CONF-871179--1 Vugraphs

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TRANSPORT SCALING IN THE COLLISIONLESS-DETRAPPING REGIME

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- For the collisionless-detrapping or ν-regime, we have determined stellarator transport scalings with:

E_ρ	(electric field)
ϵ_t and ϵ_h	(geometry)
ν	(collision frequency)

From numerical solutions to the DKE.

- We find a new geometrical scaling $\propto \epsilon_t^{3/2}$, independent of the helical ripple ϵ_h. Compare with Galeev-Sagdeev scaling $\epsilon_t \epsilon_h^{1/2}$

- We present integral expressions for Γ and Q that can be used in transport simulations, along with self-consistent determination of E_ρ, to obtain, for example, confinement times for specific devices

OUTLINE

- Brief description of numerical techniques
- Deduction of the scalings from the numerical results.
- Expressions for Γ and Q
- Comparisons with other scalings

NUMERICAL TECHNIQUE

- Use DKES (Drift Kinetic Equation Solver) code to solve linearized DKE for f_1, the deviation from the Maxwellian

$$\vec{V}_d \cdot \nabla f_1 + \dot{\alpha}\,(\partial f_1/\partial \alpha) - C(f_1) = S,$$

where

$$\vec{v} = v\cos\alpha\,\hat{n} + \underline{E_\rho \nabla\rho \times \vec{B}/\langle B^2\rangle}$$

$$v_\parallel/v = \cos\alpha,\quad \hat{n} = \vec{B}/B,\quad E_\rho = -d\Phi/d\rho$$

$$\dot{\alpha} = -\frac{v}{2}\sin\alpha\,\vec{B}\cdot\nabla(VB)$$

$$S = f_M\left[-\vec{V}_d\cdot\nabla\rho\,(A_1 + x A_2) - Bv\cos\alpha\,A_3\right]$$

$$A_1 = \frac{n'}{n} - \frac{3T'}{2T} - \frac{eE_\rho}{T}$$

$$A_2 = \frac{T'}{T}$$

$$A_3 = -\frac{e\langle\vec{E}\cdot\vec{B}\rangle}{T\langle B^2\rangle}$$

$$x = Mv^2$$

- No resonant superbanana orbits since the [redacted] curvature and ∇B drift terms in $\vec{v} \cdot \nabla f_1$ neglected.

- Poloidal $\underline{\vec{E}_0 \times \vec{B}}$ drift retained for these calculations of collisionless detrapping/retrapping of particles in helically trapped orbits.

- $C(f_1)$ is a pitch-angle scattering operator

- The treatment of the boundary layer between trapped and circulating particles is exact; __no__ assumptions regarding boundary conditions on the distribution are made.

- Solve the linearized DKE in terms of Fourier-Legendre series for f_1 at a fixed value of the normalized energy x for a number of values of ν and E_ρ on a flux surface.

- Then obtain fluxes:

$$\langle \vec{\Gamma} \cdot \nabla\rho \rangle = \left\langle \int d^3v \, \vec{V}_d \cdot \nabla\rho \, f_1 \right\rangle = -\sum_{n=1}^{2} L_{1n} A_n$$

$$\left\langle \frac{\vec{Q} \cdot \nabla\rho}{T} \right\rangle = \left\langle \int d^3v \, \vec{V}_d \cdot \nabla\rho \, x \, f_1 \right\rangle = -\sum_{n=1}^{2} L_{2n} A_n$$

(Ignore A_3) [redacted]

- We use L_{22}, the coefficient of $A_2 = \frac{T'}{T}$ in the heat flux for illustrative purposes. Same scalings from L_{11} and

MAGNETIC SPECTRUM

- Use truncated spectrum on a flux surfa

  $$B = B_0\left[1 - \epsilon_t \cos\theta + \epsilon_h \cos(\ell\theta - m\zeta)\right]$$

  representative of vacuum B of ATF($\ell = 2$, $m = 12$)

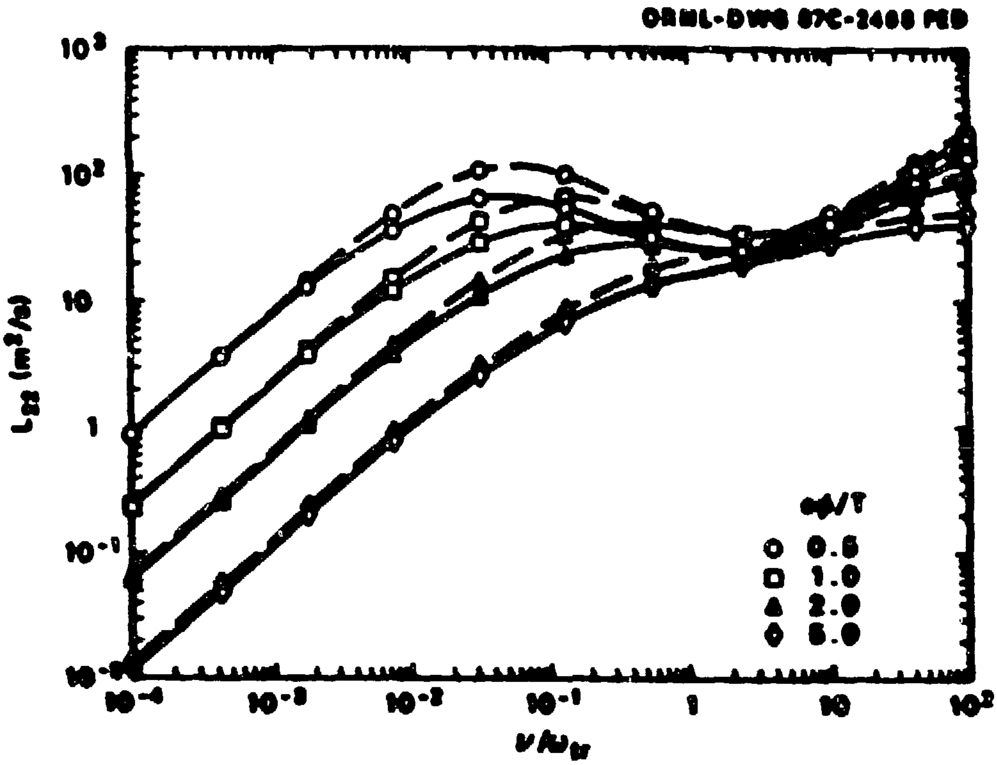
- Investigated effects of multiple helicity

  using a more complete, eight-term spectr

- NOTE: Truncated spectrum inappropriate

  for transport-optimized stellarators.

EFFECTS OF MULTIPLE HELICITY



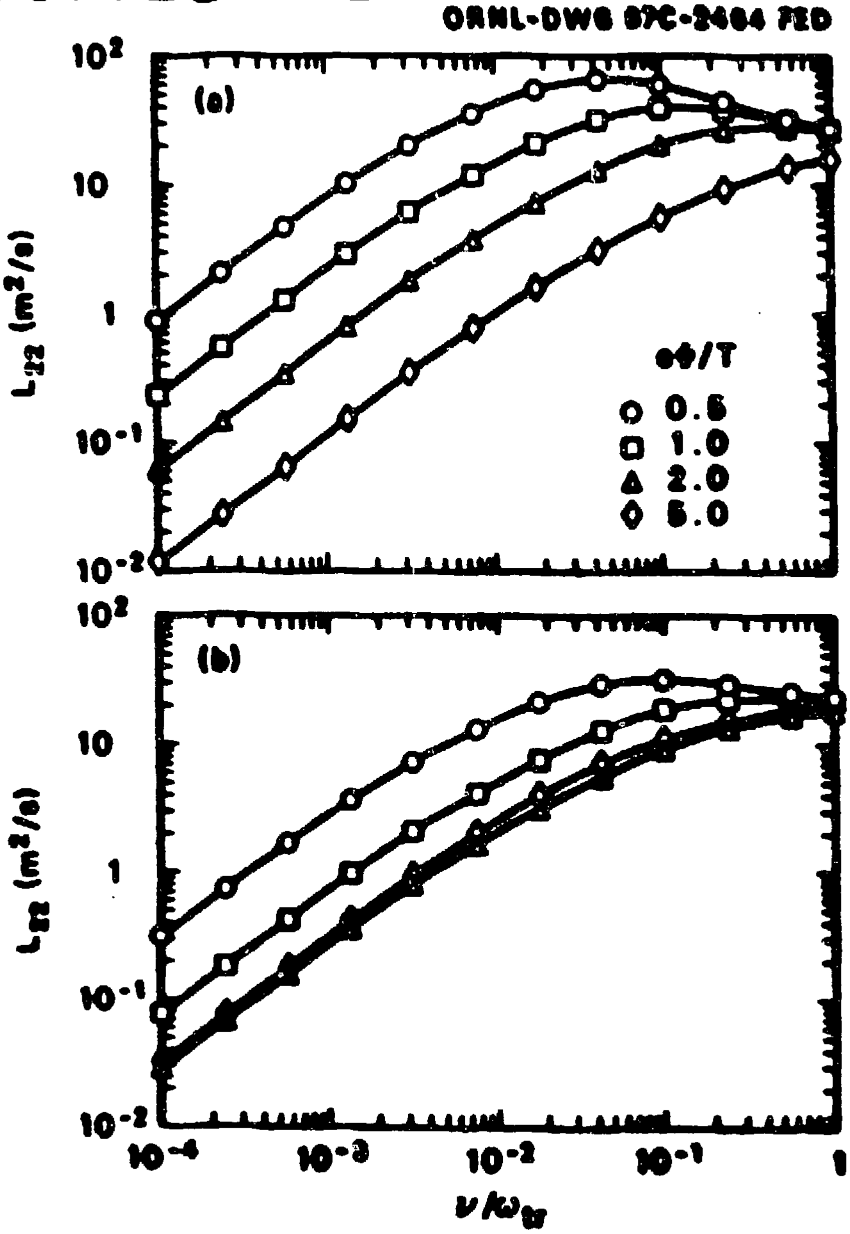
——— Truncated (three-term) spectrum

— — Eight-term spectrum

$\bar{a} = 0.3$ m

- Effects not very important in ν-regime
- Factor of two for small $\frac{e\bar{\Phi}}{T}$ in $1/\nu$ regime
- Less for larger $\frac{e\bar{\Phi}}{T}$ and larger ν.

ELECTRIC FIELD SCALING AND RESONANCE



$\epsilon_t = 0.12$

$\epsilon_h = 0.14$

$\iota = 0.68$

$\epsilon_t = 0.071$

$\epsilon_h = 0.066$

$\iota = 0.45$

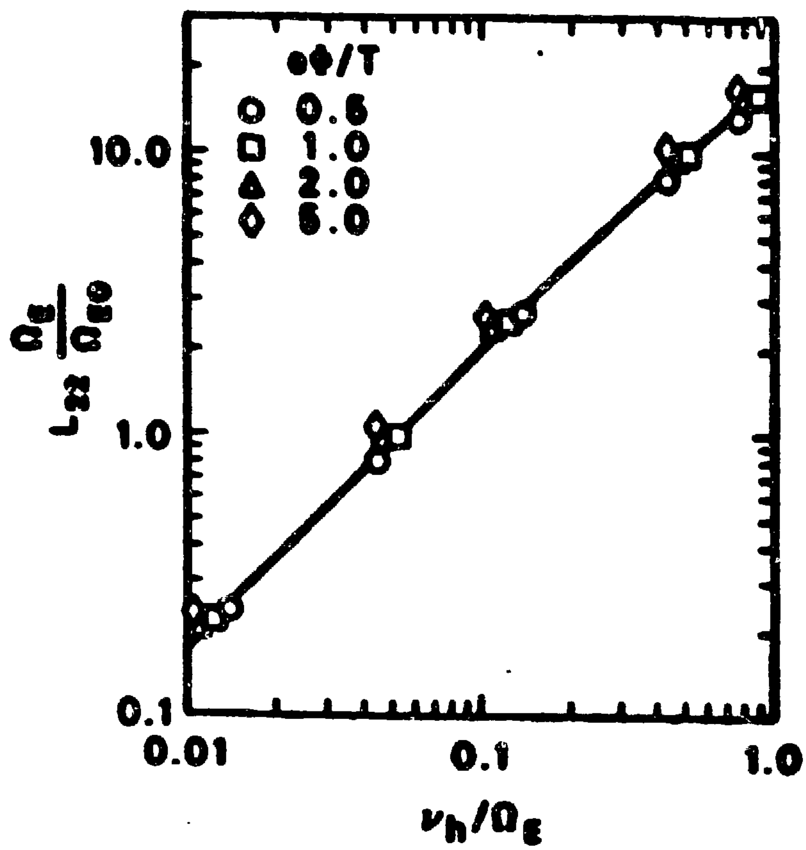
- In (a), for $\frac{\nu}{\omega_{E}} \lesssim 10^{-3}$, $\underline{L_{22} \propto 1/\Phi^2}$

- In (b), resonance between $v_{\parallel}$ and poloidal $\vec{E}_0 \times \vec{B}$ drift causes scaling to deviate from $1/\Phi^2$ for $\frac{e\Phi}{T} \gtrsim 5$

- To obtain scalings, we avoided this resonance.

COLLISION FREQUENCY SCALING

ORNL-DWG 87C-2465 FED



ϵ_t and ϵ_h fixed

Ω_E = poloidal $\vec{E}\times\vec{B}$ frequency.

$\Omega_{E0} = \Omega_E(e\Phi/T = 1)$.

$\nu_h = \nu/\epsilon_h$

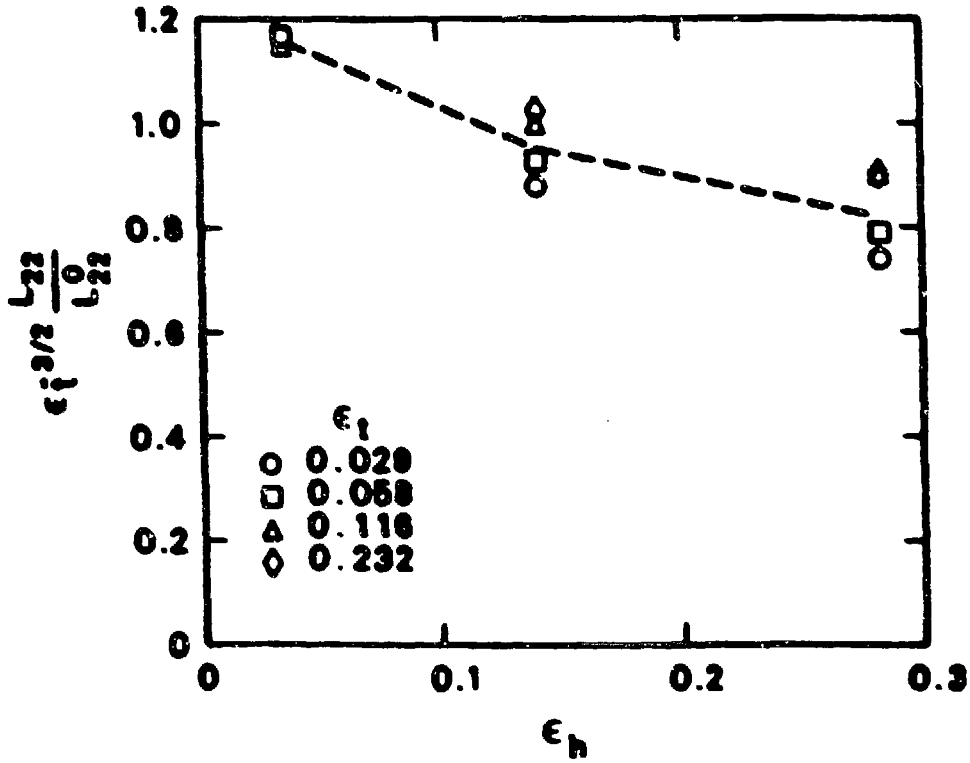
- $L_{22} \propto \nu$ from unit slope

- $L_{22} \propto 1/\Omega_E^2 \propto 1/\Phi^2$ confirmed.

  from clustering of points.

GEOMETRICAL SCALING

ORNL-DWG 87C-2486 FED



$$L_{22}^{0} = L_{22}(\epsilon_t = 0.12\ ;\ \epsilon_h = 0.14)$$

- $\epsilon_t^{-3/2} \dfrac{L_{22}}{L_{22}^{0}}$ not very sensitive to ϵ_h

∴ dominant geometrical dependence is $L_{22} \propto \epsilon_t^{3/2}$

- Summarizing, the scaling we find is

$$L_{22}^{DKES} \propto \nu\, \epsilon_t^{3/2}\, G(\epsilon_t, \epsilon_h)/\Omega_E^2$$

where $G(\epsilon_t, \epsilon_h)$ is a weak function of ϵ_t, ϵ_h

ESTIMATION OF f, THE FRACTION OF PARTICLES INVOLVED IN THE TRANSPORT PROCESS

- In a random walk argument

$$D \sim f\,(\Delta r)^2/\Delta t$$

- In the ν-regime, the step size Δr is determined by the helically trapped particles

$$\Delta r = \frac{V_d}{\Omega_E} = \frac{\epsilon_t T}{M\Omega r \Omega_E}$$

- The effective time between collisions Δt is

$$\Delta t = (\nu/f^2)^{-1}$$

- Therefore,

$$D \sim \nu\, \epsilon_t^2 T^2 / M^2 \Omega^2 r^2 \Omega_E^2 f$$

and, using the scaling found above

$$f \simeq \sqrt{\epsilon_t}\; G^{-1}(\epsilon_t, \epsilon_h)$$

EXPRESSIONS FOR THE FLUXES

- Shaing has given integral expressions for fluxes

$$\begin{bmatrix} \Gamma_a \\ Q_a \end{bmatrix} = -\epsilon_t^2 \sqrt{\epsilon_h}\, v_a^2 \begin{bmatrix} n_a \\ n_a T_a \end{bmatrix} \int_0^\infty dx_a \begin{bmatrix} x_a^{3/2} \\ x_a^{5/2} \end{bmatrix} e^{-x_a} \frac{\nu_{ad}(x_a)(A_{1a} + x_a A_{2a}}{\omega_a^2(x_a)}$$

where: $v_a = \frac{V_{da}}{\epsilon_t}$

$$\nu_{ad}(x_a) = \begin{cases} \dfrac{\nu_{ii}(x_i)}{\epsilon_h} & \text{ions} \\ \dfrac{\nu_{ee}(x_e) + \nu_{ei}(x_e)}{\epsilon_h} & \text{electrons} \end{cases}$$

- Here, recalling ∇B drift not included in $\vec{v} \cdot \nabla f_a$

$$\omega_a^2(x_a) = 1.5 \sqrt{\epsilon_t / \epsilon_h}\, \Omega_E^2 + 3 \nu_{ad}^2(x_a)$$

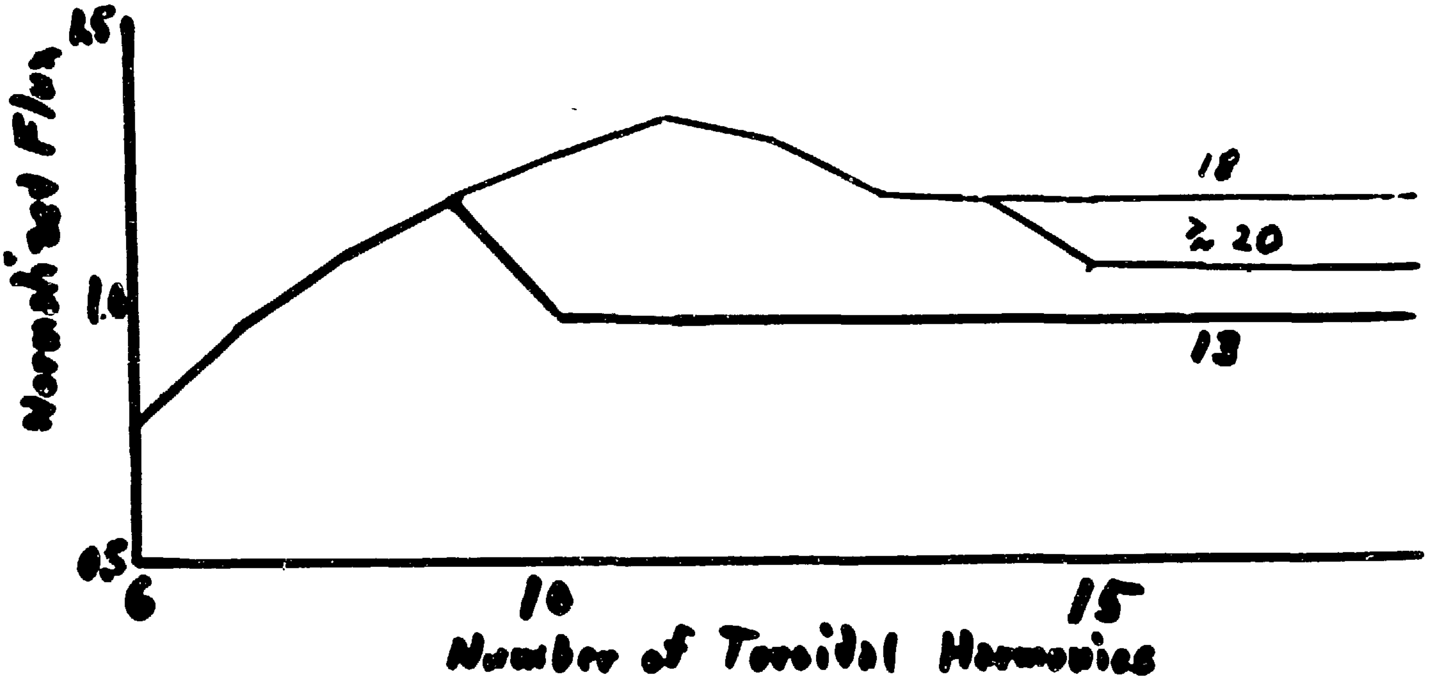
Previously, Galeev-Sagdeev geometrical scaling

$$\left[\omega_a^2(x_a)\right]_{old} = 1.67 \frac{\epsilon_t}{\epsilon_h} \Omega_E^2 + 3 \nu_{ad}^2(x_a)$$

- **NOTE:** Superbanana terms must be added to $\omega_a^2(x_a)$ for device transport simulations

CONVERGENCE STUDIES

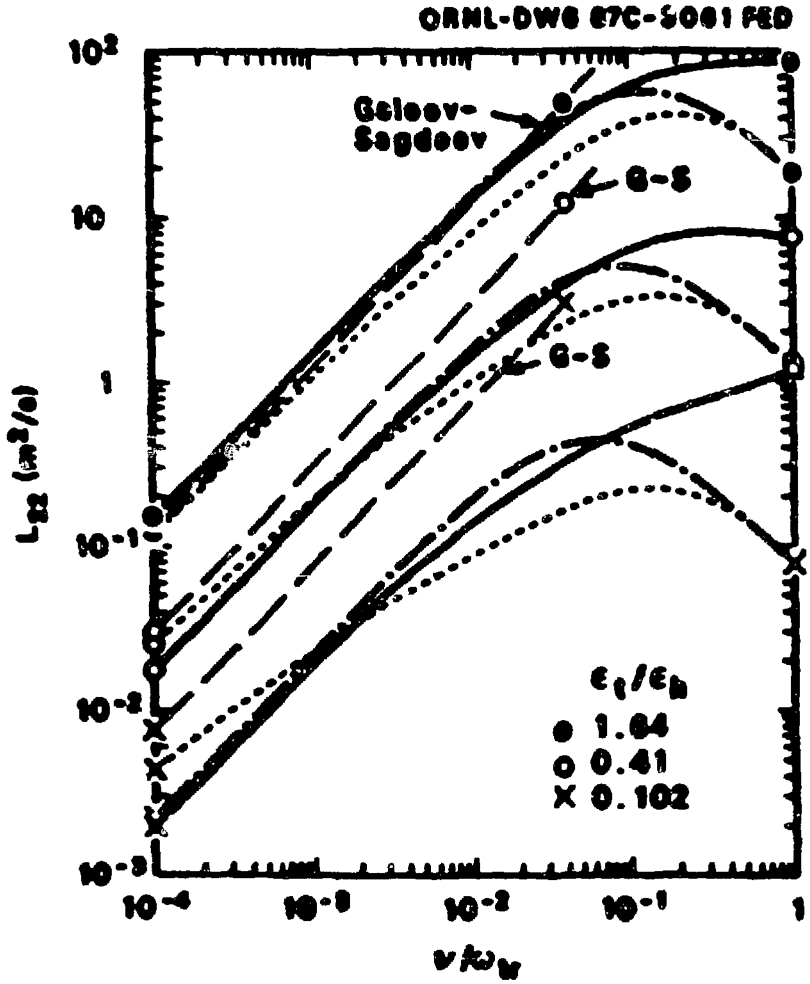
- To ascertain the reliability of our numerical results, we have run convergence studies in which the number of poloidal, toroidal, and pitch-angle harmonics were varied.



- Conclude that scalings with ν and E_p accurate to a few percent, scalings with ϵ_t and ϵ_h accurate to variation illustrated.

- Coefficient of 1.5 accurate to 10-20%.

COMPARISON OF SCALINGS



ϵ_h fixed

- For $\nu/\omega_{tr} \lesssim 10^{-2}$ agreement between DKES (— and integral expressions (——) very good. For larger ν/ω_{tr} DKES has plateau-Pfirsch-Schlüter
- Galeev-Sagdeev results (——) show same collision frequency scaling, but not ϵ_t scaling.
- Beidler et al. results (---) [excluding symmetric] show different ν and geometries scalings, but within a factor of 2 for these cases

CONCLUSIONS

- From numerical solutions to the full, linearized DKE, we find a geometrical scaling of transport coefficients in the collisionless-detrapping regime different from scalings found using other analytic or numerical treatments.

- The discrepancies probably result from approximate boundary conditions imposed in other treatments; <u>no</u> such conditions are imposed here

- The new scaling is in the direction of providing more flexibility in stellarator design, since it is independent of the magnitude of the helical ripple.

- The results are incorporated into integral expressions for particle and heat fluxes useful for transport simulations.