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DETECTION OF A CHIRPING ELECTROMAGNETIC SIGNAL

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ABSTRACT

A matched chirp transform (MCT) method for detecting a dispersive electromagnetic pulse is described. The unique feature of this transform is that it gives a distribution of signal amplitude over time rather than frequency, and thereby simplifies signal detection and identification in the case described here.

In the MCT method, the incoming signal is matched to a set of signal segments that chirp in accordance with an expected model of the dispersive medium. The performance of the MCT method is compared with that of a standard periodogram method of frequency measurement.

1. Introduction

In this paper we consider the problem of detecting an electromagnetic pulse that has traveled through a dispersive medium and arrived at a receiver. For our purposes the medium is assumed to be characterized by a previously-described model [1] in which the transfer function through the medium is defined to have unit gain and nonlinear delay. The general model of the delay of the dispersive medium used here is

$$d = \frac{C}{f^{1/a}} \qquad (1)$$

d = time delay (samples)
C = constant; C>0
f = frequency (Hz-s)
a = constant; 0<a<1

Since a is a positive constant, the received signal chirps from high to low frequencies. From Eq. 1, the chirp rate (derivative of f with respect to d) is

$$\dot{f} = -aC^{a}d^{-(1+a)} \quad \text{Hz-s/sample} \qquad (2)$$

The result of passing an impulse through a medium described by Eqs. 1 and 2 is shown in Fig. 1. For this figure the time step (T) between samples and the other parameters in Eq. 1 are

T = 2.5 ns, C = 4.E4T, a = 0.5

The amplitude spectrum and group delay of the signal in Fig. 1 are shown in Fig. 2. (The group delay is found by differentiating the phase spectrum.) These verify the unit gain of the medium and the delay given in Eq. 1. The problem considered here is the detection of a noisy version of this type of signal, and we will use Fig. 1 with noise as an example.

2. Detection Methods

The detection process must ultimately work in real time, and must work at a high data rate such as in the example above. Therefore, the detection method must be reasonably simple and not involve extensive computation. Several such detection methods were considered.

The classical approach to real-time frequency measurement involves using the fast Fourier transform (FFT). The incoming waveform is partitioned into overlapping N-sample segments. Figure 3 illustrates such a partitioning of a portion of the waveform in Fig. 1, with segments overlapping 75%. As each segment arrives, the FFT is computed using a window sequence $[w_k]$ as follows:

$$X_m = \sum_{k=0}^{N-1} w_k x_k e^{-j2\pi mk/N}; \quad 0 \le m \le M \tag{3}$$

The FFT is treated as if the segment were monochromatic, and the segment frequency is taken to be the frequency of the largest squared FFT magnitude (periodogram) component.

Suppose the signal, x_k, comprising a data segment is a sinusoidal signal given by

$$x_k = e^{j2\pi k(f_0 + k\dot{f}/2)}; \quad 0 \le k < N \tag{4}$$

That is, the sequence $[x_k]$ begins at frequency f_0 and chirps at a constant rate. Then it is not difficult to show that $[X_m]$, the FFT of $[x_k]$, is given by

$$[X_m] = \text{FFT}[e^{j\pi k^2 \dot{f}}]; \quad m \leftarrow (m - m_0) \tag{5}$$

where m_0 is the index of frequency f_0. Thus, unless the derivative of f (f dot) is zero, $[X_m]$ is smeared around the base frequency, f_0, in proportion to f dot. This smearing causes an uncertainty in the location of the peak periodogram component, which in turn results in degraded performance of the FFT frequency detector in the presence of noise.

Another detection method that works with chirping signals is the "dechirping" method described by Li [2], which in turn results in a fast chirp transform (FCT). The FCT is derived as follows. We view an FFT component, X_m, as the correlation of the N-sample sequence $[x_k]$ with a complex sinusoidal sequence with constant frequency, f_m, and linear phase, θ_m, given by

$$f_m = m/N \quad \text{Hz-s}$$
$$\theta_m(k) = 2\pi\int f_m dk = 2\pi km/N \quad \text{rad} \tag{6}$$

Similarly, an FCT component is the correlation of $[x_k]$ with a complex sinusoid having a frequency chirping around f_m in Eq. 6 at a constant rate, that is, with frequency and phase given by

$$f_m(k) = m/N + (k-N/2)\dot{f} \quad \text{Hz-s}$$
$$\theta_m(k) = 2\pi[mk/N + (k^2-Nk)\dot{f}/2] \quad \text{rad} \tag{7}$$

In other words, the FCT of a sequence $[x_k]$ with N samples is the FFT of a dechirped sequence and is given by

$$FCT[x_k] = FFT[x_k e^{-j\pi(k^2-Nk)\dot{f}}] \tag{8}$$

The FCT is really a two-dimensional transform over a set of values of the chirp rate, f dot, as well as the frequency index, m, and the resulting large amount of computation is a disadvantage. Also, in the present situation, we are detecting signals with nonlinear chirp rates as in Eq. 2, which the FCT does not match precisely. The FCT and FFT methods are examined further in a Sandia Labs report [3], which amounts to this paper plus more details. We show that the two methods perform similarly with noisy versions of Fig. 1, but not as well as the method described in the next section.

A third method applicable to the detection and tracking of a chirping signal uses the adaptive predictor or line enhancer to track a narrowband signal in noise. Different versions of the adaptive line enhancer have been described by Griffiths [4] and Hush [5], and have been analyzed by Morgan [6] and others. We have not yet tried this approach, which could produce useful results, in the present study.

A fourth method, which we wish to apply in the present case, uses a matched chirp transform (MCT). Since we have a model of the time delay, that is, we know that we are searching for a signal similar to Fig. 1 in noise, we do not need all of the degrees of freedom in the FCT. Instead, we can search only for waveform components that are chirping in the time-dependent manner given by Eq. 2. To perform such a search, we can correlate each segment $[x_k]$ with sinusoidal components, just as described above, but now chirping in accordance with Eq. 2. This approach is described in the next section.

3. A Matched Chirp Transform

We now develop a matched chirp transforn (MCT) in which components chirping in accordance with Eq. 2 are correlated with each waveform segment $[x_k;\ 0 \le k < N]$. As in the FFT and FCT methods described above, the MCT component with maximum squared magnitude is taken to designate the most-likely frequency and chirp rate of the segment. Since Eq. 2 is obtained by differentiating Eq. 1, an MCT component also designates a specific time delay, that is, a specific time interval following the arrival of a signal through the dispersive medium. In effect, the frequency and time domains are made dependent by Eq. 1. The dependence is nonlinear and therefore evenly spaced points in one domain do not map into evenly spaced points in the other domain. We will derive MCT components that are spaced evenly in the time domain rather than in the frequency domain. The even time domain spacing produces the preferable detection procedure described below.

To specify the time domain spacing, we define d_0 as the minimum delay corresponding with the highest detectable frequency, 0.5 Hz-s, and we define d_1 as the maximum delay corresponding with the lowest frequency of interest, f_{min}, with $f_{min} > 0$. From Eq. 1, we have

$$d_o = C(0.5)^{-1/a}; \quad d_1 = C(f_{min})^{-1/a} \text{ samples} \quad (9)$$

The "time domain" for the MCT is now defined to be the domain from d_0 to d_1 samples, and we divide this domain into M equal intervals specified by

$$d_m = d_o + m(d_1 - d_o)/M \text{ samples}; \quad 0 \le m \le M \quad (10)$$

(We assume that (d_1-d_o) is a multiple of M here.) The frequency of a signal initially at the frequency corresponding with d_m amd chirping in accordance with Eq. 1 is

$$f_m(k) = \left[\frac{CM}{(M-m)d_o + md_1 + Mk}\right]^a \quad \text{Hz-s}; \qquad \begin{array}{l} 0 \le m \le M \\ 0 \le k \end{array} \qquad (11)$$

We define the mth MCT component to be the correlation of the N-sample sequence $[x_k]$ defined above with a complex sinusoid chirping in accordance with Eq. 11. The phase of the complex sinusoid is given by

$$\begin{aligned} \theta_m(k) &= 2\pi \int f_m(k)dk \\ &= \frac{2\pi(CM)^a}{M(1-a)}[(M-m)d_o + md_1 + Mk]^{1-a} \qquad (12) \end{aligned}$$

$$\begin{array}{l} 0 \le m \le M \\ 0 \le k < N \end{array}$$

The MCT of $[x_k]$ has the form of the other transforms above, that is,

$$MCT[x_k] = \sum_{k=0}^{N-1} x_k e^{-j\theta_m(k)}; \qquad 0 \le m \le M \qquad (13)$$

Unlike the FCT in Eq. 8, the MCT cannot be written as the FFT of a dechirped sequence, because of the form of $\theta_m(k)$ in Eq. 12. However, if all N(M-1) values of $\exp[-\theta_m(k)]$ in Eq. 13 are tabulated, the MCT does not require the excessive computing of the FCT, and its computation in real time is feasible in our applications.

4. Application of the MCT

An example of the MCT of a single segment is shown along with the corresponding FFT in Fig. 4. The segment has length N=256 samples and is taken from the waveform in Fig. 1 beginning at 1000 samples or 2.5 μs, where the signal frequency is, in accordance with Eq. 1, 0.3162 Hz-s or 126.5 MHz. Like the FFT, the MCT is computed over M=128 delay intervals so that it has the number of degrees of freedom of the waveform segment. As expected, the MCT amplitude is concentrated into a band that is narrow compared with the FFT band.

We apply the MCT in essentially the manner described at the beginning of section 2 above.

The incoming waveform, containing a signal similar to Fig. 1 in noise, is partitioned into segments. The segments may be overlapping. (Currently we are using an overlap of 75%, as illustrated in Fig. 3.) The MCT of each segment is taken without windowing as the segment arrives, and the delay (d) value of the segment is taken to be the delay corresponding with the largest MCT magnitude.

An example comparing the MCT and FFT is shown in Fig. 5. Both transforms were applied to the detection of the waveform of Fig. 1 added to a large amount of broadband (20-180 MHz) Gaussian noise. The signal begins at 15.0 μs. The MCT parameters were the same as in Fig. 4. Segment overlap was 75% for both transforms. The FFT of course gives the frequency of each segment, while the MCT gives the delay. The MCT produces a clearer detection of the signal in this example, as it does generally.

The linearity of the MCT delay plot in Fig. 5 is an advantage in detection for several reasons. First, a simple detection criterion can be established by fitting a least-squares line to the delay plot and placing a threshold on the squared error. Also, by extrapolating the line back to its intercept with the time axis, one can extablish an accurate estimate of the signal arrival (15.0 μs in Fig. 5).

Furthermore, suppose that the constant C used in the MCT is not chosen correctly, and that $\hat{C}$ is used instead. In Eq. 1 we see that this produces a scaling of the delay (d) by the factor $\hat{C}/C$, which is indicated by the slope of the delay-vs-time line. Thus we can establish C from

$$C = \frac{\hat{C}}{\text{slope of delay-vs-time line}} \qquad (14)$$

The other chirp parameter, a, affects the linearity of the delay-vs-time plot, and must be known accurately for the MCT method to work.

In conclusion, the MCT appears to be a tool that is useful in the detection of a chirping electromagnetic signal.

5. References

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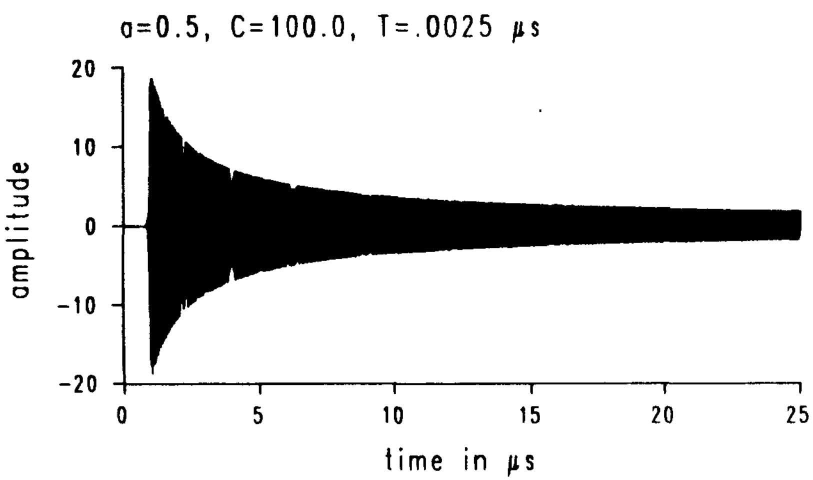

Fig. 1. Impulse response of dispersive medium

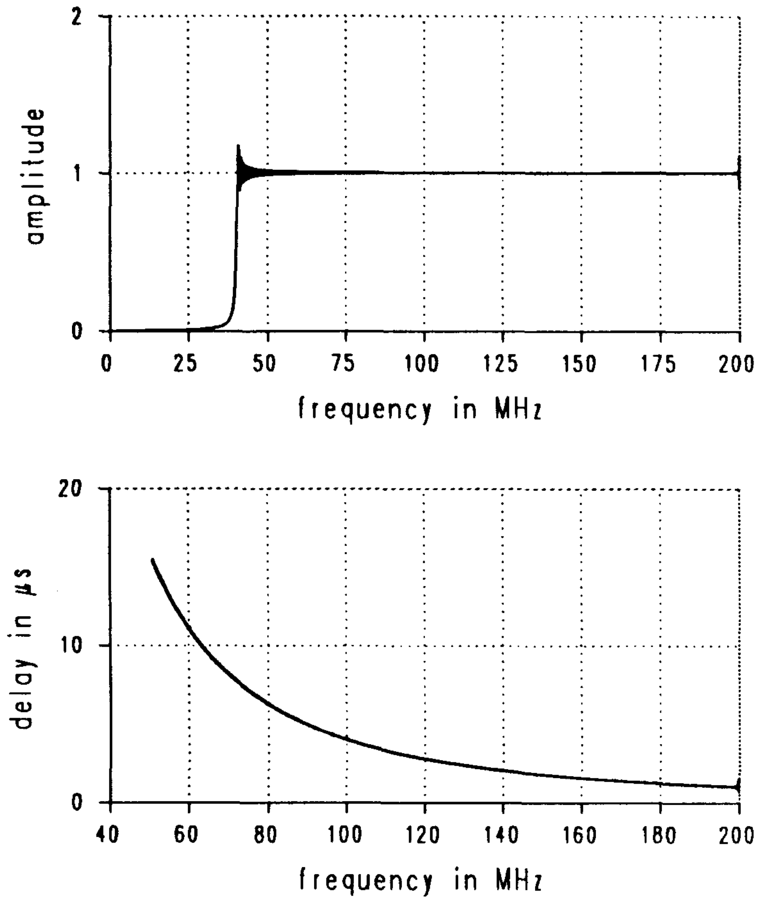

Fig. 2. Amplitude and group delay spectra of the impulse response in Fig. 1.

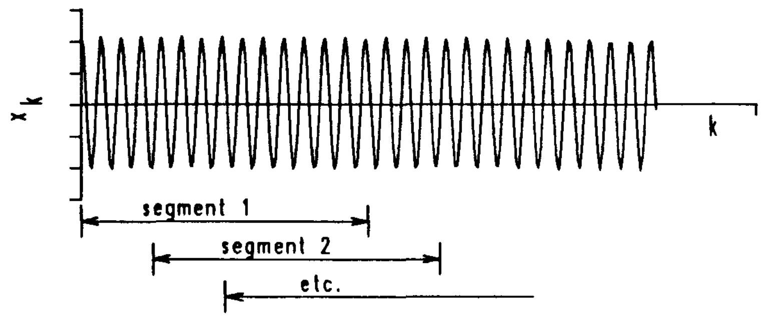

Fig. 3. Time-domain segments overlapping 75%.

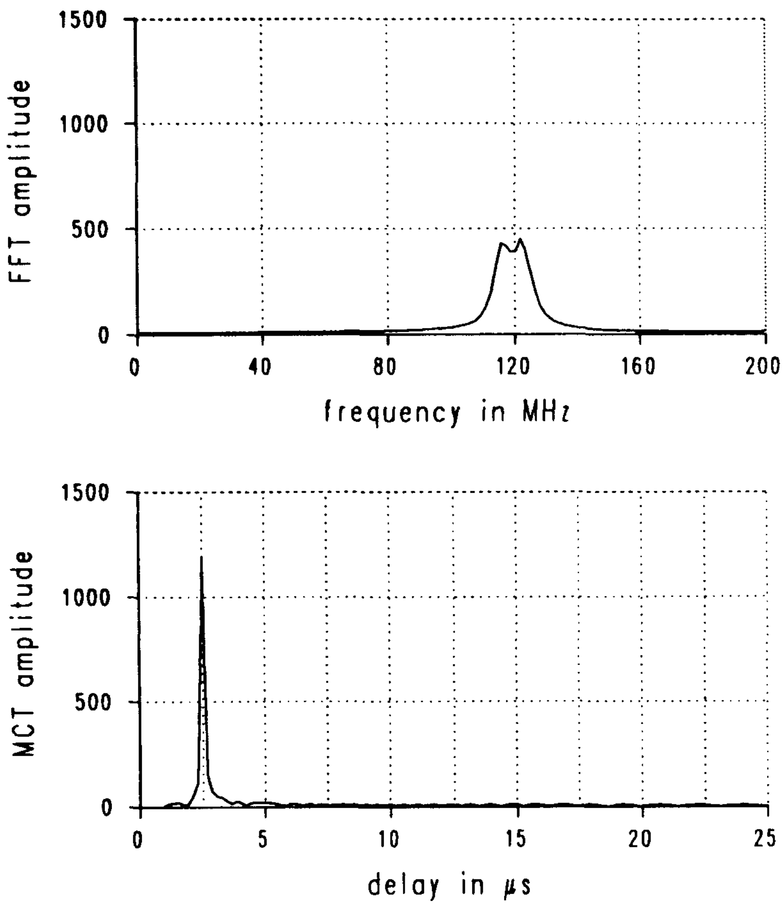

Fig. 4. Comparison of FFT and MCT amplitudes of a segment beginning at 2.5 microseconds in in Fig. 1. N=256, M=128, 400<d<10000.

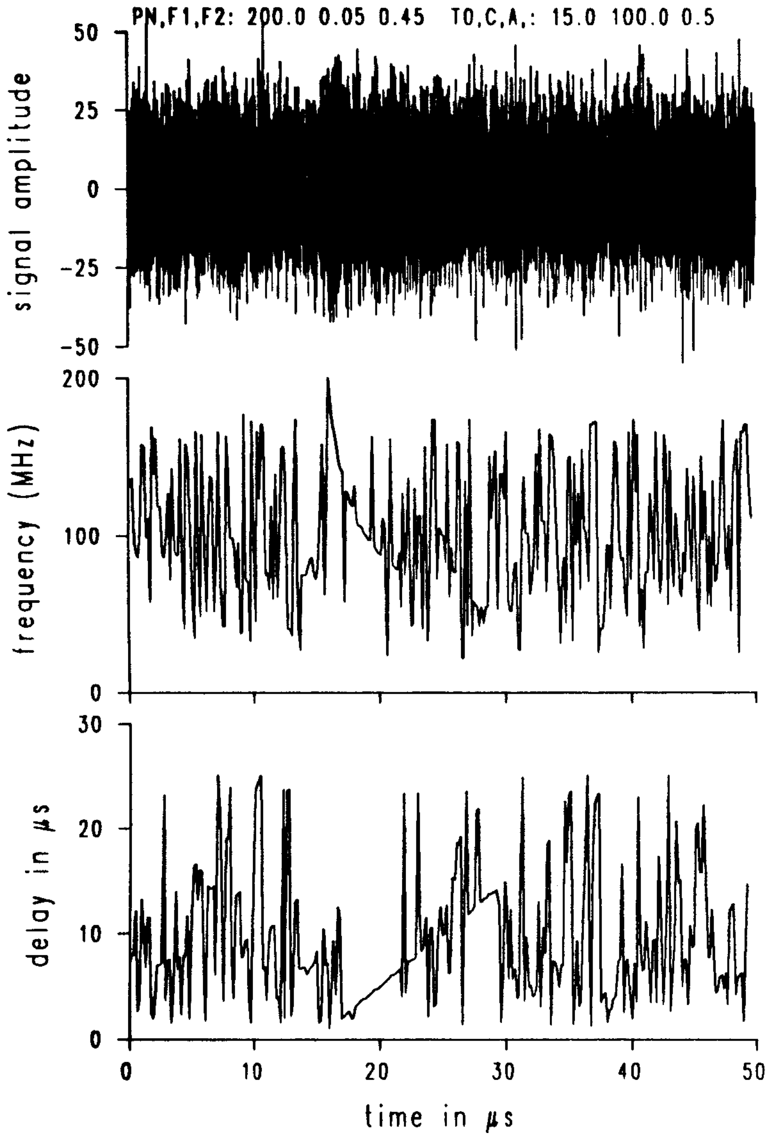

Fig. 5. Detection using the FFT and MCT methods on the signal in Fig. 1 plus noise.