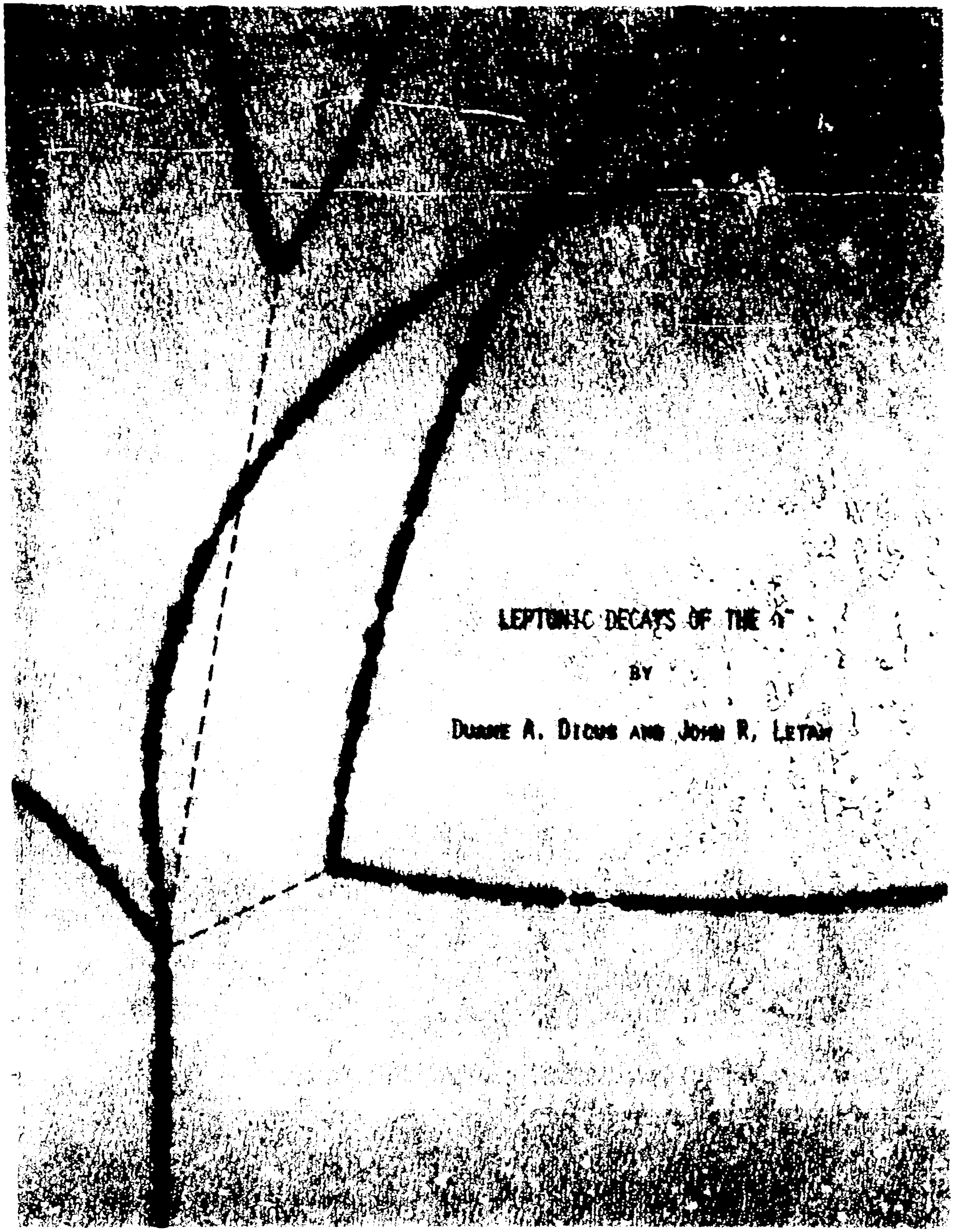


LEPTONIC DECAYS OF THE Ω^-

BY

DUANE A. DICUS AND JOHN R. LETAW

Leptonic Decays of the Ω^-

Duane A. Dicus* and John R. Let[illegible]w

Center for Particle Theory
University of Texas
Austin, Texas 78712

*Work supported in part by the Department of Energy.

Abstract

Branching ratios, widths, and lepton energy spectra are derived for the leptonic decays of the Ω^- into Ξ^o and Ξ^{o*}. The calculations are made by hypothesizing that higher order multipoles in an expansion of the baryon current are zero. The axial vector coupling is determined using the bag model.

I. Introduction

The leptonic decays,

$$\Omega \to \Xi\, e\bar{\nu}$$
$$\to \Xi\, \mu\bar{\nu}$$
$$\to \Xi^* e\bar{\nu}$$
$$\to \Xi^* \mu\bar{\nu} \quad ,$$

are rare but observable. The branching ratio for these modes is expected to be of the order of 10^{-2}. The CERN SPS charged hadron beam has been in operation for two years during which several thousand Ω^- decays have been detected. Thus, even though the branching ratio is small, the time seems right to attempt a careful calculation of the semi-leptonic modes listed above. Several attempts at calculating the $\Xi\ell\nu$ modes were made shortly after the Ω^- was discovered.[2,3,4,5] The results are incomplete because of an ignorance of the axial vector current form factors. Only a rough estimate of the widths for the $\Xi^*\ell\nu$ modes has been made.[2]

To attempt to improve on these previous calculations we will work in a specific model designed for the decay of higher spin particles. The model is based on the idea that the weak interaction actually takes place between s-wave, spin 1/2 quarks. Thus if the hadronic matrix element of the weak current is expanded in multipole moments, those moments with $J > 1$ will be zero. Together with CVC or PCAC this will serve to determine most of the matrix element. This strong assumption was also used by Mathews[5] for the vector part of the $\Xi\ell\nu$ decays. Although we will offer a specific criticism of this assumption in Sec. III we propose to extend it to the other semi-leptonic modes. In addition we now have models which allow accurate calculations of static quantities such as the axial vector coupling constant. We will use the M.I.T. bag model for this purpose.[6] Combining these things will allow us to make definite predictions for the vector width of the $\Xi\ell\nu$ mode, the axial vector width of the $\Xi\ell\bar{\nu}$ mode, the electron energy spectrum for $\Xi e\bar{\nu}$, the muon energy spectrum for $\Xi\mu\bar{\nu}$, the axial vector width for $\Xi^*\ell\bar{\nu}$, and an upper bound on the vector width for $\Xi^*\ell\bar{\nu}$.

II. Kinematics and Widths

The weak decays are calculated with the usual V-A current-current interaction

$$H_I = \frac{G}{\sqrt{2}} \; g_{\alpha\beta} \; J^{\alpha}_{Baryon} \, J^{\beta}_{Lepton} \tag{1}$$

The lepton current is

$$J^{\beta}_{Lepton} = \bar{u}(s)\, \gamma^{\beta}\, (1 - \gamma^5)\, v(t) \tag{2}$$

The baryon currents are written most generally as

$$J^{\alpha}_{Baryon}\,(\Omega\to\Xi\ell\bar{\nu}) = \bar{u}(p)\, \{(f_1+g_1\gamma_5)g^{\alpha\nu} + (f_2+g_2\gamma_5)\gamma^{\alpha}q^{\nu} + (f_3+g_3\gamma_5)q^{\alpha}q^{\nu} + (f_4+g_4\gamma_5)P^{\alpha}q^{\nu}\}u_{\nu}(P) \tag{3}$$

$$J^{\alpha}_{Baryon}\,(\Omega\to\Xi^*\ell\bar{\nu}) = \bar{u}_{\mu}(p)\, \{(G_1+F_1\gamma_5)g^{\mu\nu}\gamma^{\alpha} + (G_2+F_2\gamma_5)g^{\mu\nu}q^{\alpha} + (G_3+F_3\gamma_5)g^{\mu\nu}P^{\alpha} + (G_4+F_4\gamma_5)g^{\mu\alpha}q^{\nu} + (G_5+F_5\gamma_5)g^{\nu\alpha}q^{\mu} + (G_6+F_6\gamma_5)q^{\mu}q^{\nu}q^{\alpha} + (G_7+F_7\gamma_5)q^{\mu}q^{\nu}P^{\alpha}\}u_{\nu}(P) \tag{4}$$

We use the notation of Mathews.[5] The lepton, neutrino, Ξ (or Ξ^*), and Ω four momenta are labeled s, t, p, and P respectively. The four momentum transfer at the hadron vertex is $q = P - p$. The Rarita-Schwinger[7] spinor, u_{μ} is used for spin 3/2 particles. The vector (g_i and G_i) and axial vector (f_i and F_i) form factors are in general arbitrary complex functions of q^2.

A naive attempt at writing the most general current for the spin 3/2 to spin 3/2 transition yields an additional term

$$(G_8+F_8\gamma_5)\bar{u}_\mu q^\mu q^\nu \gamma^\alpha u_\nu \quad .$$

This term is not independent; it is related to the others by the non-trivial identities

$$\bar{u}_\mu(p)\{\tfrac{1}{2}[(M+m)^2-q^2]g^{\mu\nu}\gamma^\alpha + M\, g^{\mu\nu}q^\alpha \qquad (5)$$

$$- (M+m)g^{\mu\nu}P^\alpha - M\, g^{\mu\alpha}q^\nu + m\, g^{\nu\alpha}q^\mu + q^\mu q^\nu \gamma^\alpha\}\; u_\nu(P) = 0$$

$$\bar{u}_\mu(p)\gamma_5\{\tfrac{1}{2}[(M-m)^2-q^2]g^{\mu\nu}\gamma^\alpha + M\, g^{\mu\nu}q^\alpha \qquad (6)$$

$$- (M-m)g^{\mu\nu}P^\alpha - M\, g^{\mu\alpha}q^\nu - m\, g^{\nu\alpha}q^\mu + q^\mu q^\nu \gamma^\alpha\}\; u_\nu(P) = 0$$

where $M = \Omega^-$ mass, $m = \Xi^{o*}$ mass. These identities have been obtained by others[8] in a less general form applicable only to electromagnetic transitions.

The unknowns in the baryon current are the form factors. They are real because of time reversal invariance. Since the q^2 dependence is unknown, the widths are calculated for two extremes which put bounds on the effects of non-zero momentum transfer. The lower limit is no q^2 dependence,

$$F(q^2) = 1$$

The upper limit is gotten by analogy with beta decay; each form factor is given a q^2 dependence

$$F(q^2) = \frac{1}{(1-q^2/m_K^2)^2}$$

where m_κ is approximately the mass of a strange vector meson which we take to be 1 GeV. The form factors at $q^2 = 0$ remain as the only unknowns, their values are discussed in the remaining sections.

The widths for these processes are

$$\Gamma = \int (2\pi)^4 \delta(P-p-s-t) \frac{1}{2M} \frac{d^3p}{(2\pi)^3 2E} \frac{d^3s}{(2\pi)^3 2s_0} \frac{d^3t}{(2\pi)^3 2t_0} \frac{1}{4} \sum_{spins} |H|^2 . \quad (7)$$

Given the interaction (1), the currents (2-4), and the assumed q^2 dependence of the form factors; the width can be displayed as a function of the form factors at $q^2 = 0$.

$$\Gamma(\Omega^- \to \Xi^\circ \ell \bar{\nu}) = \sum_{i,j=1}^{4} a_{ij} f_i f_j + v_{ij} g_i g_j \quad (8)$$

$$\Gamma(\Omega^- \to \Xi^{\circ *} \ell \bar{\nu}) = \sum_{i,j=1}^{7} A_{ij} F_i F_j + V_{ij} G_i G_j \quad (9)$$

The integrals a_{ij}, v_{ij}, A_{ij}, V_{ij} are given in tables in Appendix A. These integrals are, of course, independent of our model; their accuracy is limited only by uncertainty in the baryon masses. Errors reflecting this are given in the tables. All masses and other experimental quantities have been taken from the 1978 Review of Particle Properties.

We also calculate the energy spectra of the charged lepton, $\frac{d\Gamma}{ds^0}$, for the Ξ decays.

III. The Multipole Hypothesis

The baryon current operator may be expanded in a series of irreducible tensor operators[9] transforming as representations of the rotation group. For example, the time component is

$$\hat{J}^\circ = \sum_{J=0}^{\infty} \hat{T}_{J,0} \quad . \quad (10)$$

When evaluated between spin eigenstates using the Wigner-Eckart theorem the usual multipole expansion is generated. This definition may be generalized to non-static transitions by evaluating the current operator between helicity eigenstates.

$$\begin{aligned} \langle s',\lambda'|\hat{J}^{o}|s,\lambda\rangle &= \sum_{J=0}^{\infty} \langle s',\lambda'|\hat{T}_{J,0}|s,\lambda\rangle \\ &= (-1)^{s'-\lambda'} \sum_{J=0}^{\infty} \begin{pmatrix} s' & J & s \\ -\lambda' & 0 & \lambda \end{pmatrix} \langle s' \| \hat{T}_{J,0} \| s\rangle \qquad (11) \\ &= (-1)^{s'-\lambda'} \sum_{J=0}^{\infty} \begin{pmatrix} s' & J & s \\ -\lambda' & 0 & \lambda \end{pmatrix} Q_J \end{aligned}$$

In the limit of a static transition only Q_0 and Q_1 are allowed. When the baryons are in relative motion, however, Q_2 and Q_3 can be non-zero.

The multipoles for the other components of the current are similarly defined[10]

$$\langle s',\lambda'|\hat{J}^{3}|s,\lambda\rangle = (-1)^{s'-\lambda'} \sum_{J=0}^{\infty} \begin{pmatrix} s' & J & s \\ -\lambda' & 0 & \lambda \end{pmatrix} L_J \qquad (12)$$

$$\langle s',\lambda'|\hat{J}^{+}|s,\lambda\rangle = (-1)^{s'-\lambda'} \sum_{J=0}^{\infty} \begin{pmatrix} s' & J & s \\ -\lambda' & 1 & \lambda \end{pmatrix} (E_J + M_{J+1}) \qquad (13)$$

$$\langle s',\lambda'|\hat{J}^{-}|s,\lambda\rangle = (-1)^{s'-\lambda'} \sum_{J=0}^{\infty} \begin{pmatrix} s' & J & s \\ -\lambda' & -1 & \lambda \end{pmatrix} (E_J - M_{J+1}) \qquad (14)$$

where $\hat{J}^{\pm} = \pm(\hat{J}^1 \pm i\hat{J}^2)$. Q_J, L_J, E_J, M_J are the Coulomb, Longitudinal, Transverse Electric, and Transverse Magnetic multipoles respectively. The expansions have polar four vector parity when J is even and axial four vector parity when J is odd.

The multipole E_0 is always zero. Multipoles not satisfying

$$|s' - s| \leq J \leq s' + s \qquad (15)$$

are also zero. Before any assumptions are made the non-zero multipoles for the $\Xi \ell \bar{\nu}$ decay are

$$Q_1, Q_2, L_1, L_2, E_1, E_2, M_1, M_2$$

while for the $\Xi^* \ell \bar{\nu}$ decay the non-zero multipoles are

$$Q_0, Q_1, Q_2, Q_3, L_1, L_2, L_3,$$

$$E_1, E_2, E_3, M_1, M_2, M_3$$

In the static limit, multipoles with $J \geq 2$ are zero because the intermediate boson carrying the weak interaction has only vector and scalar parts. The higher order multipoles arise because spin is not a Lorentz invariant.

Now we invoke our picture of baryons as three independent spin 1/2 quarks. The Ω^- decay is then a transition from a strange quark in a s-wave state to an up quark in an s-wave state. The multipoles with $J>1$ must then be zero because (15) is not satisfied. The limitation of this multipole hypothesis can be seen in a simple illustration: Mathews[5] argues that because the E_2 contribution to the width of $\Delta \to p \gamma$ is small the vector form factors g_2 and g_4 are related. Indeed if, as we also would assume, $\Gamma_{E2}/\Gamma_{M1} = 0$ then $g_2 = -Mg_4$.[5] However, if $\Gamma_{E2}/\Gamma_{M1} = .015$, which is certainly experimentally allowed, then Mg_4/g_2 equals 2 or 0. Thus justification for our multipole hypothesis cannot be looked for in experiment. The assumption is that the values of the form factors derived by assuming the higher multipoles are precisely zero gives accurate values for the rates even in the real world where there may be a small contribution from the higher multipoles. (Changing Mg_4/g_2 by a factor of two introduced only a 1.5% contribution from the higher multipole in the illustration above.)

We now examine the conditions that result from this assumption on the multipoles. Expressions for the current operators between various helicity eigenstates are found using explicit representations of the Dirac and Rarita-Schwinger spinors to evaluate the different components of the currents (equations (3) and (4)). We choose a coordinate system where the Ω is at rest and the Ξ moves along the z axis. In this frame, the Rarita-Schwinger spinors are expressed as combinations of helicity 1 and 1/2 eigenstates.[11]

The resulting expressions for the higher order multipoles are

$$Q_2 \sim g_2 + g_3 (M-E) + g_4 M \tag{16}$$

$$L_2 \sim g_1 + g_2 (E+m) - g_3 (E^2-m^2) \tag{17}$$

$$E_2 \sim g_1 + g_2 (E+m) \tag{18}$$

$$M_2 \sim f_2 \tag{19}$$

for the $\Xi \ell \bar{\nu}$ modes. E is the energy of the Ξ. The $\Xi^* \ell \bar{\nu}$ multipoles are

$$Q_2 \sim G_1 + G_2 (M-E) + G_3 M - (E+m) \{G_4 + G_6 M (M-E) + G_7 M^2\} \tag{20}$$

$$L_2 \sim (E-m) \{G_1 - G_2 (E+m)\} - (E+m) \{G_4 E + G_5 M - G_6 M (E^2-m^2)\} \tag{21}$$

$$E_2 \sim mG_4 + MG_5 \tag{22}$$

$$M_3 \sim (E-m) G_1 - (E+m) \{mG_4 - MG_5\} \tag{23}$$

$$Q_3 \sim F_1 + F_2 (M-E) + F_3 M - (E+m) \{F_4 + F_6 M (M-E) + F_7 M^2\} \tag{24}$$

$$L_3 \sim F_1 - F_2 (E-m) - F_4 E - F_5 M + F_6 M (E^2-m^2) \quad (25)$$

$$E_3 \sim 2F_1 - mF_4 - MF_5 \quad (26)$$

$$M_2 \sim mF_4 - MF_5 \quad (27)$$

Because there are 8 form factors to fix in the $\Xi\ell\bar{\nu}$ decay and 14 to fix in the $\Xi^*\ell\bar{\nu}$ decay, the multipole hypothesis is not sufficient to determine the widths. Further relations are developed in the next section.

IV. More Relations on the Form Factors

The Ademollo-Gatto theorem indicates that the vector current is almost conserved, even for strangeness changing decays. Thus we use

$$q_\mu \, J^\mu_{\text{Vector}} = 0 \quad . \quad (28)$$

This requirement on the $\Xi\ell\bar{\nu}$ current yields a relation[5] at $q^2 = 0$

$$g_1 + (M+m) g_2 + \frac{1}{2} (M^2-m^2)\, g_4 = 0 \quad (29)$$

Requiring that the $\Xi^*\ell\bar{\nu}$ vector current be conserved gives

$$G_1 + \frac{1}{2} (M+m)\, G_3 = 0 \quad (30)$$

$$G_4 + G_5 + \frac{1}{2} (M^2-m^2)\, G_7 = 0 \quad (31)$$

The CVC statements are not independent of the multipole hypothesis. Equation (29) relates Q_2 and L_2 and contains no information because these are zero. Equations (30) and (31) relate Q_0 to L_0 and Q_2 to L_2, therefore only one of these is independent of the requirement that Q_2 and L_2 equal zero.

The assumption of a partially conserved axial vector current gives the relation[5]

$$f_1 + f_2\,(M-m) + \frac{1}{2}\,(M^2-m^2)\; f_4 = g_{\Omega K\Xi}\;\gamma_K \tag{32}$$

for the $\Xi\ell\bar{\nu}$ decays. γ_K is calculated[5] from the observed width of the decay $K \to \mu\bar{\nu}$

$$\gamma_K \simeq .0354 \quad . \tag{33}$$

$g_{K\Omega\Xi}$ is the strong coupling at the $\Omega\Xi K$ vertex. This is taken to be equal to the coupling at the $\Delta p\pi^+$ vertex found from the Δ^{++} lifetime,

$$g_{K\Omega\Xi} \simeq g_{\Delta p\pi^+} \simeq 15.5 \tag{34}$$

An identical argument for the $\Xi^*\ell\bar{\nu}$ current using

$$J_K^\mu = \frac{G}{\sqrt{2}}\; g_{K\Omega\Xi^*}\; \bar{u}_\alpha\gamma_5\, g^{\alpha\beta}u_\beta\, \frac{1}{q^2-m_K^2}\,\gamma_K q^\mu \tag{35}$$

for the meson pole contributions to the axial vector current yields at $q^2 = 0$

$$(M+m)\; F_1 + \frac{1}{2}\,(M^2-m^2)\; F_3 = g_{K\Omega\Xi^*}\;\gamma_K \tag{36}$$

$$F_4 + F_5 + \frac{1}{2}\,(M^2-m^2)\; F_7 = 0 \tag{37}$$

The $K\Omega\Xi^*$ coupling can in principle be found from the $\Delta\pi\Delta$ coupling. This coupling is not well determined. Dicus, et al.[12] find

$$g_{\Delta\pi\Delta} = 22. \pm 6.$$

from an analysis of $\pi^- p \to \pi^+\pi^- n$ scattering data while a prediction based on SU(6) symmetry gives

$$g_{\Delta\pi\Delta} = 40.$$

Using the mean between these values we find

$$g_{K\Omega\Xi^*} \sim g_{\Delta^{++}\pi^+\Delta^+} = \sqrt{\frac{2}{5}}\, g_{\Delta\pi\Delta} = 20. \tag{38}$$

One of the axial vector form factors for each decay mode can be found using the MIT bag model. SU(6) Clebsch-Gordon coefficients[13] relate the baryon currents to currents for weak transitions between quarks.

$$<1/2,\ 1/2|\hat{J}_{Axial}|3/2,\ 1/2> = \sqrt{\frac{2}{3}}\, \frac{G}{\sqrt{2}} \sin\theta_c (<u\uparrow|\hat{J}^3_{Axial}|s\uparrow> - <u\downarrow|\hat{J}^3_{Axial}|s\downarrow>) \tag{39}$$

$$<3/2,\ 3/2|\hat{J}^3_{Axial}|3/2,\ 3/2> = \sqrt{3}\, \frac{G}{\sqrt{2}} \sin\theta_c <u\uparrow|\hat{J}^3_{Axial}|s\uparrow> \tag{40}$$

$\theta_c = .26$ is the Cabbibo angle.

The axial vector current operator between the quarks is

$$\hat{J}^3_{Axial} = \left(\int_{bag} \bar{\psi}(-\gamma^\mu \gamma_5)\psi \; dV\right)_{\mu = 3}$$

$$= - \int_{bag} \psi^\dagger \sigma_3 \psi = - g_A \sigma_3$$

where ψ is the wave function for a Dirac particle confined to a spherical bag. This integral has been evaluated by DeGrand, et al.[6]

$$g_A = .707 \tag{41}$$

The LHS of equations (39) and (40), calculated using explicit representations of the spinors normalized to one, are

$$\sqrt{\frac{2}{3}}\sqrt{\frac{E+m}{2m}}\,[f_1 + (E-m)\,f_2 - (E^2-m^2)\,f_3] = -2\sqrt{\frac{2}{3}}\;g_A \sin\theta_c \tag{42}$$

$$\sqrt{\frac{E+m}{2m}}\,[F_1 - F_2\,(E-m)] = -\sqrt{3}\;g_A\,\sin\theta_c \tag{43}$$

The bag model results are strictly true only for static transitions; thus relations (42) and (43) are evaluated at E=m

$$f_1 = -2g_A\,\sin\theta_c \tag{44}$$

$$F_1 = -\sqrt{3}\,g_A\,\sin\theta_c \tag{45}$$

The bag model can also be used to relate several of the vector form factors in the $\Xi^*\ell\bar{\nu}$ decay.

$$\langle 3/2,\,3/2|\hat{J}^0_{Vector}|3/2,\,3/2\rangle = \frac{G}{\sqrt{2}}\sin\theta_c\,\sqrt{3}\,\langle u\uparrow|\hat{J}^0_{Vector}|s\uparrow\rangle \tag{46}$$

the current operator between the quarks is

$$\hat{J}^0_{Vector} = \int_{bag} \bar{\psi}\,\gamma^\circ\,\psi\,dV$$

$$= \int_{bag} \psi^+\,\psi\,dV = g_v$$

We have evaluated g_v and found, in the notation of DeGrand, et al.[6]

$$g_v = \frac{2x_i x_j}{x_i^2-x_j^2}\;\frac{\lambda_i - \lambda_j}{\sqrt{2\alpha_i(\alpha_i-1)+\lambda_i}\;\sqrt{2\alpha_j(\alpha_j-1)+\lambda_j}} \tag{47}$$

For the $\Xi^*\ell\bar{\nu}$ decays

$$g_v = .972 \tag{48}$$

Explicit calculation of the LHS of equation (46) gives for static transitions

$$-\left(G_1 + G_2\,(M-m) + G_3 M\right) = \sqrt{3}\,\sin\theta_c\, g_v \tag{49}$$

A final relation on the form factors for the $\Xi\ell\bar{\nu}$ mode is found by relating the experimental amplitude, $\tilde{A}_{3/2}$, for the helicity 3/2 component of the $\Delta \to p\gamma$ transition to $<1/2,\ 1/2|\hat{J}^-_{Vector}|3/2,\ 3/2>$ by the generalized CVC hypothesis. A calculation of the width for $\Delta \to p\gamma$ shows

$$\sqrt{4Mm}\ <1/2,\ 1/2|\hat{J}^-_{Vector}|3/2,\ 3/2> = \sqrt{\frac{3}{2}}\ \frac{G\sin\theta_c}{e}\ 2\sqrt{\frac{m_p(m_\Delta^2-m_p^2)}{m_\Delta}}\ \tilde{A}_{3/2} \tag{50}$$

The Review of Particle Properties[15] gives $\tilde{A}_{3/2} = -0.256$. We have

$$<1/2,1/2|\hat{J}^-_{Vector}|3/2,\ 3/2> = -\sqrt{\frac{E+m}{2m}}\ \frac{P}{E+m}\, g_1 \quad . \tag{51}$$

Thus, at $q^2 = 0$,

$$g_1 = 1.47$$

V. Results

Combining the relations of the previous two sections fixes most of the form factors:

$\Omega^- \to \Xi^o + \ell + \bar{\nu}$

$g_1 = 1.47$	$f_1 = -.364$
$g_2 = -.551\ GeV^{-1}$	$f_2 = 0.0\ GeV^{-1}$
$g_3 = 0.0\ GeV^{-2}$	$f_3 =$ undetermined
$g_4 = .330\ GeV^{-2}$	$f_4 = .403\ GeV^{-2}$

$\Omega^- \to \Xi^{o*} + \ell + \bar{\nu}$

G_1 = undetermined	$F_1 = -.315$
$G_2 = .312G_1 - 3.08\ \text{GeV}^{-1}$	$F_2 = 21.1\ \text{GeV}^{-1}$
$G_3 = 0.624G_1\ \text{GeV}^{-1}$	$F_3 = 7.64\ \text{GeV}^{-1}$
$G_4 = 6.27 \times 10^{-4}G_1\ \text{GeV}^{-1}$	$F_4 = -0.206\ \text{GeV}^{-1}$
$G_5 = -5.74 \times 10^{-4}G_1\ \text{GeV}^{-1}$	$F_5 = -0.188\ \text{GeV}^{-1}$
$G_6 = 2.55 \times 10^{-3}G_1 - 0.601\ \text{GeV}^{-3}$	$F_6 = 1.28\ \text{GeV}^{-3}$
$G_7 = -2.34 \times 10^{-4}G_1\ \text{GeV}^{-3}$	$F_7 = 1.75\ \text{GeV}^{-3}$

The form factor f_3 is not fixed in this discussion. Its contribution to the width is negligible, however, because it is the coefficient of q^μ which, when multiplied into the lepton factor (2), becomes the lepton mass. This can be checked by looking at the coefficients of f_3 in Table I. Thus the widths for the Ξ decays are completely determined,

	Axial Vector (s^{-1})	Vector (s^{-1})	Total (s^{-1})
$\Gamma(\Omega^- \to \Xi^\circ + e + \bar{\nu})$	$6.7\text{-}9.1 \times 10^7$	$3.2\text{-}5.7 \times 10^6$	$7.0\text{-}9.7 \times 10^7$
$\Gamma(\Omega^- \to \Xi^\circ + \mu + \bar{\nu})$	$4.9\text{-}6.9 \times 10^7$	$2.8\text{-}3.6 \times 10^6$	$5.1\text{-}7.3 \times 10^7$

The $\Xi^*\ell\bar{\nu}$ decays are not fully described in this discussion. The vector form factors G_2 and G_6 are not important because their coefficients in Table 2 are always small. The factors G_4, G_5, and G_7 are very small relative to G_1 allowing these terms to be neglected in the width calculation. The widths for the vector part of these decays can then be expressed as a function of G_1. The results are limited by uncertainties in the tabulated integrals. We find

$$\Gamma_{\text{Vector}}(\Omega^- \to \Xi^* e\nu) \quad < 9 \times 10^5\, G_1^2\ s^{-1}$$

$$\Gamma_{\text{Vector}}(\Omega^- \to \Xi^* \mu\bar{\nu}) \quad < 1 \times 10^5\, G_1^2\ s^{-1}$$

The axial vector contribution to the width depends only on F_1 and F_3 since the coefficients of axial vector form factors F_2, F_4, F_5, F_6, F_7 in Table 2 are small. The partial widths are

$\Gamma_A(\Omega^- \rightarrow \Xi^{o*} + e + \bar{\nu})$	$8.1 - 8.4 \times 10^5\ s^{-1}$
$\Gamma_A(\Omega^- \rightarrow \Xi^{o*} + \mu + \bar{\nu})$	$4.3 - 4.6 \times 10^4\ s^{-1}$

The lifetime of the Ω^- is not known exactly. The 1978 Review of Particle Properties[14] lists $\tau = 1.1 \times 10^{-10}$ seconds while the recent experimental analysis of Gaillard[1] finds $\tau = 0.82 \times 10^{-10}$ seconds. The accuracy of branching ratios is limited by the accuracy of the total lifetime measurements. We find the branching ratios for the two cases are

	$\tau = 1.1 \times 10^{-10}$ sec	$\tau = 0.82 \times 10^{-10}$ sec
$\Omega^- \rightarrow \Xi^o e\bar{\nu}$	$0.8 - 1.1 \times 10^{-2}$	$0.6 - 0.8 \times 10^{-2}$
$\Omega^- \rightarrow \Xi^o \mu\bar{\nu}$	$0.6 - 0.8 \times 10^{-2}$	$0.4 - 0.6 \times 10^{-2}$
$\Omega^- \rightarrow \Xi^{o*} e\bar{\nu}$	$8.9 - 9.2 \times 10^{-5}$	$6.7 - 6.9 \times 10^{-5}$
$\Omega^- \rightarrow \Xi^{o*} \mu\bar{\nu}$	$4.7 - 5.1 \times 10^{-6}$	$3.5 - 3.8 \times 10^{-6}$

The Ξ^* branching ratios are for the axial vector contribution only. The vector contribution is comparable or smaller.

Since we were able to determine all the necessary form factors for the Ξ decays it seemed worthwhile to determine the energy spectra of the outgoing electron or muon. These are shown in Figures 1 and 2. The change of form factor momentum dependence, $F(q^2)$, has little effect on either spectrum. The shapes of the spectra differ substantially from what would be expected on the basis of pure phase space considerations. For example the maxima occur at approximately 20 MeV above that of the phase space distributions.

Captions

Figure 1. The electron energy distribution in $\Omega^- \rightarrow \Xi^\circ + e + \bar{\nu}$. The solid line is $F(q^2) = 1$, the dashed line is $F(q^2) = (1-q^2/m^2)^{-2}$ the dot-dashed line is result if the matrix element is independent of the energy (pure phase space).

Figure 2. The muon energy distribution of $\Omega^- \rightarrow \Xi^\circ + \mu + \nu$. The various lines are the same cases as in Fig. 1.

References

1. J.M. Gaillard, Proc. of SLAC Summer Institute on Particle Physics, (1978) p. 431.

2. S.L. Glashow and R.H. Socolow, Phys. Lett. 10, (1964) 143.

3. J. Yellin, Phys. Rev. 135, (1964) B1203.

4. V. DeSantis, Phys. Rev. Lett. 13, (1964) 217.
   V. DeSantis, Phys. Rev. 137, (1964) B645.

5. J. Mathews, Phys. Rev. 137, (1964) B444.

6. T. DeGrand, et. al., Phys. Rev. D 12, (1975) 2060.

7. W. Rarita and J. Schwinger, Phys. Rev. 60, (1941) 61.

8. M. Gourdin and J. Micheli, Il Nuovo Cimento 40, (1965) 225;
   M.D. Scadron, Phys. Rev. 165, (1968) 1640.

9. See for example L.I. Schiff, Quantum Mechanics, McGraw Hill, New York, 1968.

10. L. Durand, P.D. DeCelles and R.B. Marr, Phys. Rev. 126, (1962) 1882.

11. P.R. Auvil and J.J. Brehm, Phys. Rev. 145 (1966) 1152.

12. D.A. Dicus, et. al., U. of Texas preprint;
    R. Arndt, et. al., Virgina Polytechnic Institute preprint.

13. W. Thirring, Acta Physica Austriaca, Su-p. 11, (1966) 183.

14. Particle Data Group, Phys. Lett. 75B, (1978) 1.

15. Particle Data Group, Rev. Mod. Phys. 48, (1976) 1.

Appendix

The widths for the leptonic decays are given in (8) and (9) of the text as functions of the form factors f_i, g_i, F_i, and G_i.

$$\Gamma(\Omega \to \Xi \ell \bar{\nu}) = \sum_{i,j=1}^{4} a_{ij} f_i f_j + v_{ij}\, g_i g_j$$

$$\Gamma(\Omega \to \Xi^* \ell \bar{\nu}) = \sum_{i,j=1}^{7} A_{ij} F_i F_j + V_{ij}\, G_i G_j$$

The purpose of this appendix is to tabulate the integrals a_{ij}, v_{ij}, A_{ij}, and V_{ij}. They are listed here according to lepton type and form factor dependence. They are symmetric in the indices. Uncertainties in the last two decimal places are shown in parenthesis. These uncertainties arise entirely from uncertainty in the baryon masses.

$F(q^2) = 1$

	v_{ij}		a_{ij}	
ii	Electron	Muon	Electron	Muon
11	1.60(03)E-5	8.89(20)E-6	2.17(03)E-3	1.44(02)E-3
12	3.60(07)E-5	1.92(04)E-5	2.38(04)E-4	1.32(03)E-4
13	2.50(03)E-12	3.07(07)E-8	2.43(03)E-10	3.82(07)E-6
14	3.49(08)E-6	1.61(04)E-6	3.42(05)E-4	1.91(04)E-4
22	1.24(02)E-4	6.78(15)E-5	5.72(11)E-5	2.98(07)E-5
23	7.45(14)E-12	9.24(22)E-8	8.68(13)E-11	1.37(03)E-6
24	1.01(02)E-5	4.64(12)E-6	8.46(16)E-5	4.47(10)E-5
33	7.06(17)E-14	1.49(04)E-9	8.81(16)E-12	2.14(05)E-7
34	1.37(03)E-12	1.72(05)E-8	1.34(02)E-10	2.15(04)E-6
44	1.41(03)E-6	6.18(18)E-7	1.26(02)E-4	6.77(15)E-5

$F(q^2) = (1 - q^2)^{-2}$

	v_{ij}		a_{ij}	
11	1.94(04)E-5	1.14(03)E-5	2.84(04)E-3	1.97(03)E-3
12	4.31(08)E-5	2.44(06)E-5	2.81(05)E-4	1.66(03)E-4
13	2.82(05)E-12	3.78(09)E-8	2.84(04)E-10	4.84(09)E-6
14	3.94(09)E-6	1.94(05)E-6	4.01(07)E-4	2.39(05)E-4
22	1.50(03)E-4	8.65(20)E-5	6.46(12)E-5	3.62(08)E-5
23	8.41(16)E-12	1.13(03)E-7	1.01(02)E-10	1.73(04)E-6
24	1.14(03)E-5	5.60(15)E-6	9.54(18)E-5	5.44(12)E-5
33	8.59(22)E-14	1.89(06)E-9	1.12(02)E-11	2.82(07)E-7
34	1.54(03)E-12	2.11(06)E-8	1.57(03)E-10	2.72(06)E-6
44	1.55(04)E-6	7.33(22)E-7	1.42(03)E-4	8.23(19)E-5

Electron

$F(q^2) = 1$

ij	A_{ij}		V_{ij}	
11	4.54×10^{-5}	(11)	73×10^{-5}	(07)
12	1.85×10^{-12}	(05)	1.27×10^{-10}	(02)
13	1.06×10^{-6}	(03)	4.37×10^{-5}	(11)
14	-1.34×10^{-8}	(05)	1.24×10^{-6}	(04)
15	-1.34×10^{-8}	(05)	-1.32×10^{-6}	(04)
16	-5.24×10^{-15}	(18)	-5.00×10^{-13}	(15)
17	-3.01×10^{-9}	(12)	-2.05×10^{-7}	(07)
22	3.25×10^{-15}	(11)	7.11×10^{-12}	(18)
23	1.31×10^{-13}	(04)	2.06×10^{-10}	(04)
24	-1.64×10^{-15}	(06)	-3.56×10^{-12}	(09)
25	-1.64×10^{-15}	(06)	-3.56×10^{-12}	(09)
26	-7.16×10^{-18}	(32)	-2.00×10^{-14}	(07)
27	-3.72×10^{-16}	(15)	-8.11×10^{-13}	(24)
33	33×10^{-8}	(19)	6.99×10^{-5}	(17)
34	-9.31×10^{-10}	(37)	-2.02×10^{-6}	(06)
35	-9.49×10^{-10}	(38)	-2.07×10^{-6}	(06)
36	-3.72×10^{-16}	(15)	-8.11×10^{-13}	(24)
37	-1.64×10^{-10}	(08)	-3.29×10^{-7}	(12)
44	2.28×10^{-10}	(10)	1.79×10^{-7}	(06)
45	1.82×10^{-10}	(08)	1.03×10^{-7}	(04)
46	4.99×10^{-17}	(23)	3.34×10^{-14}	(12)
47	2.85×10^{-11}	(14)	1.90×10^{-8}	(08)
55	2.47×10^{-10}	(11)	1.97×10^{-7}	(07)
56	4.99×10^{-17}	(23)	3.34×10^{-14}	(12)
57	2.89×10^{-11}	(15)	1.94×10^{-8}	(08)
66	1.79×10^{-19}	(10)	1.46×10^{-16}	(07)
67	1.13×10^{-17}	(06)	7.59×10^{-15}	(31)
77	5.28×10^{-12}	(30)	3.36×10^{-9}	(15)

Electron

$F(q^2) = (1 - q^2)^{-2}$

ij	A_{ij}		V_{ij}	
11	4.71×10^{-5}	(12)	2.79×10^{-5}	(07)
12	1.89×10^{-12}	(05)	1.31×10^{-10}	(03)
13	1.09×10^{-6}	(03)	4.47×10^{-5}	(11)
14	-1.37×10^{-8}	(05)	-1.26×10^{-6}	(04)
15	-1.37×10^{-8}	(05)	-1.35×10^{-6}	(04)
16	-5.34×10^{-15}	(19)	-5.12×10^{-13}	(15)
17	-3.06×10^{-9}	(12)	-2.09×10^{-7}	(07)
22	3.37×10^{-15}	(12)	7.44×10^{-12}	(19)
23	1.35×10^{-13}	(04)	2.13×10^{-10}	(04)
24	-1.67×10^{-15}	(06)	-3.64×10^{-12}	(09)
25	-1.67×10^{-15}	(06)	-3.64×10^{-12}	(09)
26	-7.37×10^{-18}	(34)	-2.08×10^{-14}	(07)
27	-3.78×10^{-16}	(15)	-8.30×10^{-13}	(25)
33	5.42×10^{-8}	(19)	7.15×10^{-5}	(18)
34	-9.48×10^{-10}	(38)	-2.07×10^{-6}	(06)
35	-9.66×10^{-10}	(39)	-2.12×10^{-6}	(06)
36	-3.78×10^{-16}	(15)	-8.30×10^{-13}	(25)
37	-1.67×10^{-10}	(08)	-3.35×10^{-7}	(12)
44	2.32×10^{-10}	(11)	1.84×10^{-7}	(06)
45	1.86×10^{-10}	(08)	1.05×10^{-7}	(04)
46	5.07×10^{-17}	(23)	3.40×10^{-14}	(12)
47	2.89×10^{-11}	(15)	1.94×10^{-8}	(08)
55	2.52×10^{-10}	(12)	2.03×10^{-7}	(08)
56	5.07×10^{-17}	(23)	3.40×10^{-14}	(12)
57	2.94×10^{-11}	(15)	1.97×10^{-8}	(08)
66	1.83×10^{-19}	(10)	1.51×10^{-16}	(07)
67	1.15×10^{-17}	(06)	7.73×10^{-15}	(31)
77	5.35×10^{-12}	(30)	3.41×10^{-9}	(16)

Muon

$F(q^2) = 1$

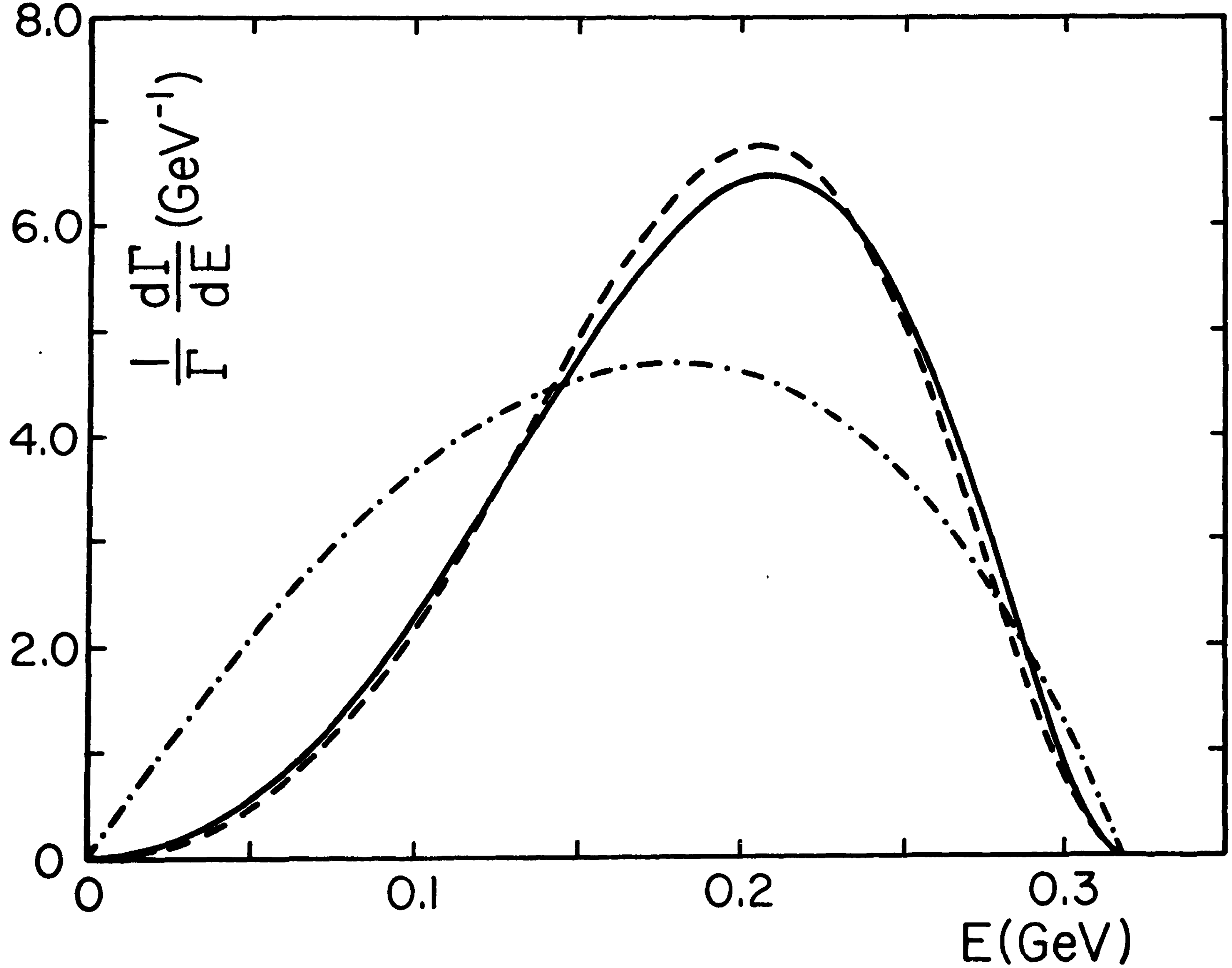
ij	A_{ij}		V_{ij}	
11	1.64×10^{-6}	(12)	9.87×10^{-7}	(74)
12	3.76×10^{-10}	(35)	9.38×10^{-8}	(68)
13	1.26×10^{-8}	(12)	1.63×10^{-6}	(12)
14	-4.86×10^{-11}	(57)	-1.42×10^{-8}	(14)
15	-4.86×10^{-11}	(57)	-1.57×10^{-8}	(15)
16	-3.20×10^{-13}	(39)	-1.01×10^{-10}	(10)
17	-1.10×10^{-11}	(14)	-1.95×10^{-9}	(20)
22	1.81×10^{-12}	(18)	1.10×10^{-8}	(08)
23	2.73×10^{-11}	(27)	1.56×10^{-7}	(11)
24	-9.99×10^{-14}	(121)	-7.21×10^{-10}	(68)
25	-9.99×10^{-14}	(121)	-7.21×10^{-10}	(68)
26	-1.48×10^{-15}	(19)	-1.12×10^{-11}	(11)
27	-2.32×10^{-14}	(29)	-1.68×10^{-10}	(17)
33	5.20×10^{-10}	(54)	2.68×10^{-6}	(20)
34	-3.38×10^{-12}	(42)	-2.38×10^{-8}	(23)
35	-3.50×10^{-12}	(43)	-2.47×10^{-8}	(24)
36	-2.32×10^{-14}	(29)	-1.68×10^{-10}	(17)
37	-4.73×10^{-13}	(62)	-3.20×10^{-9}	(33)
44	8.18×10^{-13}	(105)	1.97×10^{-9}	(20)
45	5.15×10^{-13}	(67)	6.11×10^{-10}	(66)
46	9.82×10^{-16}	(145)	2.04×10^{-12}	(25)
47	3.39×10^{-14}	(51)	6.92×10^{-11}	(85)
55	9.45×10^{-13}	(122)	2.29×10^{-9}	(23)
56	9.82×10^{-16}	(145)	2.04×10^{-12}	(25)
57	3.51×10^{-14}	(53)	7.16×10^{-11}	(88)
66	1.41×10^{-17}	(22)	3.02×10^{-14}	(38)
67	2.28×10^{-16}	(35)	4.75×10^{-13}	(60)
77	4.87×10^{-15}	(77)	9.69×10^{-12}	(127)

Muon

$F(q^2) = (1 - q^2)^{-2}$

ij	A_{ij}		V_{ij}	
11	1.76×10^{-6}	(13)	1.05×10^{-6}	(08)
12	4.00×10^{-10}	(38)	1.00×10^{-7}	(07)
13	1.34×10^{-8}	(13)	1.74×10^{-6}	(13)
14	-5.15×10^{-11}	(61)	-1.51×10^{-8}	(14)
15	-5.16×10^{-11}	(61)	-1.67×10^{-8}	(16)
16	-3.40×10^{-13}	(41)	-1.08×10^{-10}	(11)
17	-1.17×10^{-11}	(14)	-2.07×10^{-9}	(22)
22	1.93×10^{-12}	(20)	1.18×10^{-8}	(09)
23	2.90×10^{-11}	(29)	1.66×10^{-7}	(12)
24	-1.06×10^{-13}	(13)	-7.68×10^{-10}	(72)
25	-1.06×10^{-13}	(13)	-7.68×10^{-10}	(72)
26	-1.57×10^{-15}	(20)	-1.19×10^{-11}	(12)
27	-2.46×10^{-14}	(31)	-1.79×10^{-10}	(18)
33	5.53×10^{-10}	(57)	2.86×10^{-6}	(21)
34	-3.58×10^{-12}	(44)	-2.53×10^{-8}	(24)
35	-3.71×10^{-12}	(46)	-2.62×10^{-8}	(25)
36	-2.46×10^{-14}	(31)	-1.79×10^{-10}	(18)
37	-5.02×10^{-13}	(66)	-3.40×10^{-9}	(35)
44	8.68×10^{-13}	(112)	2.09×10^{-9}	(21)
45	5.47×10^{-13}	(72)	6.49×10^{-10}	(71)
46	1.04×10^{-15}	(16)	2.17×10^{-12}	(26)
47	3.59×10^{-14}	(54)	7.34×10^{-11}	(91)
55	1.00×10^{-12}	(13)	2.43×10^{-9}	(25)
56	1.04×10^{-15}	(15)	2.17×10^{-12}	(26)
57	3.71×10^{-14}	(56)	7.60×10^{-11}	(94)
66	1.49×10^{-17}	(23)	3.21×10^{-14}	(41)
67	2.42×10^{-16}	(37)	5.04×10^{-13}	(64)
77	5.16×10^{-15}	(82)	1.03×10^{-11}	(14)

#1



#2