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ABSTRACT

The mean field theory of the chiral invariant σ-model is outlined. Bound states (solitons) of valence quarks are obtained self-consistently using a hedgehog shape for the pion field. A schematic model for the coupled fermion-boson fields is presented. Renormalization is worked out for the fermion one-loop corrections and numerical results presented for the purely scalar-field case. The interpretation of the baryon number of the perturbed vacuum is considered.

INTRODUCTION

Several speakers at this workshop will be discussing models which may or may not be appropriate to hadrons, based on either the Skyrme[1] Lagrangian density

$$\mathscr{L} = -\frac{f_\pi^2}{4}\,\mathrm{Tr}[L_\mu L_\mu] - \frac{\varepsilon^2}{4}\,\mathrm{Tr}[L_\mu, L_\nu]^2$$

$$L_\mu = U^+ \partial_\mu U\ , \quad U = \frac{1}{f_\pi}\left[\sigma(x) + i\vec{\tau}\cdot\vec{\pi}(x)\right] \qquad (1)$$

or else on the Hamiltonian density

$$H = \psi^+\left\{\vec{\alpha}\cdot\vec{p}c + g\beta(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right\}\psi$$

$$+\frac{1}{\hbar c}\left\{\frac{(\nabla\sigma)^2}{2} + \frac{(\nabla\vec{\pi})^2}{2} + W(\sigma^2 + \vec{\pi}^2)\right\} \qquad (2)$$

The latter, the σ-model introduced by Gell-Mann and Levy[2] in 1960 to describe the interactions between nucleons, has played an important role through the years, appearing in many guises. Most recently the above, related models have been used as effective field theories involving quarks rather than nucleons[3,4,5] and hopefully will describe the low mass hadron spectrum and low energy hadron interactions.

The correct theory, QCD, for which these effective theories presumably stand in, is a gauged vector theory. On the other hand, intuition based on the Dirac equation tells us that confinement requires the presence of an average scalar field, or the nonvanishing of average values of quadratic fermion operators and consequently implies the breaking of the chiral symmetry possessed by massless, fermion fields interacting with vector bosons. The simplest extension restoring chiral symmetry is realized in (2) with a transformation amongst the 4 scalar fields $\pi^\alpha = (\sigma, \pi^i)$ acting as counterpoint to the fermion transformation

$$\psi'_{R,L} = U_{R,L}\psi_{R,L} \tag{3}$$

The Lagrangian for (2) yields two conserved currents, a vector

$$J^\mu_\alpha(x) = \frac{1}{2}\,[\bar{\psi},\gamma^\mu\tau^\alpha\psi] + \varepsilon^{\alpha\beta\gamma}\pi_\beta\,\partial_\mu\pi_\gamma \tag{4a}$$

and an axial vector

$$J^\mu_{A\alpha}(x) = \frac{1}{2}\,[\bar{\psi},\gamma^\mu\gamma_5\tau^\alpha] + \;[\sigma\partial_\mu\pi^\alpha - \pi^\alpha\partial_\mu\sigma] \tag{4b}$$

One standard procedure for introducing a π mass, and breaking chiral symmetry, is by adding a term of the form $a\sigma$, which in turn yields

$$\partial_\mu J^\mu_{A\alpha} = -a\,\pi^\alpha \tag{5}$$

and from which flow many important low energy results.

One may also break chiral invariance spontaneously through an effective potential

$$W(\sigma^2 + \vec{\pi}^2) = \lambda(\sigma^2 + \vec{\pi}^2 - \sigma_o^2)^2 \tag{6}$$

which possesses a minimum for nonvanishing values of $\sigma(\vec{r}),\vec{\pi}(\vec{r})$. For sufficiently large λ one would in a classical approximation be close to the nonlinear σ-realization

$$\sigma^2 + \vec{\pi}^2 = \sigma_o^{\;2} \tag{7}$$

This spontaneous symmetry breaking is achieved quantum mechanically through introduction of the coherent state

$$\sigma(\vec{r})\;|\sigma'\rangle = \sigma'(\vec{r})\,|\sigma'\rangle\;,\quad \sigma' \text{ a c-number field} \tag{8}$$

and by minimization of the energy functional

$$E[\sigma'] = \langle\sigma',S\left|H\right|\sigma',S\rangle \qquad (9)$$

Where for example S may represent a filled Dirac Sea + n valence fermions. In the mean field approximation $E[\sigma',S]$ is quadratic in the Dirac field and an effective single particle Hamiltonian,

$$h = \vec{\alpha}\cdot\vec{p} + g\beta(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5)$$

which is straightforwardly diagonalized, completely describes the Dirac sector. Presumably the lowest energy occurs for a constant field $\sigma'(r) = \sigma_0$, $\vec{\pi}' = 0$ and thus for free fermions of mass $m = g\sigma_0$. Solutions describing fermions bound in localized fields $[\sigma(r),\vec{\pi}(r)]$ also exist[6,7,9,10]. We discuss the numerical search for these localized, chiral solitons, in stages, considering first only valence fermions, later treating also the perturbed Dirac Sea.

We note the similarity in spirit to the work of Friedberg and Lee[9], of Goldflam and Wilets[10] and even earlier work by Creutz[11], all attempting to lay field theoretic foundations for the various bag models. These latter treatments demonstrated the existence of localized self-consistent solutions even in a purely scalar formulation. In recent work by Birse and Banerjee[7] and by the present authors[6] it is shown the pion field radically alters the fermion and boson dynamics. One must point out here the pioneering work of Rho, Brown and collaborators[12], of Chodos and Thorn[13], indicating the importance of chiral symmetry to the bag models. These latter authors made signal use of the prototype soliton solution, the hedgehog, for which isospin and space are intimately coupled. In what follows we will successively develop the mean field solutions using the hedgehog ansatz: first with only valence quarks, then introduce one loop corrections to the energy and densities and finally consider a treatment of baryon number within this symmetry breaking approximation.

SOLITONS WITH VALENCE QUARKS

(A) It is convenient to treat the Hamiltonian in (1) in terms of dimensionless distances and fields defined by

$$\vec{x} = \frac{g\sigma_0}{\hbar c}\,\vec{r},\ \phi = \frac{\sigma}{\sigma_0},\ \vec{\chi} = \frac{\vec{\pi}}{\sigma_0} \qquad (10)$$

whence

$$\frac{H}{g\sigma_o} = \int d^3x \quad \psi^\dagger \left[\frac{\vec{\alpha} \cdot \nabla_x}{i} + \beta(\phi + i\,\gamma_5 \vec{\chi}\cdot\vec{\tau})\right]\psi$$

$$+ \frac{1}{g^2} \int d^3\vec{x} \left[\frac{(\nabla_x \phi)^2}{2} + \frac{(\nabla_x \vec{\chi})^2}{2} + \kappa(\phi^2 + \chi^2 - 1)^2 \right]$$

$$\kappa = \left(\frac{\hbar c}{g^2}\right)^2 \lambda \qquad (11)$$

If we impose the mean field approximation inherent in Eqs. [8,9] for some fixed number of valence quarks then searching for eigenstates of H reduces to solving

$$h\,|\lambda\rangle = \left[-i\vec{\alpha}\cdot\nabla_x + \beta(\phi + i\gamma_5 \vec{\tau}\cdot\vec{\chi})\right]|\lambda\rangle = \varepsilon_\lambda |\lambda\rangle \qquad (12)$$

and then minimizing

$$\frac{E}{g\sigma_o} = N\varepsilon_\lambda + \frac{1}{g^2}\int d^3x \left\{ \frac{(\nabla_x \phi)^2}{2} + \frac{(\nabla_x \vec{\chi})^2}{2} + \kappa(\phi^2 + \vec{\chi}^2 - 1)^2 \right\} \qquad (13)$$

Both the fermion level energies ε_λ and the total energy E are functionals of the chiral fields

$$\phi^a = [\phi, \chi^i]$$

Equivalently one can simultaneously solve Eq. (12) and the boson equations

$$-\frac{1}{g^2}\nabla_x^2\phi + 4\kappa\phi(\phi^2 + \vec{\chi}^2 - 1) + N\,\langle\lambda|\vec{x}\rangle\,\beta\,\langle\vec{x}|\lambda\rangle = 0 \qquad (13a)$$

$$-\frac{1}{g^2}\nabla_x^2\chi^i + 4\kappa\chi^i(\phi^2 + \vec{\chi}^2 - 1) + iN\,\langle\lambda|\vec{x}\rangle\beta\gamma_5\tau_i\,\langle\vec{x}|\lambda\rangle = 0 \qquad (13b)$$

We recover the physical vacuum, a state of free Dirac particles of mass $m = g\sigma_o$ at spatial infinity through

$$\phi(\vec{r}) \to 1, \quad \chi^i(\vec{r}) \to 0 \quad \text{as} \quad x = |\vec{x}| \to \infty \qquad (14)$$

It is always possible from Eq. (13) to achieve a bound or "confined state" of quarks + fields if:

(a) the potentials are sufficiently strong to bind single particle
states i.e. $|\lambda\rangle$ exists so that $\varepsilon_\lambda < 1$

and (b) $1/g^2$ is sufficiently small

Once such solutions exist one can scale the energy and size consistent with Eq. (10).

The solution with minimum energy found here, and by others[1,3,5,7] in a similar context, seems to possess the symmetry breaking "hedgehog" form

$$\phi(\vec{r}) = \phi(r) \ , \quad \vec{\chi}(\vec{r}) = \frac{\vec{r}}{r}\,\chi(r) = \hat{r}\chi(r) \tag{15}$$

The Dirac equation is then most easily handled in a representation diagonalizing $\vec{K} = \vec{J} + 1/2\vec{\tau}$, with for example K = 0 "s-wave" states having the form

$$\langle x|\lambda\rangle = \psi_\lambda(\vec{r}) = \begin{bmatrix} ix^{-1}F_\lambda(x)|\nu\rangle \\ \\ x^{-1}G_\lambda(x)\hat{x}\cdot\vec{\sigma}|\nu\rangle \end{bmatrix} \tag{16}$$

with $\vec{K}|\nu\rangle = 0$, $\langle\nu|\nu\rangle = 1$. Radial forms for $F_\lambda(x)$, $G_\lambda(x)$ are discussed in reference [6] in terms of which

$$\varepsilon_\lambda = \int_0^\infty dx\ \{\phi(F_\lambda^2 - G_\lambda^2) - 2\chi F_\lambda G_\lambda - F_\lambda(\partial_x + \frac{1}{x})G_\lambda$$
$$+ G_\lambda(\partial_x - \frac{1}{x})F_\lambda\} \tag{17a}$$

and

$$\frac{E}{g\sigma_o} = N\varepsilon_\lambda + \frac{4\pi}{g^2}\frac{1}{2}\int_0^\infty dx\ \{(\frac{\partial\phi}{\partial x})^2 x^2 + (\frac{\partial\chi}{\partial x})x^2 + 2\chi^2\}$$
$$+ \frac{4\pi}{g^2}\kappa\int_0^\infty dx\ \chi^2(\phi^2 + \chi^2 - 1)^2] \tag{17b}$$

(B) Restriction to the "Chiral Circle"

Soliton like solutions for the purely scalar theory $\vec{\chi} = 0$ were, of course, demonstrated to exist by Friedberg and Lee. In this latter case the explicit form of the rather arbitrary potential $W(\sigma^2)$ is crucial. One must penetrate the potential from $\sigma = \sigma_o$ towards $\sigma = 0$ (or perhaps to $\sigma = -\sigma_o$). For sufficiently large λ in Eq. (6) however, one is restricted to the chiral circle

$$\sigma^2 + \vec{\pi}^2 = \sigma_o^2 \tag{18a}$$

and the dynamics is quite different. With the obvious parametrization

$$\phi(r) = \cos\Theta(r), \quad \chi(r) = \sin\Theta(r) \tag{18b}$$

subject to $\theta(x) \to 0$ as $x \to \infty$, minimization of the energy may be obtained by simply varying the form of $\theta(x)$ along (17). The energy now reads

$$\frac{E}{g\sigma_o} = N\varepsilon_\lambda + \frac{4\pi}{g^2}\frac{1}{2}\int_0^\infty dx \left\{x^2 \left(\frac{d\theta}{dx}\right)^2 + 2 \sin^2\theta\right\} \tag{19}$$

with $\varepsilon_\lambda = \varepsilon_\lambda [\theta(x)]$ but <u>independent</u> of g. The field equation for θ is

$$-\frac{d}{dx}\left(x^2 \frac{d\theta}{dx}\right) + \sin 2\theta + \frac{Ng^2}{2\pi}\frac{d\varepsilon_\lambda}{d\theta} = 0 \tag{20}$$

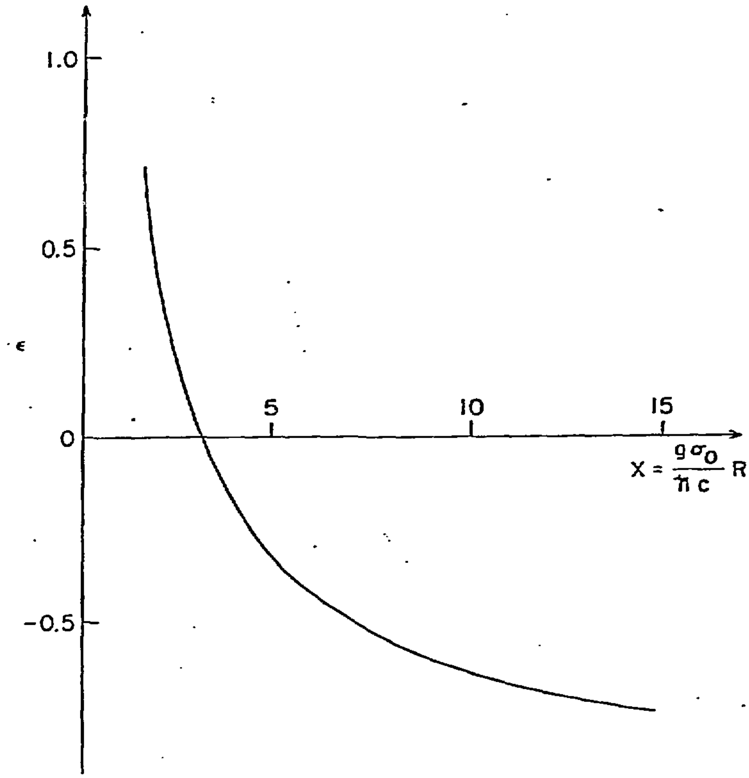

Fig. 1

Figure 1: Variation of the valence orbital energy ε with the distance X in the soluble model. The curve is fitted by Eq. (5.1).

From the latter it is easily shown

$$\Theta(r) \to x - n\pi \quad \text{for } x \to 0$$
$$\text{and } \Theta(r) \to \frac{b}{x^2} \text{ for } x \to \infty \tag{21}$$

Self-consistent solutions of our coupled field equations are found on the chiral circle by expanding

$$\Theta(x) = \sum_n C_n \, j_o(\frac{n\pi x}{\Delta}) \tag{22}$$

within a spherical box of radius Δ, solving for the Dirac states in a finite basis as functionals of C_n and finally by minimizing the energy with respect to the C_n. A schematic calculation for a simple analytic form of $\Theta(x)$ satisfying the boundary conditions (21), and possessing many features of the self-consistent solution, is given by the choice (see Fig. 1)

$$\Theta(x) = \frac{\pi(X - x)}{x} \tag{23}$$

This yields

$$\varepsilon_o = \frac{3.12}{X} - 0.94, \text{ valid for } 2 \lesssim X \lesssim 12 \tag{24}$$

and an effective mass formula (see Figs. 2 and 3)

$$\frac{E}{g\sigma_o} = \frac{3.12N}{X} - 0.94N + \frac{4\pi}{g^2}\left(\frac{1}{2} + \frac{\pi^2}{6}\right) X \tag{25}$$

Curiously there is no volume dependence of the energy in Eq. (25) as is generally found in the MIT-like bag models[14] or purely scalar soliton theories[9,10]. A volume energy would result, however, if for example one permits fluctuations in the boson fields away from these strong coupling solutions on the chiral circle. The specific form of the potential $W(\sigma^2 + \pi^2)$ then becomes of importance, the fluctuations fighting against the slopes of the wall at $\phi^2 + \chi^2 = 1$. Chiral invariance is broken throughout space in the present treatment; no sharp division between two phases being evident (at least for finite $g\phi_o$). Fig. 2 indicates the dependence of orbital energies on the size parameter X and in particular exhibits the important asymmetry between positive and negative energies.

A minimum energy in Eq. (25) obtains for

$$X = 0.59\ g \tag{26}$$

and takes the value

$$E_{min} = N\sigma_o\ [10.6 - 0.94\ g] \tag{27}$$

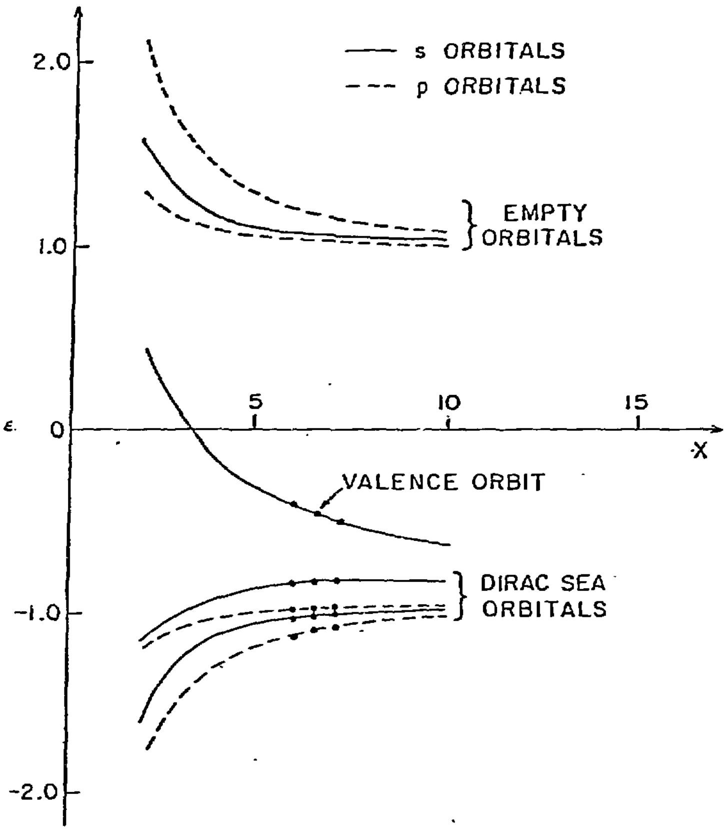

Fig. 2

Figure 2: Energies of the s (full line) and p (dashed line) orbitals as a function of the distance X in the soluble model. The valence and Dirac Sea orbitals are filled by a maximum number of N = 3 quarks each. The figure shows that no orbits cross despite the possible negative energy for the valence orbital. Only s and p orbitals for spin and isospin coupled to zero are shown. The orbitals are calculated by diagonalizing the *Dirac Hamiltonian* (12) in a finite basis. Because of this energies of unbound orbitals $|\varepsilon\rangle$ have no physical meaning.

In this schematic calculation the coupling must not exceed $g_{max} \approx 10.6/0.94$ if the vacuum is to remain stable with respect to the addition of quarks.

Because of the scaling discussed previously we can find values of σ_o,g so as to obtain a given radius and equilibrium energy. Table I displays the energies and sizes of solitons resulting from arbitrarily fixing the equilibrium energy to be $E_{min} = 1$ GeV. The values of $\sigma_o = f_\pi$ are also shown. Table II is an equivalent presentation for the self-consistent solution under the same assumption for the total energy. Included in this table are the "size" for the quark density defined by

$$\langle x^2 \rangle = \int d^3x\, \rho(x)\chi^2, \quad \int d^3x\, \rho(x) = N = 3.$$

Fig. 3 illustrates some properties of the self-consistent densities and pion fields for particular choice of g, σ_o, while Fig. 4 displays the emergence of quark bound states from the Dirac (positive and negative) continua as a function of the coupling constant g. For the self-consistent model the single particle energy plunges near $g \approx 10$.

ONE LOOP CORRECTIONS: THE DIRAC SEA

If one treats the physical vacuum in the limit $\pi^i(r) = 0$, $\sigma(r) = \sigma_o$ and fills the negative energy states then the vacuum energy is

$$E_o = -\sum_{k_n}^{\Lambda} E_o(k_n)$$

with $E_o(k_n) = \sqrt{k_n^2+(g\sigma_o)^2}$ in a plane wave limit or in a spherical box with appropriate boundary conditions which render the k_n discrete. Λ is an ultraviolet cutoff. Perturbing this vacuum of free fermions with a non-zero pion field changes the energy of the valence free state to

$$E = -\sum_{n^-}^{\Lambda} E_n$$

with $E_\lambda = m\varepsilon_\lambda$ the eigenvalues arising from solutions to Eq. (12). One then expects as a correction to the total energy

$$-\Delta E = -\sum_{n^-} E_n + \sum_{k^-} E_o(k). \qquad (28)$$

This energy is the fermion one-loop correction obtained from diagonalization of

$$h = \vec{\alpha}\cdot\vec{p} + g\beta(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5) = ho + \beta\hat{V} \qquad (29)$$

Table I Results of the soluble model for successive values of the coupling constant g. The value of σ_o is always chosen so as to make the energy E in Eq. (27) equal to 1 GeV. The second column gives the equilibrium values of X beyond which the chiral angle vanishes. The third column gives the valence orbital energy in dimensionless units. The last column gives $R = (hc/g\sigma_o)X$ in fm.

g	X	ε	$\frac{E}{g\sigma_o}$	σ_o MeV	$g\sigma_o$ GeV	R fm
5	2.95	0.12	3.53	60	0.28	2.05
6	3.54	-0.06	2.48	70	0.40	1.73
7	4.13	-0.18	1.72	80	0.58	1.40
8	4.71	-0.28	1.15	110	0.87	1.07
9	5.30	-0.35	0.71	160	1.41	0.74
10	5.89	-0.41	0.36	280	2.80	0.42
11	6.48	-0.46	0.07	1330	14.61	0.09

Table II Results of the self-consistent calculation for successive values of the coupling constant g. The value of σ_o is always chosen so as to make the total energy E in Eq. (19) equal to 1 GeV. The second and seventh columns give the quark density mean square radius in dimensionless units (2.2) and in fm, respectively. The last column gives the chiral field energy (second term of 19) in dimensionless units.

g	$\sqrt{\langle x^2\rangle}$	ε	$\frac{E}{g\sigma_o}$	σ_o MeV	$g\sigma_o$ GeV	$\sqrt{\langle r^2\rangle}$ fm	$\frac{E(\phi,\chi)}{g\sigma_o}$
4	2.40	+0.42	3.71	67	0.27	1.76	2.42
5	2.75	+0.15	2.60	77	0.39	1.41	2.14
6	3.14	-0.037	1.78	94	0.56	1.10	1.89
7	3.55	-0.17	1.16	122	0.85	0.82	1.68
8	3.95	-0.27	0.69	180	1.44	0.54	1.51
9	4.38	-0.35	0.32	352	3.17	0.27	1.37
10	4.78	-0.41	.0082	12144	120	0.0078	1.25

and is exactly given by

$$\Delta E = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \operatorname{tr} \ln[(h - \omega)\frac{1}{h_0 - \omega}] \tag{30a}$$

$$= \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \operatorname{tr} \ln[1 + \frac{1}{-\gamma p + m} \hat{V}] \tag{30b}$$

with

$$p^{\mu} = (\omega, -i\nabla)$$

and

$$\hat{V} = V - g\sigma_o \tag{30c}$$

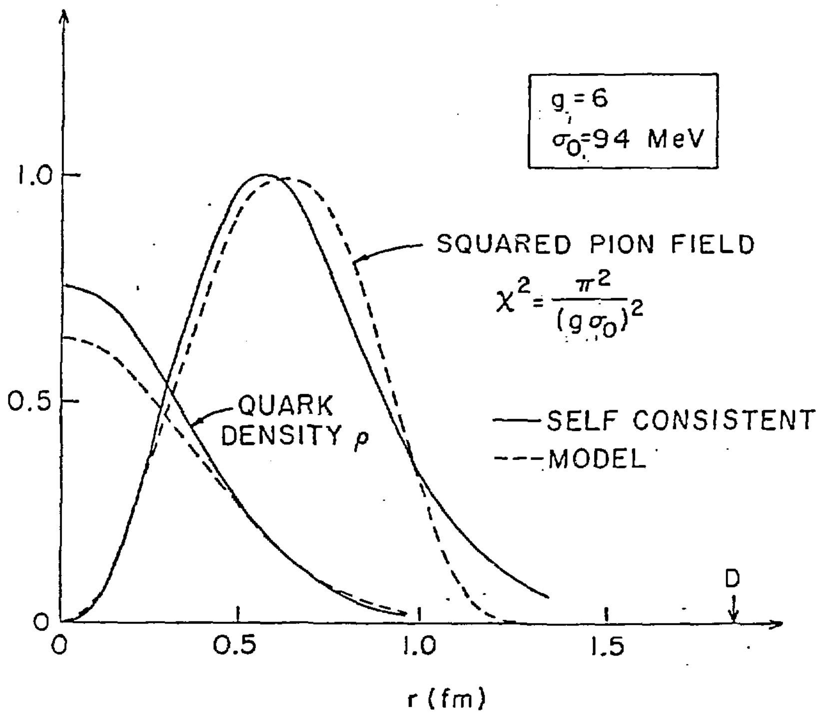

Fig. 3

Figure 3: Comparison between the model (dashed line) and self-consistent (full line) calculations. The quark densities and squared pion fields are compared.

The trace in Eqs. (30) is taken over all Dirac variables, SU(2) isotopic variables, colour if included explicitly and over a complete set of configuration states. ΔE is a functional of the perturbing fields ϕ^a, adds a contribution to the effective potential $W(\phi^a\phi_a)$ and hence also to the energy to be minimized. Of course, as it stands ΔE is not defined, being ultraviolet divergent. It is, however, a straightforward task in a plane wave theory to introduce counter terms to renormalize ΔE. In our finite spherical box, with a truncated Hilbert space $\{\lambda\}$ some care must be exercised. Our procedure is to calculate ΔE by diagonalization in a suitably truncated space and to use the

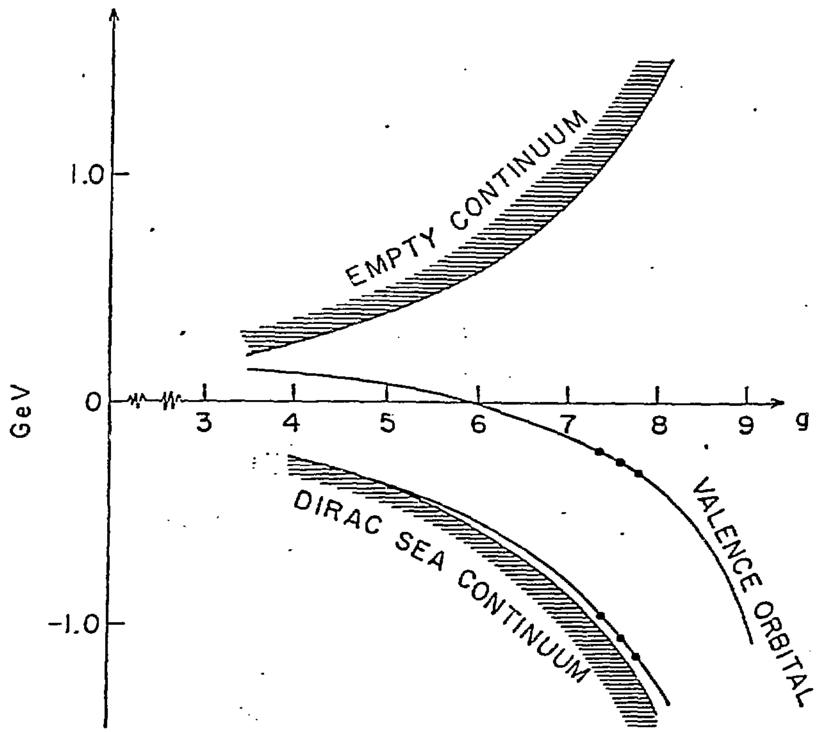

Fig. 4

Figure 4: The emergence of bound orbitals from the Dirac continuum as the coupling constant g increases (self-consistent calculation). Only s and p orbitals were calculated and only s orbitals yield a bound state.

one-loop expression to extract the infinite parts of ΔE. One may then stop with the fourth order term in the perturbation V. Schematically the quadratic divergence in an effective plane wave cutof Λ is cancelled in the difference in Eq. (28), leaving divergences of order Λ^2 and $\ln\Lambda$.

Indeed one finds $(\Delta E)_\infty$

$$(\Delta E)_\infty = \sum_{n^-} \Big\{ -\frac{1}{E_0(k)} \langle n^- \big| [mg\hat{\sigma} + \frac{1}{2} g^2(\hat{\sigma}^2 + \vec{\pi}^2)] \big| n^- \rangle$$

$$+ \frac{1}{E_0(k)^3} \langle n^- \big| [\frac{m^2}{2} g^2 \hat{\sigma}^2 + g^2 (\nabla\sigma)^2 + g^2 (\nabla\vec{\pi})^2 \qquad (31)$$

$$+ \frac{mg^3}{2} \hat{\sigma}(\hat{\sigma}^2 + \vec{\pi}^2) + \frac{g^4}{8} (\hat{\sigma}^2 + \vec{\pi}^2)^2] \big| n^- \rangle \Big\}$$

with $\hat{\sigma} = \sigma - \sigma_0$.

In appropriate units ΔE reduces to

$$(\Delta E)_\infty = -\frac{1}{2} \sum_{n^-} \frac{1}{\varepsilon_0(k)} \langle n \big| (\phi^2 + \vec{\chi}^2 - 1) \big| n^- \rangle$$

$$+ \frac{1}{8} \sum_{n^-} \frac{1}{\varepsilon_0(k)^3} \langle n \big| [(\phi^2 + \vec{\chi}^2 - 1)^2 + \big((\nabla\phi)^2 + (\nabla\vec{\chi})^2 \big) \big| n \rangle \qquad (32)$$

The latter expression may then be used as a counterterm. An additional finite renormalization proportional to the gradients of ϕ^a is removed to properly set the renormalized value of the coupling constant g. Several points are of interest:

(1) For the quadratic divergence ($\sim 1/\varepsilon_0$) one must treat $\phi^a \phi_a$ as matrix products in the truncated space.

(2) The non-gradient terms leading to infinite quantities vanish formally on the chiral circle. To exploit this vanishing it is probably best to perform the local rotation

$$\psi' = e^{\frac{i\theta(r)\vec{\tau}\cdot\vec{\pi}\gamma_5}{2}} \psi \qquad (33)$$

on the Dirac single particle equation.

At present we have not completed the numerical evaluation of (31), (32) except for the pure scalar case. Fig. 5 shows a graph of the convergence of ΔE with an effective cut-off in the case of a Lee-Friedberg soliton, and indicates the possible destabilization of this soliton.

One, of course, expects similar boson one-loop corrections due to vacuum fluctuations in the scalar and pseudo-scalar fields. It is possible that an equivalence of the theory described here with the Skyrme theory of Witten and collaborators[3], and Rho and Brown[5] may be achieved by expanding the one-loop (and higher order) corrections in the appropriate derivatives of V. We note finally in this section that the authors of these latter references regard their theories as arising from the large N_c limit of QCD.

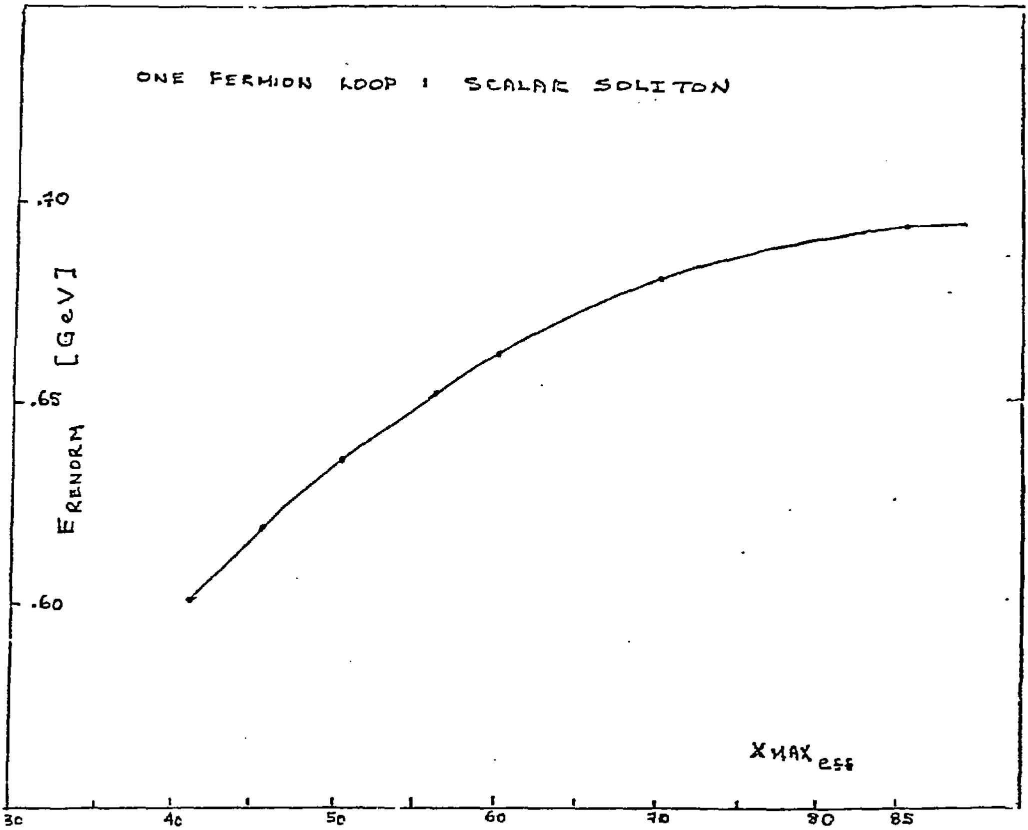

Fig. 5

Figure 5: The renormalized energy

$$-\sum_{n^-} E_n + \sum_{n^-} E_o(k) - \text{CTRM}$$

near a soliton solution in a scalar field theory with one fermion loop additions. No correction has been made here for finite gradient renormalization nor have one-boson-loops been included.

BARYON NUMBER IN THE CHIRAL MODELS

A very interesting feature of the one-loop addition to the purely valence quark theory is the implied effect on quark density. The correction ΔE might be referred to as the energy of an empty bag. Should a soliton solution still obtain in the presence of one-loop corrections to the valence quarks one might naively expect the detailed density distribution of quarks to be locally altered by vacuum polarization but the total quark nu... ...r to remain the same. One might expect for example in the perturbed vacuum obtained from the physical vacuum by the perturbation

$$\hat{V} = V - g\sigma_o = g(\sigma - \sigma_o) + ig\gamma_5\vec{\tau}\cdot\vec{\pi} \tag{34}$$

that

$$B = \int d^3x \langle J^o(x)\rangle = 0 \tag{35}$$

Several authors, Goldstone and Wilczek[15], Goldstone and Jaffe[16] have suggested otherwise. These authors obtain for the present chiral σ-model of Eqs. (2, 18a), treating the bosons as external fields,

$$\begin{aligned}\langle J^\mu(x)\rangle &= \frac{1}{12\pi^2}\,\varepsilon^{\mu\alpha\beta\gamma}\,\varepsilon_{dabc}\,\phi^d\,\partial_\alpha\phi^a\partial_\beta\phi^b\partial_\gamma\phi^c \\ &= \frac{1}{12\pi^2}\,G^\mu(x)\end{aligned} \tag{36}$$

This result given in reference [15] may be derived by appropriate functional differentiation of the one loop energy (30b) and by retaining the first non-vanishing (finite) term in an expansion in the derivatives $\partial_\alpha\phi^a$. For our truncated spaces

$$\langle J^\mu(x)\rangle = \frac{20}{2^6}\frac{\nu}{\Omega}\sum_{n^-}\frac{1}{\varepsilon_o(k)^7}\,G^\mu(x) \tag{37}$$

with ν the degeneracy of filled Dirac states. Moreover it would also seem naively

$$B = \int d^3x \langle J^o(x)\rangle = \int d^3x \sum_{n^-}\left\{[\Psi_n^\dagger\Psi_n]_x - (\psi_n^\dagger\psi_n)_x\right\} \tag{38}$$

where Ψ_n are the perturbed states and ψ_n the "physical " vacuum states. Whence a non-zero result is possible only if the integral and summation in Eq. (38) are not interchangeable. Goldstone and Jaffe [16] obtain a result for B in the case of a bag with a modified chiral boundary condition. The results in this latter reference and in the work on the Skyrme Lagrangian[3,4,5] suggest a strong topological connection of the baryon number with the chiral angle θ through

$$B = \chi\frac{1}{\pi}\left[\Theta(r) - \frac{1}{2}\sin 2\Theta(r)\right]_o^\infty$$

These very interesting speculations clearly so important for specifying the total quark number and the topological nature of the soliton solutions continue to be examined by many authors.

One last comment about our numerical approach seems relevant to the density problem. The solutions we found for $\Theta(r)$ satisfy $\Delta\theta/\pi = 1$ yet the energy level diagram in Fig. 2 indicates a crossing to negative energy of a valence orbit only for sufficiently large X (in the schematic description). Goldstone and Jaffe [16] find to the contrary the magnitude of $\Delta\theta$ controls the energy level position more directly, producing a crossing at $\Delta\Theta = \pi/2$. If the chiral soliton σ-model and the chiral bag model are to agree their convergence is to be found in the finite part $(\Delta E)_{renorm}$.

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