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# PLASMA PHYSICS

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# STABILITY ANALYSIS OF CYLINDRICAL VLASOV EQUILIBRIA

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# STABILITY ANALYSIS OF CYLINDRICAL VLASOV EQUILIBRIA

R.W. Short

A method is presented for the fully kinetic, nonlocal stability analysis of cylindrically symmetric equilibria. Applications to the lower hybrid drift instability and the modes associated with a finite-width relativistic E-layer are discussed.

## I. INTRODUCTION

A fully kinetic stability analysis of a Vlasov equilibrium usually proceeds as follows: the Vlasov equation is integrated over the unperturbed particle orbits to obtain the perturbed distribution function, which is in turn integrated over velocity space to obtain the perturbed charge and current densities. Substituting these densities into linearized field equations then yields a closed set of equations for the perturbed fields. If we Laplace transform and set the initial value terms to zero, we get a dispersion relation for the complex frequencies of the normal modes.

For inhomogeneous plasmas in bounded configurations the most difficult part of this calculation is usually the integrations over the unperturbed orbits and over velocity space. The purpose of this paper is to present an efficient method of performing such calculations for systems with cylindrical symmetry, taking advantage of the fact that in such geometries the particle motion must be periodic in the radial coordinate. The method is first described in general, then an application to the stability analysis of a relativistic E-layer is presented.

## II. DESCRIPTION OF METHOD

We assume cylindrical symmetry, with coordinate system as shown in Fig. 1, so that r is the only non-ignorable coordinate. The perturbed potentials will be expanded in eigenfunctions of the field operators; so that these eigenfunctions will be discrete we assume the plasma to be surrounded by a conducting cylinder of radius R and impose periodicity in the z-direction with period L. These assumptions are made for mathematical convenience; if they are inappropriate to the problem at hand, they may be removed by taking $R \to \infty$, $L \to \infty$, resulting in a continuous dispersion matrix.

Denoting the equilibrium scalar and vector potentials by $\phi_o(r)$, $\underline{A}_o(r)$, the equilibrium fields have the form

$$\underline{E}_o = E_o(r)\hat{r},$$
$$\underline{B}_o = B_{o\theta}(r)\hat{\theta} + B_{oz}(r)\hat{z}. \qquad (1)$$

The equilibrium distribution functions will depend only on the particle constants of motion:

$$f_{oj}(\underline{r},\underline{v}) = f_{oj}(H,P_\theta,P_z) \qquad (2)$$

where the index "j" denotes particle species and the constants of the motion are the energy and the momenta conjugate to the ignorable coordinates θ and z:

$$H = \frac{m_j}{2}(v_r^2 + v_\theta^2 + v_z^2) + e_j\phi_o(r), \qquad (3)$$

$$P_\theta = m_j r v_\theta + \frac{e_j}{c} r A_\theta^0(r) \qquad (4)$$

$$P_z = m_j v_z + \frac{e_j}{c} A_z^0(r). \qquad (5)$$

To simplify the description of the method we consider the electrostatic approximation. It is not difficult to extend the approach to the electromagnetic case,[1] and the example to be discussed later is fully electromagnetic.

The plasma is described by the Vlasov-Poisson equations:

$$\left[\frac{\partial}{\partial t} + \underline{v} \cdot \nabla + \frac{e_j}{m_j}\left(\underline{E} + \frac{\underline{v}\times\underline{B}}{c}\right) \cdot \frac{\partial}{\partial \underline{v}}\right] f_j(\underline{r},\underline{v},t) = 0 \qquad (6)$$

$$\nabla^2\phi(\underline{r},t) = -\sum_j 4\pi e_j \int d^3v f(\underline{r},\underline{v},t) \tag{7}$$

We linearize these equations, writing

$$f(\underline{r},\underline{v},t) = f_o(r,\underline{v}) + f_1(\underline{r},\underline{v},t),$$

$$\phi(\underline{r},t) = \phi_o(r) + \phi_1(\underline{r},t),$$

where $f_1$ and $\phi_1$ represent a small perturbation of the equilibrium quantities $f_o$ and $\phi_o$. In the electrostatic approximation the magnetic field is not perturbed. The linearized Vlasov-Poisson equations read

$$\left(\frac{\partial}{\partial t} + \mathcal{L}_o\right) f_{1j}(\underline{r},\underline{v},t) = -\frac{e_j}{m_j}\underline{E}_1(\underline{r},t) \cdot \frac{\partial}{\partial \underline{v}} f_{oj}(\underline{r},\underline{v}) = \frac{e_j}{m_j}[\nabla\phi_1(\underline{r},t)] \cdot \frac{\partial f_{oj}}{\partial \underline{v}} \tag{8}$$

$$\nabla^2\phi_1(\underline{r},t) = -\sum_j 4\pi e_j \int d^3v f_{1j}(\underline{r},\underline{v},t) \tag{9}$$

where we have defined the equilibrium Liouville operator

$$\mathcal{L}_o = \underline{v} \cdot \nabla + \frac{e_j}{m_j}[\underline{E}_o + \frac{1}{c}(\underline{v} \times \underline{B}_o)] \cdot \frac{\partial}{\partial \underline{v}}.$$

From (3)-(5) we have the relations

$$\frac{\partial H}{\partial \underline{v}} = m_j\underline{v}, \quad \frac{\partial P_\theta}{\partial \underline{v}} = m_j r\hat{\theta}, \quad \frac{\partial P_z}{\partial \underline{v}} = m_j\hat{z},$$

so that (8) may be written

$$\left(\frac{\partial}{\partial t} + L_o\right)f_{1j} = e_j\left[\frac{\partial f_{oj}}{\partial H}\underline{v}\cdot\nabla + \frac{\partial f_{oj}}{\partial P_\theta}\frac{\partial}{\partial\theta} + \frac{\partial f_{oj}}{\partial P_z}\frac{\partial}{\partial z}\right]\phi_1 \qquad (10)$$

We assume $e^{-i\omega t}$ time dependence for all perturbed quantities, with $\mathrm{Im}(\omega) > 0$ since we are interested in instabilities. Thus we write

$$\phi_1(\underline{r},t) = \phi_1(\underline{r})e^{-i\omega t}, \quad f_{1j}(\underline{r},\underline{v},t) = f_{1j}(\underline{r},\underline{v})e^{-i\omega t}$$

The operator $\left(\frac{\partial}{\partial t} + L_o\right)$ in (10) represents the total time derivative along a particle trajectory, and so the perturbed distribution function is obtained by integrating over time along the unperturbed orbit:

$$f_{1j}(\underline{r},\underline{v}) = \int_{-\infty}^{t} dt' e^{i\omega(t-t')}\left[\frac{\partial f_{oj}}{\partial H}\frac{d}{dt'} + \frac{\partial f_{oj}}{\partial P_\theta}\frac{\partial}{\partial\theta'} + \frac{\partial f_{oj}}{\partial P_z}\frac{\partial}{\partial z'}\right]\phi_1[\underline{r}'(t')] \qquad (11)$$

Here $\underline{r}'(t')$ is the trajectory of a particle in the equilibrium fields with initial condition

$$\underline{r}'(t' = t) = \underline{r}, \quad \underline{v}'(t' = t) = \underline{v}. \qquad (12)$$

Next we resolve $\phi_1(\underline{r})$ into its Fourier components in the ignorable coordinates $\theta$ and $z$:

$$\phi_1(\underline{r}) = \sum_{\ell,k}\phi_{\ell,k}(r)e^{i(\ell\theta+kz)}, \qquad (13)$$

where $k = 2\pi n/L_z$, $n$ an integer. Using (11) and (13), Poisson's equation becomes

$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}\right) \sum_{\ell, k} \phi_{\ell, k}(r) e^{i(\ell\theta + kz)}$$

$$= - \sum_j 4\pi e_j^2 \int d^3v \int_{-\infty}^{t} dt' e^{i\omega(t-t')} \left[\frac{\partial f_{oj}}{\partial H}\frac{d}{dt'} + \frac{\partial f_{oj}}{\partial P_\theta}\frac{\partial}{\partial \theta'} + \frac{\partial f_{oj}}{\partial P_z}\frac{\partial}{\partial z'}\right] \tag{14}$$

$$\cdot \sum_{\ell', k'} \phi_{\ell', k'}(r') e^{i(\ell'\theta' + k'z')}$$

Note that although the right side of (14) contains t, it is actually independent of the value of t. In fact, we could remove t altogether by defining a new variable $\tau = t' - t$ and replacing $\int_{-\infty}^{t} dt'$ by $\int_{-\infty}^{0} d\tau$, (and this usually done). We shall retain the formal t "dependence," however, as it will prove useful later.

Gr. tau

To isolate one Fourier coefficient on the left side of (14) we multiply by $(1/2\pi)\exp[-i(\ell\theta+kz)]$ and integrate over θ and z. From (12), and the fact that θ and z are ignorable, we see that the quantities $\theta' - \theta$ and $z' - z$ are independent of θ and z, respectively, since for fixed t' and t a change in θ changes θ' by the same amount, and similarly for z. Using the identities

$$k'z' - kz = k'(z' - z) + (k' - k)z,$$

$$\ell'\theta' - \ell\theta = \ell'(\theta' - \theta) + (\ell' - \ell)\theta$$

we obtain

$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{\ell^2}{r^2} - k^2\right)\phi_{\ell,k}(r) =$$

$$- \Sigma 4\pi e_j^2 \int d^3v \int_{-\infty}^{t} dt' e^{i\omega(t-t')}\left[\frac{\partial f_{oj}}{\partial H}\frac{d}{dt'} + \frac{\partial f_{oj}}{\partial P_\theta}\frac{\partial}{\partial \theta'} + \frac{\partial f_{oj}}{\partial P_z}\frac{\partial}{\partial z'}\right] \tag{15}$$

$$\cdot\ \phi_{\ell,k}(r') e^{i[\ell(\theta'-\theta) + k(z'-z)]}$$

Next we expand the radial dependence in eigenfunctions of the field operator

$$\phi_{\ell,k}(r) = \sum_n \alpha_n \phi_n(r), \tag{16}$$

where $\phi_n(r)$ satisfies the eigenvalue equation

$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{\ell^2}{r^2}\right)\phi_n(r) = -\lambda_n^2 \phi_n(r). \tag{17}$$

Here we have suppressed the indices $\ell$ and k on $\phi_n$ and $\alpha_n$. The eigenfunctions may be written

$$\phi_n(r) = A_n J_\ell(\lambda_n r),$$

where $J_\ell$ is the $\ell^{th}$ order Bessel function, $A_n = \sqrt{2}/[RJ_{\ell+1}(\lambda_n R)]$ is a normalization constant, and $\lambda_n$ is the $n^{th}$ root of the equation $J_\ell(\lambda_n R) = 0$. The functions $\phi_n(r)$ satisfy the orthonormality relation

$$\int_o^R dr r \phi_{n'}(r)\phi_n(r) = \delta_{nn'}. \tag{18}$$

Substituting (16) into (15), multiplying by $r\phi_{n'}(r)$, and integrating over r yields a linear relation in the coefficients $\alpha_n$, which we may write as

$$\sum_{n'} (\lambda_n^2 + k^2) D_{nn'}(\omega)\alpha_{n'} = 0, \tag{19}$$

where

$$D_{nn'}(\omega) = \delta_{nn'} - \sum_j \frac{4\pi e_j^2}{k^2+\lambda_n^2} \int_o^R dr r \int d^3v \phi_n(r) e^{-i(\ell\theta+kz-\omega t)}$$
$$\cdot \int_{-\infty}^{t} dt' e^{-i\omega t'} \left[\frac{\partial f_{oj}}{\partial H} \frac{d}{dt'} + i\ell \frac{\partial f_{oj}}{\partial P_\theta} + ik \frac{\partial f_{oj}}{\partial P_z}\right] \phi_{n'}(r') e^{i(\ell\theta'+kz')} \tag{20}$$

Stability is now determined by truncating the infinite dispersion matrix $D_{nn'}(\omega)$ in a suitable way (which will be discussed below) and solving the dispersion relation

$$\det[D(\omega)] = 0 \tag{21}$$

for the complex mode frequency $\omega$. If $\mathrm{Im}(\omega) > 0$ the mode is unstable, and $\mathrm{Im}(\omega)$ is the growth rate. The eigenvector of $D(\omega)$ associated with eigenvalue zero then gives the expansion coefficients $\alpha_n$ for the potential associated with the mode.

The remaining problem, then, is the evaluation of the matrix elements $D_{nn'}(\omega)$. The main purpose of this paper is to present an efficient way of calculating these elements.

## III. ORBIT AND PHASE SPACE INTEGRALS

First the particle orbits in the unperturbed fields must be determined. The Hamiltonian depends only on r, $P_r$, $P_\theta$, and $P_z$ and is a constant equal to the particle energy:

$$H(r,P_r,P_\theta,P_z) = E.$$

We may then solve for $P_r$ as a function of $r$, which gives $\dot{r}$ as a function of $r$:

$$\dot{r} = \frac{\partial H}{\partial P_r} = \dot{r}(r,E,P_\theta,P_z).$$

If the particle motion is bounded and non-asymptotic, a particle which is at $r_o$ at some time $t$ must return to $r_o$ at some later time $t + T$. When the particle returns to $r_o$ it must have the same radial velocity $\dot{r}(r_o)$. Thus the motion in $r$ is periodic with period $T(P_\theta,P_z,E)$. Now $\dot{\theta} = \partial H/\partial P_\theta$ and $\dot{z} = \partial H/\partial P_z$ depend on $t$ only through $r$, so they too must be periodic, and we may write

$$\theta(t) = \eta t + \tilde{\theta}(t) + \theta_o,$$

$$z(t) = \sigma t + \tilde{z}(t) + z_o,$$

where $\eta$ and $\sigma$ are constants, $\tilde{\theta}$ and $\tilde{z}$ are periodic, and $\theta_o$ and $z_o$ are chosen so $\tilde{\theta}(t = 0) = \tilde{z}(t = 0) = 0$.

The expansion functions for the potential may now be written

$$\phi_n(r')e^{i(\ell\theta'+kz')} = A_n e^{i(\ell\eta+k\sigma)t'} J_\ell(\lambda_n r') e^{i\ell\tilde{\theta}'} e^{ik\tilde{z}'} e^{i(\ell\theta_o+kz_o)}$$

This expression can be written in the form of a Fourier series:

$$\phi_n(r')e^{i(\ell\theta'+kz')} = \sum_{m=-\infty}^{\infty} G_m(E,P_\theta,P_z)e^{i(\ell\eta+k\sigma+m\Omega)t'} \qquad (22)$$

where $\Omega = 2\pi/T(E,P_\theta,P_z)$ and the coefficients $G_m$ may be calculated in a straightforward manner using Graf's addition theorem and other Bessel function identities.[1] Using (22) we may now perform the t' integration in (20) and obtain

Gr. delta

$$D_{nn'}(\omega) = \delta_{nn'} - \sum_j \frac{4\pi e_j^2}{k^2+\lambda_n^2} \int_0^R drr \int d^3v \phi_n(r) e^{-i(\ell\theta+kz)}$$

$$\cdot\; i\left[(\ell\eta + k\sigma + m\Omega)\frac{\partial f_{oj}}{\partial H} + \ell\frac{\partial f_{oj}}{\partial P_\theta} + k\frac{\partial f_{oj}}{\partial P_z}\right] \tag{23}$$

$$\cdot\; \frac{\sum_m G_m e^{i(\ell\eta+k\sigma+m\Omega)t}}{i[\ell\eta+k\sigma+m\Omega-\omega]}$$

Next we make use of the t dependence of the integrand in (23). If we regard $(r,\theta,z)$ as functions of t in the same manner that $(r',\theta',z')$ are functions of t', then increasing t simply advances each particle along its orbit. Since we start with an equilibrium our result for the dispersion matrix must thus be independent of t. However, advancing t also advances the integrand along an orbit, and we can use this fact to perform the integral over the non-ignorable coordinate r. We shall see that writing the potential exponsion function in the form (22) is very helpful in this calculation.

First we transform the variables of integration from the velocities to the momenta:

$$\frac{\partial(P_r,P_\theta,P_z)}{\partial(v_r,v_\theta,v_z)} = m_j^3 r, \quad \text{so} \quad rdrd^3v = \frac{1}{m_j^3} drdP_r dP_\theta dP_z$$

Next we change the integration variables r, $P_r$ to t, H using $drdP_r = dtdH$. The result is

$$D_{nn'}(\omega) = \delta_{nn'} - \sum_j \frac{4\pi e_j^2}{(k^2+\lambda_n^2)m_j^3} \int dH dP_\theta dP_z \int_0^T dt \sum_{m'} G^*_{m'} e^{-i(\ell\eta+k\sigma+m'\Omega)t}$$

$$\cdot \left[(\ell\eta+k\sigma+m\Omega)\frac{\partial f_{oj}}{\partial H} + \ell\frac{\partial f_{oj}}{\partial P_\theta} + k\frac{\partial f_{oj}}{\partial P_z}\right]\sum_m \frac{G_m e^{i(\ell\eta+k\sigma+m\Omega)t}}{\ell\eta+k\sigma+m\Omega-\omega} \tag{24}$$

$$= \delta_{nn'} - \sum_j \frac{4\pi e_j^2}{(k^2+\lambda_n^2)m_j^3} \int dH dP_\theta dP_z T(H,P_\theta,P_z)$$

$$\cdot \sum_m \left[(\ell\eta+k\sigma+m\Omega)\frac{\partial f_{oj}}{\partial H} + \ell\frac{\partial f_{oj}}{\partial P_\theta} + k\frac{\partial f_{oj}}{\partial P_z}\right]\frac{|G_m(H,P_\theta,P_z)|^2}{\ell\eta+k\sigma+m\Omega-\omega}$$

We are now left with only the integration over the constants of the motion to perform, which must in general be done numerically.

To give some idea of the simplifications afforded by the above approach as compared to the usual method, we consider the problem of a rigid rotor distribution function, which has the form

$$f_{oj}(\underline{r},\underline{v}) = f_{oj}(H_\perp - \omega_j P_\theta, v_z),$$

where $H_\perp = \frac{1}{2}m(v_x^2 + v_y^2) + e\phi$ and $\omega_j$ is a constant for each species. This problem has been treated by R.C. Davidson,(2) who shows that for a uniform magnetic field $\underline{B}_o = B_o\hat{z}$ the equilibrium electric field is proportional to radius, $\underline{E}_o = \varepsilon_o r\hat{r}$, $\varepsilon_o$ a constant. It may be shown that in a reference frame rotating with angular velocity $\omega_j$ the velocity distribution for species j is isotropic. Thus due to the $\underline{E} \times \underline{B}$ drift each species rotates in the fluid approximation with angular frequency $\omega_j$, and if two species are present with differing $\omega_j$'s, their relative drift may give rise to instability.

If we define

$$\omega_j^{\pm} = -\frac{\omega_{cj}}{2} \pm \frac{1}{2}\sqrt{\omega_{cj}^2 - \frac{4e_j\varepsilon_o}{m_j}}\,, \quad V_x = v_x + \omega_j y,\ V_y = v_y - \omega_j x,\ V_\perp^2 = V_x^2 + V_y^2,$$

$$D_{nn'}(\omega) = \delta_{nn'} + \sum_j \chi^j_{nn'}(\omega),$$

we may write Davidson's result for the dispersion matrix elements as

$$\chi^j_{nn'}(\omega) = -\frac{4\pi e_j^2}{m_j(k^2+\lambda_n^2)} A_n A_{n'} \int_o^R drr J_\ell(\lambda_n r) J_\ell(\lambda_{n'} r) \int d^3V \frac{1}{V}\frac{\partial f_{oj}}{\partial V_\perp}$$

$$+ \frac{4\pi e_j^2}{m_j(k^2+\lambda_n^2)} A_n A_{n'} \int_o^R drr \sum_{p=-\infty}^{\infty} J_\ell(\lambda_n r) J_p\left(\frac{\omega_j - \omega_j^-}{\omega_j^+ - \omega_j^-}\lambda_{n'} r\right) J_{\ell-p}\left(\frac{\omega_j^+ - \omega_j}{\omega_j^+ - \omega_j^-}\lambda_{n'} r\right)$$

$$\cdot \int d^3V \sum_{m=-\infty}^{\infty} J_m^2\left(\frac{\lambda_n V_\perp}{\omega_j^+ - \omega_j^-}\right) \frac{\left[k\left(\frac{\partial}{\partial v_z} - \frac{v_z}{V_\perp}\frac{\partial}{\partial V_\perp}\right) + \frac{\omega - \ell\omega_j}{V_\perp}\frac{\partial}{\partial V_\perp}\right] f_{oj}}{\omega_j - \ell\omega_j^- - (p+m)(\omega_j^+ - \omega_j^-) - kv_z} \tag{25}$$

This may be compared with the result obtained using the method of this paper (as discussed in detail in Ref. 1):

$$\chi^j_{nn'}(\omega) = -\frac{8\pi^2 e_j^2}{m_j(k^2+\lambda_n^2)} A_n A_{n'} \sum_{m=-\infty}^{\infty} \int_o^R da \int_o^{R-a} db \int_{-\infty}^{\infty} dv_z$$

$$\cdot \left\{ m_j \frac{\partial f_{oj}}{\partial H_\perp}\left[\ell(\omega_j^- - \omega_j) - m(\omega_j^+ - \omega_j^-)\right] + k\frac{\partial f_{oj}}{\partial v_z} \right\}$$

$$\cdot \left[(\omega_j^+ a + \omega_j^- b)^2 + (\omega_j^+ a + \omega_j^- b)\omega_{cj}(a+b) + \frac{e_j\varepsilon_o}{m_j}(a+b)^2\right]$$

$$\cdot \frac{J_{\ell+m}(\lambda_n b) J_m(\lambda_n a) J_{\ell+m}(\lambda_{n'} b) J_m(\lambda_{n'} a)}{\omega + m(\omega_j^+ - \omega_j^-) - kv_z - \ell\omega_j^-} \tag{26}$$

Here a and b are the distance of a particle gyrocenter from the symmetry axis and the gyroradius of the particle, respective. The expression (26) is somewhat simpler than (25) (it involves one less integration) due to the fact that we were able to carry out the integral corresponding to the r integral in (25) analytically. Another advantage of the form (26) is that the limits are chosen so $a + b < R$, i.e., only those particles whose orbits lie entirely inside the cylinder are included in the integral. By contrast, the expression (25) includes particles whose orbits cross the cylinder wall; for this reason it is not strictly correct, and also for this reason it is not analytically identical to (26). Finally, we note that the method used to derive (25) works only for rigid rotor distribution functions, while the method presented here may be applied to an arbitrary equilibrium distribution. Further discussion of these results and a numerical application to the lower hybrid drift instability may be found in Ref. 1.

## IV. ELECTROMAGNETIC CASE: INTERACTION OF EXTRAORDINARY MODE WITH RELATIVISTIC E-LAYER

The Vlasov-Maxwell equations may be treated in a similar manner to that presented above for the electrostatic equations, though the algebra is lengthier. For ease of presentation we treat the specific problem of the interaction of a relativistic E-layer with a warm background plasma and omit most of the calculational details. These details, along with a treatment of the general Vlasov-Maxwell problem, may be found in Ref. 1.

The mode we are interested in was invoked by Striffler and Kammash[3] to explain radiation observed in Astron near the upper hybrid frequency. In

Astron-type devices a relativistic E-layer rotates within a warm background plasma (Fig. 2). Striffler and Kammash, using the local approximation, showed that the extraordinary electromagnetic mode of the background plasma may be driven unstable by resonant interaction with the E-layer particles. We now present the results of a non-local calculation for these modes, which illustrate some of the features of our method for the electromagnetic case.

The linearized Vlasox-Maxwell equations in the Lorentz gauge are

$$\left(\frac{\partial}{\partial t} + L_o\right) f_{1j}(\underline{r},\underline{v},t) = \frac{e_j}{m_j}\left\{\nabla\phi_1(\underline{r},t) - \frac{1}{c}v \times [\nabla \times \underline{A}_1(\underline{r},t)] + \frac{1}{c}\frac{\partial}{\partial t}\underline{A}_1(\underline{r},t)\right\} \cdot \frac{\partial f_{oj}}{\partial \underline{v}} \quad (27)$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\underline{A}_1(\underline{r},t) = -\sum_j \frac{4\pi e_j}{c}\int d^3v\underline{v}f_{1j}(\underline{r},\underline{v},t) \quad (28)$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi_1(\underline{r},t) = -\sum_j 4\pi e_j\int d^3v f_{1j}(\underline{r},\underline{v},t) \quad (29)$$

with gauge condition

$$\nabla \cdot \underline{A}_1 + \frac{1}{c}\frac{\partial\phi_1}{\partial t} = 0 \quad (30)$$

We again assume a time dependence $e^{-i\omega t}$, with $\mathrm{Im}(\omega) > 0$.

The Lorents condition (30) does not uniquely specify $\underline{A}_1$, and by a further restricted gauge transformation[4] we may require $\underline{A}_1$ and $\phi_1$ to satisfy the boundary conditions

$$\nabla \cdot \underline{A}_1\Big|_{r=R} = \phi_1\Big|_{r=R} = 0; \quad (31)$$

where R is again the radius of the conducting cylinder.

Using the gauge condition (30) we eliminate $\phi_1$ from the equations:

$$\phi = -\frac{ic}{\omega}\nabla \cdot \underline{A},$$

where we now drop the "1" on perturbed quantities. The mode we are interested in has $\underline{E}$ in the x-y plane and $\underline{B}$ in the z-direction, so we take $k_z = A_z = 0$ and define $A^{\pm} = A_x \pm iA_y$. A suitable expansion for the vector potential in terms of cylindrical harmonics is then found to be

$$A^{+}(\underline{r}) = \sum_{\ell,n}[\alpha_{n\ell}J_{\ell+1}(\lambda'_{n\ell}r) + \beta_{n\ell}J_{\ell+1}(\lambda_{n\ell}r)]e^{i(\ell+1)\theta}$$

$$A^{-}(\underline{r}) = \sum_{\ell,n}[\alpha_{n\ell}J_{\ell-1}(\lambda'_{n\ell}r) - \beta_{n\ell}J_{\ell-1}(\lambda_{n\ell}r)]e^{i(\ell-1)\theta} \qquad (32)$$

Here $\alpha_{n\ell}$, $\beta_{n\ell}$ are the expansion coefficients representing the two independent components of $\underline{A}$, $\lambda_{n\ell}$ is the $n^{th}$ root of $J_\ell(\lambda R) = 0$, and $\lambda'_{n\ell}$ is the $n^{th}$ root of $J'_\ell(\lambda R) = 0$. It is straightforward to verify that (32) satisfies the correct boundary conditions at the conducting cylinder.

Next we integrate the Vlasov equations along the unperturbed trajectories to obtain the perturbed distribution function, which on integration over velocity space yields the perturbed current density. Substituting this current density into the linearized field equations we then obtain the dispersion relation. As in the electrostatic case a considerable simplification is achieved by using the fact that the particle motion is periodic in the non-ignorable coordinate to do both the orbit integral and the radial part of the phase space integral analytically.

The result is a linear algebraic equation in the expansion coefficients $\alpha_{n\ell}$ and $\beta_{n\ell}$. As in the preceding section it is useful to express the result in terms of a dispersion matrix $D(\omega)$:

$$\sum_{n'} D_{nn'}(\omega)\begin{pmatrix}\alpha_{n'\ell}\\ \beta_{n'\ell}\end{pmatrix} = \begin{pmatrix}D^{\alpha\alpha}(\omega) & D^{\alpha\beta}(\omega)\\ D^{\beta\alpha}(\omega) & D^{\beta\beta}(\omega)\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = 0,$$

where $\alpha$ represents the $\alpha_{n\ell}$'s, $\beta$ the $\beta_{n\ell}$'s, and we divide $D(\omega)$ into four corresponding submatrices, as shown. The dispersion relation is then

$$\det[D(\omega)] = 0.$$

For numerical evaluation the matrix $D(\omega)$ must be truncated, of course, as discussed below. The expressions for the matrix elements in the general case are very complicated (though straightforward to evaluate), so we give them here only for the special case of the E-layer problem.

To simplify the calculation we treat the E-layer and background plasma electrons as different species and calculate their contribution to the dispersion matrix separately. We are interested in a high frequency mode, so the ions may be ignored. Since $k_z = 0$ we ignore the z-dependence of the distribution function and take $f_o$ for the background plasma to be a two dimensional Maxwellian:

perpendicular symbol

$$f_o = \frac{m\hat{n}}{2\pi T}e^{-H_\perp/T}$$

where $\hat{n}$ and T are the density and temperature, respectively, and

$$H_\perp = \frac{1}{2}mv_\perp^2.$$

It is convenient to write the dispersion matrix elements in the form

$$D^{\alpha\alpha}_{nn'}(\omega) = (\frac{\omega^2}{c^2} - \lambda'^2_{n\ell})\frac{R^2}{2}[1 - (\frac{\ell}{\lambda'_{n\ell}R})^2]\, J^2_{\ell}(\lambda'_{n\ell}R)\delta_{nn'} + X^{\alpha\alpha}_{nn'}(\omega) + Y^{\alpha\alpha}_{nn'}(\omega),$$

$$D^{\alpha\beta}_{nn'}(\omega) = X^{\alpha\beta}_{nn'}(\omega) + Y^{\alpha\beta}_{nn'}(\omega)$$

$$D^{\beta\alpha}_{nn'}(\omega) = X^{\beta\alpha}_{nn'}(\omega) + Y^{\beta\alpha}_{nn'}(\omega)$$

$$D^{\beta\beta}_{nn'}(\omega) = (\frac{\omega^2}{c^2} - \lambda^2_{n\ell})\lambda_{n\ell}\frac{R^2}{2}\, J^2_{\ell+1}(\lambda_{n\ell}R)\delta_{nn'} + X^{\beta\beta}_{nn'}(\omega) + Y^{\beta\beta}_{nn'}(\omega)$$

where the X's represent the contribution of the background plasma and the Y's the contribution of the E-layer. The first term in $D^{\alpha\alpha}$ and $D^{\beta\beta}$ is the contribution of the field operator, which is diagonal in the expansion functions. Note that the matrix is diagonal in $\ell$.

The $(\alpha\alpha)$ contribution of the background plasma is then found to be:

$$X^{\alpha\alpha}_{nn'}(\omega) = \frac{4\pi e^2 n}{m_e c^2} \sum_m \frac{\omega}{m\omega_c - \omega}\{-2(\frac{T}{m_e\omega_c^2})[\frac{\lambda'^2_{n\ell}+\lambda'^2_{n'\ell}}{2}\, I'_m(\frac{\lambda'_{n\ell}\lambda'_{n'\ell}T}{m_e\omega_c^2})$$

$$-\lambda'_{n\ell}\lambda'_{n'\ell}I_m(\frac{\lambda'_{n\ell}\lambda'_{n'\ell}T}{m_e\omega_c^2})] + m^2(\frac{m_e\omega_c^2}{\lambda'_{n\ell}\lambda'_{n\ell}T})\, I_m(\frac{\lambda'_{n\ell}\lambda'_{n'\ell}T}{m_e\omega_c^2})\}B^{\alpha\alpha}_{nn'm}\exp[-\frac{(\lambda'^2_{n\ell}+\lambda'^2_{n'\ell})T}{2m_e\omega_c^2}]$$

where $\omega_c$ is the electron cyclotron frequency, $I_m$ is the $m^{th}$ order modified Bessel function, and $B^{\alpha\alpha}_{nn'm}$ is a normalization constant:

$$B^{\alpha\alpha}_{nn'm}(\omega) = \begin{cases} \frac{R}{\lambda'^2_{n\ell}-\lambda^2_{n'\ell}}[\lambda'_{n\ell}J_{\ell+m+1}(\lambda'_{n\ell}R)J_{\ell+m}(\lambda'_{n\ell}R) - \lambda'_{n'\ell}J_{\ell+m}(\lambda'_{n\ell}R)J_{\ell+m+1}(\lambda'_{n'\ell}R)] & n \neq n', \\ \frac{R}{2}[J^2_{\ell+m}(\lambda'_{n\ell}R) - J_{\ell+m-1}(\lambda'_{n}R)J_{\ell+m+1}(\lambda'_{n\ell}R)], & n = n'. \end{cases}$$

Similar results are obtained for the elements of the other submatrices.[1]

We take the E-layer to consist of particles with gyrocenter at the origin and gyroradii uniformly distributed between $r_{min}$ and $r_{max}$. This gives a uniform density E-layer of thickness $t_{EL} = r_{max} - r_{min}$. In actuality, of course, the finite width of the E-layer is due both to the spread of energies and to the distribution of E-layer gyrocenters about the origin. We neglect the latter effect here in order to come as close as possible in cylindrical geometry to the infinite homogeneous cold beam approximation used by Striffler and Kammash and thus elucidate by comparison the non-local effects. Off-center E-layer orbits could be included in the formalism with little additional difficulty, but since they would not be in resonance with the modes they would not contribute significantly to instability.

The gyrofrequency varies with radius for the relativistic E-layer particles, so we expect the modes to be localized radially, a feature absent from the local approximation calculation.

We calculate the contribution of particles at each radius $r_b$ separately, integrating the resulting susceptibilities over $r_b$ from $r_{min}$ to $r_{max}$ to obtain the total E-layer contribution. The result for an E-layer of density $\hat{n}_b$ is:

$$Y^{\alpha\alpha}_{mm'}(\omega) = \frac{4\pi e^2 \hat{n}_b}{\gamma m_b c^2} \int_{r_{min}}^{r_{max}} \left( - \frac{\omega}{\ell\frac{\omega_c}{\gamma} + \omega} \frac{\partial}{\partial r_b} [r_b J'_\ell(\lambda'_{n\ell} r_b) J'_\ell(\lambda'_{n'\ell} r_b)] \right.$$

$$+ \frac{\omega^2}{c^2} r_b J'_\ell(\lambda'_{n\ell} r_b) J'_\ell(\lambda'_{n'\ell} r_b) \frac{\omega_c^2 r_b^3}{\gamma^2} \frac{1}{(\ell\frac{\omega_c}{\gamma} + \omega)^2}$$

$$+ r_b^2 \{ \frac{\lambda'_{n\ell}}{2} [J'_{\ell-1}(\lambda'_{n\ell} r_b) J_{\ell-1}(\lambda'_{n'\ell} r_b) + J'_{\ell+1}(\lambda'_{n\ell} r_b) J_{\ell+1}(\lambda'_{n'\ell} r_b)] - \lambda'_{n'\ell} J'_\ell(\lambda'_{n\ell} r_b) J_\ell(\lambda'_{n'\ell} r_b) \}$$

Gr. gamma

$$-\frac{\omega_c r_b^3}{\gamma}\,\frac{\lambda'_{n\ell}\lambda'_{n'\ell}}{2}\,\Big[\frac{J'_{\ell-1}(\lambda'_{n\ell}r_b)J_{\ell-1}(\lambda'_{n'\ell}r_b)}{(\ell-1)\frac{\omega_c}{\gamma}+\omega} - \frac{J'_{\ell+1}(\lambda'_{n\ell}r_b)J'_{\ell+1}(\lambda'_{n'\ell}r_b)}{(\ell+1)\frac{\omega_c}{\gamma}+\omega}\Big]),$$

where $\gamma$ is the relativistic mass factor for the E-layer particles. Again, similar results are obtained for the elements of the other submatrices.

For the numerical calculations we first consider parameters appropriate to the Astron device and the calculations of Striffler and Kammash.(3) Thus $r_{min}$ = 30 cm, $r_{max}$ = 50 cm, $t_{EL}$ = 20 cm, R = 70 cm, and B $\simeq$ 380 G. We take the temperature of the background plasma to be one eV and the relativistic mass factor for the E-layer particles to be 9.0 at r = 40 cm. A range of plasma densities was considered, with $\omega_{PP}/\omega_{CP}$ ranging from .1 to .9, where $\omega_{PP}$ is the plasma frequency and $\omega_{CP}$ the cyclotron frequency of the background plasma. Following Striffler and Kammash, we take the density of the E-layer to be given by $\omega_{CP}/\omega_{CB}$ = .3, where $\omega_{CB}$ is the (relativistic) cyclotron frequency of the E-layer electrons.

The dispersion matrix was truncated for this case at n = 20, so that the matrix was 40 × 40 (it multiplies a vector of 20 $\alpha$'s and 20 $\beta$'s). We shall see that for n $\gtrsim$ 10 the coefficients $\alpha_n$ and $\beta_n$ become negligible for most of the unstable modes, so that this is a large enough truncation to give an accurate representation of these modes.

From the dispersion relation for the extraordinary electromagnetic mode in a uniform plasma we expect the frequencies of the unstable mode to be near the upper hybrid frequency of the plasma: $\omega_H = \sqrt{\omega_{PP}^2+\omega_{CP}^2}$. The mode frequency will therefore be above the background plasma frequency, but for the range of r where the mode is in resonance with the E-layer particles it will be below the E-layer plasma frequency in a reference frame moving with that part of the E-layer. Consequently we expect a larger perturbation in the charge density in that region of the E-layer which is in resonance with the mode than in the rest of the plasma.

Figures 3(a-e) show some of the unstable modes found for $\ell = 8$, $\omega_{PP}/\omega_{CP} = .1$. In the upper graph the absolute value of the charge density is plotted against radius, while the lower graph shows the absolute values of the coefficients $\alpha_n$ and $\beta_n$ plotted against n (the squares represent $\alpha_n$ and the circles $\beta_n$). We see that the perturbed charge density is in fact localized near that value of the radius where the $\ell^{th}$ harmonic of the E-layer gyrofrequency is in resonance with the real part of the mode frequency. Note also that for n near 20 the coefficients $\alpha_n$ and $\beta_n$ become essentially zero, indicating that these modes are accurately represented by the first 20 $\alpha$'s and $\beta$'s. Figures 4(a-d) show similar modes with $\omega_{PP}/\omega_{CP}$ increasing from .3 to .9. Note that as the background plasma density increases it contributes more to the perturbed charge density, as expected.

The growth rates for these modes are approximately an order of magnitude smaller than those obtained by Striffler and Kammash using the local approximation. This is primarily due to the finite width of the E-layer and the fact that due to the variation of the relativistic cyclotron frequency with radius only a small part of the E-layer can be near resonance with the mode.

A preliminary calculation has also been carried out for the device described in Reference (5), which can be regarded for our purposes as a miniature version of Astron, with $r_{min} \doteq 2.75$ cm, $r_{max} = 3.25$ cm, $t_{EL} = .5$ cm, R = 10 cm. The background plasma has a temperature of one eV, a plasma frequency of $1.88 \times 10^{11}$ sec$^{-1}$, and a cyclotron frequency of $2.46 \times 10^{10}$ sec$^{-1}$. The E-layer has a density of $10^{11}$ particles/cm and a relativistic mass factor of $\gamma = 3.3$. The modes of interest for this device have wavelengths larger than $t_{EL}$, so that the local approximation is not valid.

Since $\omega_H/(\omega_c/\gamma) = 25.5$ for this device, it was decided to look for instabilities of the $\ell = 26$ modes. Figure 5(a-b) show two modes obtained with the dispersion matrix truncated at $n = 100$. Due to the short wavelengths involved the modes would be more accurately represented by using an even larger truncation, but the general features of the modes are apparent. They both have frequencies near the upper hybrid frequency and have perturbed charge densities peaked in the vicinity of the E-layer. Consequently, we expect that in this device too the extraordinary electromagnetic mode should be unstable, and this may give rise to the radiation observed in this experiment.(5)

## V. CONCLUSIONS

In this paper we have presented a general method for the fully kinetic, nonlocal stability analysis of Vlasov equilibria with cylindrical symmetry. Using the fact that the particle motion must be periodic in the radial coordinate, we were able to carry out both the integration over unperturbed orbits and the radial part of the phase space integral analytically, leaving only the integration over the constants of the motion to be done numerically.

The method has been successfully applied to a calculation of the lower hybrid drift instability in the electrostatic approximation and a fully electromagnetic analysis of a relativistic, finite width E-layer interacting with a warm background plasma. In the electrostatic problem the method was seen to yield results which are both simpler than the usual method of integration over unperturbed orbits and easier to adapt to the correct boundary conditions for a finite plasma. In the E-layer problem we saw that nonlocal effects cause a significant reduction in the growth rate compared to a calculation based on the local approximation.

ACKNOWLEDGMENTS

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## FIGURE CAPTIONS

1. Geometry and coordinate system.
2. E-layer model.
3. Modes for $\ell = 8$, $\omega_{PP}/\omega_{CP} = .1$.
4. Modes for increasing $\omega_{PP}/\omega_{CP}$.
5. Two modes found for the device of Ref. (3).

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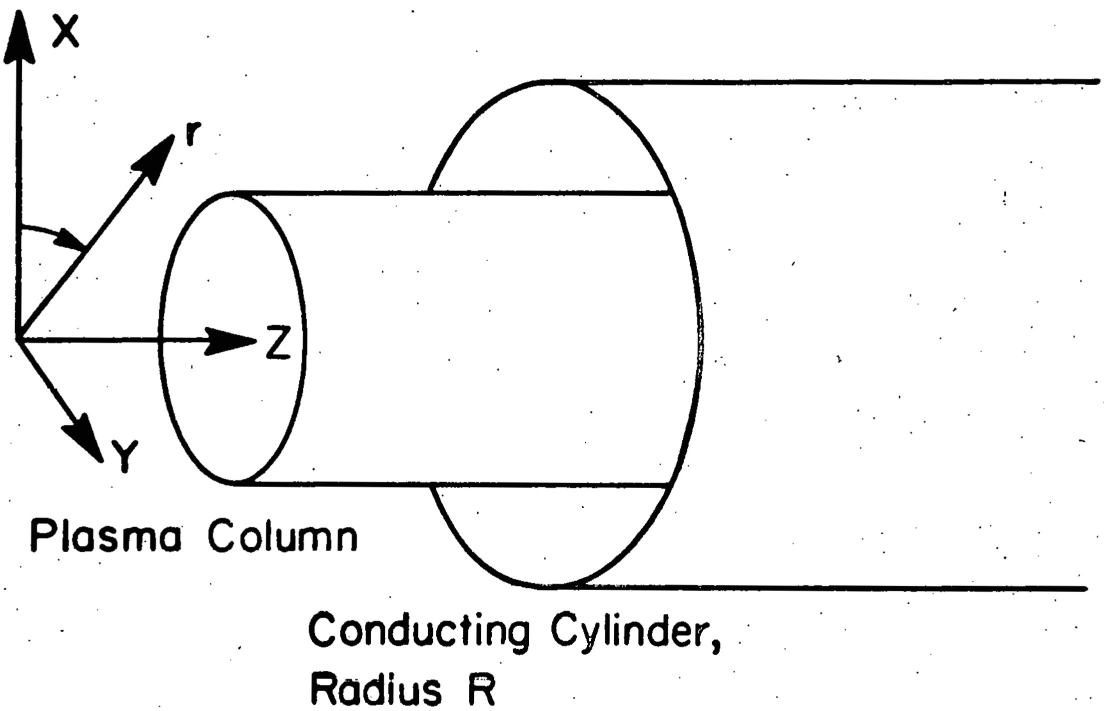
Fig. 1



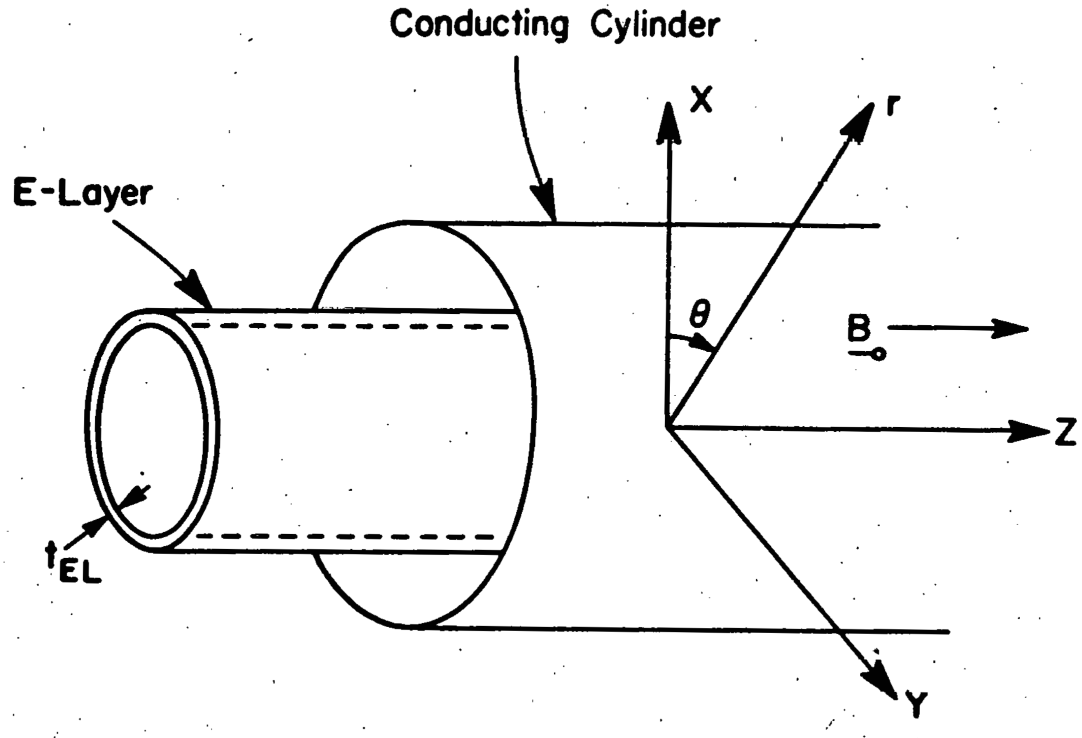
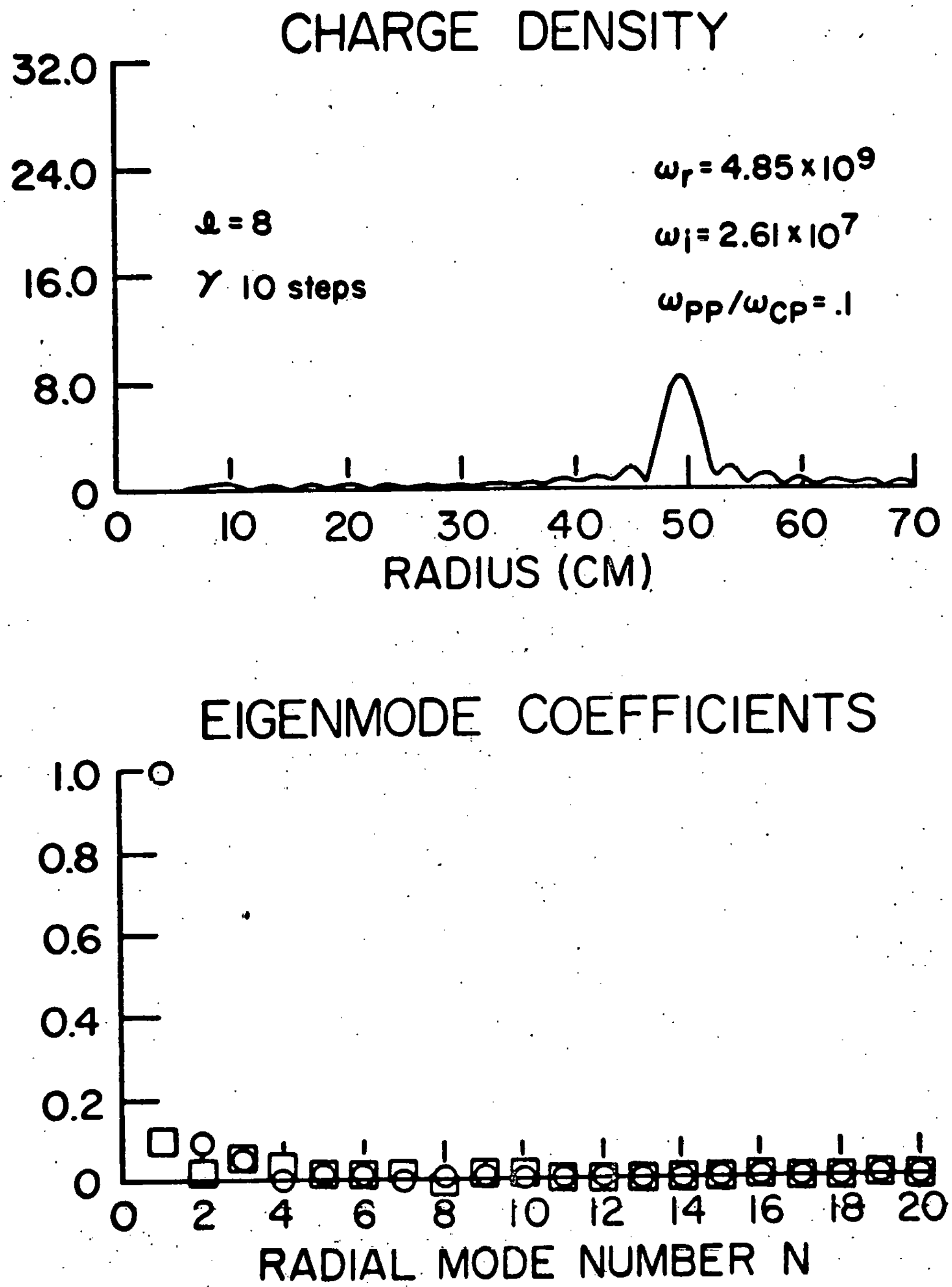



Fig. 2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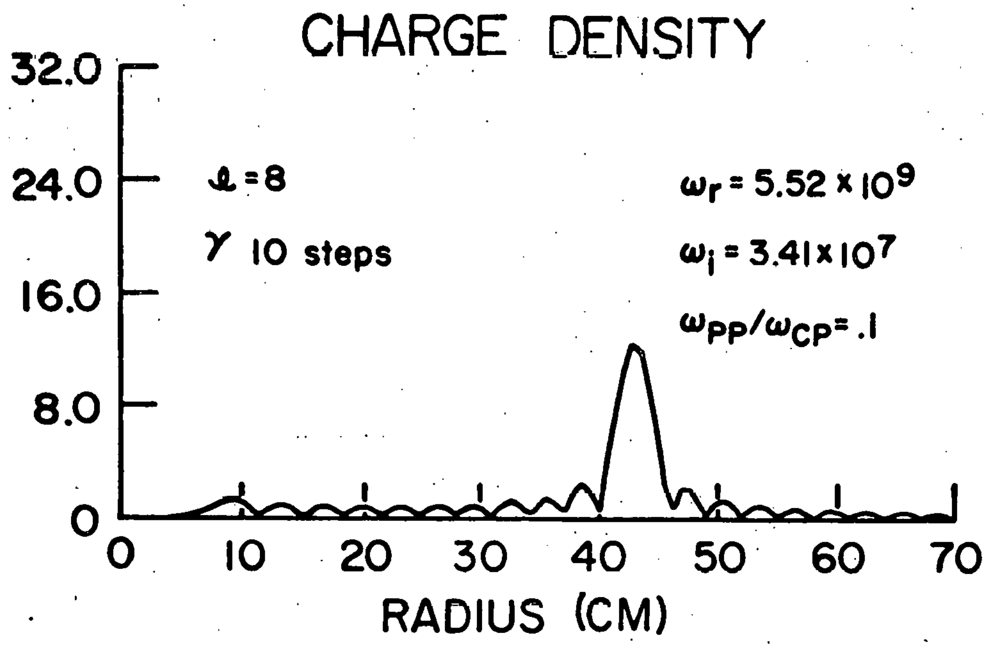
24.0
16.0
8.0
0
ℓ = 8
γ 10 steps
ωr = 4.85 x 10^9
ωi = 2.61 x 10^7
ωpp/ωcp = .1
0 10 20 30 40 50 60 70
RADIUS (CM)
EIGENMODE COEFFICIENTS
1.0
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N


CHARGE DENSITY
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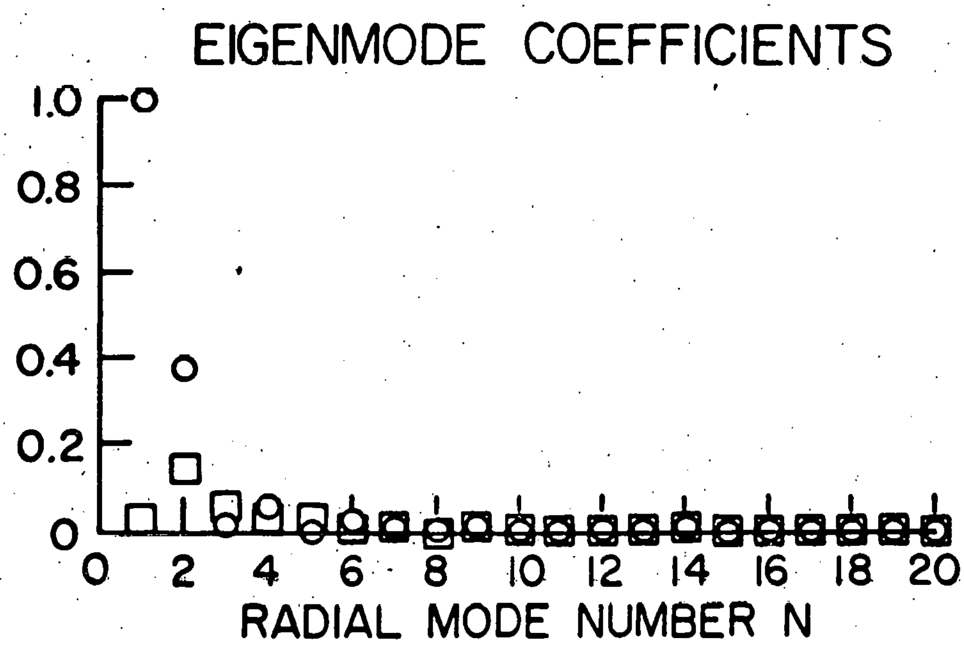
24.0
16.0
8.0
0
ℓ=8
γ 10 steps
ω_r = 5.52 × 10^9
ω_i = 3.41 × 10^7
ω_pp/ω_cp = .1
0 10 20 30 40 50 60 70
RADIUS (CM)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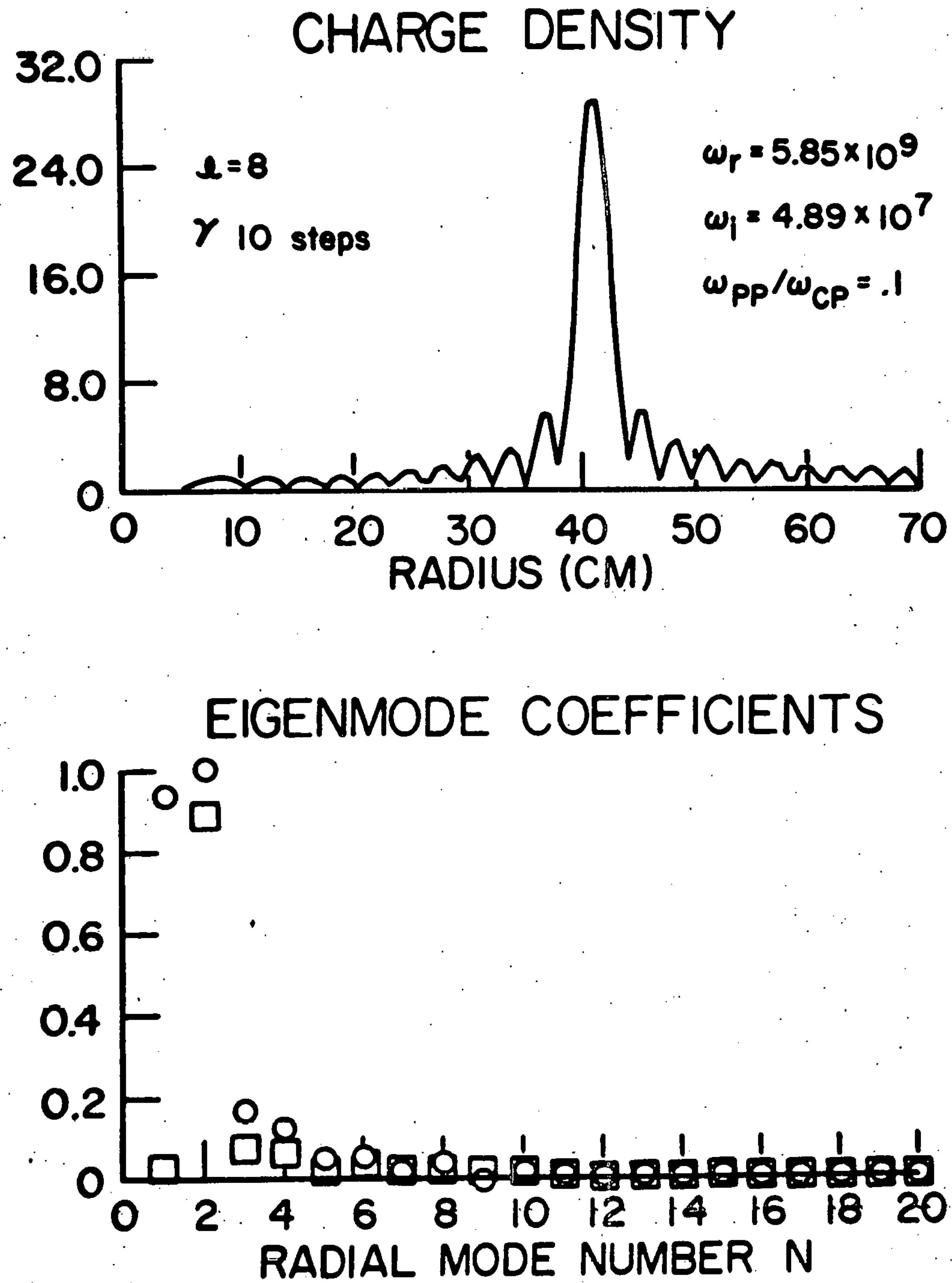
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N


CHARGE DENSITY
32.0
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16.0
8.0
0
ℓ=8
γ 10 steps
ωr = 5.85×10^9
ωi = 4.89×10^7
ωpp/ωcp = .1
0 10 20 30 40 50 60 70
RADIUS (CM)
EIGENMODE COEFFICIENTS
1.0
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N

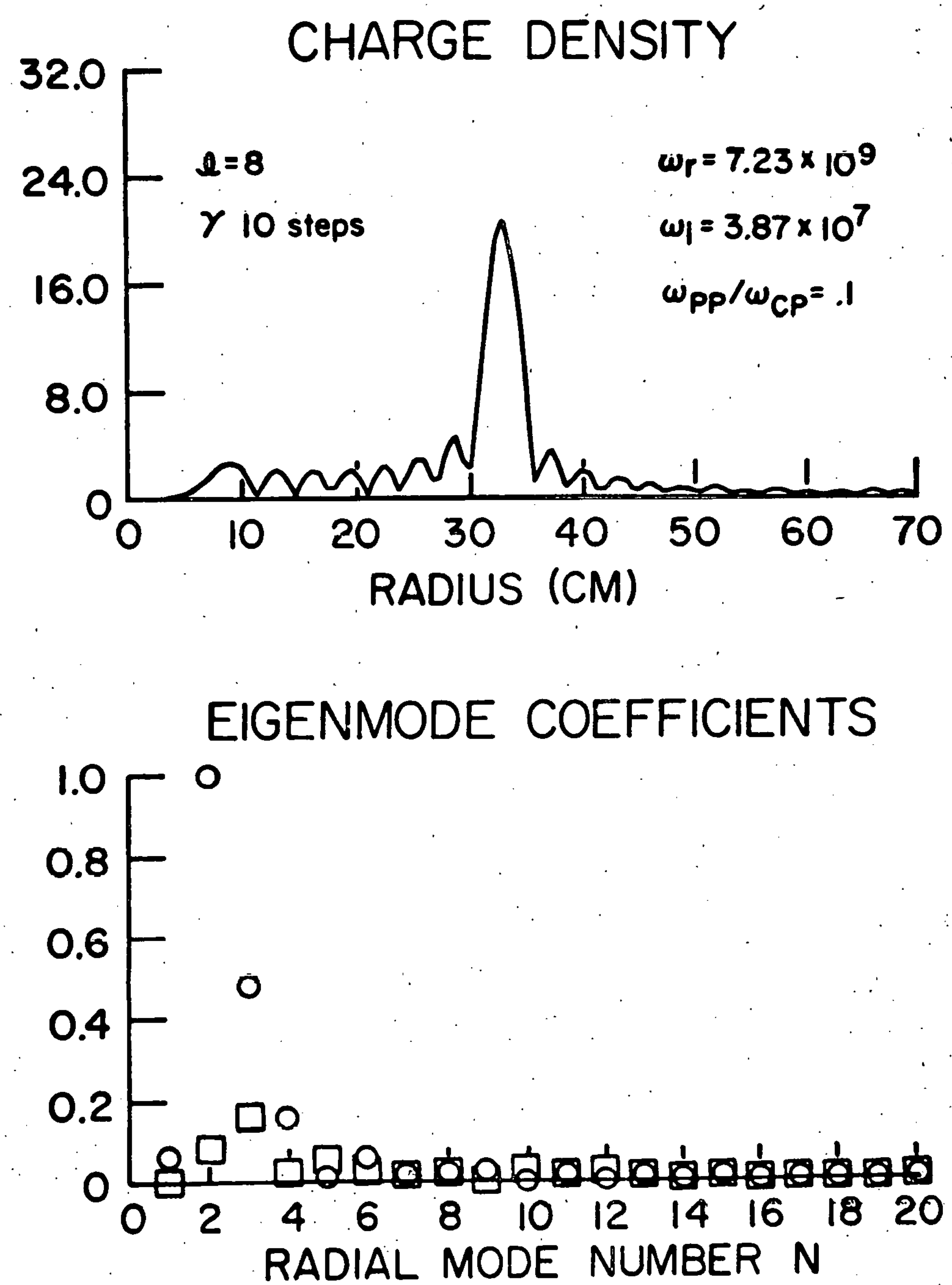
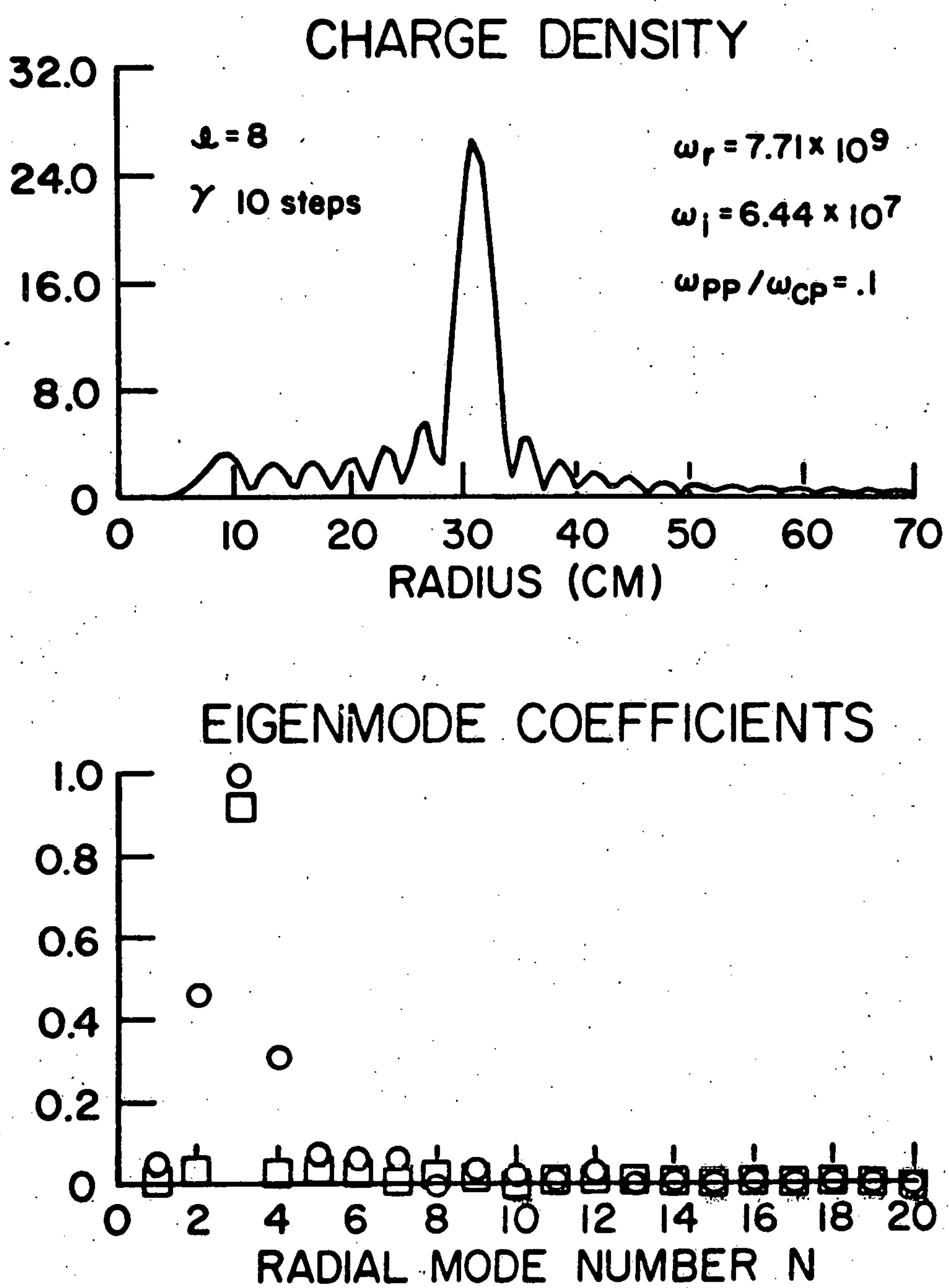

CHARGE DENSITY
32.0
24.0
16.0
8.0
0
ℓ=8
γ 10 steps
ωr = 7.23 x 10^9
ωI = 3.87 x 10^7
ωpp/ωcp = .1
0 10 20 30 40 50 60 70
RADIUS (CM)
EIGENMODE COEFFICIENTS
1.0
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N

Fig. 3e



CHARGE DENSITY
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24.0
16.0
8.0
0
ℓ = 8
γ 10 steps
ω_r = 7.71 × 10^9
ω_i = 6.44 × 10^7
ω_pp / ω_cp = .1
0 10 20 30 40 50 60 70
RADIUS (CM)
EIGENMODE COEFFICIENTS
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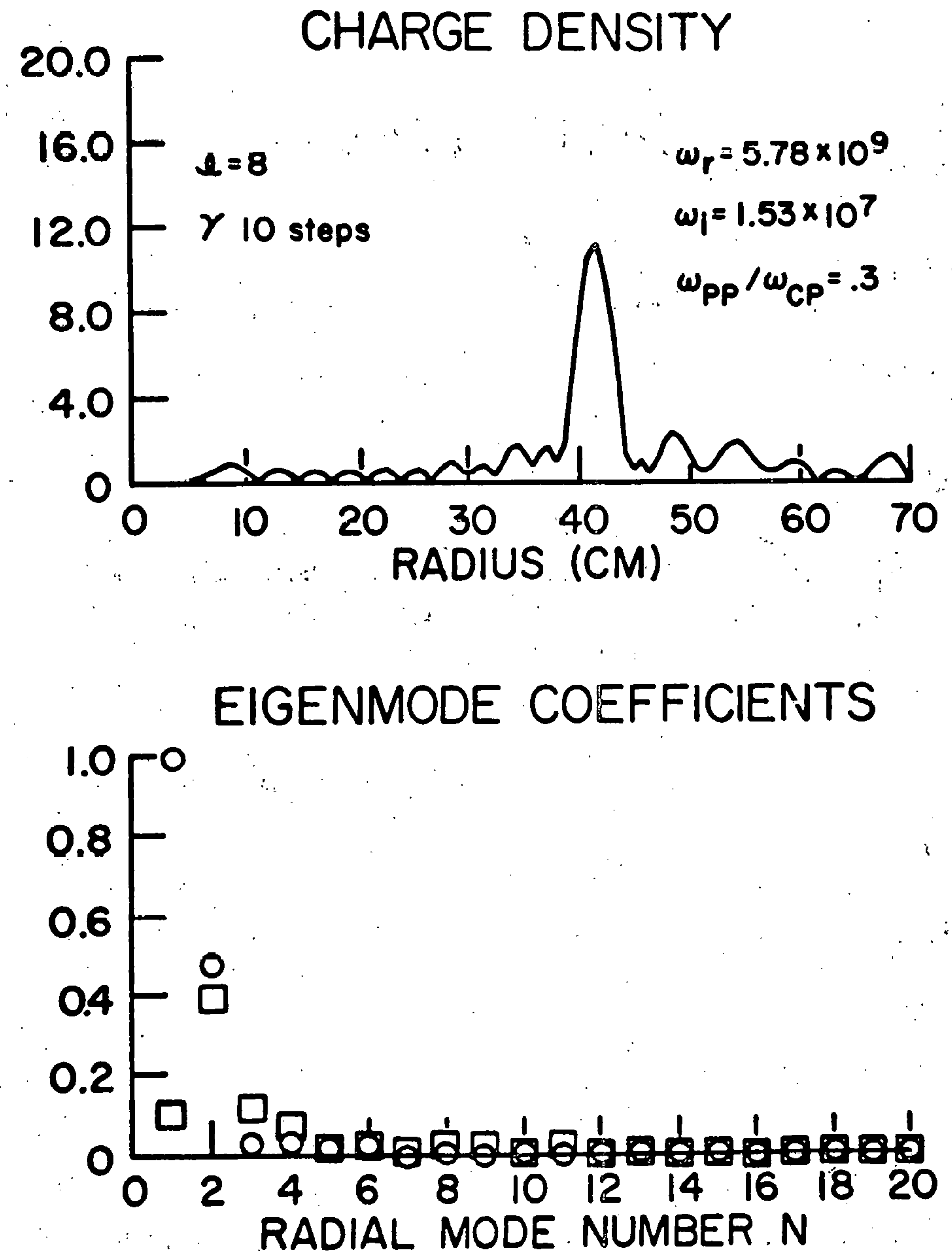
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N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12.0
8.0
4.0
0
ℓ=8
γ 10 steps
ωr = 5.78 x 10^9
ωI = 1.53 x 10^7
ωpp/ωcp = .3
0 10 20 30 40 50 60 70
RADIUS (CM)
EIGENMODE COEFFICIENTS
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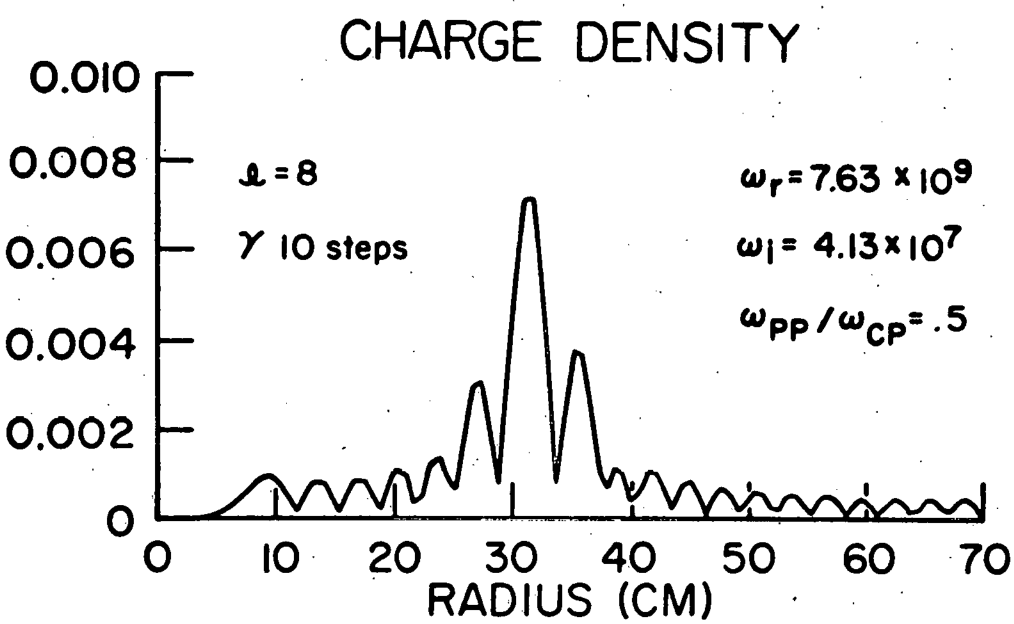
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N

Fig. 4b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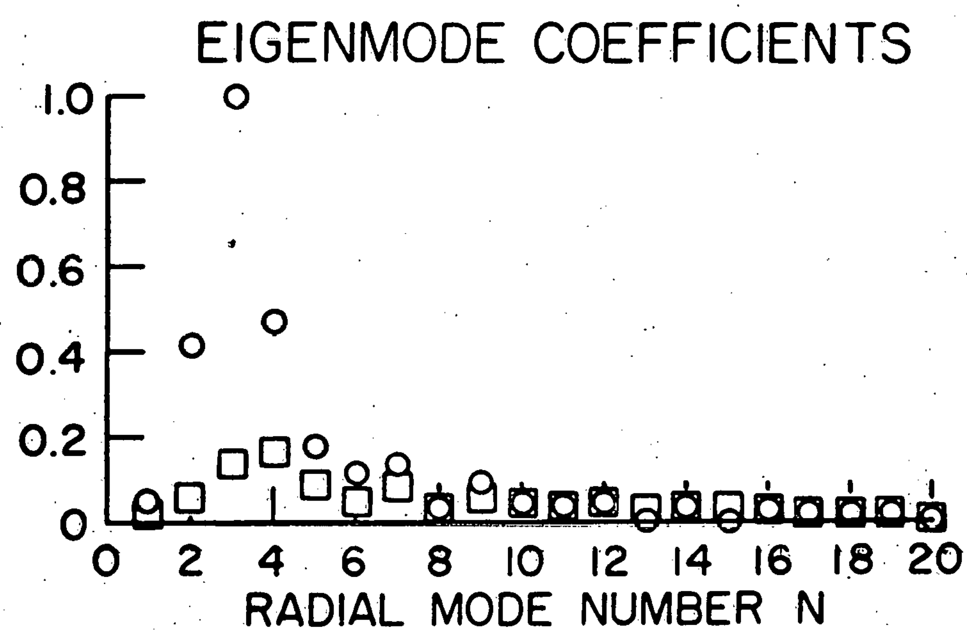
0.010
0.008
0.006
0.004
0.002
0
ℓ = 8
γ 10 steps
$\omega_r = 7.63 \times 10^9$
$\omega_I = 4.13 \times 10^7$
$\omega_{pp}/\omega_{cp} = .5$
0 10 20 30 40 50 60 70
RADIUS (CM)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0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N

Fig. 4c

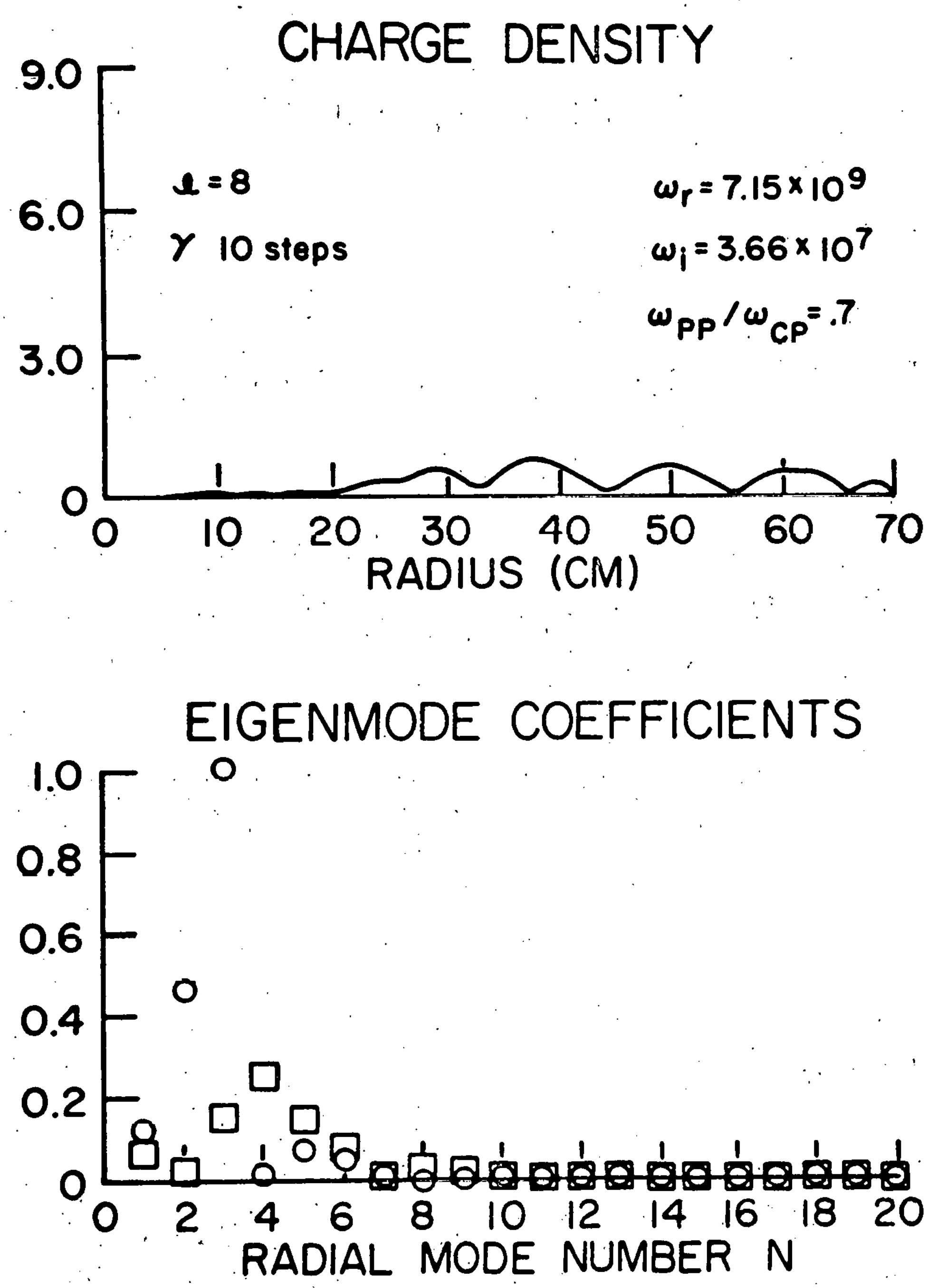
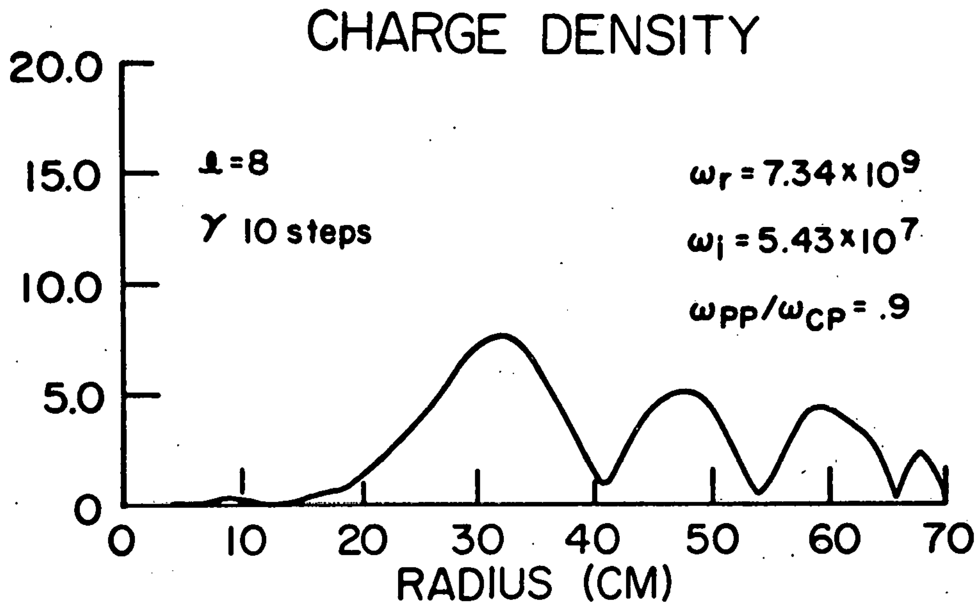

CHARGE DENSITY
9.0
6.0
3.0
0
ℓ = 8
γ 10 steps
$\omega_r = 7.15 \times 10^9$
$\omega_i = 3.66 \times 10^7$
$\omega_{pp}/\omega_{cp} = .7$
0 10 20 30 40 50 60 70
RADIUS (CM)
EIGENMODE COEFFICIENTS
1.0
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N

Fig. 4d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0
ℓ=8
γ 10 steps
ωr = 7.34×10^9
ωi = 5.43×10^7
ωpp/ωcp = .9
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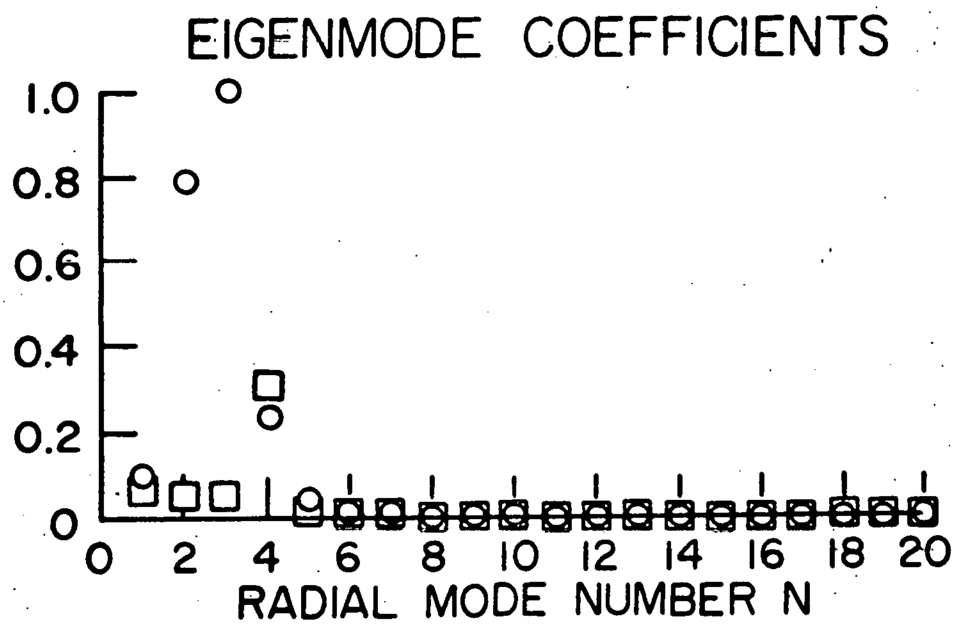
RADIUS (CM)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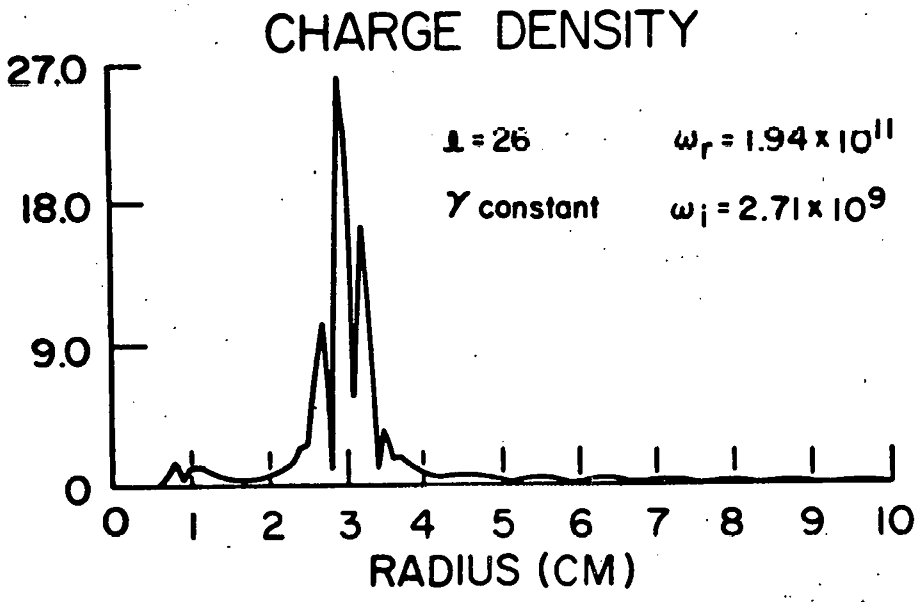
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
RADIAL MODE NUMBER N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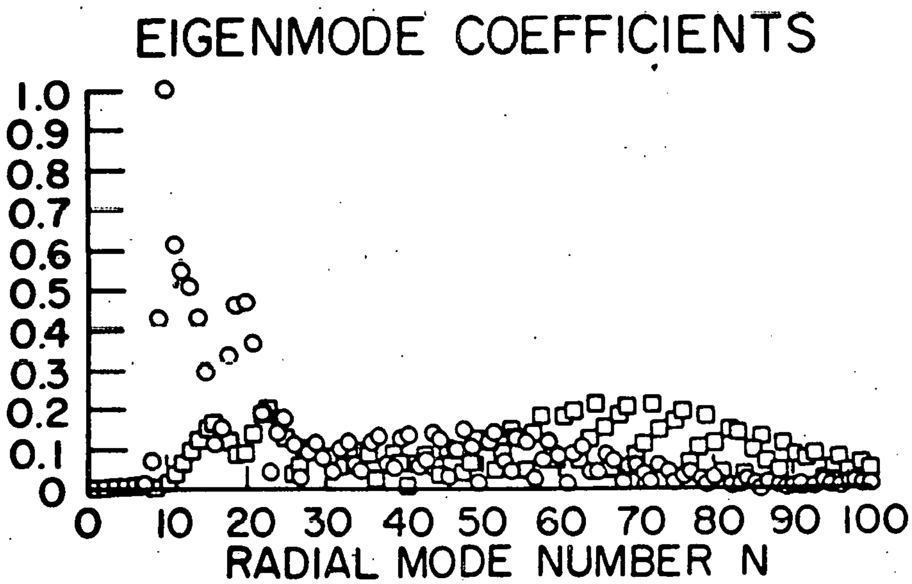
18.0
9.0
0
ℓ = 26
γ constant
ω_r = 1.94 × 10^11
ω_i = 2.71 × 10^9
0 1 2 3 4 5 6 7 8 9 10
RADIUS (CM)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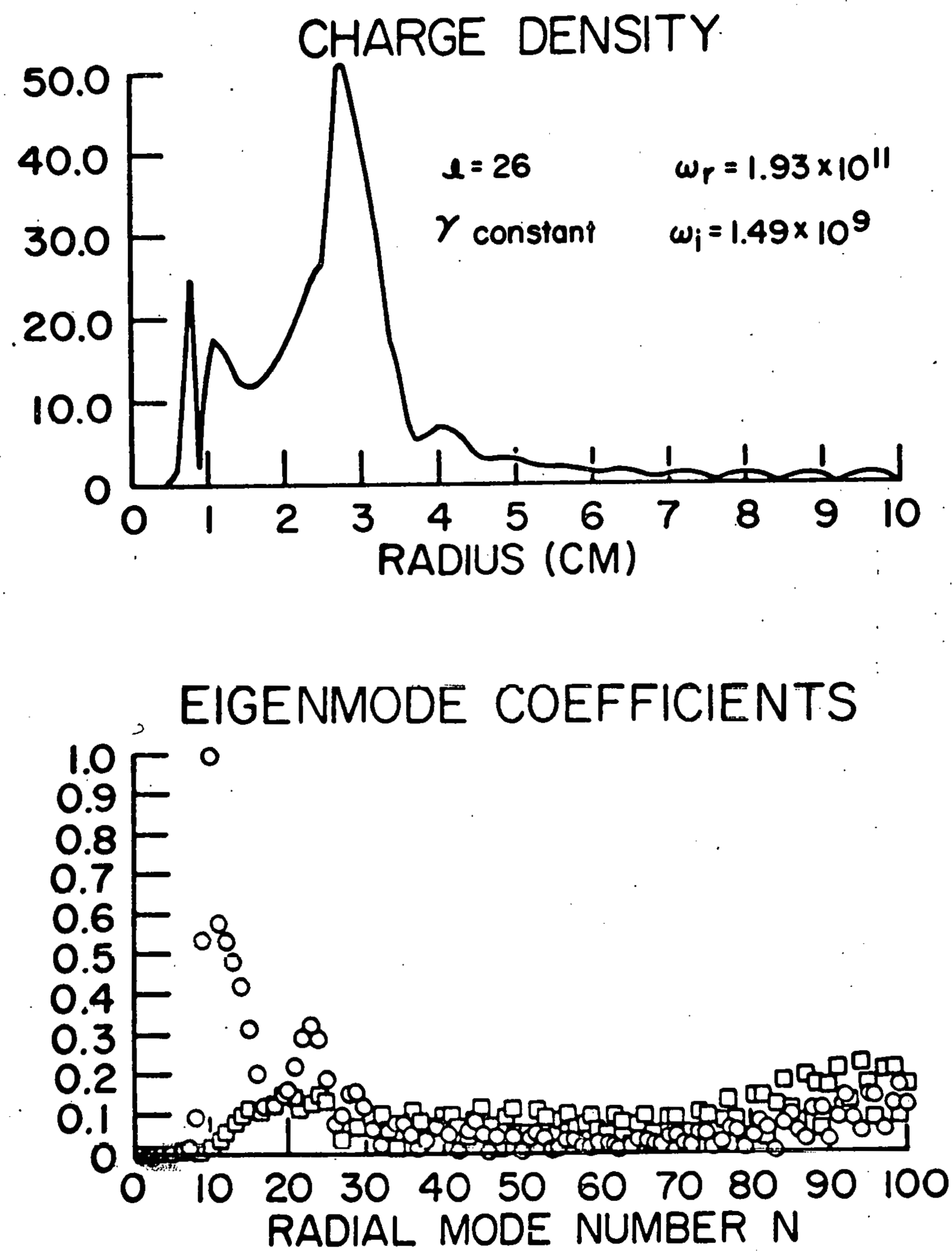
RADIAL MODE NUMBER N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30.0
20.0
10.0
0
ℓ = 26
γ constant
ωr = 1.93 x 10^11
ωi = 1.49 x 10^9
0 1 2 3 4 5 6 7 8 9 10
RADIUS (CM)
EIGENMODE COEFFICIENTS
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 10 20 30 40 50 60 70 80 90 100
RADIAL MODE NUMBER N

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