ORIGIN AND PROPERTIES OF THE HIGGS FIELD*

Kerson Huang

Institute of High Energy Physics, Academia Sinica
P.O. Box 918, Beijing, China

and

Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

*This work is supported in part through funds provided by the U.S. DEPARTMENT OF ENERGY (DOE) under contract EY-76-C-02-3069.

Contribution to the 1980 Guangchou (Canton) Conference on Theoretical Particle Physics, January 5-14, 1980, Guangchou, China

November, 1979
CTP# 823

MASTER

COO-3069- 748
OSP- 77906
CONF - 800120--1

Origin and Properties of the Higgs Field

Kerson Huang (黄克孫)

Institute of High Energy Physics, Academia Sinica
P.O. Box 918, Beijing, China
and
Laboratory for Nuclear Science and Physics Department
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA02139

Abstract

It is suggested that the Higgs field in the Weinberg-Salam theory is not fundamental, but made of $\bar{e}_R e_L$ pairs, which are analogous to Cooper pairs in superconductivity. The pair $\bar{e}_R e_L$ forms a bound state by virtue of the Coulomb interaction. Thus, there will be no Higgs bosons, only vortex ring excitations. By a variational calculation, the mass of a vortex ring containing the Z^0 field is estimated to be greater than 1000 GeV/c^2. Its decay will be spectacular.

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Contribution to the 1980 Guangchou (Canton) Conference on Theoretical Particle Physics, January 5-14, 1980, Guangchou, China

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Higgs场的起源及性质

黄 克 孙

北京高能物理所及

美国麻省理工学院

摘 要

此文作以下建议：Weinberg-Salam 理论里的Higgs场不是基本的，而是 $\bar{e}_R e_L$ 的近似平均场。电子对 $\bar{e}_R e_L$ 由库论力组成束缚态，与超导现象的Cooper 对类似。这样，Higgs玻色子就不存在。但旋涡环激发态总是有的。用变分方法，含Z°场的旋涡环的质量估计在1000Gev/c²以上。它的衰变是很壮观的。

One of the open questions in the Weinberg-Salam theory(1) of unified electromagnetic and weak interactions concerns the nature of the Higgs field: Is it an independent field, or a phenomenological device? We have no evidence for or against either view, because no dynamical effect of the Higgs field has been observed so far.

I favor the phenomenological view, for two reasons. First, it is always desirable to reduce the number of independent fields. Secondly, the only Higgs field for which we have evidence is that in superconductivity, and there it is a mean-field description of bound electron pairs —— Cooper pairs.

I suggest that the Higgs field in the Weinberg-Salam theory is a mean-field description of bound $\bar{e}_R e_L$ pairs, where e_R and e_L denotes respectively a right-handed and left-handed electron. The force responsible for the binding is the Coulcmb attraction between the particles. In the following I give some qualitative arguments.

Let the generators of SU(2) x U(1) be $\vec{t}$ and $y/2$, and the corresponding gauge fields be $\vec{W}_\mu$ and B_μ. The covariant derivative is given by

$$D^\mu = \partial^\mu + ig\vec{W}^\mu\cdot\vec{t} + \frac{1}{2}\, ig' B^\mu y \qquad (1)$$

The photon field A^μ and the neutral field Z^μ are defined by

$$\begin{aligned} B^\mu &= A^\mu \cos\theta_W - Z^\mu \sin\theta_W , \\ W_3^\mu &= A^\mu \sin\theta_W + Z^\mu \cos\theta_W , \end{aligned} \qquad (2)$$

where θ_W is the Weinberg angle, given experimentally by(2)

$$\xi \equiv \sin^2\theta_W = 0.25 \pm 0.05 \qquad (3)$$

Thus,

$$D^\mu = \partial^\mu + ig(W_1^\mu t_1 + W_2^\mu t_2) + i[(g\sin\theta_w)t_3 + (g'\cos\theta_w)\, y/2]A^\mu + i[(g\cos\theta_w)t_3 - (g'\cos\theta_w)\, y/2]Z^\mu. \qquad (4)$$

The correct electromagnetic interaction is obtained by requiring

$$g\sin\theta_w = g'\cos\theta_w = e\ , \qquad (5)$$

$$Q = t_3 + \tfrac{1}{2}y\ , \qquad (6)$$

where e is the absolute value of the electronic charge ($e^2/4\pi = 1/137$), and Q is the electric charge operator in units of e. Thus,

$$D^\mu = \partial^\mu + ig(W_1^\mu t_1 + W_2^\mu t_2) + ieQA^\mu + ie'Q'Z^\mu, \qquad (7)$$

where

$$\frac{e'}{e} = \frac{1}{\sin\theta_w \cos\theta_w}\ , \qquad (8)$$

$$Q' = t_3 \cos^2\theta_w - \tfrac{1}{2}\, y \sin^2\theta_w\ . \qquad (9)$$

Some eigenvalues of Q and Q' are listed in Table I.

Table I. Electric charge Q (in units of e), and Neutral charge Q' (in units of e')
$e'/e = (\sin\theta_w\cos\theta_w)^{-1}$, $\xi = \sin^2\theta_w$

Particle	Q	Q'	(e'/e)Q' for $\xi = 1/4$
ν_L	0	$\frac{1}{2}$	$2/\sqrt{3}$
e_L	-1	$\xi - \frac{1}{2}$	$-1/\sqrt{3}$
e_R	-1	ξ	$1/\sqrt{3}$
Higgs field ϕ_0	0	$-\frac{1}{2}$	$-2/\sqrt{3}$
Higgs field ϕ_+	1	$\frac{1}{2} - \xi$	$1/\sqrt{3}$

We see from Table I that, for arbitrary values of the Weinberg angle,

$$\bar{e}_R e_L \sim \phi_0\ , \quad \bar{e}_R \nu_L \sim \phi_+\ , \qquad (10)$$

where $\sim$ means having the same Q and Q'. Thus, $\bar{e}_R e_L$ at least has the right quantum numbers to be the Higgs field ϕ_0. The field ϕ_+ need not be considered separately, because it can be transformed away by a gauge transformation.

Let us take

$$\xi = \sin^2\theta_w = \frac{1}{4}\ , \qquad (11)$$

which is consistent with experiments. Then,

$$\bar{e}_R \text{ and } e_L \text{ have } \begin{cases} \text{opposite electric charge } \pm e, \\ \text{equal neutral charge } e/\sqrt{3}. \end{cases} \qquad (12)$$

Hence there is a net attractive force between $\bar{e}_R$ and e_L. Incidentally, (11) implies that all the neutral charges in Table I are integer multiples of a smallest unit $e/\sqrt{3}$ (3).

In a massless theory without fundamental Higgs fields, the attraction between $\bar{e}_R$ and e_L should populate the bare vacuum with a condensate of $\bar{e}_R e_L$ bound pairs. This would be a concrete example of "dynamical symmetry breaking" (4),(5). If this happens, one can describe the low-lying states of the system by introducing the Higgs field as a mean-field approximation to the quantum field $\bar{e}_R e_L$, in a manner similar to the Ginsburg-Landau theory of superconductivity (6).

To study the proposed mechanism, work is in progress to calculate the effective potential for the field $\bar{e}_R e_L$ in a massless gauge theory. The object is to show that the effective potential has a lowest minimum at $\langle \bar{e}_R e_L \rangle \neq 0$. The method is an extension of that of Coleman and Weinberg[7], who consider the case of a scalar field. Results of this calculation will be published elsewhere[8].

If the Higgs field is phenomenological in the sense described above, it should not be quantized; there would be no Higgs bosons. However, there would still be dynamical manifestations, such as vortex rings. I now turn to a description of these excitation.

By analogy with superconductivity, one might expect a vortex line excitation, in which a quantized amount of the magnetic flux of the Z^0 field is trapped in a long tube, in which the Higgs field falls to below its vacuum value [9]. Such a vortex line, however, must have infinite energy, because the flux lines cannot end. A vortex ring, on the other hand, will have finite energy. If the Z^0 field is treated classically, the vortex ring will be stable, by virtue of flux quantization, which is a requirement for minimum energy. Its decay is an effect involving quantum mechanical tunneling, and is ignored in this discussion.

The size and the energy of a vortex ring can be estimated by a variational calculation. For static fields the classical energy is the negative action:

$$E = \int d^3x \left\{ \tfrac{1}{2} |\nabla \times \vec{Z}|^2 + |(\nabla - iq\vec{Z})\phi|^2 + V(\phi) \right\} , \qquad (13)$$

$$q = -\frac{2}{\sqrt{3}} e ,$$

where $\phi(\vec{x})$ is the complex Higgs field, and

$$V(\phi) = \lambda(\phi^*\phi - \phi_0^2)^2 ,$$

$$\phi_0 = 11.4\, m_p , \quad (m_p = \text{proton mass}). \qquad (14)$$

We have put $W^\mu = A^\mu = 0$, and ignored leptons. A field configuration that minimizes E is a solution to the classical equations of motion. We seek a solution corresponding to a circular vortex ring, symmetric about the z-axis, as shown in Fig. 1.

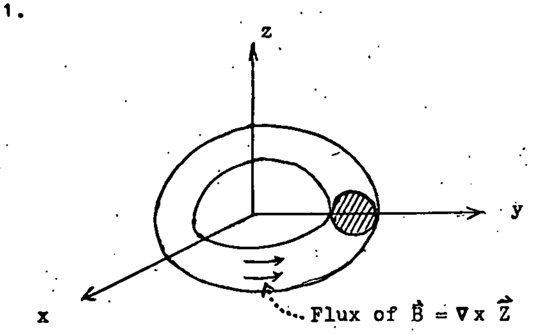

Fig.1 Vortex ring consisting of magnetic flux of the Z^0 field trapped in a tube, in which the Higgs field has values below its vacuum value ϕ_0.

To minimize the energy, we must have

$$(\nabla - iq\vec{Z})\, \phi(\vec{x}) = 0 ,$$
$$\text{whereever} \quad \vec{B} = \nabla \times \vec{Z} = 0 . \qquad (15)$$

This leads to the flux-quantization condition

$$\oint d\vec{s} \cdot \vec{Z} = \frac{2\pi n}{q} , \quad (n = 0, \pm 1, \pm 2, \ldots), \qquad (16)$$

where the line integral is taken over any closed path along which $\vec{B} = 0$. Taking the closed path to be the z-axis plus an infinite semicircle, and assuming that $\vec{Z}$ vanishes at infinity fast enough, we have

$$\int_{-\infty}^{\infty} dz\, Z_z = \frac{2\pi n}{q} \quad , \ (n=0,\pm1,\pm2,\dots). \tag{17}$$

The variational calculation [8] is formulated in terms of toroidal coordinates [10] μ, θ, φ:

$$x = \frac{a}{\tau} \sinh\mu \cos\varphi ,$$
$$y = \frac{a}{\tau} \sinh\mu \sin\varphi , \tag{18}$$
$$z = \frac{a}{\tau} \sin\theta ,$$

where a is an arbitrary scale parameter,

$$0 \leq \mu < \infty , \quad 0 \leq \theta \leq 2\pi , \quad 0 \leq \varphi \leq 2\pi ,$$
$$\tau \equiv \cosh\mu - \cos\theta \tag{19}$$

The volume element is

$$dx\, dy\, dz = \frac{a^3}{\tau^3} \, d(\cosh\mu)\, d\theta\, d\varphi. \tag{20}$$

Curves of constant μ are toruses. Those of constant θ are spheres. Their intersections with the z-x plane are shown in Fig.2.

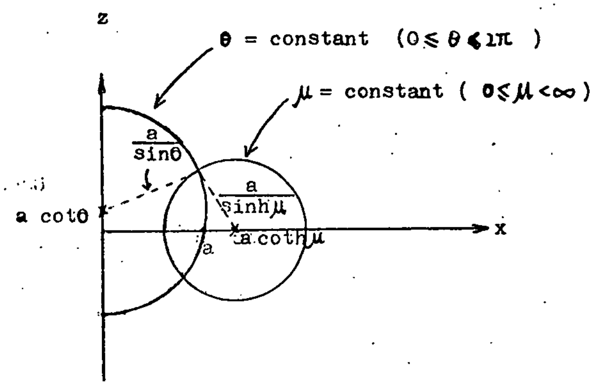

Fig.2 Toroidal coordinates. There is rotational symmetry about the z-axis. The surface of a vortex ring corresponds to $\mu=\mu_0$. The two variational parameters are a and μ_0.

A trial solution that satisfies all the conditions is

$$\phi(\mu,\theta) = \phi_0\, G(\mu)\, e^{-in\theta} , \quad (n=\pm1,\pm2,\cdots), \tag{21}$$
$$\vec{Z}(\mu,\theta) = \hat{\theta}\, \frac{n\tau}{qa} F(\mu) .$$

The conditions on the two real arbitrary functions $G(\mu)$ and $F(\mu)$ are

$$G(0) = 1 , \quad G'(0) = 0 .$$
$$F(0) = 1 , \quad F'(0) = 0 , \quad F(\infty) = 0 . \tag{22}$$

Using (21), we obtain

$$\vec{\nabla}\cdot\vec{Z} = \frac{\tau^3}{a^2} \left(\frac{n}{q}\right) F(\mu) \sin\theta ,$$
$$\vec{B} = -\hat{\phi} \frac{\tau^2}{a^2} \left(\frac{n}{q}\right) F'(\mu) . \tag{23}$$

The energy now reads

$$E = (2\pi)^2 \int_0^\infty d\mu \Big\{ \frac{1}{4a} \left(\frac{n}{q}\right)^2 (\sinh 2\mu)\, (F')^2$$
$$+ a\,\phi_0^2 \left[(G')^2 + n^2 (1-F)^2 G^2 \right]$$
$$+ a^3 \lambda\, \phi_0^4 \left(\frac{\cosh^2\mu + 1/2}{\sinh^4\mu} \right) (G^2-1)^2 \Big\} . \tag{24}$$

To make the calculation simple, choose

$$G(\mu) = F(\mu) = \begin{cases} e^{-2(\mu-\mu_0)} & (\mu > \mu_0) \\ 1 & (\mu < \mu_0) \end{cases} \tag{25}$$

Then

$$\vec{B} = \begin{cases} \hat{\phi}\, \frac{2\tau^2}{a^2} \left(\frac{n}{q}\right) e^{-2(\mu-\mu_0)} & (\mu > \mu_0) \\ 0 & (\mu < \mu_0) \end{cases} \tag{26}$$

The two variational parameters are <u>a</u> and μ_0, in which <u>a</u> describes the average radius of the vortex ring, and μ_0 describes its cross-sectional radius. Define a dimensionless radius R, and an energy scale E_0 :

$$R = ma\,, \quad m \equiv \left|\sqrt{2}\, q\phi_0\right|\,, \qquad E_0 = \frac{\pi^2 m}{q^2} = \sqrt{\frac{3}{2}}\,\frac{\pi^2\phi_0}{e} = 400\ \mathrm{GeV}/c^2. \tag{27}$$

Then

$$\frac{E}{E_0} = \frac{n^2}{R}\left(\alpha - \frac{1}{3\alpha}\right) + 2R\left(1+\frac{n^2}{24}\right) + \left(\frac{\lambda}{q^2}\right)R^3K(\alpha)\,, \qquad \alpha \equiv e^{2\mu_0}\,, \tag{28}$$

where

$$K(\alpha) \equiv \int_{\mu_0}^{\infty} d\mu\,\frac{\cosh^2\mu + 1/2}{\sinh^4\mu}\left[e^{-4(\mu-\mu_0)} - 1\right]^2 = 16\left[8\alpha^3+4\alpha^2+\frac{2}{3}\alpha+1+\frac{1}{\alpha-1}+8\alpha^2\left(\alpha^2-\frac{1}{4}\right)\ln\left(1-\frac{1}{\alpha}\right)\right]. \tag{29}$$

Variation with respect to R at fixed α gives

$$\frac{E}{E_0} = \frac{4}{3}\left[\frac{n^2}{R}\left(\alpha - \frac{1}{3\alpha}\right) + \left(1+\frac{n^2}{24}\right)R\right], \tag{30}$$

where

$$R^2 = \frac{1+n^2/24}{(3\lambda/q^2)K(\alpha)}\left\{\left[1+\left(\frac{n}{1+n^2/24}\right)^2\left(\alpha-\frac{1}{3\alpha}\right)\frac{3\lambda}{q^2}K(\alpha)\right]^{1/2}-1\right\}. \tag{31}$$

Variation with respect to α is done numerically.

Final results depend on the ratio of dimensionless coupling constants λ/q^2, whose limiting values have the following correspondence with superconductors:

$$\lambda/q^2 \longrightarrow \begin{cases} \infty & \text{Type II superconductor,} \\ 0 & \text{Type I superconductor.} \end{cases} \tag{32}$$

The results for n=1 are represented in Figs. 3-5.

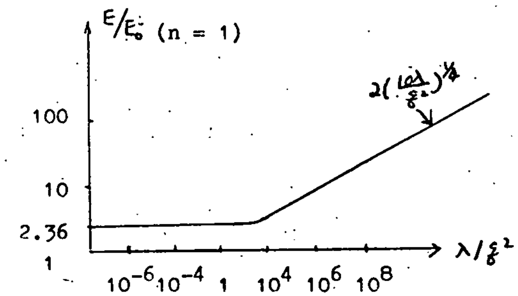

Fig3 Energy as function of λ/q^2

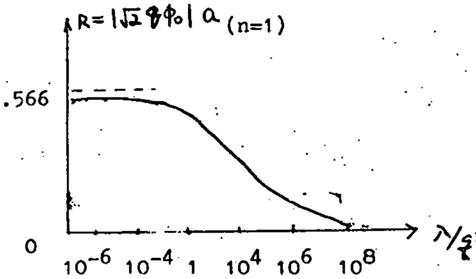

Fig.4 Mean radius parameter as function of λ/q^2

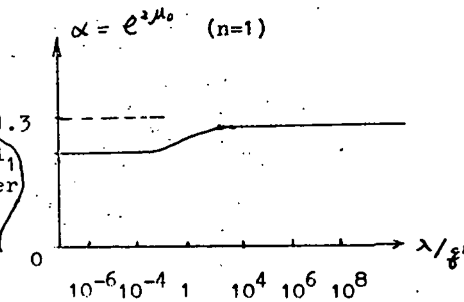

Fig.5 Cross-sectional radius parameter as function of λ/q^2

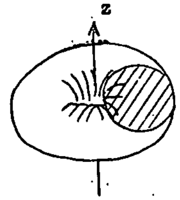

Fig. 6 The vortex ring is fat

Throughout the range of λ/q^2, α remains close to unity, which means μ_0 is relatively small. The vortex ring is always "fat", as depicted in Fig.6. The energy remains close to $2.36E_0$ for $\lambda/q^2<1$. For $\lambda/q^2>1$, it increases very slowly, like $(\lambda/q^2)^{1/4}$. The mass of the vortex ring is therefore greater than $1000 GeV/c^2$. When quantum effects are taken into account, it can decay into W's, Z^{0}'s, photons, lepton pairs, and hadron pairs. In particular, it can decay into hadronic jets. Other excitations include vortex rings with trapped leptons and quarks, and vortex rings made with the W field.

There is a wide range of high energy phenomena to be investigated, apart from and independent of the question whether the Higgs field is fundamental. Details of this and further calculations will be published elsewhere.

Footnotes

1. S.Weinberg, Phys. Rev.Lett.19,1264 (1967); A.Salam, in Elementary Particle Theory, ed. N.Svartholm (Almquist, Forlag AB, Stockholm,1968).
2. Experiment of W.Mo *et al* at Fermilab, 1979.
3. Perhaps the reason for $\sin^2\theta_W=\frac{1}{4}$ is that, like electric charge, neutral charge is quantized, owing to reasons yet unknown, such as the possibility of imbedding SU(2) x U(1) in a compact gauge group.
4. R.Jackiw and K.Johnson, Phys.Rev. D8, 2386 (1973).
5. J.M.Cornwall and R.E.Norton, Phys.Rev. D 8, 3338 (1973).
6. V.L.Ginsburg and L.D.Landau, Zh.Exp.Teor.Fiz. 20,1064 (1958).
7. S.Coleman and E.Weinberg, Phys.Rev. D 7, 1888 (1973).
8. in collaboration with my graduate student at M.I.T., Robert Tipton.
9. Vortex rings containing the flux of the charged W fields will not be considered here.
10. P.M.Morse and H.Feshbach, Methods of Theoretical Physics (McGraw Hill, New York) p.1301.