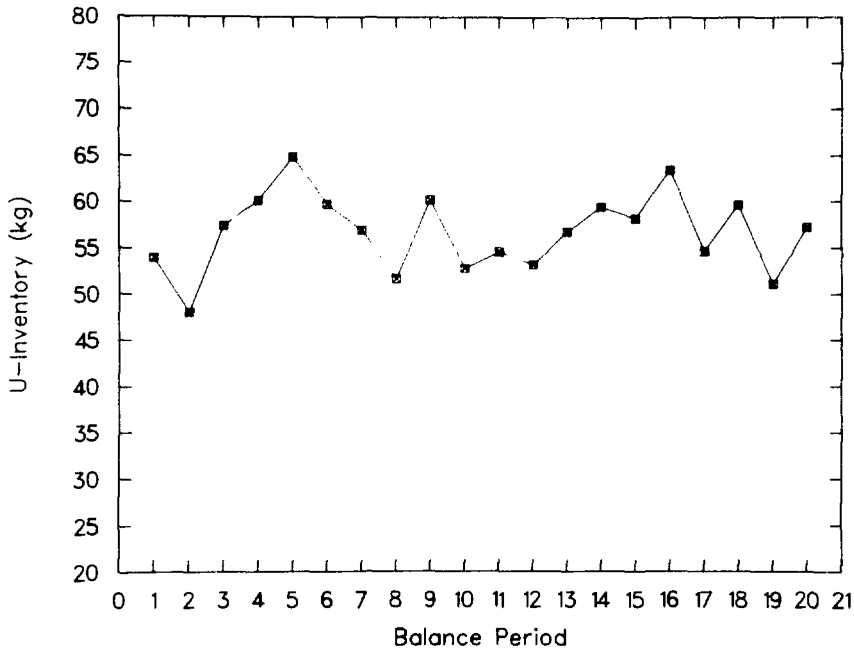


Fig. 20. **Modified uranium inventories for 19 balance periods where 5 hourly averages are used and measurement errors in column 3A are corrected.**

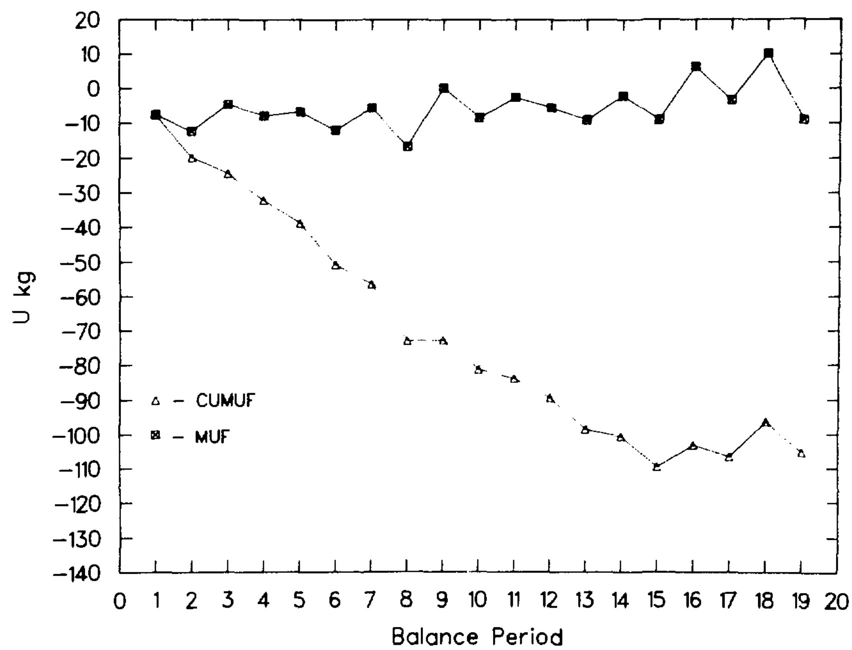


Fig. 21. **Results for materials balances and cumulative materials balances based on the modified uranium inventories of Fig. 20.**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 338.956 | | | | | | | | | |
| 140.843 | 369.089 | | | | | | | | |
| 6.282 | -194.914 | 446.610 | | | | | | | |
| 5.436 | 12.118 | -215.301 | 504.529 | | | | | | |
| 8.328 | 5.255 | 7.738 | -256.054 | 500.583 | | | | | |
| 7.546 | 6.686 | 8.216 | 7.758 | -214.564 | 439.822 | | | | |
| 8.267 | 5.064 | 7.587 | 6.820 | 9.109 | -194.087 | 383.453 | | | |
| 3.395 | 13.397 | 10.256 | 11.238 | 5.013 | 6.317 | -162.148 | 410.192 | | |
| 9.474 | 4.433 | 7.845 | 6.803 | 10.291 | 9.324 | 10.216 | -221.858 | 417.062 | |
| 6.239 | 9.961 | 9.610 | 9.730 | 7.569 | 7.914 | 7.430 | 9.282 | -166.724 | 374.814 |
| 8.299 | 9.280 | 10.288 | 9.989 | 9.625 | 9.523 | 9.492 | 8.691 | 10.316 | -176.704 |
| 380.530 | | | | | | | | | |
| 6.942 | 12.734 | 11.750 | 12.066 | 8.612 | 9.216 | 8.438 | 11.831 | 8.673 | 10.917 |
| 165.666 | 397.739 | | | | | | | | |
| 6.106 | 10.766 | 10.048 | 10.282 | 7.522 | 8.001 | 7.373 | 10.017 | 7.608 | 9.360 |
| 10.002 | -189.911 | 437.122 | | | | | | | |
| 8.479 | 9.625 | 10.604 | 10.314 | 9.850 | 9.766 | 9.713 | 9.010 | 10.543 | 10.069 |
| 11.647 | 11.958 | -210.641 | 450.201 | | | | | | |
| 5.757 | 13.103 | 11.345 | 11.900 | 7.427 | 8.284 | 7.250 | 12.150 | 7.205 | 10.453 |
| 10.692 | 12.976 | 11.055 | -200.601 | 481.726 | | | | | |
| 11.109 | 4.972 | 9.073 | 7.818 | 12.047 | 10.871 | 11.965 | 4.788 | 13.761 | 8.987 |
| 12.023 | 10.020 | 8.788 | 12.286 | -243.636 | 460.101 | | | | |
| 6.986 | 14.423 | 12.851 | 13.349 | 8.852 | 9.676 | 8.658 | 13.375 | 8.755 | 11.877 |
| 12.370 | 14.672 | 12.509 | 12.766 | 14.369 | -176.696 | 431.484 | | | |
| 10.506 | 4.519 | 8.476 | 7.263 | 11.376 | 10.231 | 11.302 | 4.352 | 13.028 | 8.402 |
| 11.307 | 9.353 | 8.201 | 11.554 | 7.683 | 15.329 | -213.188 | 404.610 | | |
| 5.166 | 13.204 | 11.095 | 11.758 | 6.828 | 7.797 | 6.651 | 12.230 | 6.478 | 10.177 |
| 10.210 | 12.716 | 10.814 | 10.553 | 12.678 | 7.415 | 14.153 | -156.635 | 385.777 | |

**Fig. 22. Lower triangular form of the dispersion matrix for the MUF series based on modified data.**

and output data in Figs. 10-11. They show on the average a higher reading for the outputs compared with the inputs, which finally leads to a negative trend in the CUMUFs. It should be mentioned that the systematic error meaning is still to be questioned. By the way, the systematic error of the inventories does not have a significant influence on the dispersion matrix for the MUFs. This can be seen by looking at Fig. 24, where the standardized dispersion matrix for 0% systematic error for the inventories is illustrated.

Furthermore, Fig. 23 shows that the random measurement errors of the inventories dominate the correlations of the materials balances. PROSA will now be applied to the data in Figs. 21-22, and we again assume a 5% false alarm probability for the evaluation of the 19 balance periods. The results in Figs. 25-27 show indeed that we do not get an alarm this time, and the data may be explained by the measurement model.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.0000 | -0.3982 | 0.0161 | 0.0131 | 0.0202 | 0.0195 | 0.0229 | 0.0091 | 0.0252 | 0.0175 |
| 0.0231 | 0.0189 | 0.0159 | 0.0217 | 0.0142 | 0.0281 | 0.0183 | 0.0284 | 0.0143 | |
| -0.3982 | 1.0000 | -0.4801 | 0.0281 | 0.0122 | 0.0166 | 0.0135 | 0.0344 | 0.0113 | 0.0268 |
| 0.0248 | 0.0332 | 0.0268 | 0.0236 | 0.0311 | 0.0121 | 0.0361 | 0.0117 | 0.0350 | |
| 0.0161 | -0.4801 | 1.0000 | -0.4536 | 0.0164 | 0.0185 | 0.0183 | 0.0240 | 0.0182 | 0.0235 |
| 0.0250 | 0.0279 | 0.0227 | 0.0236 | 0.0245 | 0.0200 | 0.0293 | 0.0199 | 0.0267 | |
| 0.0131 | 0.0281 | -0.4536 | 1.0000 | -0.5095 | 0.0165 | 0.0155 | 0.0247 | 0.0148 | 0.0224 |
| 0.0228 | 0.0269 | 0.0219 | 0.0216 | 0.0241 | 0.0162 | 0.0286 | 0.0161 | 0.0267 | |
| 0.0202 | 0.0122 | 0.0164 | -0.5095 | 1.0000 | -0.4573 | 0.0208 | 0.0111 | 0.0225 | 0.0175 |
| 0.0221 | 0.0193 | 0.0161 | 0.0207 | 0.0151 | 0.0251 | 0.0190 | 0.0253 | 0.0155 | |
| 0.0195 | 0.0166 | 0.0185 | 0.0165 | -0.4573 | 1.0000 | -0.4726 | 0.0149 | 0.0218 | 0.0195 |
| 0.0233 | 0.0220 | 0.0182 | 0.0219 | 0.0180 | 0.0242 | 0.0222 | 0.0243 | 0.0189 | |
| 0.0229 | 0.0135 | 0.0183 | 0.0155 | 0.0208 | -0.4726 | 1.0000 | -0.4088 | 0.0255 | 0.0196 |
| 0.0248 | 0.0216 | 0.0180 | 0.0234 | 0.0169 | 0.0285 | 0.0213 | 0.0287 | 0.0173 | |
| 0.0091 | 0.0344 | 0.0240 | 0.0247 | 0.0111 | 0.0149 | -0.4088 | 1.0000 | -0.5364 | 0.0237 |
| 0.0220 | 0.0293 | 0.0237 | 0.0210 | 0.0273 | 0.0110 | 0.0318 | 0.0107 | 0.0307 | |
| 0.0252 | 0.0113 | 0.0182 | 0.0148 | 0.0225 | 0.0218 | 0.0255 | -0.5364 | 1.0000 | -0.4217 |
| 0.0259 | 0.0213 | 0.0178 | 0.0243 | 0.0161 | 0.0314 | 0.0206 | 0.0317 | 0.0162 | |
| 0.0175 | 0.0268 | 0.0235 | 0.0224 | 0.0175 | 0.0195 | 0.0196 | 0.0237 | -0.4217 | 1.0000 |
| -0.4679 | 0.0283 | 0.0231 | 0.0245 | 0.0246 | 0.0216 | 0.0295 | 0.0216 | 0.0268 | |
| 0.0231 | 0.0248 | 0.0250 | 0.0228 | 0.0221 | 0.0233 | 0.0248 | 0.0220 | 0.0259 | -0.4679 |
| 1.0000 | -0.4258 | 0.0245 | 0.0281 | 0.0250 | 0.0287 | 0.0305 | 0.0288 | 0.0266 | |
| 0.0189 | 0.0332 | 0.0279 | 0.0269 | 0.0193 | 0.0220 | 0.0216 | 0.0293 | 0.0213 | 0.0283 |
| -0.4258 | 1.0000 | -0.4555 | 0.0283 | 0.0296 | 0.0234 | 0.0354 | 0.0233 | 0.0325 | |
| 0.0159 | 0.0268 | 0.0227 | 0.0219 | 0.0161 | 0.0182 | 0.0180 | 0.0237 | 0.0178 | 0.0231 |
| 0.0245 | -0.4555 | 1.0000 | -0.4748 | 0.0241 | 0.0196 | 0.0288 | 0.0195 | 0.0263 | |
| 0.0217 | 0.0236 | 0.0236 | 0.0216 | 0.0207 | 0.0219 | 0.0234 | 0.0210 | 0.0243 | 0.0245 |
| 0.0281 | 0.0283 | -0.4748 | 1.0000 | -0.4308 | 0.0270 | 0.0290 | 0.0271 | 0.0253 | |
| 0.0142 | 0.0311 | 0.0245 | 0.0241 | 0.0151 | 0.0180 | 0.0169 | 0.0273 | 0.0161 | 0.0246 |
| 0.0250 | 0.0296 | 0.0241 | -0.4308 | 1.0000 | -0.5175 | 0.0315 | 0.0174 | 0.0294 | |
| 0.0281 | 0.0121 | 0.0200 | 0.0162 | 0.0251 | 0.0242 | 0.0285 | 0.0110 | 0.0314 | 0.0216 |
| 0.0287 | 0.0234 | 0.0196 | 0.0270 | -0.5175 | 1.0000 | -0.3966 | 0.0355 | 0.0176 | |
| 0.0183 | 0.0361 | 0.0293 | 0.0286 | 0.0190 | 0.0222 | 0.0213 | 0.0318 | 0.0206 | 0.0295 |
| 0.0305 | 0.0354 | 0.0288 | 0.0290 | 0.0315 | -0.3966 | 1.0000 | -0.5102 | 0.0347 | |
| 0.0284 | 0.0117 | 0.0199 | 0.0161 | 0.0253 | 0.0243 | 0.0287 | 0.0107 | 0.0317 | 0.0216 |
| 0.0288 | 0.0233 | 0.0195 | 0.0271 | 0.0174 | 0.0355 | -0.5102 | 1.0000 | -0.3965 | |
| 0.0143 | 0.0350 | 0.0267 | 0.0267 | 0.0155 | 0.0189 | 0.0173 | 0.0307 | 0.0162 | 0.0268 |
| 0.0266 | 0.0325 | 0.0263 | 0.0253 | 0.0294 | 0.0176 | 0.0347 | -0.3965 | 1.0000 | |

**Fig. 23. Standardized dispersion matrix of matrix shown in Fig. 22.**

## VI. CONCLUSION

It was shown that the uranium data of a bulk-handling reprocessing facility could be used for a statistical analysis of NRTA data where PROSA was a helpful tool. The most interesting part was the development of the statistical measurement model for the collection of the materials balance data. PROSA could be used to test whether the underlying assumptions about measurement errors agree with the materials balance data. Adjustments for the inventory data of the pulsed columns led to a considerable reduction of the random error. What the systematic error for the pulsed column inventory means is still an unanswered question. It was demonstrated that measurement models, that is, dispersion matrices, for flow sheet data or real process data might differ considerably. Furthermore, it was explained

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.0000 | -0.3925 | 0.0188 | 0.0175 | 0.0156 | 0.0169 | 0.0176 | 0.0177 | 0.0181 | 0.0194 |
| 1.0218 | 0.0226 | 0.0186 | 0.0206 | 0.0192 | 0.0200 | 0.0233 | 0.0199 | 0.0207 | |
| -0.3925 | 1.0000 | -0.4878 | 0.0218 | 0.0194 | 0.0209 | 0.0218 | 0.0220 | 0.0224 | 0.0242 |
| 0.0271 | 0.0281 | 0.0231 | 0.0256 | 0.0240 | 0.0249 | 0.0291 | 0.0249 | 0.0258 | |
| 0.0188 | -0.4878 | 1.0000 | -0.4561 | 0.0182 | 0.0196 | 0.0205 | 0.0207 | 0.0211 | 0.0228 |
| 0.0255 | 0.0265 | 0.0217 | 0.0241 | 0.0226 | 0.0234 | 0.0274 | 0.0235 | 0.0243 | |
| 0.0175 | 0.0218 | -0.4561 | 1.0000 | -0.5080 | 0.0183 | 0.0191 | 0.0193 | 0.0197 | 0.0212 |
| 0.0238 | 0.0247 | 0.0202 | 0.0225 | 0.0210 | 0.0218 | 0.0255 | 0.0218 | 0.0226 | |
| 0.0156 | 0.0194 | 0.0182 | -0.5080 | 1.0000 | -0.4602 | 0.0170 | 0.0172 | 0.0175 | 0.0188 |
| 0.0211 | 0.0219 | 0.0180 | 0.0199 | 0.0186 | 0.0193 | 0.0226 | 0.0193 | 0.0201 | |
| 0.0169 | 0.0209 | 0.0196 | 0.0183 | -0.4602 | 1.0000 | -0.4762 | 0.0185 | 0.0188 | 0.0203 |
| 0.0227 | 0.0235 | 0.0194 | 0.0215 | 0.0201 | 0.0208 | 0.0243 | 0.0208 | 0.0216 | |
| 0.0176 | 0.0218 | 0.0205 | 0.0191 | 0.0170 | -0.4762 | 1.0000 | -0.4050 | 0.0197 | 0.0212 |
| 0.0237 | 0.0246 | 0.0202 | 0.0224 | 0.0210 | 0.0218 | 0.0254 | 0.0218 | 0.0226 | |
| 0.0177 | 0.0220 | 0.0207 | 0.0193 | 0.0172 | 0.0185 | -0.4050 | 1.0000 | -0.5321 | 0.0214 |
| 0.0240 | 0.0249 | 0.0204 | 0.0227 | 0.0212 | 0.0220 | 0.0257 | 0.0220 | 0.0228 | |
| 0.0181 | 0.0224 | 0.0211 | 0.0197 | 0.0175 | 0.0188 | 0.0197 | -0.5321 | 1.0000 | -0.4214 |
| 0.0244 | 0.0254 | 0.0208 | 0.0231 | 0.0216 | 0.0224 | 0.0262 | 0.0224 | 0.0233 | |
| 0.0194 | 0.0242 | 0.0228 | 0.0212 | 0.0188 | 0.0203 | 0.0212 | 0.0214 | -0.4214 | 1.0000 |
| -0.4677 | 0.0273 | 0.0224 | 0.0249 | 0.0233 | 0.0241 | 0.0282 | 0.0242 | 0.0251 | |
| 0.0218 | 0.0271 | 0.0255 | 0.0238 | 0.0211 | 0.0227 | 0.0237 | 0.0240 | 0.0244 | -0.4677 |
| 1.0000 | -0.4255 | 0.0251 | 0.0279 | 0.0261 | 0.0271 | 0.0316 | 0.0271 | 0.0281 | |
| 0.0226 | 0.0281 | 0.0265 | 0.0247 | 0.0219 | 0.0235 | 0.0246 | 0.0249 | 0.0254 | 0.0273 |
| -0.4255 | 1.0000 | -0.4576 | 0.0289 | 0.0271 | 0.0281 | 0.0329 | 0.0282 | 0.0292 | |
| 0.0186 | 0.0231 | 0.0217 | 0.0202 | 0.0180 | 0.0194 | 0.0202 | 0.0204 | 0.0208 | 0.0224 |
| 0.0251 | -0.4576 | 1.0000 | -0.4746 | 0.0222 | 0.0230 | 0.0269 | 0.0231 | 0.0239 | |
| 0.0206 | 0.0256 | 0.0241 | 0.0225 | 0.0199 | 0.0215 | 0.0224 | 0.0227 | 0.0231 | 0.0249 |
| 0.0279 | 0.0289 | -0.4746 | 1.0000 | -0.4307 | 0.0256 | 0.0299 | 0.0256 | 0.0266 | |
| 0.0192 | 0.0240 | 0.0226 | 0.0210 | 0.0186 | 0.0201 | 0.0210 | 0.0212 | 0.0216 | 0.0233 |
| 0.0261 | 0.0271 | 0.0222 | -0.4307 | 1.0000 | -0.5150 | 0.0280 | 0.0240 | 0.0248 | |
| 0.0200 | 0.0249 | 0.0234 | 0.0218 | 0.0193 | 0.0208 | 0.0218 | 0.0220 | 0.0224 | 0.0241 |
| 0.0271 | 0.0281 | 0.0230 | 0.0256 | -0.5150 | 1.0000 | -0.3932 | 0.0249 | 0.0258 | |
| 0.0233 | 0.0291 | 0.0274 | 0.0255 | 0.0226 | 0.0243 | 0.0254 | 0.0257 | 0.0262 | 0.0282 |
| 0.0316 | 0.0329 | 0.0269 | 0.0299 | 0.0280 | -0.3932 | 1.0000 | -0.5076 | 0.0302 | |
| 0.0199 | 0.0249 | 0.0235 | 0.0218 | 0.0193 | 0.0208 | 0.0218 | 0.0220 | 0.0224 | 0.0242 |
| 0.0271 | 0.0282 | 0.0231 | 0.0256 | 0.0240 | 0.0249 | -0.5076 | 1.0000 | -0.3916 | |
| 0.0207 | 0.0258 | 0.0243 | 0.0226 | 0.0201 | 0.0216 | 0.0226 | 0.0228 | 0.0233 | 0.0251 |
| 0.0281 | 0.0292 | 0.0239 | 0.0266 | 0.0248 | 0.0258 | 0.0302 | -0.3916 | 1.0000 | |

**Fig. 24. Standardized dispersion matrix of MUF in Fig. 21 with zero per cent systematic error for the inventory measurements.**

that the structure of the dispersion matrix can be used for a first estimate about the performance of the NRTA analysis. The procedure of NRTA data analysis explained in this report might be used as an example for the evaluation of other facility data.

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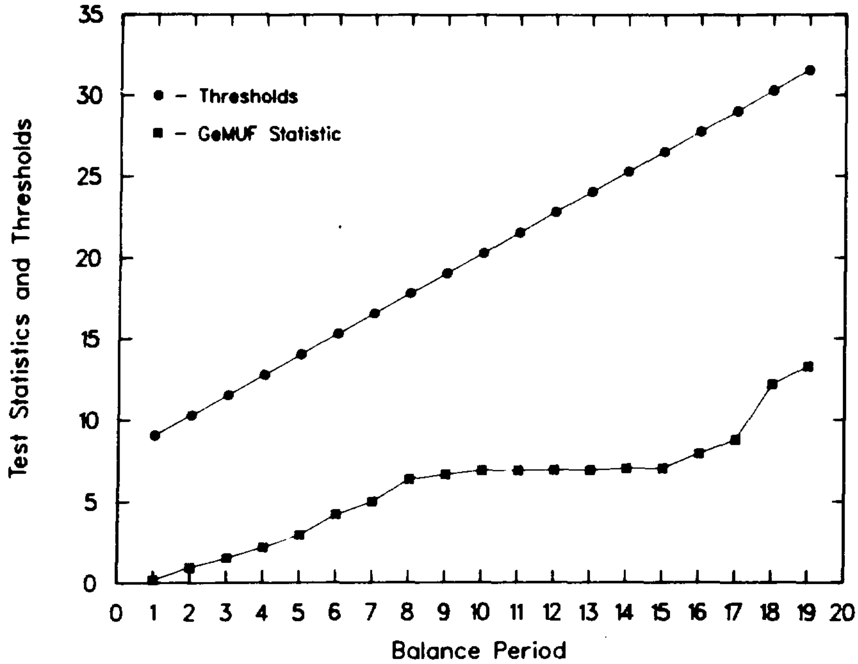


**Fig. 25. PROSA results for GeMUF Test using data of Figs. 21-22; no alarm.**

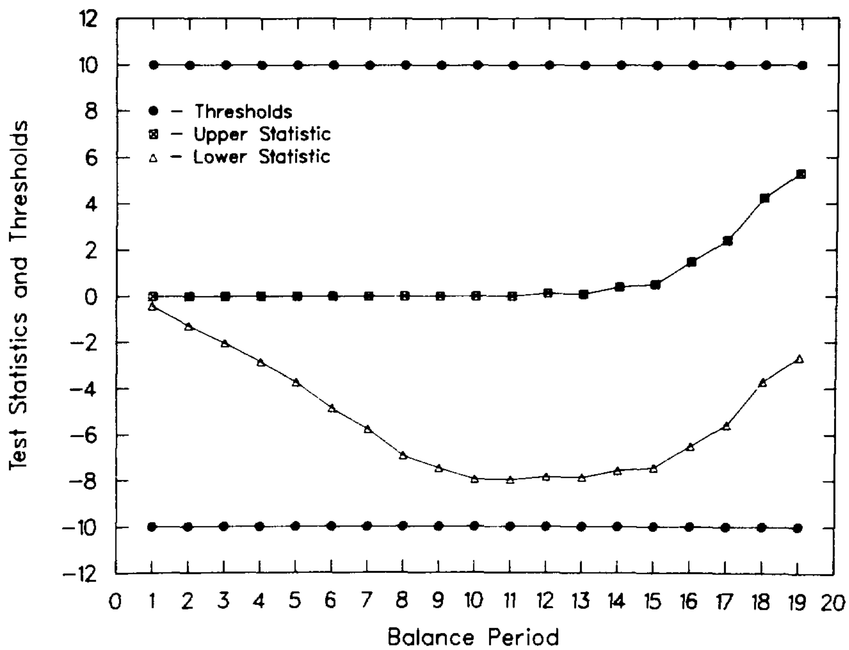


**Fig. 26. PROSA results for Page's Test on modified data of Fig. 25; no alarm.**

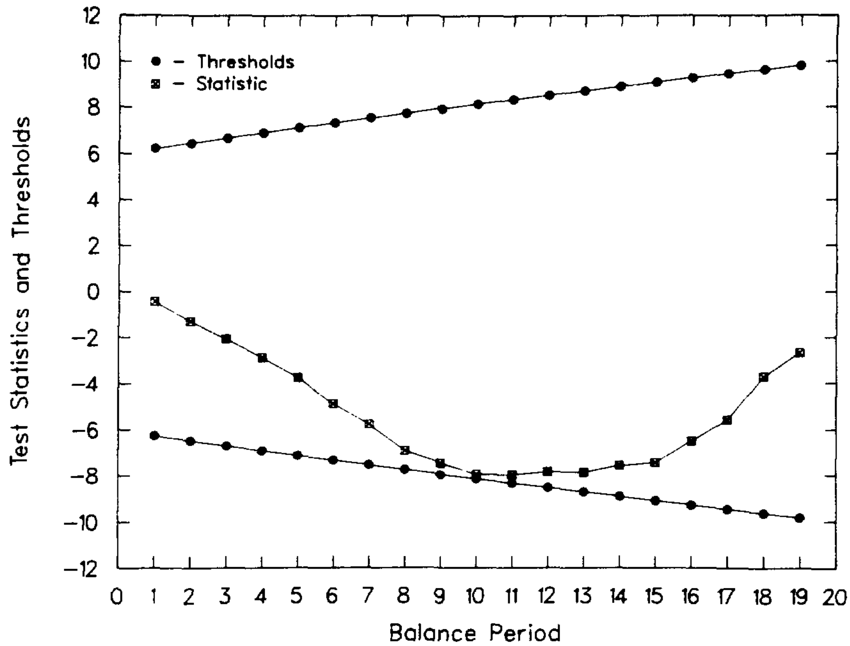


**Fig. 27. PROSA results for Power One Test on modified data of Fig. 25; no alarm.**

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