LBL--27467

August 9, 1989 DE91 014714

How to Search for an Electric Dipole Form Factor of the τ Lepton at a τ–Charm Factory.* †

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Abstract

We investigate some CP–odd correlations which can be used to search for an electric dipole form factor $d_\tau(s)$ of the τ in the reaction $e^+e^- \to \tau^+\tau^-$ at c.m. energies $\sqrt{s}$ relevant for a τ–charm factory. These observables require measurement of the τ polarizations. Using the channel $e^+e^- \to \tau^+\tau^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$ one should be able to measure d_τ at $\sqrt{s} = 4$ GeV with an accuracy $\delta(d_\tau) \simeq 2.5 \times 10^{-16} e\ cm$, assuming the production of 10^7 τ pairs. The same accuracy can be reached with 10^6 τ pairs at $\sqrt{s} = 10$ GeV, the typical c.m. energy of a B–factory.

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*Contribution to the τ–charm workshop, SLAC, Stanford, May 1989.

†This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

‡On leave of absence from Universität Heidelberg, F.R.G. Supported by Heisenberg–Programm of the D.F.G.

I. Introduction

As is well-known CP violation in the lepton sector is expected to be unobservably small in the standard model (SM) of particle physics – even in extensions of the model incorporating massive neutrinos [1]. Therefore, evidence for CP–violating effects involving leptons only would signal a new interaction. Among the searches for such effects are the measurements of electric dipole moments (EDMs) of leptons. (A nonzero EDM of a particle would imply breakdown of time reversal invariance which, if the CPT theorem applies, also means CP violation.) An upper limit exists on the EDM of the muon and more stringent one on the EDM of the electron [2]. (The sensitivity to the electron's EDM is likely to be improved by several orders of magnitude in the near future [3].) As to the τ lepton practically no direct information on its EDM exists so far. Of course, a sizeable EDM d_τ of the τ would lead to a considerable deviation ($\sim d_\tau^2$) of the measured cross section for $e^+e^- \to \tau^+\tau^-$ from its SM value, which has not been observed. In this way Barr and Marciano [1] deduce $|d_\tau| < 10^{-16}\ e\ cm$. But this argument is an indirect one and cancellations of the EDM contribution with other possible new physics terms in the cross section cannot be excluded. Nevertheless this number indicates the order of magnitude in accuracy which direct experiments should try to surpass.

It has been pointed out that the reaction $e^+e^- \to \tau^+\tau^-$ is well suited to study two CP–violating parameters of the τ: namely, its electric dipole form factor (EDF) $d_\tau(s)$ [4,5] and its weak dipole form factor $\tilde{d}_\tau(s)$ [5] at c.m. energy $\sqrt{s}$ which may be present in the $\tau\overline{\tau}$ photon and $\tau\overline{\tau}Z$ boson vertex, respectively. (The EDM of the τ is given by $d_\tau(s=0)$.) The purpose of this note is to investigate the sensitivity to the EDF $d_\tau(s)$ in the above reaction at a "low-energy" τ–charm factory assuming unpolarized e^+e^- beams. Here, low energy means $s << m_Z^2$, where m_Z is the Z mass. Appropriate CP–odd observables, some of which were already discussed in [5], are given and their expectation values are calculated. Here we investigate only the channel $e^+e^- \to \tau^+\tau^- \to \pi^+\overline{\nu}_\tau\pi^-\nu_\tau$ at c.m. energies around $\sqrt{s} = 4$ GeV, an energy relevant for the proposed SLAC τ–charm factory. We also estimate the sensitivity to d_τ using this channel at $\sqrt{s} = 10$ GeV, a typical energy of a B – factory, and recapitulate the possibilities to measure d_τ in the vicinity of the Z resonance [4], respectively $\tilde{d}_\tau$ at $\sqrt{s} = m_Z$ [5]. Finally we emphasize that our observables can also be used to search for CP violation in any reaction $e^+e^- \to$ fermion antifermion and $p\bar{p} \to$ fermion antifermion.

2. CP–violating interactions

The scattering amplitude of the reaction

$$e^+(p_+) + e^-(p_-) \to \tau^+(k_+) + \tau^-(k_-) \tag{1}$$

can be decomposed into 1–particle irreducible vertices (with respect to the particles of the SM, Higgs exchange being neglected) as shown in Fig. 1, all of which can be affected by contributions from CP–violating couplings. As stated in the introduction these must come from new interactions beyond the SM if observable CP–violating effects are present in (1).

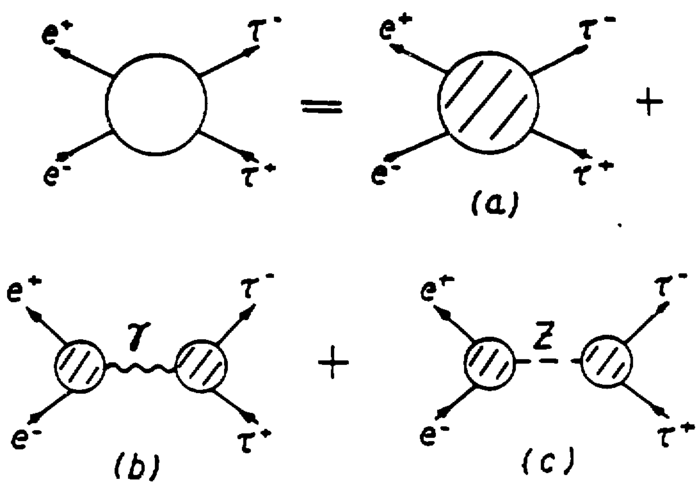

Figure 1. Decomposition of the amplitude for (1) into 1–particle irreducible parts (a), (b), (c).

Let us assume that these new CP–violating interactions are characterized by a scale $\Lambda_{CP} >> \sqrt{s}$. We can then make an effective Lagrangian analysis; i.e., we can classify the CP–violating interactions by local operators of dimension d with couplings proportional to Λ_{CP}^{4-d}. If one assumes unpolarized e^+e^- beams, sets the electron mass $m_e = 0$, and keeps only CP–odd contract terms with $d \leq 6$ it is straightforward to show [5,6] that one is left with the $d = 5$ electric and weak dipole interactions

$$L_{CP} = -\frac{i}{2}\bar{\tau}\sigma^{\mu\nu}\gamma_5\tau[d_\tau F_{\mu\nu} + \tilde{d}_\tau Z_{\mu\nu}], \tag{2}$$

as the only sources of CP violation in the reaction (1). Here $F_{\mu\nu}$ is the electromagnetic field strength tensor and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$. Note that the electric dipole interaction is C–even, P–odd. The weak dipole interaction is CP–odd, but has no definite transformation properties with respect to C or P alone since no definite values of these quantum numbers can be assigned to the Z. In a form factor decomposition of the $\gamma\bar{\tau}\tau$ $(Z\bar{\tau}\tau)$ vertex, $d_\tau(\tilde{d}_\tau)$ corresponds to a C–even, P–odd (CP-odd) electric (weak) dipole form factor at c.m. energy $\sqrt{s}$. The form factors $d_\tau(s), \tilde{d}_\tau(s)$ may have absorptive parts, which we neglect, however, in this paper.

In many extensions of the SM, such as left–right symmetric models, supersymmetric models, or multi–Higgs models, CP–violation in the lepton sector occurs quite naturally [1] and non–zero form factors $d_\tau(s), \tilde{d}_\tau(s)$ are generated. A priori d_τ and $\tilde{d}_\tau$ are not related but in such models one expects them to be of the same order of magnitude. Probably the most sizeable potential source of CP–violation involving τ leptons exists in Higgs models of CP–violation. In particular, in Weinberg–type models [7] there are neutral spin 0 bosons ϕ, some of which may solely couple to leptons ℓ via

$$L_I = (\sqrt{2}G_F)^{\frac{1}{2}} m_\ell(\xi_s \bar{\ell}\ell + i\xi_p \bar{\ell}\gamma_5\ell)\phi. \tag{3}$$

Here ξ_s, ξ_p are dimensionless coupling constants and G_F is the Fermi constant. If ξ_s and $\xi_p \neq 0$ the interaction (3) generates $d_\ell, \tilde{d}_\ell \sim m_\ell^3$ [8,5] unless m_ℓ^2 is much larger than m_ϕ^2. That is, in such models one expects

$$d_\tau \approx \tilde{d}_\tau \approx \left(\frac{m_\tau}{m_\mu}\right)^3 d_\mu \approx \left(\frac{m_\tau}{m_e}\right)^3 d_e. \tag{4}$$

Using the present experimental limits [2] on $d_e(|d_e| < 3 \cdot 10^{-24} e\ cm)$ and on d_μ $(|d_\mu| \lesssim 10^{-18} e\ cm)$ eq. (4) implies that sizeable values of $d_\tau, \tilde{d}_\tau$, say of the order of $10^{-17} e\ cm$, are conceivable.

In the kinematic regime $s << m_Z^2$, in which we are primarily interested here, the Z boson exchange contributions to (1) can be neglected. CP–violating effects will be generated by the interference of the Born amplitudes of Fig. 2. On the other hand, at or in the close vicinity of the Z resonance γ–exchange amplitudes are of order α compared to Z exchange, and CP–odd observables are primarily sensitive to the weak dipole form factor $\tilde{d}_\tau(m_Z)$.

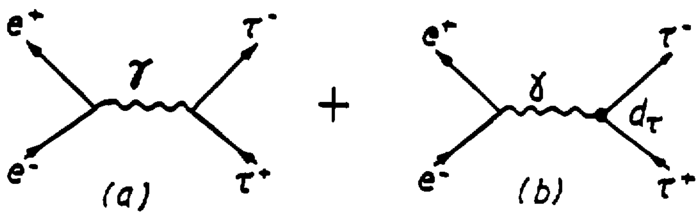

Figure 2. CP–even (a) and CP–odd (b) Born amplitudes

3. CP–odd observables

Let us now investigate CP–odd final state observables for the reaction (1). We shall work in the c.m. frame. As already emphasized the e^+ and e^- beams are assumed to be unpolarized. The initial state is then described by a CP–invariant density matrix. In this situation any nonzero expectation value of a CP–odd observable constructed from final state variables is evidence for CP–violation – with no caveats whatsoever. Final state variables at our disposal are the c.m. unit momentum vector $\hat{\mathbf{k}}_+ = \mathbf{k}_+/|\mathbf{k}_+|$ of the τ^+ and the τ^+ and τ^- spin operators $\boldsymbol{\sigma}_+ \equiv \boldsymbol{\sigma} \otimes 1$ and $\boldsymbol{\sigma}_- \equiv 1 \otimes \boldsymbol{\sigma}$ acting in the product space of the τ^+ and τ^- spin spaces. Under C and P transformations we have

$$\begin{aligned} \mathrm{C} &: \hat{\mathbf{k}}_\pm \to \hat{\mathbf{k}}_\mp = -\hat{\mathbf{k}}_\pm, \;\; \boldsymbol{\sigma}_\pm \to \boldsymbol{\sigma}_\mp \\ \mathrm{P} &: \hat{\mathbf{k}}_\pm \to -\hat{\mathbf{k}}_\pm, \;\; \boldsymbol{\sigma}_\pm \to \boldsymbol{\sigma}_\pm. \end{aligned} \tag{5}$$

where we use $\mathbf{k}_+ + \mathbf{k}_- = 0$ in the c.m. system. This allows the classification of the spin–momentum correlation observables of the final state into CP–even and odd ones. As to CP–odd observables: Since we consider $s << m_Z^2$ we are primarily interested in the effects generated by the interference of the SM Born amplitude Fig. 2a, which is C– and P–even, with the C–even P–odd Born amplitude Fig. 2b. Such effects can be detected with C–even, P–odd CPT–even correlations. A general analysis [5] shows that, restricting ourselves to rank 0 and 2 tensors* there are 3 linearly independent observables having this property:

$$A = \hat{\mathbf{k}}_+ \cdot (\boldsymbol{\sigma}_+ \times \boldsymbol{\sigma}_-), \tag{6}$$

$$B_{ij} = \hat{k}_{+i}(\boldsymbol{\sigma}_+ \times \boldsymbol{\sigma}_-)_j + (i \leftrightarrow j) - \frac{2}{3}\delta_{ij}A, \tag{7}$$

$$C_{ij} = A(\hat{k}_{+i}\hat{k}_{+j} - \frac{1}{3}\delta_{ij}), \tag{8}$$

where $1 \leq i,j \leq 3$ are Cartesian vector indices. Taking into account the diagrams of Fig. 2 we find for the expectation values of (6), (7), and (8):

$$\langle A \rangle = 4(d_\tau/e)m_\tau w, \tag{9}$$

$$\langle B_{ij} \rangle = \frac{-12}{5}(d_\tau/e) \cdot \sqrt{s}\, w \cdot (1 + 4m_\tau/(3\sqrt{s}))s_{ij}, \tag{10}$$

$$\langle C_{ij} \rangle = -\frac{8}{5}(d_\tau/e)m_\tau w s_{ij}, \tag{11}$$

where $e > 0$ is the positron charge and

$$w = (1 - 4m_\tau^2/s)^{\frac{1}{2}}/(1 + 2m_\tau^2/s), \tag{12}$$

$$s_{ij} = \frac{1}{2}(\hat{p}_{+i}\hat{p}_{+j} - \frac{1}{3}\delta_{ij}) = diag(-\frac{1}{6}, -\frac{1}{6}, \frac{1}{3}). \tag{13}$$

The second equation in (13) holds if the positron unit momentum vector $\hat{\mathbf{p}}_+$ is chosen along the z–axis.

Measurement of the observables (6), (7), (8) requires the measurement of the τ^+ and τ^- polarizations. Polarization information can be obtained from τ decays, assuming standard V–A interaction; most easily through $\tau \to \pi\nu_\tau$. Moreover, for (6)-(8) we need to know the momentum directions of the τ's, event by event. This will not be easy. (Due to radiation $\hat{\mathbf{k}}_- = -\hat{\mathbf{k}}_+$ will not always hold. For CP studies one should replace in (6), (7), (8) $\hat{\mathbf{k}}_+ \to (\hat{\mathbf{k}}_+ - \hat{\mathbf{k}}_-)/2$.)

*Rank 1 tensors, i.e. vectors are not useful here since for unpolarized e^+e^- -beams the diagram Fig. 2a generates no vector polarization for the virtual photon.

CP–odd observables which are easier to measure are composed of momenta of the final-state decay products only. Let us discuss for the channel

$$e^+e^- \to \tau^+\tau^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau \tag{14}$$

the following two CP–odd tensor observables:

$$T_{ij} = (q_{+i} - q_{-i})(\mathbf{q}_+ \times \mathbf{q}_-)_j + (i \leftrightarrow j), \tag{15}$$

$$\hat{T}_{ij} = (\hat{q}_{+i} - \hat{q}_{-i})\frac{(\hat{\mathbf{q}}_+ \times \hat{\mathbf{q}}_-)_j}{|\hat{\mathbf{q}}_+ \times \hat{\mathbf{q}}_-|} + (i \leftrightarrow j), \tag{16}$$

where $\mathbf{q}_\pm, \hat{\mathbf{q}}_\pm = \mathbf{q}_\pm/|\mathbf{q}_\pm|$ are the momenta, respectively unit momenta of $\pi^\pm$ in the overall c.m. frame. Considering again the diagrams Fig. 2a,b and using the standard V-A decay distribution for $\tau \to \pi\nu_\tau$ we obtain for the expectation values of (15), (16):

$$\langle T_{ij}\rangle = c\sqrt{s}(d_\tau/e)s_{ij}, \tag{17}$$

$$\langle \hat{T}_{ij}\rangle = \hat{c}\sqrt{s}(d_\tau/e)s_{ij}, \tag{18}$$

where the values for c and $\hat{c}$ are given in Table 1 for various c.m. energies. Moreover, the expectation values of T_{33}^2 and $\hat{T}_{33}^2$, calculated with the SM Born amplitude of the reaction (14), are given. These values are needed to estimate the accuracy with which d_τ can be measured by means of (17), (18). From these

$\sqrt{s}$ [GeV]	c [GeV^3]	$\hat{c}$	$(< T_{33}^2 >)^{\frac{1}{2}}$ [GeV^3]	$(< \hat{T}_{33}^2 >)^{\frac{1}{2}}$
3.8	.08	.08	.65	.86
4.0	.16	.13	.82	.95
4.2	.24	.18	.99	.92
4.6	.40	.25	1.38	1.02
10.0	3.95	.44	9.74	1.42

Table 1:Values for $c, \hat{c}$ defined in (17), (18) and expectation values of T_{33}^2 and $\hat{T}_{33}^2$ (see (15), (16) for the reaction (14) at various c.m. energies.

numbers we obtain the following estimates: Assuming the production of $10^7\tau^+\tau^-$ pairs at $\sqrt{s} = 4$ GeV it should be possible, using (16), to measure $d_\tau(s)$ with an accuracy $\delta(d_\tau) = 2.5 \times 10^{-16} e\ cm$ (1 s.d.). At $\sqrt{s} = 10$ GeV the accuracy obtainable with 10^6 τ pairs is $\delta(d_\tau) = 1.5 \times 10^{-16} e\ cm$.

The observables (15), (16) may be used for channels which have higher event rates than (14). For instance one may sum up the leptonic modes $\tau^+\tau^- \to \ell^+\nu_\ell\bar{\nu}_\tau\ell^-\bar{\nu}_\ell\nu_\tau$ or one may use all decays of $\tau^\pm$ into 1–charged prong identifying $\mathbf{q}_+(\mathbf{q}_-)$ in (15), (16) with the positively (negatively) charged particle momentum. In this way the accuracy of measuring d_τ can be increased substantially. The relevant theoretical formulae will be given in [5]. Necessary requirements for all CP tests are that the kinematic cuts are parity symmetric and that there is no "C bias" of the detector. That is, equal detection efficiencies for particles and antiparticles are necessary. The kinematic cuts may violate rotational invariance. If so the expectation values of the tensors (7), (8), (15), and (16) are no longer proportional to s_{ij} but are still perfect indicators of CP violation.

The correlations (15) and (16) were also investigated for the reaction (14) at the Z resonance [5]. For c.m. energies at or in the vicinity of m_Z Z–exchange diagrams dominate and (15), (16) are primarily sensitive to the weak dipole form factor $\tilde{d}_\tau(m_Z)$. By measuring (16) in the reaction (14) an accuracy $\delta(\tilde{d}_\tau) = 6 \times 10^{-18} e\ cm$ (1s.d.) can be reached [5] assuming the production of $10^7 Z$'s.

In [4] it was suggested to search for the electric dipole form factor d_τ by measuring the CP–odd, CPT–even correlation $\mathbf{p}_+ \cdot (\mathbf{q}_+ \times \mathbf{q}_-)$, again in the channel (14) in the vicinity of $s = m_Z^2$. Reference [4] did not consider the contribution of the weak dipole term to this correlation. A non–zero expectation value is then generated by interference of the standard Z–exchange Born amplitude with the CP–odd electric dipole amplitude Fig. 2b. In this approximation $\langle \mathbf{p}_+ \cdot (\mathbf{q}_+ \times \mathbf{q}_-) \rangle$ vanishes then at $s = m_Z^2$ (assuming d_τ to be real.) Reference [4] assumes that an integrated luminosity of 170 pb^{-1} can be collected around the Z peak (which, if spent at $s = m_Z^2$, would lead to the production of about $10^7 Z$'s) and finds the attainable accuracy $\delta(d_\tau) = 3 \times 10^{-16} e\ cm$ (1 s.d.).

4. Concluding remarks

Our study shows that CP test can be made with the reaction $e^+e^- \to \tau^+\tau^-$ at a low–energy high–luminosity τ–charm factory. The CP–odd correlations which we discussed are, if $s << m_Z^2$, sensitive to the electric dipole form factor d_τ of the τ lepton. We estimated an attainable accuracy $\delta(d_\tau) \simeq 2.5 \times 10^{-16} e\ cm$ from $\tau \to \pi\nu$ decays alone. Using also other decay channels we expect substantial improvement. Finally we note that the correlations (6), (7), and (8) can be used to look for CP violation in any other reaction $e^+e^- \to$ fermion + antifermion. (One may also use suitable analogues of (15), (16) for final state decay products.) Interesting cases might be $e^+e^- \to \Lambda\overline{\Lambda}, \Lambda_c\overline{\Lambda}_c, \ldots$ which can be investigated at a τ–charm factory or $e^+e^- \to \Lambda_b\overline{\Lambda}_b$ to be studied at a B–factory.

Acknowledgments

We would like to thank I. Bigi , S.J. Brodsky, J. Carr, D. Coward, J. Donoghue, J. Eastman, M. Perl, A. Pich, and J. Repond for useful discussions. We would like to thank the LBL and SLAC theory groups for the hospitality extended to us.

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