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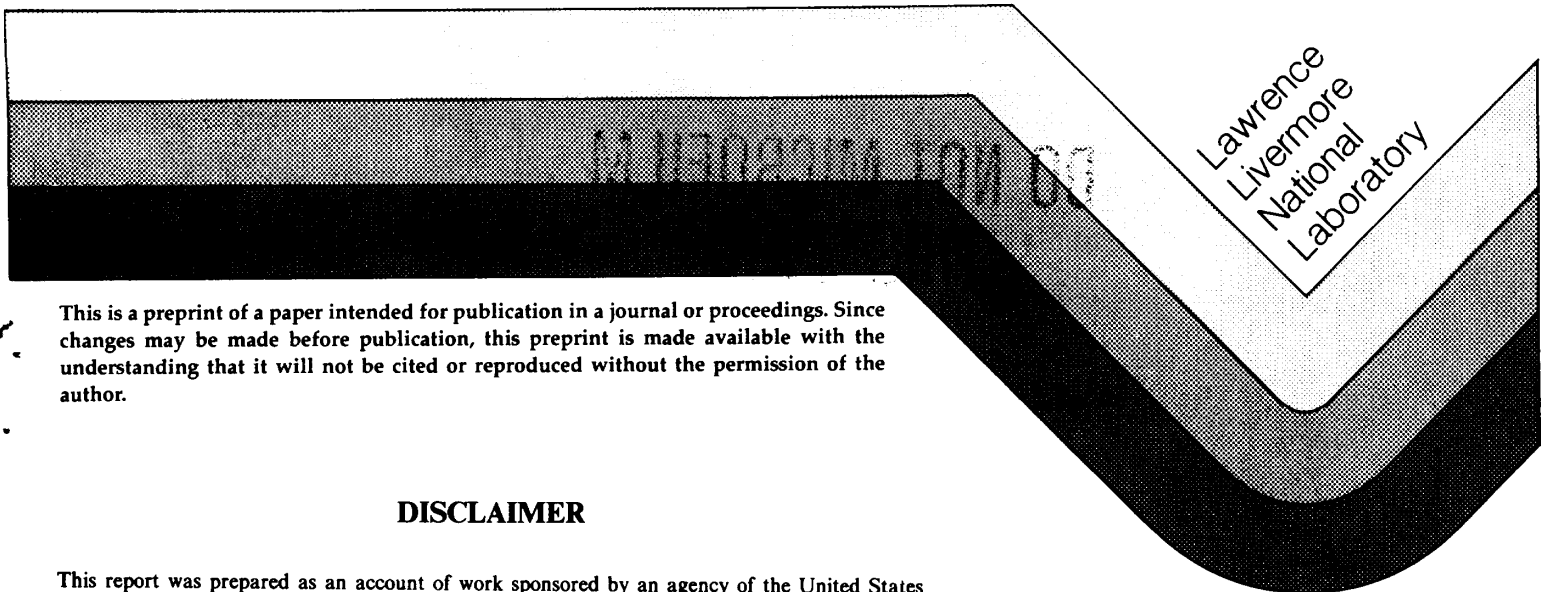
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AN OVERVIEW OF PROJECTILE PENETRATION INTO GEOLOGICAL MATERIALS, WITH EMPHASIS ON ROCKS

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ABSTRACT

An extensive study was performed of the experimental and analytical aspects of projectile penetration in rocks and soils. The experimental data base is much larger for soils than it is for rocks, in which few instrumented penetration tests have been performed.

To extrapolate experimental results, several methods of analysis and prediction have been proposed: empirical approaches, such as those of Sandia National Laboratories and the Army's Waterways Experiment Station, analytical methods, such as cavity expansion theories and the differential area force law, and numerical modeling by a variety of techniques, i.e., finite differences, finite elements, and discrete elements. This paper contains a comprehensive summary of the features of the various computer programs used for penetration modeling, in many materials and at various speeds, over the past 20 years.

Regarding rock targets, the most significant conclusions to be drawn are:

- cracks and joints are ubiquitous on most rocks, and can easily overshadow the intact rock's yield strength in influencing penetration. So, it is clear that appropriate site characterization for penetration estimates must include the geological structure at the scale of the penetration.
- for analysis, the most desirable rock strength formulation is that which describes the complete variation of shear strength with mean stress.
- at velocities of up to a few hundred metres/s, rock penetration is most dependent on shear strength, which is pressure-dependent; it is less dependent on tensile strength, and compressibility.
- it appears essential to incorporate in the models the comminution of rock and post-fracture properties of the broken material.

- the internal friction angle of the target is more important than its cohesive strength in controlling penetration.
- however, a very uncertain aspect of the penetration process is the amount of frictional force applied to the penetrator; and this is compounded by uncertainties on the values of metal/rock friction. A power law variation of friction angle with sliding velocity has been proposed and a few data have been reported for tuff, sandstone and limestone by a single investigator. For tuff, the shear stress τ is given as equal to the static coefficient of friction m multiplied by an equivalent normal stress: $\tau = \mu \sigma_{\text{eff}}$. The σ_{eff} is equal to $\sigma \cdot e^{-c/\xi}$ where σ is the actual normal stress, c is the sliding velocity, and ξ is found to be equal to 2 GPa m/s for the 3 rocks tested; interestingly, ξ is reported to be the same for a wet and a dry sandstone. Regardless of the functional form of μ , it appears advisable to recognize the velocity-dependent nature of friction and to reliably estimate the contact area between the ground and the penetrator.
- measurements of stresses and deformations in the medium are what is needed to evaluate the material models used; measuring only the penetrator deceleration and depth is not sufficient.
- the stresses induced around the penetrator diminish rapidly away from the body; an order of magnitude decay takes place over a radial distance of about 2.5 times the projectile diameter. This gives the scale of the volume of target material involved in controlling penetration.
- cavity expansion theories give higher contact stresses on the penetrator than finite element models, for example, because of artificial kinematic constraints, and lack of surface weakening.
- penetration depth for rock (and concrete) appears to scale linearly with the ratio of penetration weight over cross-sectional area.

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1. PROJECTILE PENETRATION THEORIES

The foundations of penetration theories currently used were laid at least 10 years ago, and some of them can be traced to the 1940's [B11]*, notwithstanding the work of 18th and 19th century investigators. The materials of interest have been metals, concrete, and a wide variety of geologic media (ice, rocks, boulders, soils, permafrost, snow, ...). An outstanding state-of-the-art survey was prepared in the 1970's [T9]. As for geological penetration data, a compilation of many tests results by Sandia National Laboratories (SNL), over the past 25 years, has recently been formalized in a relational data base [C5]. Also, the proceedings of a recent Earth Penetration Phenomenology Meeting contain much updated material [U2].

Today, there are essentially 3 approaches to predicting the penetration of projectiles in geological materials:

- the empirical methods, such as those of SNL, Albuquerque, NM, and of the Army's Waterways Experiment Station (WES), Vicksburg, MS.
- analytical methods such as the Cavity Expansion Theories (CET) and the Differential Area Force Law (DAFL),
- and numerical modeling by a variety of methods: finite elements, finite differences, and discrete elements. Both Lagrangian and Eulerian approaches have been used with finite elements and finite differences.

Occasionally, mixed methods are used, such as in the recent coupling of the ABAQUS finite element code for the penetrator, with a cavity expansion theory for the ground [L4], or in the linking of the Eulerian finite difference code, HULL, to a subsequent Lagrangian finite element analysis with EPIC3, for the 3-dimensional modeling of hypervelocity perforation of a plate by a rod [M1,M2].

Other methods exist, but are not presented here since they have not been used to any large extent [A1, A3, B12, T3, T8]; most are discussed in reference [T9].

*Because of the large number of references, an indexing scheme was adopted which minimizes corrections to be made when this bibliography is updated.

1.1 The Empirical Method

There are two main schools: that of SNL and that of WES.

In the SNL formulation for rock (and concrete) [Y9, Y11], the maximum penetration depth is:

$$Z = 1.14 \cdot 10^{-6} \cdot S \cdot N (W/A) (V-100) \quad \text{SI units} \quad (1)$$

where S is the target penetrability number, a measure of the rock's resistance

N is a nose performance coefficient

W is the penetrator's weight

A is the penetrator's cross-sectional area

V is the initial impact velocity.

The S number for rocks is said to be smaller than or equal to 1, depending upon rock mass strength. Note that there is no explicit mention of σ_c or of joint density, and the rock property (S) cannot be obtained through direct testing. On the other hand, the S numbers for soils are much better defined as a function of the soil type; this is a reflection of the large slant towards soils in the data base behind equation (1). Equation (1) is embodied in the SAMPLL code [Y9] and in the MOLE code [J1]. SAMPLL also contains equations for penetration in soils, ice, and marine sediments.

Other empirical formulas which do not include natural fractures are also available [A1, K1].

Two WES formulations for rock penetration [B8] were first proposed in lieu of the SNL equation, so that the rock discontinuities would be somewhat accounted for. The first one is:

$$Z = 0.2 \frac{M}{A} \cdot \frac{V}{(\rho \sigma_c)^{1/2}} \cdot \left(\frac{100}{RQD} \right)^{0.8} \quad (2) \quad \text{SI and English units}$$

where ρ is the mass density of the rock (unit weight/gravity acceleration), and M is the mass of the projectile.

RQD is the "Rock Quality Designation" of the rock mass; it is a measure of the spacing of pre-existing fractures, at the site [D2]. Other quantities are as previously defined. A number of restrictions apply to equation (2), as discussed in [B7]; among them, the equation should not be used for RQD less than 20.

The second equation is a little bit more elaborate:

$$Z = \left[\frac{M}{A} \frac{V}{b} - \frac{a}{b^2} \ln \left(1 + \frac{b}{a} V \right) \right] \quad (3), \quad \text{where}$$

$$a' = 1.6 \sigma_c (RQD/100)^{1.6} \quad (4)$$

$$b' = 3.6 (\rho \cdot \sigma_c)^{1/2} (RQD/100)^{0.8} \quad (5)$$

A comparison of equations (2) and (3)-(5) with rock penetration data is given in Figure 1 [B8]. Over the range of dimensionless parameters considered and for the various rocks penetrated, the two equations seem to be fairly close and to give credible estimates.

Shortly thereafter, an improved formula was proposed by WES [B10] as:

$$Z = \frac{M}{A} \cdot \frac{N_{rc}}{\rho} \left[\frac{V}{3} \frac{\rho^{1/2}}{\sigma_{cr}^{1/2}} - \frac{4}{9} \ln \left(1 + \frac{3}{4} V \frac{\rho^{1/2}}{\sigma_{cr}^{1/2}} \right) \right] \quad (6)$$

$$\text{with } N_{rc} = 0.863 \left[\frac{4(CRH)^2}{4CRH-1} \right]^{1/4} \quad \text{for ogives} \quad (7)$$

$$N_{rc} = 0.805 \cdot (\sin \eta_c)^{-1/2} \quad \text{for cones} \quad (8)$$

$$\sigma_{cr} = \sigma_c \cdot (RQD/100)^{0.2} \quad (9)$$

where N_c is the projectile nose performance coefficient

CRH is the Caliber-Radius-Head (ratio of radius of curvature of the tangent ogive to the diameter)

η_c is the cone half-angle

σ_{cr} is the rock mass unconfined compressive strength.

This new equation was applied to concrete as well as to the rock data of Figure 1; it appeared to give a somewhat better fit, as shown on Figure 2.

The implication of equations (2), (3), or (6), is that one of either σ_c or RQD can be back-estimated from rock penetration tests, if the other one is measured or otherwise known.

SYMBOL	TARGET	RQD	AVERAGE STRENGTH BARS	AVERAGE DENSITY gm/cm ³
Δ	WELDED TUFF	100	600	1.95
□	SANDSTONE	82	234	2.08
▲	WELDED AGGLOMERATE	60	275	1.92
●	SANDSTONE	37	489	2.12
■	SANDSTONE	32	408	2.14
▼	GRANITE	32	462	2.62

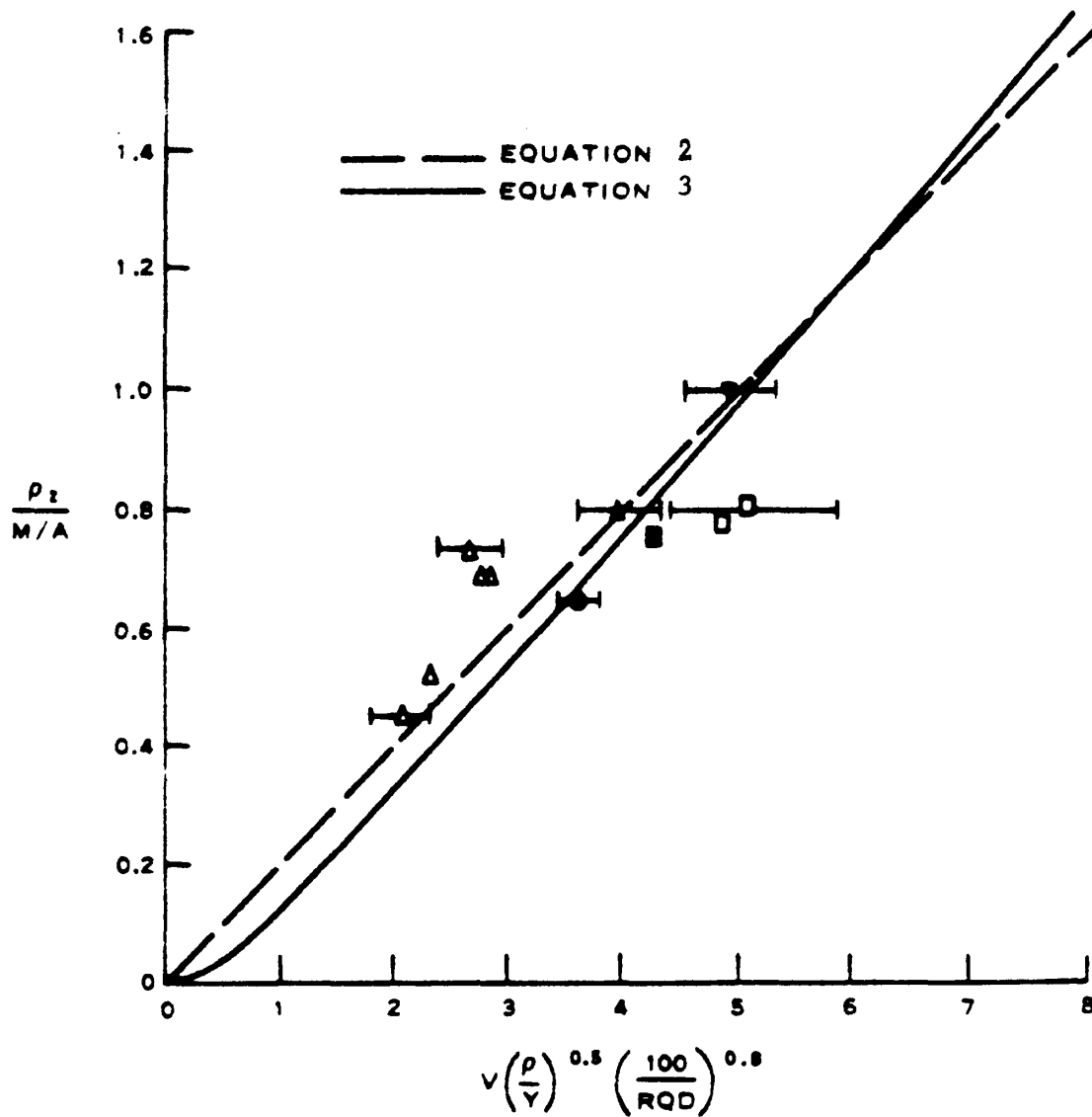


Figure 1: Comparison of Initial WES Rock Penetration Equations with Rock Penetration Data. After Bernard [B8].

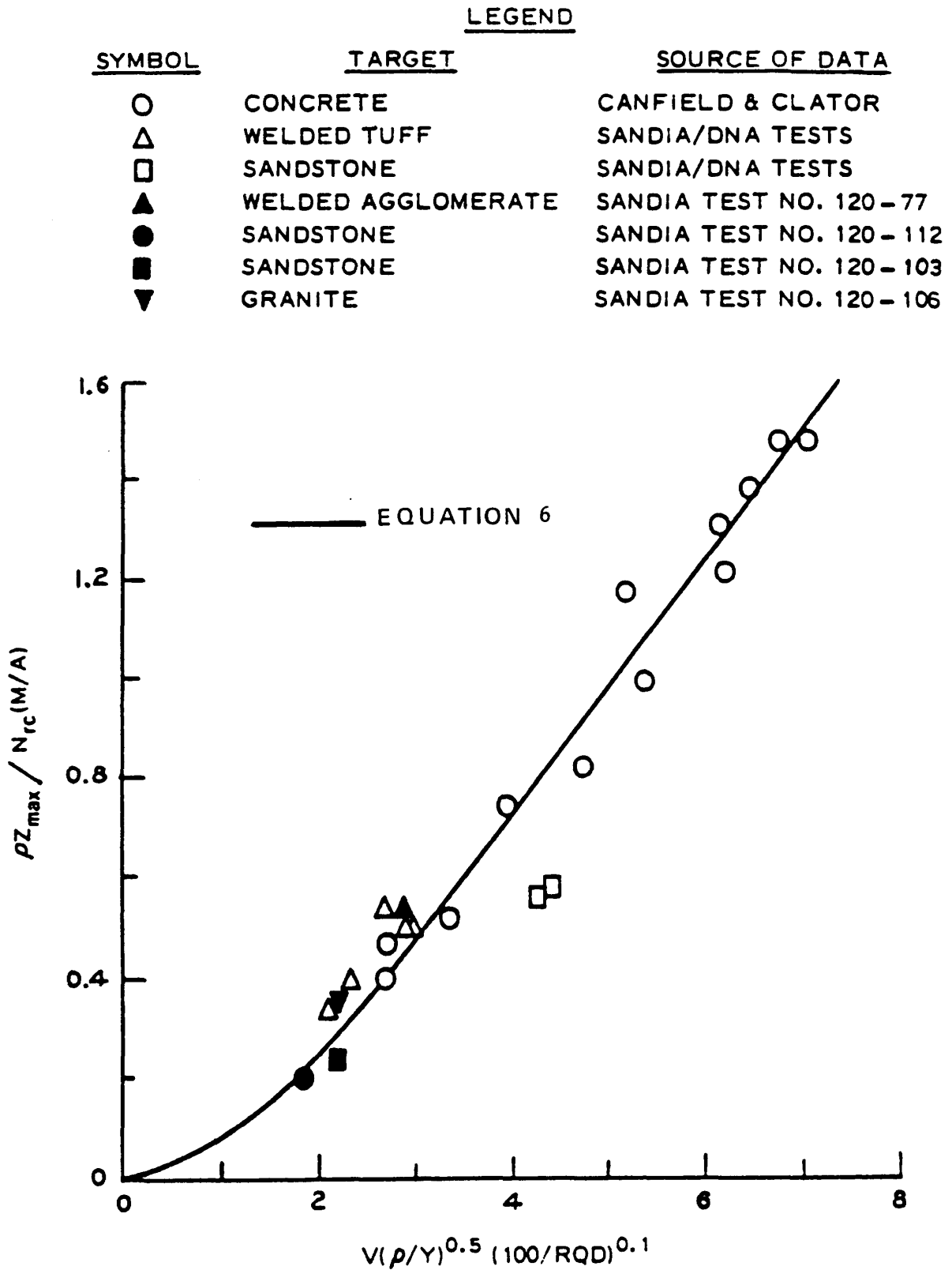


Figure 2: Comparison of Improved WES Rock Penetration Equation with Rock and Concrete Penetration Data. After Bernard and Creighton [B10].

1.2 Analytical Methods

1.2.1 Cavity expansion theories

The notion of analyzing the penetration of an object in a semi-infinite medium by simulating it as a cavity expanding in that medium was first presented over 40 years ago [B11]. Numerous developments followed, based on a spherical cavity [G3, H3, P4, R1, R4], with the material models becoming more and more representative of geological materials. The same progress was also achieved for an assumption of cylindrical cavity, which originated about 15 years ago [N3, F3 to F10, L3 to L5, Y1].

Both the Spherical Cavity Expansion Theory (SCET) and the Cylindrical Cavity Expansion Theory (CCET) assume the resistance to penetration to be the sum of the shear resistance of the target, and of the inertial effects of projectile movement in the target (also referred to as dynamic resistance). They provide closed-form solutions for the pressure on the penetrator surface. The deceleration-time history and the total depth of penetration are obtained. For example, for a locking compressible behavior of the target, in both approaches the maximum penetration depth is calculated as

$$Z = K_1 \ln(1 + K_2 V^2) \quad (10)$$

where K_1 and K_2 are somewhat lengthy expressions of penetrator characteristics, and target properties. These expressions are different for the SCET [A1, T9] and the CCET [N3]. It is noteworthy that the required target properties correspond to quantities which theoretically can be measured in tests performed on samples of the target (density, compressibility, modulus, shear strength, etc.). This may be true for snow and most soils but for rocks it certainly is subjected to the limitations of scale effects.

It is also clear that, because this type of analysis uses detailed constitutive relations for the target (failure envelope and hydrostat), it is not a practical approach for back-calculating a single quantity, such as unconfined compressive strength, from penetration tests. On the other hand, for predictive (forward) modeling of projectile penetration depth and deceleration, the CET's have been the basis for a long suite of computer programs over the years, particularly those of Sandia National Laboratories: PENAP [Y2], PENOB [Y3], RUNNOS [N6], RUNDEP [N6], FLAP [R2], FLAT

[R1], and GNOME [D1]. The succession of models reflects continued improvement in the constitutive relations adopted for the geologic targets. This is in recognition of the importance of having credible models of target shear strength.

Today, the GNOME code appears to be the most comprehensive cavity model [L4], as it includes both spherical and cylindrical theories, as well as a Mohr-Coulomb failure criterion with tension cut-off [K5].

1.2.2 Differential area force law (DAFL)

An alternate analytical method was proposed by the AVCO Corporation in the early 1970's. It provided explicit formulations for the normal stress and tangential stress at every point on the external surface of a penetrator [H1, T9]. The DAFL approach amounts to a 6-degree of freedom analysis of the rigid body motion of a penetrator. It is 3-dimensional. A limitation of the method [H1] is the fact that a total of 9 parameters is required to describe the target and the effects of the ground surface, only 2 of which (density and sonic velocity) are commonly known properties. Other parameters are empirically determined from regression and analysis of tests in the given target.

The DAFL approach was adopted and modified by the U.S. Army Waterways Experiment Station (WES), to provide a 2-dimensional theory for the analysis of oblique impacts [B10]. This theory forms the basis for WES' PENCO2D code [C7] which is in use today. PENCO has also spawned the PROPEN code [C8] for probabilistic penetration analysis. For soils, the only target strength property required is the empirical S-number, previously defined [Y6]. For rocks, s is set to zero, and equation (6) is used, which requires σ_c and RQD. In PENCO2D, the compressive resisting stresses are assumed normal to the penetrator's surface -- i.e. friction is neglected; this is a shortcoming. On the other hand, PENCO2D provides for algorithms to simulate free-surface effects and wake-separation and reattachment effects.

1.3 Numerical Methods

The heterogeneous nature of geological formations, their complex constitutive behavior, and the presence of discontinuities, are definite impediments to the application of the above empirical and analytical approaches. But this alone has not

negated their use; they are being employed with a measure of success. However, the modeling requirements ultimately extend to the structural analysis of the penetrators. There, only numerical methods are credible. In many cases the numerical models have been used both for the target penetration and the penetrator response evaluation. The three main techniques are:

- finite differences (FD): both Lagrangian [B19] and Eulerian [C6, M1, T5, T6] models have been used. Applications include metals and hypervelocity impacts, as well.
- finite elements (FE): together with their abilities for structural analysis, their great versatility in zoning has made finite element codes very attractive for penetration modeling [B5, D3, F1, F2, G5, H9, I4, J2, R6, S1, T1]. Some of the state-of-the-art programs have particularly desirable features such as erosion algorithms (DEFEL [C4], EPIC2 [J2], EPIC3 [B5]), arbitrary Lagrange-Euler capability (DEFEL, DYNA2D [F2], EFHYD [D3], EPIC2, TRIFLE [I4]), and discrete fracturing (DEFEL, EPIC2, EPIC3).
- discrete element models (DE): however sophisticated the above FD and FE models are, they cannot represent the mechanics of discontinuous media such as hard jointed rock masses or boulder fields, which contain many discrete bodies. For penetration in such media one must call upon the only technique available today -- the discrete element approach. Codes such as DECICE [M9], DIBS [W4], and PROBS [G2] offer the dynamic capability for penetration simulations.

The complete -- penetration + structural -- calculations with the above models tend to be expensive in terms of computer time. Attempts are being made at coupling analytical penetration analyses with numerical modeling [L4].

2. SUMMARY OF THE PENETRATION MODELS

For convenience, the main attributes of the various computer programs developed in the 1970's and 1980's are summarized in Table 1. The following notes apply to the Table:

1. Acronyms of institutions:

AFATL : U.S. Air Force Armament Laboratory, Eglin AFB, FL
 AFWL : U.S. Air Force Weapons Laboratory, Kirtland AFB, Albuquerque, NM
 AVCO : AVCO Corporation
 CCC : Computer Code Consultants, Chisholm, NM
 CRT : California Research and Technology, Chatsworth, CA
 DYN : Dyna East Corporation, Philadelphia, PA
 ESI : Engineering System International, Paris, France
 GAT : General Atomic, San Diego, CA
 HCS : Hibbitt, Karlsson, Sorensen Inc., Providence, RI
 HNW : Honeywell Corporation, Edina, MN
 INT : Intera Technologies, Denver, CO
 LLNL : Lawrence Livermore National Laboratory, Livermore, CA
 NWC : U.S. Naval Weapons Center, China Lake, CA
 SCUBE : S-CUBED, La Jolla, CA
 SNLA : Sandia National Laboratories, Albuquerque, NM
 WES : U.S. Army Waterways Experiments Station, Vicksburg, MS.

2. Other symbols:

ALE : Arbitrary Lagrange-Euler
 CCET : Cylindrical Cavity Expansion Theory
 DAFL : Differential Area Force Law
 DE : Discrete Element code
 DOF : Degrees-of-freedom
 FD : Finite Differences
 FE : Finite Elements
 SCET : Spherical Cavity Expansion Theory.

3. In the DAFL approach, the method is analytical, but the material coefficients are empirically determined.

4. E: Denotes an Eulerian code; others are Lagrangian.

5. 2-D+: axisymmetric, with nonaxisymmetric loading.

Table 1: Summary of Computer Programs for Penetration Modeling

	Origin	Reference		Theory						
		Date	No.	Empir	SCET	CCET	DAFL	FE	FD	DE
ABAQUS	HCS ¹	1987	H9					X		
AUTOREZ	SNLA	1980	Y4							
CET	WES	1978	B8	X ³			X			
CSQII	SNLA	1986	T5						X(E ⁴)	
CSQIII	SNLA	1988	T6						X(E ⁴)	
CTH	SNLA	1987	M4						X(E ⁴)	
DAFL	AVCO	1972	H1	X ³			X			
DECICE	INT	1987	M9							X
DEFEL	DYN	1986	F1					X		
DIBS	LLNL	1982	W4							X
DYNA2D	LLNL	1987	F2					X		
DYNA3D	LLNL	1985	R5					X		
EFHYD	ESI	1984	D3					X		
EPIC2	HNW	1986	J2					X		
EPIC3	HNW	1987	B5					X		
EXCALIBUR	CRT	1987	I4					X		
FLAP	SNLA	1983	R2			X				
FLAT	SNLA	1983	R1			X				
GNOME	SNLA	1983	D1		X	X				
HELP	SCUBE	1978	S2						X(E ⁴)	
HEMP	LLNL	1978	W5						X	
HULL	AFATL	1984	M3						X(E ⁴)	
JOY	LLNL	1983	C6						X(E ⁴)	
LASOIL	HNW	1986	M7						X(E ⁴)	
METRIC	SCUBE	1978	S2						X(E ⁴)	
MOLE	AFWL	1984	J1	X						
NORML	SNLA	1980	N4			X			X	
OBLIK	SNLA	1980	N4			X			X	
PENAP	SNLA	1978	Y2			X				
PENCO	WES	1976	B6		X					
PENCO2D	WES	1979	B10	X ³			X			
PENOB	SNLA	1979	Y3			X				
PROBS	CRT	1987	G2							X
PRONTO2D	SNLA	1986	T1					X		
PROPEN	WES	1986	C8	X			X			
RUNDEP	SNLA	1982	N6			X				
RUNNOS	SNLA	1982	N6			X				
SAMPL	SNLA	1985	Y9	X						
SCAP	SNLA	1985	R3							
SHELL SHOCK	SNLA	1984	G5					X		
TAUTQ	SNLA	1980	N4					X		
TOODY IV	SNLA	1978	S5					X		
TOOREZ	SNLA	1980	T7							
TRIDORF	CCC	1986	J4						X(E ⁴)	
TRIFLE	CRT	1987	I2					X		
TRIOIL	GAT	1986	J4						X(E ⁴)	
WAVE-L	CRT	1976	I1					X		
WHAP	NWC	1982	S1					X		
WONDY IV	SNLA	1971	L1						X	
WONDY V	SNLA	1982	K3						X	

3. SOME EVALUATIONS AND COMPARISONS

The existence of so many models and computer codes naturally raises the question of which is the best. Let us say at the outset that there is not a single best approach for any and all analyses. Valuable insights have been gained into the respective merits of the various techniques and it appears that more than one approach can give credible estimates of penetration depth and deceleration. Some important results, concerning the influence of various target properties on penetration, also have emerged from the calculations. A few studies stand out as particularly significant.

3.1 Soil Targets

- a direct comparison was performed [H1] of several methods (SNL, SCET, CCET, DAFL) against the results of a penetration test at the Watching Hill Blast Range, Ralston, Alberta, Canada. The 400 lb, 6.5-in diameter projectile impacted at about 500 ft/sec in a glacial lacustrine deposit composed of alternating thin layers of sands, silts and clays. The results of the comparison are shown in Table 2.

Table 2. Comparison of Calculations and Test Data [H1].

Method	Maximum Depth (feet)	Maximum Rigid Body Deceleration, (g's)	Remarks
Sandia empirical formula	50	133	Best deceleration
Spherical CET (WES) (1986)	50	206	Same as Nash et al.
Cylindrical CET	47	450	
DAFL	69	65	Best displacement
Test data	67.9	136 \pm 5	

The result that the best deceleration estimate was calculated by the SNL empirical approach was also obtained recently for penetration in concrete [N1]. The quality of the DAFL depth estimate is also consistent with applications of DAFL to sites where target coefficients can be first back-

calculated through regression of several tests. However, there are some doubts as to how well the DAFL approach can predict penetration a priori [T9, p 39]. For further comparison, Figure 3 shows that the predictions for rate of penetration vary quite a bit from one method to the other [H1], for the above tests at Ralston.

the same test results were also used to evaluate the capabilities of the TOODY finite difference code equipped with a rezoner [B19]. The comparison was made for predicted motions and stresses on the penetrator and in the target. The study had two shortcomings. First, an assumption was made of zero friction between target and penetrators; this does not account for friction being a controlling factor in the penetration process. Secondly, the FD model was not able to provide the final depth of trajectory as the modeling stopped far short of full penetration. This reflects a limitation of Lagrangian numerical models without automatic rezoning or ALE; such features are needed to track the complete trajectory of projectiles. Very few codes have such capabilities (Table 1).

for sites where no hands-on material characterization is obtained, the SNL empirical equation is thought to be more appropriate than the use of a CET [R5].

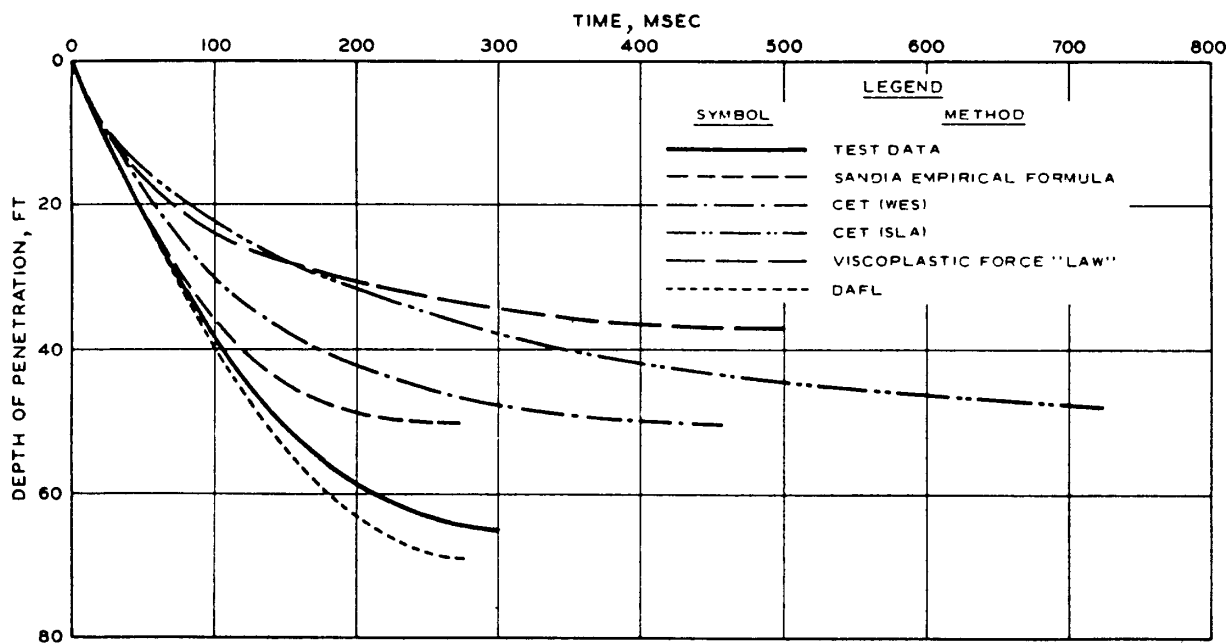


Figure 3. Comparison of Soil Penetration Calculations and Test Data, Ralston, Canada. After Hadala [H1].

as regards the effect of various properties, a series of TOODY calculations [B20] for an ogival penetration in silt showed that the effect of the soil's internal friction coefficient on deceleration increased with penetrator velocity (Figure 4). The same authors [B20], using PENAP, compared the effect on deceleration of varying several properties from a baseline case. The results are shown in Figure 5 where the baseline was: bulk modulus $k_0 = 0.23$ GPa, shear strength $\tau_0 = 4.2$ MPa, density $\rho_0 = 1.49$ Mg/m³, and friction $\mu = 0$. Friction was shown as having a strong control on deceleration, hence on penetration.

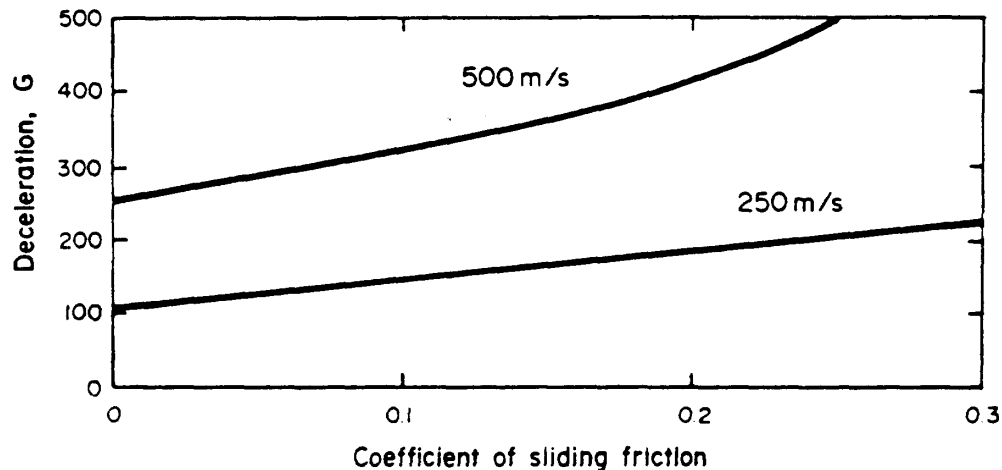


Figure 4: Effect of Internal Soil Friction on Deceleration at Different Penetration Velocities. TOODY Calculations. After Byers et al. [B20].

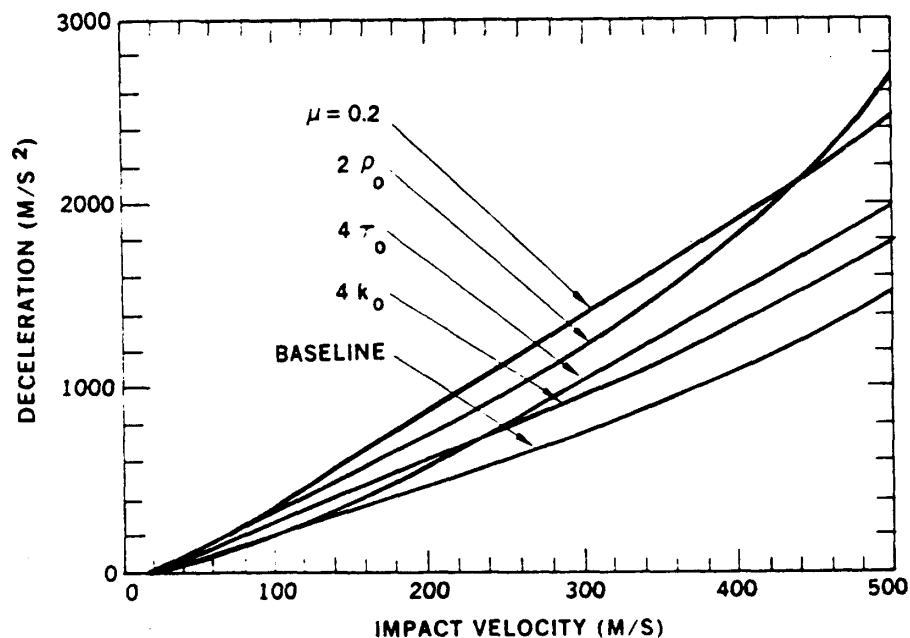


Figure 5. Comparison of Material Properties Effects on Deceleration. PENAP Calculations. After Byers et al. [B20].

3.2 Rock Targets

3.2.1 Test results and analyses

- results of 19 Sandia tests of air-delivered penetrators (20 to 25 cm in diameter and 225 to 450 kg in weight), were summarized in 1973 [P2]; 17 of them were in rocks -- i.e. welded tuff, limestone, sandstone, shale, weathered granite. The PENCO code with spherical cavity expansion theory, was used to estimate penetration depth in five of these rock targets. The calculated and measured values are compared in Figure 6 [B15, R4]. The vertical lines represent the range of estimates, given the bounds of the known or estimated geotechnical properties of the targets at the sites. It can be said that these depth estimates are reasonable. Note that most of the data shown in Figure 6 were already discussed in Figure 2 and that the empirical approach of WES [B10] seemed to give an even better fit than their SCET predictions.
- a closed-form formulation of the cylindrical cavity expansion theory was recently constructed [F10] and applied to 2 fully instrumented in-situ rock penetration tests in which both penetration-time and deceleration- time data had been acquired; both tests were in tuff from the Tonopah Test Range, Nevada. The results of the comparisons are shown in Figure 7, and the agreement is reasonably good.
- new, instrumented, rock tests were performed in the past few years by SNLA, in the context of the Pershing 2 (P2) and the Shallow Earth Penetration Weapon (SEPW) project. From these tests, deceleration and depth data in Antelope Tuff were selected as benchmark for comparison with calculations performed at SNL with several different models [H11]. The results of this comparison are summarized in Table 3. It would seem that all models give numbers reasonably close to the experimental data. It must be noted that the application of HULL at SNL is recent, and that this Eulerian code is still being adapted for penetration problems in geologic media [Y5]; for example, the inclusion of a frictional interface between the penetrator and the ground improved the calculated maximum deceleration from having an initial + 50% discrepancy with the test record, to the current + 11%.

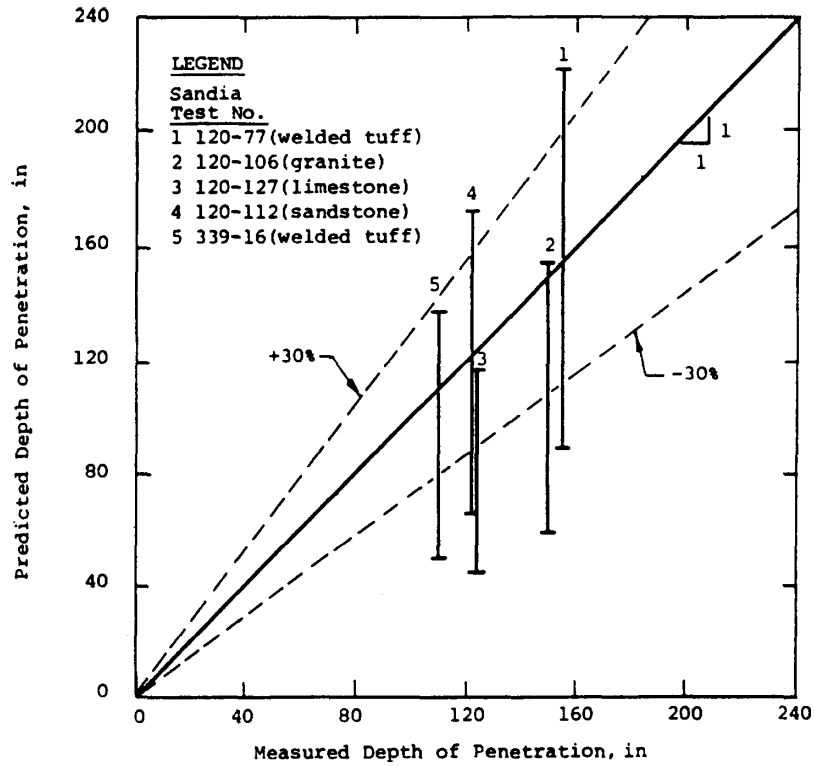
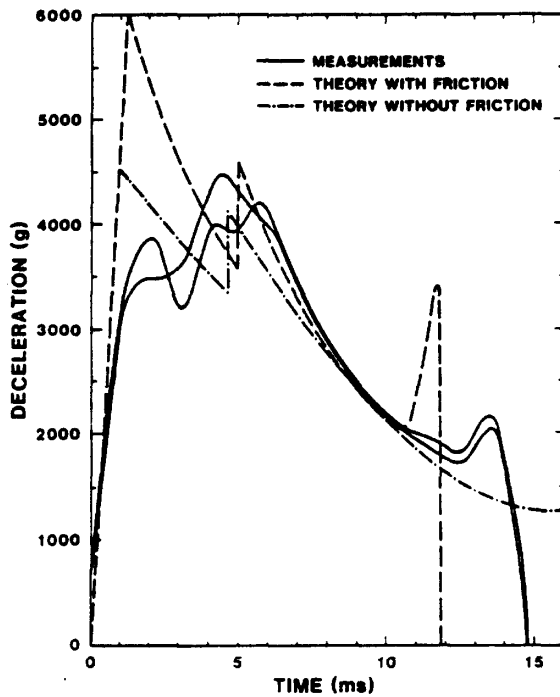
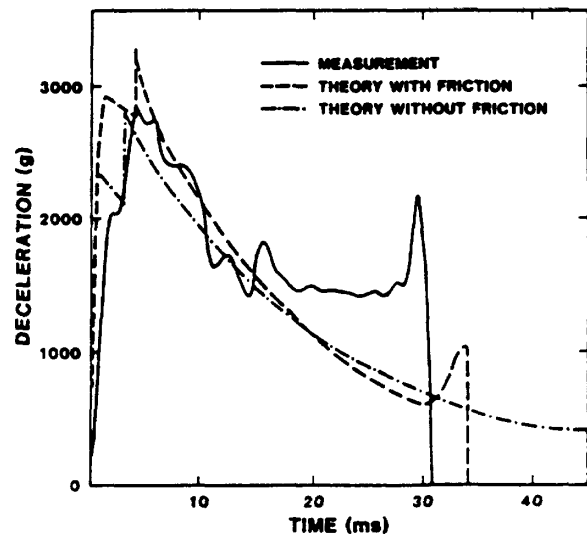


Figure 6: Penetration Depth Data vs. SCET Calculations with the WES PENCO code. After Butler [B15].



a) Mount Helen Tuff



b) Antelope Tuff.

Figure 7: Comparison of CCET Calculations with Data from Two Tests in Tuff. After Forrestal [F10].

Table 3: Differences Between Experimental Penetration Data in Antelope Tuff and Calculations. After Hightower [H11].

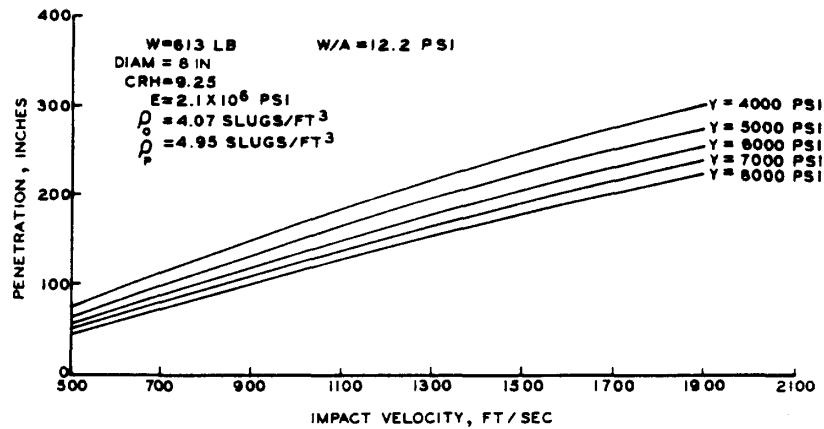
Project	Method (Calculator)	Velocity (ft/s)	Peak Rigid Body Deceleration	Penetration Depth
P2	Empirical SNL (Christensen, Young)	1600	< 5%	+ 8%
		1650	< 5%	+ 7%
		1700	+11%	+ 5%
P2	CCET/SNL, elastic- plastic model (Kipp, Longcope)	1600	-20%	-12%
		1650	-17%	- 5%
		1700	+ 9%	-16%
P2	PRONTO 2D (Chen)	1500	< 5%	-11%
		1650	- 8%	-11%
		1700	- 9%	< 5%
		1750	< 5%	-13%
SEPW	HULL (Yarrington)	2030	+11%	*

*Calculation was not run out to full penetration, as of this writing.

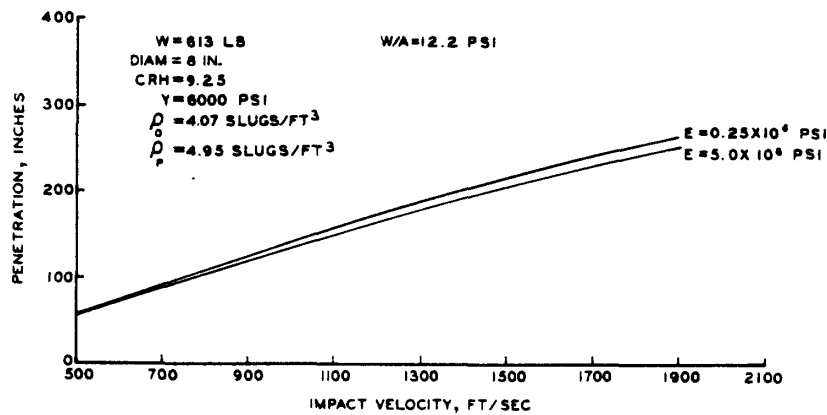
- in 1986-87 WES performed pre- and post-test simulations of penetration tests at Fort Riley, Kansas [C9, C10]. The PENCO2D and PROPEN codes were used. The geology consisted of a mix of limestone and shale overlain by soil. Path length prediction errors decreased from 30% pre-test to 10% post-test.
- in the past 12 months, about two dozen penetrator tests were made in both Sidewinder tuff and Antelope tuff, under the continuing SEPW project of SNLA. About one third of these tests were instrumented for deceleration. The analysis is ongoing [H4].

3.2.2 Constitutive laws and material properties

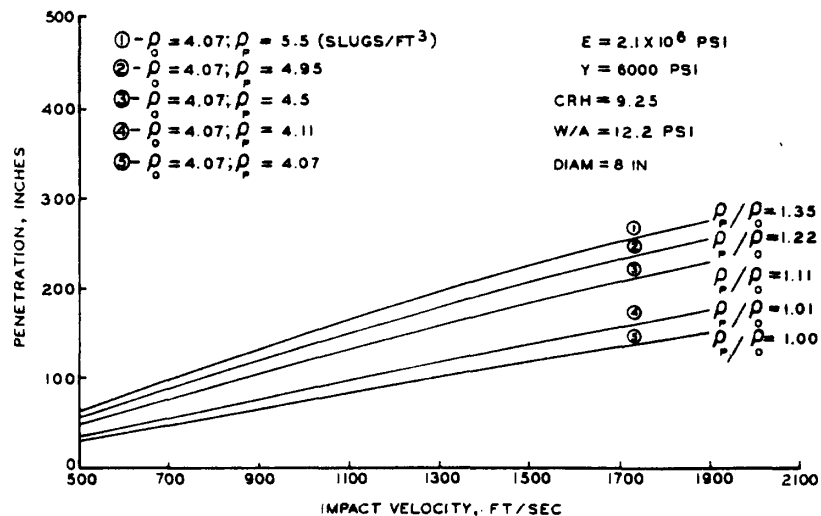
- Figure 8 shows the results of an early sensitivity analysis with the SCET approach in the PENCO code [R4]; the model does not have fractures. The figure shows how penetration depth and impact velocity are related, for different values of rock yield strength, modulus, and density. Other constitutive studies also have been made for rock targets [N5], but systematic evaluations have yet to be performed for the effect of jointing, at rock sites.



a) Rock yield strength.



b) Rock modulus of elasticity.



c) Rock density.

Figure 8: Sensitivity of Penetration vs. Impact Velocity for Various Parameters; PENCO Results, SCET Approach. After Rohani [R4].

a very uncertain aspect of the penetration process is the amount of frictional force applied to the penetrator [W1,H11]; and this is compounded by uncertainties on the values of metal/rock friction. A power law variation of friction angle with sliding velocity has been proposed and a few data have been reported for tuff, sandstone and limestone by a single investigator [G1]. Results for tuff are shown in Figure 9, where the shear stress τ is equal to the static coefficient of friction μ multiplied by an equivalent normal stress: $\tau = \mu \sigma_{\text{eff}}$. The σ_{eff} is equal to $\sigma \cdot e^{-c\sigma/\xi}$ where σ is the actual normal stress, c is the sliding velocity, and ξ is found to be equal to 2 GPa m/s for the 3 rocks tested; interestingly, ξ is reported to be the same for a wet and a dry sandstone.

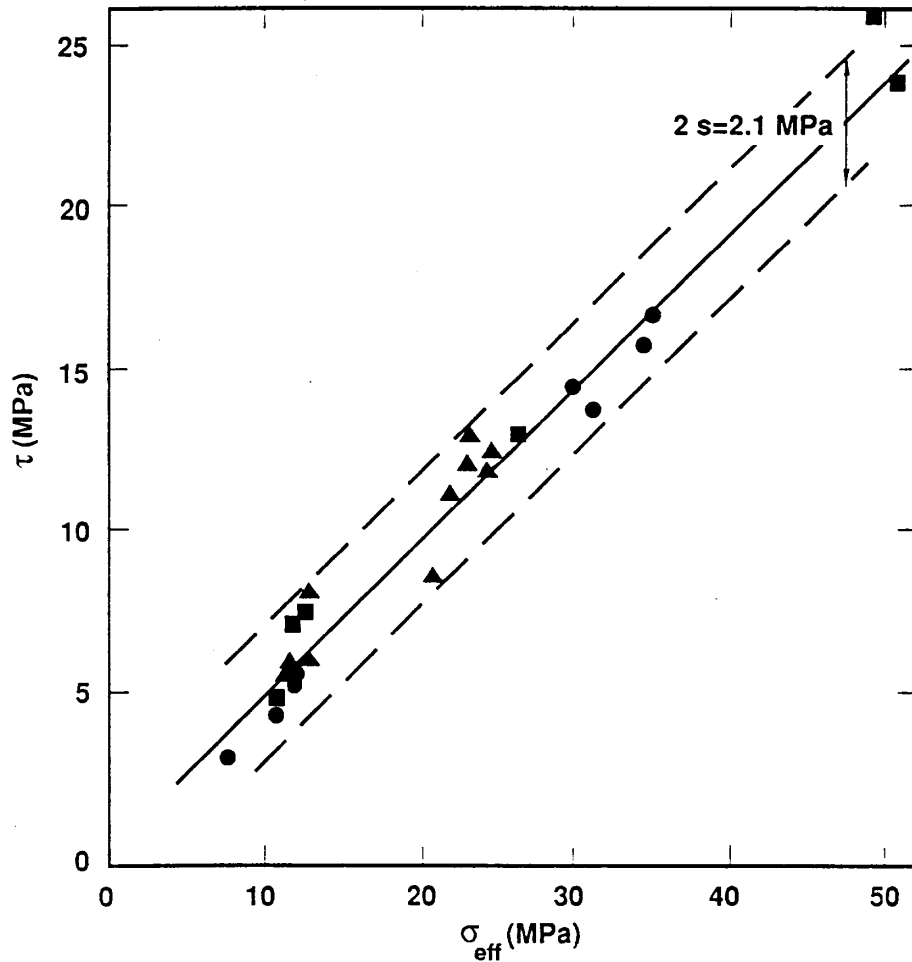


Figure 9: Frictional Data for Welded Tuff, Cast in Terms of σ_{eff} . Squares: 10 m/s; Circles: 20 m/s; Triangles: 30 m/s. After Gaffney [G1].

- recent modeling with PRONTO2D by SNL [C3], specifically addressed the effect of friction between the ground and the penetrator on the calculated penetration and velocity histories. The velocity-dependent friction coefficient was written as

$$\mu = \mu_{\infty} + (\mu_0 - \mu_{\infty})e^{-\gamma v} \quad (11)$$

where v is the sliding velocity, γ is a decay constant, μ_0 is the static coefficient of friction, and μ_{∞} is the friction value at infinite speed (usually considered to be quite low, i.e. less than 0.10). Note that equation (11) is somewhat different from the friction formulation of [G1], just discussed. The case history selected was that of an SNL steel penetrator Davis gun test in Antelope tuff (162 kg, 520 m/s). Both a constant friction and the above velocity-dependent formulation were used. Both approaches gave almost identical peak deceleration and penetration depth; but only the velocity-dependent friction could reproduce the sudden increase in deceleration which was observed in the field, prior to the projectile coming at rest.

- finally, shaped-charges also have been fired on rock, for several decades [B3,H2,H13,V1]. A recent report of tests into tuff [V1] has shown a good agreement between measured slug penetration and values predicted with the SCAP code [R3]. SCAP uses a hydrodynamic model to relate penetration to stand-off distance, for known values of jet and target densities.

4. CONCLUSIONS

The large amount of work outlined in this report has yielded a few conclusions applicable to rock penetration, upon which a consensus appears to exist.

On the subject of site characterization, it is clear that cracks and joints are dominant in jointed rocks, and can easily overshadow the intact rock's yield strength [U2].

Regarding constitutive laws and material properties:

- the most desirable strength formulation is that which describes the complete variation of shear strength with mean stress [U2].
- at velocities of up to a few hundred metres/s, rock penetration is most dependent on shear strength, which is pressure-dependent; it is less dependent on tensile strength, and compressibility [F8]. At hyper-velocities (several km/s), density dominates [F11].
- the internal friction angle of the target is more important than its cohesive strength in controlling penetration [N5]. Also, it appears to be advisable to account for the dependence of friction on contact velocity [C3].
- measurements of stresses and deformations in the medium are what is needed to evaluate the material models used [B19]; measuring only the penetrator deceleration and depth is not sufficient.
- the stresses induced around the penetrator diminish rapidly away from the body [F3]; an order of magnitude decay takes place over a radial distance of about 2.5 times the projectile diameter [B19]. This gives the scale of the volume of target material involved in controlling penetration.

As for other conclusions:

- cavity expansion theories give higher contact stresses on the penetrator than finite element models, for example, because of artificial kinematic constraints, and lack of surface weakening [K4].
- it appears essential to incorporate in the models the comminution of rock and post-fracture properties of the broken material [W1].
- penetration depth for rock (and concrete) appears to scale linearly with the ratio of the projectile weight over its cross-sectional area [N1, Y9].

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6. APPENDIX: CROSS-INDEXING OF REFERENCES

This cross-referencing on a few selected topics was performed to further enhance the usefulness to the reader of the bibliography examined for this study.

Target Materials

boulders	:	G2, I3, N2
concrete	:	A6, B9, B14, B18, F2, F12, L2, L5, N1, R4, T10
ice	:	H10, K4, R7, U1, Y8, Y10
permafrost:	:	A2, D4
rocks	:	A6, B7, B16, B17, B18, F10, G4, H5, H6, I4, K7, M8, P1, P2, R4, T2, T8, V1
snow	:	A2, Y5, Y7
soils	:	B20, C1, H7, M6, P3, S3
underwater	:	B1, B2, F7
(rock anchors)		

Numerical Method

FD	:	B19, K3, L1, S5, T5, T6, W2, W3
FE	:	B4, B5, C2, C4, J2, J3, S4, T1
DE	:	G2, I3, M9, W4

Others

conferences	:	B14, F1, H7, K6, U1, U2
hypervelocity	:	A4, B13, D3, F1, F11, H12, K2, M1, M2, Z1
reviews	:	A5, H8, J4, J5, M5, T4, T9, U3
shaped charges	:	B3, H2, H13, R3, V1

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