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APPLICATIONS AND MISAPPLICATIONS OF THE CHANNEL-CAPTURE FORMALISM OF DIRECT NEUTRON CAPTURE*

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†Eric Lynn is grateful to ORNL for its hospitality during April-May, 1985.

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Abstract: We discuss the channel-capture approximation of slow neutron direct-capture theory. We show that this approximation gives a generally good representation of the neutron capture cross sections for several electric dipole transitions in a broad range of nuclides from A = 9 to A = 136; these are mostly near-spherical nuclei. Despite this body of agreement, we examine the accuracy we can expect from the simple channel-capture theory. Comparison with calculations of the potential-capture cross section from physically more realistic optical model calculations show that, in general, the channel-capture cross section can be up to ~40% in error. In cases where the expected channel-capture cross section is much smaller than the "hard-sphere" capture cross-section estimate, the disagreement with potential capture can be much worse than this. Also, in these cases, compound-nucleus capture can be of comparable or greater magnitude. These effects have been shown to completely undermine recent attempts to determine nuclear interaction radii for targets, such as ^{12}C and ^{9}Be, by application of the channel-capture formula to capture cross-section data.

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1. Introduction

The low-energy neutron radiative capture reaction has been a rich source of data on nuclear spectroscopy for several decades, but, surprisingly perhaps, considerable uncertainty still exists in our understanding of the detailed mechanism of the reaction itself. For most heavy nuclides not in proximity to a closed shell, developing statistical estimates of radiative transition strengths, based on the spreading of the electric-dipole giant resonance into a broad range of compound nucleus states, seem to be the best that we can do. Demonstrations also have shown, however, that for a large number of light nuclides and near-closed-shell nuclides, a direct-capture mechanism dominated by single-particle transitions from the entrance channel offers a good explanation of off-resonance-capture cross sections for primary E1 transitions to final states with a certain degree of single-particle character.

The success of the channel-capture estimates of the cross sections in many such cases is now leading to its use (or misuse) as a tool for determining other nuclear quantities of interest, such as spectroscopic factors of final states, total thermal absorption cross sections, scattering lengths, and nuclear potential radii. It is therefore pertinent to ask the question of how exact we can expect the formula for the channel-capture cross section to be even in relatively ideal situations. In this study we address this question, first drawing attention to the nature of the physical approximations inherent in the derivation of the simple channel-capture expression and then to some of the numerical approximations incorporated in the present commonly used formula. Finally, we look at a few of the recent applications of the formula and draw attention to the limitations of the methodology.

2. A Simple Channel-Capture Formula

In 1960, Lane and Lynn[1] derived a simple expression for slow-neutron direct capture in the channel-capture approximation (which retains only the extranuclear contribution to the matrix element) for the cross sections of E1 transitions in terms of a cross section given by the hard-sphere approximation (extranuclear capture associated with the limiting case of hard-sphere scattering) to the direct-capture theory. This

expression has been used by several authors and, in particular, by Mughabghab, who gives the following version[2]:

$$\sigma_{\gamma f}(\text{channel}) = \sigma_{\gamma f}(\text{hard sphere}) \left[1 + \frac{R-a_s}{R} Y_f \frac{Y_f + 2}{Y_f + 3}\right]^2 , \qquad (i)$$

where

$$\sigma_{\gamma f}(\text{hard sphere}) = \frac{0.062}{R\sqrt{E_n}} \left[\frac{Z}{A}\right]^2 \mu \frac{2J_f + 1}{6(2I + 1)} S_{dp} \left[\frac{Y_f + 3}{Y_f + 1}\right]^2 Y_f^2 , \qquad (ii)$$

and $$Y_f^2 = \frac{2mE_\gamma R^2}{\hbar^2} . \qquad (iii)$$

In these equations:

R = interaction radius = $1.35 \times A^{1/3}$ fm.

a_s = coherent scattering length for channel spin s.

J_f = total spin of the final state

I = spin of the target nucleus.

S_{dp} = (d,p) spectroscopic factor.

E_γ = energy of γ-ray due to capture of an s-wave neutron feeding a final p-state.

E_n = the energy of the incident neutron = 0.0253 eV for 2200 m/sec neutrons.

μ = unity in all cases (see below).

To apply the previous expression, one needs to know the interaction radius R, the coherent scattering length a_s, and the (d,p) spectroscopic factors. Without the spectroscopic factors, the comparison with data would be jejune, but with these factors in hand, R and a_s can still be left as adjustable parameters. Tables 1 and 2 show the application of the previous expression in analyzing thermal capture data[3-16]. The overall agreement between calculation and experimental data is good. We emphasize however that when R departs significantly from either $(1.16\ A^{1/3} + 0.6)$ fm or from $(1.35 \times A^{1/3})$ fm, the agreement is somewhat contrived. So it is also when the scattering length is not known from experiment.

Table 1

Comparison between calculated and measured partial cross sections from thermal neutron capture on even targets.

Final nucleus	E(level) (keV)	(d,p) (2J + 1)S	Primary E_γ (keV)	Calc. σ_γ (mb)	Expt. σ_γ (mb)
(i) $^{12}C(n,\gamma)$ - ref. 3: Assumed R = 2.96 fm; known a_s = 6.15 fm.					
^{13}C	0	2.2	4945	2.4	2.4
	3684	0.4	1262	1.3	1.1
(ii) $^{32}S(n,\gamma)$ - ref. 4: Assumed R = 4.32 fm; known a_s = 2.74 fm.					
^{33}S	3221	1.90	5421	266	302
	4211	0.30	4431	33	25
	4918	0.09	3724	8	13
	5711	1.06	2931	74	87
	5889	0.44	2753	29	29
	6425	0.34	2217	18	13
	7188	0.18	1454	6	3
(iii) $^{34}S(n,\gamma)$ - ref. 5: Assumed R = 4.32 fm; known a_s = 3.40 fm.					
^{35}S	2348	2.04	4638	150	163
	3802	0.36	3184	18	18
	4189	0.28	2797	13	16
	4903	1.55	2083	52	46
	4963	0.87	2023	29	34
(iv) $^{36}S(n,\gamma)$ - ref. 6: Assumed R = 4.50 fm; assumed a_s = 3.20 fm.					
^{37}S	646	2.71	3657	172	161
	1992	0.22	2312	9	9
	2638	1.60	1666	46	52
	3262	0.58	1042	11	8
	3493	0.31	811	4	2
(v) $^{40}Ca(n,\gamma)$ - ref. 7: Assumed R = 4.62 fm; known a_s = 4.90 fm.					
^{41}Ca	1943	2.60	6420	119	184
	2462	0.90	5901	39	33
	3613	0.19	4750	7	9
	3730	0.02	4633	1	2
	3944	1.17	4419	43	90
	4603	0.16	3760	5	10
	4753	0.37	3610	12	33

Table 1 (continued)

Final nucleus	E(level) (keV)	(d,p) (2J + 1)S	Primary E_γ (keV)	Calc. σ_γ (mb)	Expt. σ_γ (mb)
(vi) $^{42}Ca(n,\gamma)$ - ref. 8: Assumed R = 4.69 fm; known a_s = 3.10 fm.					
^{43}Ca	593	0.16	7340	30	39
	2046	2.80	5886	106	360
	2611	0.27	5322	35	28
	2878	0.18	5054	22	16
	2943	0.19	4989	23	24
	3286	0.12	4646	13	16
	3572	0.19	4360	19	20
	4207	0.84	3725	72	56
(vii) $^{44}Ca(n,\gamma)$ - ref. 9: Assumed R = 4.77 fm; known a_s = 1.79 fm.					
^{45}Ca	1435	0.40	5980	100	94
	1900	2.20	5515	493	480
	2249	0.30	5166	62	85
	2842	0.34	4573	59	36
	3242	0.14	4173	22	21
	3419	0.49	3996	72	90
	3783	0.08	3632	10	9
	3838	0.19	3577	24	11
	4616	0.34	2799	32	30
	4999	0.36	2415	28	19
(viii) $^{128}Te(n,\gamma)$ - ref. 10: Assumed R = 6.80 fm; assumed a_s = 5.20 fm.					
^{129}Te	2040	(0.04)	4043	3	4
	2267	0.06	3815	5	5
	2361	0.57	3722	42	43
	2379	0.30	3703	22	23
	2705	0.33	3377	22	22
	3502	0.07	2579	3	3
	3558	0.05	2525	2	12
	3792	0.21	2290	9	8
(ix) $^{130}Te(n,\gamma)$ - ref. 11: Assumed R = 6.84 fm; known a_s = 5.53 fm.					
^{131}Te	2511	0.25	3418	14	13
	2582	0.87	3347	49	57
	3002	0.38	2928	19	26
	3690	0.11	2240	4	3

Table 1 (continued)

Final nucleus	E(level) (keV)	(d,p) (2J + 1)S	Primary E_γ (keV)	Calc. σ_γ (mb)	Expt. σ_γ (mb)
(x) $^{136}Xe(n,\gamma)$ - ref. 12: Assumed R = 6.94 fm; assumed a_s = 5.75 fm.					
^{137}Xe	601	1.96	3425	106	102
	986	0.68	3039	33	33
	1841	0.72	2184	25	26
	1936	0.40	2089	13	12
	2106	0.12	1829	3	3
	2490	0.60	1535	15	15
	3609	0.16	1416	4	2
(xi) $^{136}Ba(n,\gamma)$ - ref. 13: Assumed R = 6.94 fm; assumed a_s = 3.00 fm.					
^{137}Ba	2182	1.06	4723	267	291
	2663	0.36	4243	80	91
	3316	0.15	3590	26	12
	3403	0.19	3503	31	10
	3680	0.09	3226	13	9
	3778	(0.10)	3127	14	10
	3799	0.06	3106	8	4

Table 2

Comparison between calculated and measured partial cross sections from thermal neutron capture on odd targets.

Final nucleus	E(level) (keV)	(d,p) (2J + 1)S	Primary E_γ (keV)	Calc. σ_γ (mb)	Expt. σ_γ (mb)
(xii) $^{9}Be(n,\gamma)$ - ref. 14: Assumed R = 3.00 fm; known a_s = 7.01 fm.					
^{10}Be	0	2.1	6809	12.8	4.9
	3368	1.75	3444	0.41	0.86
	5959	(3.94)	853	2.0	2.0
	6180	(0.03)	632	0.016	0.018
(xiii) $^{13}C(n,\gamma)$ - ref. 3: Assumed R = 3.10 fm; known a_s = 5.47 fm.					
^{14}C	0	2.09	8174	1.16	1.15
	6589	(0.06)	1587	0.14	0.12

Table 2 (continued)

Final nucleus	E(level) (keV)	(d,p) (2J + 1)S	Primary E_γ (keV)	Calc. σ_γ (mb)	Expt. σ_γ (mb)
(xiv) $^{27}Al(n,\gamma)$ - ref. 15: Assumed R = 4.08 fm; known a_s = 3.34 fm.					
^{28}Al	3464	0.78	4262	8.2	14.2
	3592	0.90	4134	9.2	17.3
	4690	1.2	3036	9.2	16.7
	4765	1.0	2961	7.5	20.7
(xv) $^{31}P(n,\gamma)$ - ref. 16: Assumed R = 4.24 fm; known a_s = 4.99 fm.					
^{32}P	3264	1.10	4670	12.3	22.9
	4007	0.33	3927	3.5	6.5
	4036	1.30	3898	13.7	28.6
	4663	(0.40)	3271	3.9	8.9
	4878	(0.42)	3056	4.0	10.0
	5350	0.72	2584	6.4	8.8
	5509	0.24	2425	2.1	3.8
	5776	0.74	2158	6.1	11.7
	6060	(0.26)	1874	2.0	3.4
	6195	(0.20)	1739	1.5	4.3
(xvi) $^{33}S(n,\gamma)$ - ref. 17: Assumed R = 4.32 fm; known a_s = 4.68 fm.					
^{34}S	4624	0.84	6792	8.4	24.2
	5680	2.70	5737	24.7	43
	5756	1.68	5661	15.2	18.4
	6169	1.00	5248	8.7	11.8
	6342	0.80	5075	6.8	0.4
	6479	3.64	4938	30.6	22.2
	6685	1.28	4731	10.5	1.6
	6954	0.84	4462	6.7	7.9
	7110	0.52	4306	4.1	8.3
	7630	3.92	3787	28.4	26.5
	7781	1.08	3636	7.6	5.2
	8138	1.04	3279	6.9	3.2

The quantity μ in eq. (ii) was supposed to be unity or two if $I \neq 0$, depending on whether $J_f = I \pm 3/2$ or $J_f = I \pm 1/2$, respectively[2]. It follows that if the theory is valid, application of eq. (i) could lead to a spin assignment for a level as was done in ref.[12] and ref.[15]. Such a conclusion is unphysical and never intended in the formulation of the theory. The quantity μ appearing in the eq. (ii) should be removed.

3. Physical Approximations in the Channel-Capture Formula

The electric dipole radiative transition matrix element between initial i and final state f contains the factor

$$\int_0^\infty dr\, u_i(r) r w_f(r) , \qquad \text{(iv)}$$

if the motion of the system can be approximated by single-particle motion in a potential well. The radial wave function of this motion is denoted by $u_i(r)$ for the initial state and by $w_f(r)$ for the final state. In general, the internal behavior of the compound nucleus at several MeV of excitation cannot be approximated as single-particle motion, but beyond a certain radial separation of neutron and residual nucleus, denoted as the channel radius R, residual forces that cannot be described by a simple potential-energy function become negligible and the single-particle approximation is then very good. If beyond the channel radius the potential energy is also negligible, the radial wave functions assume simple analytical forms, related to spherical Bessel and Neumann functions for the initial state (describing the scattering of a neutron by the nucleus) and to spherical Hankel functions for the final state. In this case the capture cross section for (very) low-energy neutron capture can be estimated by replacing the integral (iv) by its component in the channel

$$\int_R^\infty dr\, u_i(r) r w_f(r). \qquad \text{(v)}$$

The resulting expression is[17]:

$$\sigma_{\gamma(i\to f)CH} = \frac{\bar{e}^2}{\hbar v}\,\frac{32\pi}{3}\,R^5 k_\gamma^3 \sum_{J=\left|I-1/2\right|}^{I+1/2} g_J \frac{1}{\left|1-iP_0\mathcal{R}_J\right|^2}\,\frac{f(y)}{y^4}\,W_{J_f j_J}\theta_f^2 , \qquad \text{(vi)}$$

where

$$f(y) = \left[\frac{y+3}{y+1}\right]^2 + 2\mathrm{Re}\mathcal{R}_J\,\frac{y(y+3)(y+2)}{(y+1)^2} + \left[(\mathrm{Re}\mathcal{R}_J)^2 + (\mathrm{Im}\mathcal{R}_J)^2\right] y^2 \left[\frac{y+2}{y+1}\right]^2 . \qquad \text{(vii)}$$

In these equations, $\bar{e}$ is the effective charge of the neutron ($\bar{e} = -eZ/A$, where e is the proton charge, Z is the proton number, and A is the mass

number of the target nucleus), $\hbar$ is Planck's constant divided by 2π, v is the relative velocity of neutron and target nucleus, k_γ is the wave number of the photon emitted in the transition, J is the total angular momentum of the initial scattering system, g_J is the statistical weight for this angular momentum selection [$g_J = (2J+1)/2(2I+1)$] for unpolarized particles, I is the spin of the target nucleus, P_0 is the penetration factor for s-wave neutrons (this can be expressed in terms of the wave number k for the relative motion of neutron and target nucleus as $P_0 = kR$), $\mathcal{R}_J$ is the reduced $\mathcal{R}$-function that is central to the description of the scattering function amplitude when the total angular momentum of the scattering function is J, and $y = \kappa R$, κ being the reciprocal attenuation length of the channel wave function of the final state. In terms of the binding energy E_f of the final state and the reduced mass μ of the (neutron + target nucleus) system, $\kappa = (2\mu E_f)^{1/2}/\hbar$. If the neutron scattering length a_{sJ} of the target nucleus is known at the incident neutron energy, the real part of $\mathcal{R}_J$ is determined (in the absence of strong neutron capture) by the simple expression $\mathrm{Re}\ \mathcal{R}_J = 1 - (a_{sJ}/R)$. (Note that this substitution then closely relates eqs. (vi) and (vii) with eq. (i) if $\mathrm{Im}\ \mathcal{R}_J = 0$ and the scattering lengths are not J-dependent.) The quantity $W_{J_f j_J}$ is a spin-coupling factor depending on the angular momentum J_f of the final state and on j, the coupled spin and orbital momentum of the single-particle motion of the neutron in the final state as well as on J. The value $W_{J_f j_J}$ can be written in terms of the Wigner W-coefficients and vector-coupling coefficients, and numerical values of $W_{J_f j_J}$ have been tabulated in ref. [17]. Finally, the quantity θ_f^2 is the spectroscopic factor for the fractionation of the single-particle bound state into the final state f.

The ideal situation in which eq. (vi) would be a very accurate representation of the cross section for the transition i→f would be (a) if the nucleus were very sharp-edged, having very little interaction of any kind with the incident neutron outside the channel radius R and showing full effect of the overall potential field just within it; (b) if the neutron energy were strongly off-resonance; and (c) if the nucleus were very large and the nuclear potential deep (compared to the final-state binding energy) so that the radial wave function of the single-particle final

state within the nucleus had many nodes. Condition (a) would guarantee that the analytic forms of the radial wave functions used in deriving eq. (vi) are fully accurate and that the integrand of eq. (iv) becomes highly oscillatory and therefore largely self-cancelling for $r < R$ (or alternately strongly damped). Condition (b) would imply that in any case the contribution to the integral [eq. (iv)] from the region $r = 0$ to $r = R$ would be of small amplitude. Condition (c) would guarantee that the approximate value of the final-state wave function at the channel radius, namely $w_f^2(R) = 2/R$, employed in deriving eq. (vi) is accurate.

4. Center-of-Mass Factors in Channel-Capture Formulas and Computations

If eq. (i) is going to be employed willy-nilly in analyzing experimental results, it is important to consider center-of-mass factors in greater detail. We need to begin with the expression for hard-sphere capture [eq. (9) of Ref. [17]].

$$\sigma_{HS} = \frac{\bar{e}^2}{\hbar v} \frac{32\pi}{3} \frac{R^5 k_\gamma^3}{y^4} \left[\frac{y+3}{y+1}\right]^2 , \qquad \text{(viii)}$$

where

$$y = \kappa R \text{ and } \kappa = \sqrt{\frac{2\mu E_f}{\hbar^2}} = \sqrt{\frac{M}{M+m}} \sqrt{\frac{2m E_f}{\hbar^2}} .$$

Here μ is reduced mass, M target mass, m neutron mass, E_f binding energy of the final state, and v relative velocity of incident neutron and target. Suppose y is expressed without its $(M/M+m)^{1/2}$ factor. The error introduced in the $[(y+3)/(y+1)]^2$ term is small. But the y^{-4} term will require a factor

$$\left(\frac{M+m}{M}\right)^2 .$$

Eq. (viii) is usually developed for numerical work by recognizing that at low neutron energy $E_\gamma \approx E_f$. Hence $k_\gamma = E_\gamma/\hbar c = E_f/\hbar c$, and eq. (viii) becomes

$$\sigma_{HS} = \frac{\bar{e}^2\hbar^2}{c^2} \frac{4\pi}{3} \frac{1}{v} \frac{1}{\mu^3} \frac{1}{R} y^2 \left[\frac{y+3}{y+1}\right]^2 . \qquad \text{(ix)}$$

The above universal formula [eq. (ix)] will now contain the factor

$$\underbrace{\left(\frac{M+m}{M}\right)^3}_{\text{from } \mu^{-3}} \quad \underbrace{\left(\frac{M}{M+m}\right)}_{\text{from } y^2} = \left(\frac{M+m}{M}\right)^2 \text{ as before.}$$

In the formula for channel capture, additional terms in y will appear with different powers. These terms often cancel each other and hence depend critically on the precise value of y. It is important therefore to use the exact value of y in this formula, that is,

$$y = R\sqrt{\frac{M}{M+m}}\sqrt{\frac{2m\,E_f}{\hbar^2}}\,. \tag{x}$$

Eq. 17(a) of ref.[17] is then, in terms of laboratory neutron energy,

$$\sigma_{\gamma(i\to f)(CH)} = \frac{\bar{e}^2}{\hbar}\,\frac{32\pi}{3}\,\frac{R^5\,E_f{}^3}{\hbar^3c^3}\sqrt{\frac{m}{2E_{lab}}}\,\frac{1}{y^4}$$

$$\times \sum_{J=|I-1/2|}^{I+1/2} g_J \frac{1}{|1-iP_0\mathcal{R}_J|^2}\left[\left(\frac{y+3}{y+1}\right)^2 + 2\mathrm{Re}\mathcal{R}_J\,\frac{y(y+3)(y+2)}{(y+1)^2} + |\mathcal{R}_J|^2 y^2\left(\frac{y+2}{y+1}\right)^2\right] W_{J_f j_J}\theta_f^2 \tag{xi}$$

If $E_f{}^3$ is written in terms of κ, i.e., $E_f = \hbar^2\kappa^2/2\mu = \frac{\hbar^2\kappa^2}{2m}\left(\frac{M+m}{M}\right)$, we obtain eq. 17(b) of ref.[17]:

$$\sigma_{\gamma(i\to f)(CH)} = \frac{0.06137}{R\sqrt{E_{lab}}}\left(\frac{Z}{A}\right)^2\left(\frac{M+m}{M}\right)^3 \sum_J g_J \frac{1}{|1-iP_0\mathcal{R}_J|^2}\left[y^2\left(\frac{y+3}{y+1}\right)^2 + 2\mathrm{Re}\mathcal{R}_J\,\frac{y^3(y+1)(y+2)}{(y+1)^2} + |\mathcal{R}_J|^2 y^2\left(\frac{y+2}{y+1}\right)^2\right] W_{J_f j_J}\theta_f^2\,. \tag{xii}$$

We emphasize that the factor in square bracket contains the exact value of y given by eq. (x) above.

5. Optical Model Comparisons with the Channel-Capture Formula

The limitations of the channel-capture formula in its application to neutron capture by real nuclei can be demonstrated at one level by comparison with the study of neutron capture within the framework of the optical model. This was briefly discussed in ref. 1 and more extensively in ref.[18]. Recent explicit work on the comparison of channel capture with the fuller version of slow-neutron direct capture (which is known as potential capture) as calculated in the optical model framework, with special reference to the sulfur isotopes, is given in ref.[17]. Briefly, within the optical model the radial dipole integral [eq. (iv)] is calculated numerically for the full range of the radial variable by integrating the Schrödinger equation for initial (scattering) and final (bound) states using a realistic form (such as Woods-Saxon) for the potential-energy function; within the internal region of the nucleus, the effect of residual interactions that remove strength from the incident nucleon channel is represented crudely by the inclusion of an imaginary potential-energy term. Thus, the delimiting conditions (a), (b), and (c) in the last paragraph of sect. 3 for ensuring the accuracy of the channel-capture formula are largely relaxed by explicitly calculating the potential-capture cross section. Direct comparisons in the A = 30-70 range of the potential-capture cross section calculated in this manner to the channel-capture cross section have been given in ref.[17].

The ratio of potential-capture cross section to channel-capture cross section for a given set of conditions (final single-particle-state binding energy E_f, channel radius identified with the potential-well radius R for the purpose of calculating the channel-capture cross section in conjunction with the potential scattering radius R' substituted for the scattering length a_s) is termed C_{opt}. The behavior of C_{opt} as a function of final-state binding energy for various parameterizations of the Woods-Saxon surface diffuseness d and the imaginary component of the optical model potential is shown in figs. (1) and (2), reproduced from ref.[17]. It should be noted that a change in the final-state binding energy implicitly governs a change also in the potential-radius R and hence in the scattering length. For fairly realistic values of the optical model parameters, C_{opt} usually differs from unity by less than ~40%. This is a

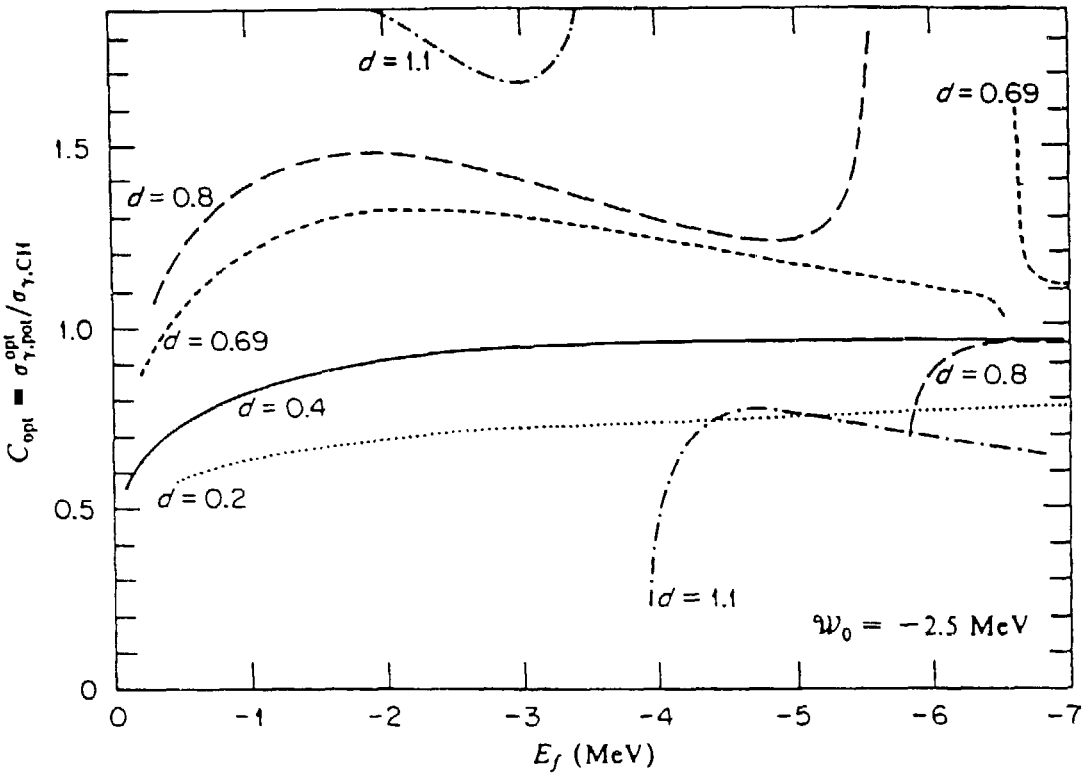

Fig. 1. Dependence of the ratio of potential-capture cross section to channel capture cross section on the surface diffuseness parameter d. Calculations have been made for $j_f = 3/2$ and the volume absorption model. [Reproduced from ref.[17)]].

measure, therefore, of the accuracy we can reasonably expect from the channel-capture formula in the absence of other capture mechanisms and is a quite remarkable vindication of its considering the nature of the approximations inherent in it. We note however that some of the approximations leading to the channel-capture expression are clearly partially cancelling. Thus, on taking small values of the surface diffuseness parameter, the channel-capture formula underestimates the potential-capture cross section, contrary to condition (a). This can be explained as a result of the approximation $w_f^2(R) = 2/R$ being a poor one for a p-wave state in a square potential-well with small radius.

Although figs. (1) and (2) demonstrate the general usefulness (within ~40%) of the channel-capture expression it is also clear that in certain situations this expression can be highly inaccurate, with C_{opt} going

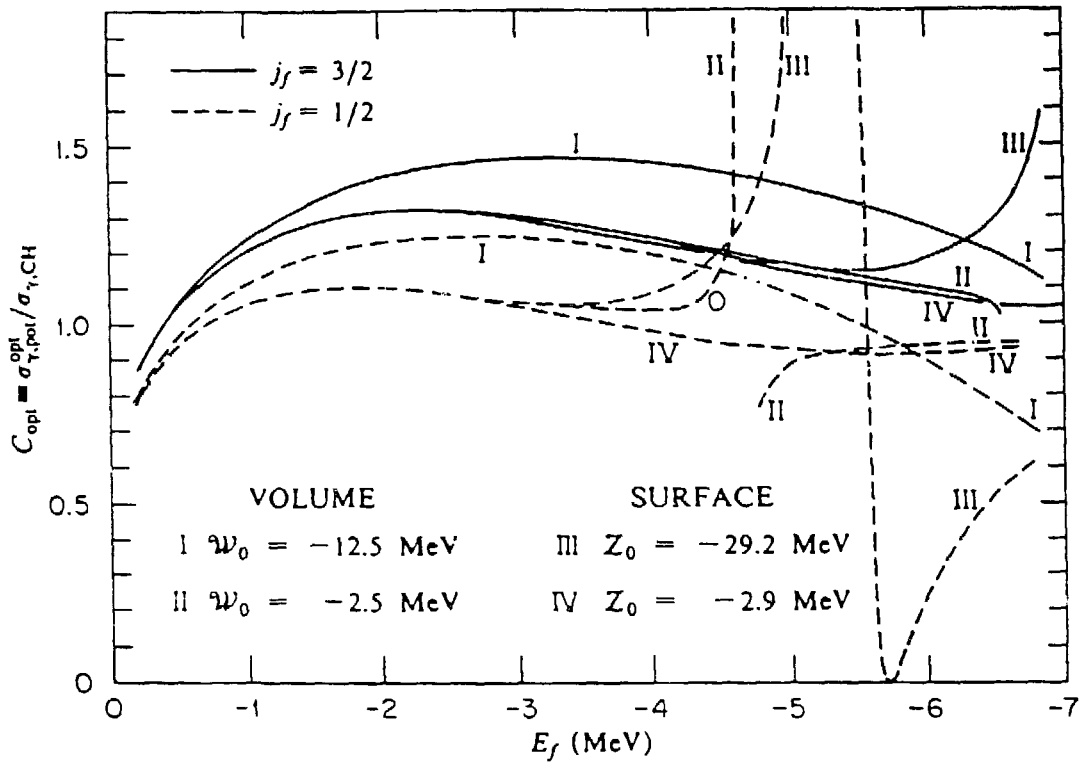

Fig. 2. Dependence of the ratio of potential capture cross section to channel capture cross section on the magnitude of the imaginary potential. Calculations have been made for surface and volume absorption and for both values of spin-orbit coupling. The surface diffuseness parameter is fixed at d = 0.69 fm. [Reproduced from ref.[17]].

through a major fluctuation from very large values to very small values for certain narrow ranges of the final-state binding energy. This fluctuation is associated with destructive interference of the terms in f(y) of eq. (vii) (occurring when Re R_J is negative, i.e., $a_{SJ} > R$) giving rise to a very small value of the channel-capture cross section. In this case the individual terms of f(y) will not be estimated accurately enough from the assumed simple analytic forms of the radial wave functions in the external region; furthermore, the ignored contribution to the radial dipole integral from the internal region becomes of comparable importance. Thus, it is dangerous to use the channel-capture formula as a precise estimate when this formula gives rise to a value of the capture cross section that is very much smaller than the estimate of the hard-sphere capture cross section of eq. (ix).

6. Compound-Nucleus Contributions

The previously discussed limitations of the channel-capture formula apply even when no other mechanisms contributing to the capture process exist. However, in the neutron-nucleus interaction, the target nucleus cannot be described as a totally inert core, simply providing a potential field for the scattering of the incident nucleon. One aspect of the residual nucleon interactions in the system is implied by the use of the spectroscopic factor θ_f^2 to give the single-particle purity of the final state. The remaining configurations of the final states can be connected in the E1 transition strength to configurations mixed into the initial state by excitation of highly complicated states of the compound nucleus. Even at off-resonance energies of the incident neutron, small components of such compound-nucleus states will be mixed into the initial state; and therefore, it is necessary to make some estimate of the interfering compound-nucleus contribution to the radiative capture cross section.

An overall survey of the radiation widths for individual transitions from low-energy compound nucleus resonances was carried out by Cameron[19] with the overall result

$$\Gamma_{\gamma,CN}(\text{in meV}) = 0.33 \times 10^{-6} E_\gamma^3(\text{in MeV}) A^{2/3} D(\text{in eV}), \qquad \text{(xiii)}$$

where D is the mean resonance spacing (for resonances of single spin and parity). An alternative estimate of the compound-nucleus radiation width can be derived using Brink's method from the giant dipole resonance model. This gives

$$\Gamma_{\gamma,CN} = \frac{4}{3\pi} \frac{NZ}{A} \frac{e^2}{\hbar c} \frac{(1+0.8x)}{mc^2} \frac{\Gamma_G E_\gamma^4}{(E_\gamma^2 - E_G^2)^2 + (\Gamma_G E_\gamma)^2} \cdot D , \qquad \text{(xiv)}$$

where E_G is the energy of the electric dipole giant resonance, Γ_G is its width, and x is the fraction of exchange force present in the nuclear force. Equation (xiv) probably gives a more nearly accurate representation of the gamma-ray energy dependence of the partial radiative width, but it has been established from the study of total radiation widths that it overestimates the bulk of radiative transitions with energies in the

range 2 to 3 MeV from neutron resonances by a factor of the order of 2 to 4. On the other hand, eq. (xiii) underestimates the average strength of high energy transitions ($E_\gamma \gtrsim 5$ MeV). Hence, this equation can be taken as giving a conservative estimate of the average width for the typical compound-nucleus component of radiative transitions. An estimate of the compound-nucleus contribution to the cross section may be obtained from eq. (xiii) by assuming that for each channel spin J a single nearby compound nucleus level of energy $E_{\lambda J}$ is responsible for the major part. Then,

$$\sigma_{\gamma(i\to f)CN} = \pi\lambda\!\!\!^{-2} \sum_J g_J \frac{\Gamma_{\lambda(n)}\Gamma_{\gamma(i\to f)CN}}{(E_{\lambda J}-E)^2}$$

$$= \pi\lambda\!\!\!^{-2} \sum_J g_J \frac{2P_0 \mathrm{Re}R_J}{E_{\lambda J}-E} \cdot \Gamma_{\gamma(i\to f)CN} , \qquad \text{(xv)}$$

where P_0 is the s-wave neutron penetration factor, $P_0 = R/\lambda\!\!\!^{-}$. The amplitude of this contribution to the radiative transition can be taken to interfere either constructively or destructively with the channel-capture contribution, giving a final estimate for the overall capture cross section of

$$\sigma_{\gamma(i\to f)} \approx \left| \sigma^{1/2}_{\gamma(i\to f)CH} \pm \sigma^{1/2}_{\gamma(i\to f)CN} \right|^2 . \qquad \text{(xvi)}$$

The actual value of $\sigma_{\gamma(i\to f)CN}$ is subject to Porter-Thomas fluctuations inherent in the partial radiative width $\Gamma_{\gamma(i\to f)CN}$, so the two extreme values of $\sigma_{\gamma(i\to f)}$ given by eq. (xvi) can be taken as a measure of the range of variation that can be expected in the observed value of the cross section. Numerical examples of the possible effect of the compound-nucleus contributions are given in sects. 7 and 8 on the application of the channel-capture theory to some specific data.

7. Capture by ^{12}C

In low-energy neutron capture by ^{12}C, significant primary transitions to the ground state and 3.686-MeV excited state of ^{13}C (gamma-ray energies of 4.945 MeV and 1.262 MeV, respectively) occur; the cross

sections for neutrons of energy 0.0253 eV are 2.38 ± 0.05 mb and 1.14 ± 0.02 mb, respectively. The spectroscopic factors of the final state are 1.1 and 0.1, respectively. The scattering length of thermal neutrons interacting with ^{12}C is 6.15 fm. The channel-capture cross section calculated from eq. (xii) for these two transitions is shown as a function of potential radius R in fig. 3. Note that these calculations are 27% higher than those given in ref.[3]) because of the $[(M+m)/M]^3$ factor discussed in sect. 4.

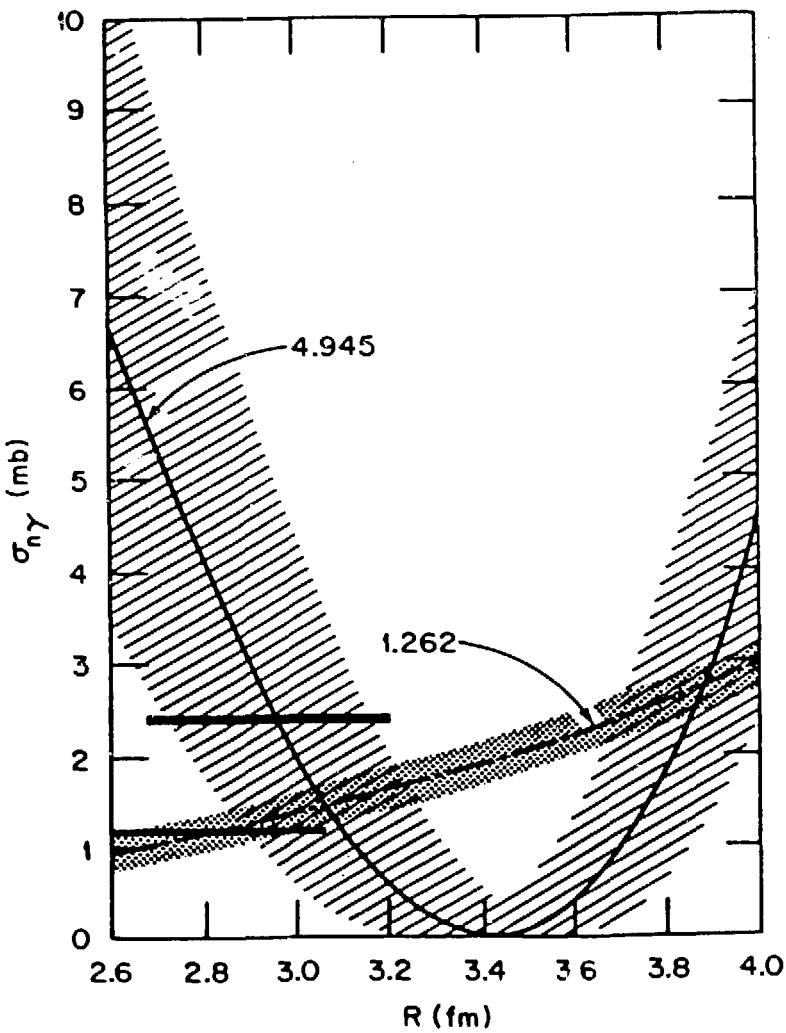

Fig. 3. Calculated channel-capture cross section as a function of the nuclear potential radius for the 4.945-MeV and 1.262-MeV transitions from the $^{12}C(n,\gamma)$ reaction. Shaded regions show the effect of including a small compound-nucleus contribution. Measured cross sections are shown by horizontal bars.

is quite insensitive to the values of the potential radius, provided that the well depth is adjusted to reproduce the binding energy of the final state. Allowing for interference with a valency component from the bound state at -2.02 MeV, these authors obtain a realistic estimate of the modified channel-capture cross section which is close to the measured value; and again their estimate is quite insensitive to the value of the potential radius.

8. Capture by ^{9}Be

Four electric dipole transitions have been observed following slow neutron capture by ^{9}Be. The final state, their spectroscopic factors, gamma-ray energies, and measured capture cross sections used in ref.[14] for analysis of these transitions within the framework of the channel-capture theory are shown in table 2, together with the estimate of the capture cross section from eq. (xii), using a potential radius of $R = 3.00$ fm and a slow neutron scattering length $a = 7.01$ fm (essentially independent of channel spin). The theoretical estimates differ from the experimental values of the cross sections by a factor of up to about 2.5 in either direction. These discrepancies have been overcome in ref.[14] by adjustment of the nuclear radius R, separately for each incident channel spin J, i.e., eq. (vii) is modified by defining $y_J = \kappa R_J$. In addition, allowance is made for mixing of the two different spin-orbit configuration $j = 1/2$ and $3/2$ in the final state f by using the factor $|\sum_j W_{J_f j_J} \cdot \theta_{f,j}|^2$, the phase in the product of Wigner 6-j symbols and vector coupling coefficient being implied in the process of taking the square root. The available data and calculation on the spin-orbit mixing of the 3.368- and 5.959-MeV state of ^{10}Be imply that the major contributions to the channel-capture cross section come from opposite incident channel spins in the two cases and allow an apparently clean separation of the "spin-dependent potential radii," R_J. The analysis of the data in this way leads to a quoted difference $R_{J=1} - R_{J=2} = 1.12$ fm. From this difference, a spin-dependent term $V_I^+ \cdot \vec{\sigma}$ (where $\vec{\sigma}$ denotes the neutron spin) in the optical potential for ^{9}Be+ n is deduced.

From our discussion in sect. 6, it is clear that the development given in ref.[14] is unnecessary in order to explain the experimental

capture cross section data and, indeed, that the deduction of a channel-spin-dependent nuclear potential radius, if that development is followed, must be a very imprecise procedure. The reason is that conditions are such that large interference terms are again present (the hard-sphere capture cross sections calculated for the first three transitions of table 2 are 25, 13, and 10 mb, respectively, i.e., 5 to 15 times greater than the experimental quantities). Thus, the C_{opt} factor resulting from a more realistic optical model calculation of the direct-capture cross section could differ very considerably from unity, and the factor of up to 2.5 noted in the preceding paragraph is not an unreasonable expectation for C_{opt}. Furthermore, the interfering effect of the compound-nucleus mechanism, as described in some detail for $^{12}C(n,\gamma)$ in sect. 7, could also be substantial here. A compound-nucleus contribution based on the Cameron estimate (eq. xiii), a bound s-wave level at $E_\lambda = -0.85$ MeV, and a level spacing conservatively estimated at $D \sim 1$ MeV, would give a range of cross sections of 3.8 to 26 mb, 0.005 to 1.4 mb, and 1.8 to 2.2 mb, respectively, for the first three transitions in ^{10}Be of table 2, thus easily encompassing the experimental data.

9. Conclusion

We have pointed out in this paper that a commonly used formulation of the theory of slow neutron direct capture, namely the channel-capture formula of Lane and Lynn[1], although appearing surprisingly accurate in reproducing a large body experimental data, is based on idealization of the scattering function that can lead to an analytically tractable and simple expression. The degree of accuracy that can be expected from the channel-capture formula has been estimated elsewhere[17] by comparisons with numerical computations of the potential-capture cross section from physically realistic optical models of neutron scattering. We have pointed out that, generally speaking, the channel-capture formula is accurate to better then 40% in the mass number range $A \approx 30$ to 70 (but usually underestimating the direct-capture cross section) if the spectroscopic factors are known reliably and accurately. It is worthwhile to stress that these factors (derived from comparisons of experimental results with DWBA predictions) generally carry an irreducible overall

uncertainty of 20%. We have also shown that for light nuclei certain mass-dependent factors not normally included in the channel-capture formula can be numerically significant.

In certain situations where there is large destructive interference among the terms entering the channel-capture formula, the inaccuracy inherent in the formula can be much greater than 40%. Furthermore, in such situations the contribution of the compound-nucleus mechanism to the capture amplitude from the tails of neutron resonance states can be comparable to or greater than the direct component.

It follows that the application of the channel-capture formula can be very unreliable in such cases. In particular, we have examined its application to the capture cross-section data of ^{12}C in order to determine the nuclear potential radius [ref.[3]] and to the data of ^{9}Be in order to determine channel-spin-dependent potential radii (ref.[14])] and hence the magnitude of a possible spin-spin term in the optical potential. We find in both cases that the likely magnitude of compound-nucleus contributions is sufficient to prevent the determination of such quantities with any worthwhile degree of precision. Because both cases also involve large destructive interference among the terms entering the channel-capture formula, the very application of this formula to treat these data is questionable.

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