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TITLE **ALGEBRAIC DESCRIPTION OF INTRINSIC MODES IN NUCLEI**

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ALGEBRAIC DESCRIPTION OF INTRINSIC MODES IN NUCLEI

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ABSTRACT

We present a procedure for extracting normal modes in algebraic number-conserving systems of interacting bosons relevant for collective states in even-even nuclei. The Hamiltonian is resolved into intrinsic (bandhead related) and collective (in-band related) parts. Shape parameters are introduced through non-spherical boson bases. Intrinsic modes decoupled from the spurious modes are obtained from the intinsic part of the Hamiltonian in the limit of large number of bosons. Intrinsic states are constructed and serve to evaluate electromagnetic transition rates. The method is illustrated for systems with one type of boson as well as with proton-neutron bosons.

KEYWORDS

Intrinsic collective resolution, normal modes, Goldstone bosons, intrinsic states.

INTRODUCTION

The interacting boson model (IBM) (Iachello *et al.*, 1987a) has been empirically successful in describing a wide range of data on low lying collective states in even-even nuclei (Casten *et al.*, 1988). The building blocks of the model are a monopole boson $s^\dagger$ ($J = 0^+$) and a quadrupole boson $d^\dagger_\mu$ ($J = 2^+$, projection quantum number μ). The bosons are regarded as images of correlated monopole and quadrupole pairs of identical valence nucleons (Iachello *et al.*, 1987b). As such the total number of bosons N is conserved and is taken as the sum of valence neutron and proton particle or hole pairs counted from the nearest closed shell. The Hamiltonian consists of rotational invariant, hermitian, number conserving one- and two- body interactions among the bosons. The presence of six building blocks and a conserved total boson number confers on the model a group structre of U(6). The group structure provides both analytic solutions (for specific choice of interactions) and bases for diagonalization of the Hamiltonian in the general (and more realistic) case. Electromagnetic static and dynamic moments are calculated with number conserving transition operators of appropriate ranks.

Typical spectra obtained in the IBM is shown in Fig. 1. In Fig. (1a) the spectrum of ^{148}Sm displays the familiar sequence of levels 0^+, 2^+, $(0^+, 2^+, 4^+)$ of a spherical nucleus excited by quadrupole excitations. In Fig. (1c) the IBM spectrum of ^{168}Er displays a characteristic pattern of an axially deformed nucleus with well developed rotational bands. The clearly visible ground, γ, β and higher bands have a typical rotor $J(J+1)$ splitting.

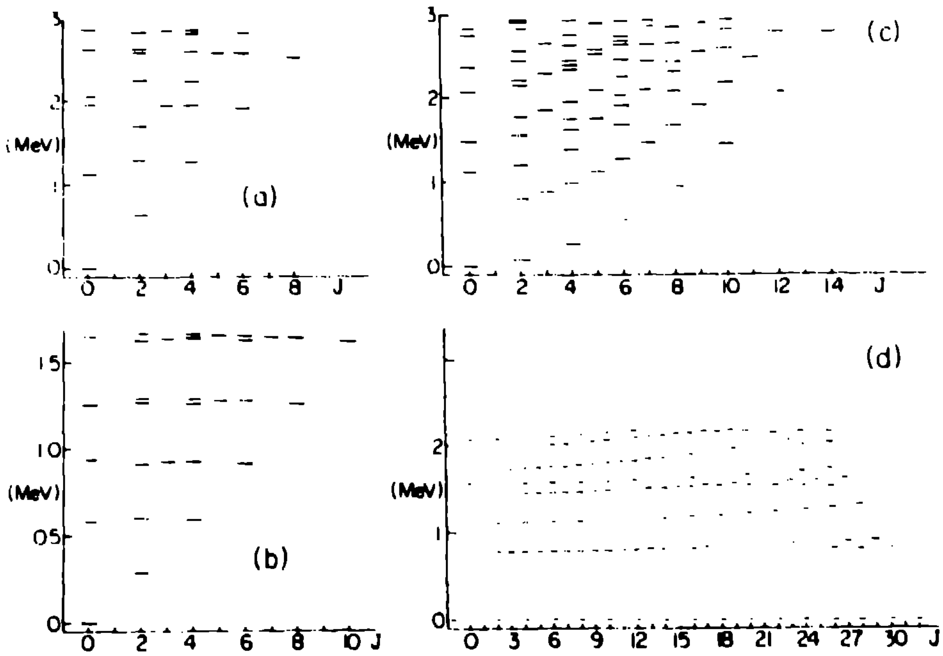

Fig. 1. Spectra of (a) $H_{IBM}(^{148}Sm)$, below 3 MeV; (b) $H_{int}(^{148}Sm)$, below 1.7 MeV; (c) $H_{IBM}(^{168}Er)$, below 3 MeV; (d) $H_{int}(^{168}Er)$, below 2.2 MeV. Taken from (Kirson *et al.*, 1985).

The IBM Hamiltonian that give rise to such diverse spectra is

$$H = \epsilon_s s^\dagger s + \epsilon_d d^\dagger \cdot \tilde{d} + u_0 (s^\dagger)^2 s^2 + u_2 s^\dagger d^\dagger \cdot \tilde{d} s + v_0 \left[(s^\dagger)^2 \tilde{d} \cdot \tilde{d} + d^\dagger \cdot d^\dagger s^2 \right] + v_2 \left[s^\dagger d^\dagger \cdot (\tilde{d}\tilde{d})^{(2)} + (d^\dagger d^\dagger)^{(2)} \cdot \tilde{d} s \right] + \sum_{L=0,2,4} c_L (d^\dagger d^\dagger)^{(L)} \cdot (\tilde{d}\tilde{d})^{(L)} \quad (1)$$

Here $\tilde{d}_\mu = (-)^\mu d_{-\mu}$, the dot implies scalar product and standard notation of angular momentum coupling is used. The coefficients in front of the interaction terms are determined from fits to the experimental data. The algebraic second-quantized form of the Hamiltonian (1) is rather abstract. It does not reveal how a given interaction affects the spectrum nor what are the model's excitation modes. To address such issues one needs a geometrical realization for the abstract algebraic model. This can be acheived through a concept of an intrinsic state which is a condensate of N bosons (Ginocchio *et al.*, 1980; Dieperink *et al.*, 1980)

$$|N, \beta, \gamma > = (N!)^{-1/2} (b_c^\dagger)^N |0> \quad (2a)$$

$$b_c^\dagger = (1+\beta^2)^{-1/2} [\beta \cos\gamma \cdot d_0^\dagger + \beta \sin\gamma \cdot \frac{1}{\sqrt{2}} (d_2^\dagger + d_{-2}^\dagger) + s^\dagger] \quad (2b)$$

Each condensate boson is a mixture of s- and d bosons and depends on both quadrupole deformation parameters $\beta > 0$ and $0 \leq \gamma \leq \frac{1}{3}\pi$. the expectation value of the Hamiltonian in the state (2) defines an energy surface

$$E_N(\beta, \gamma) = < N; \beta, \gamma | H | N; \beta, \gamma > \quad (3)$$

which depends on N, β, γ and on parameters of the Hamiltonian. Minimizing the energy surface with respect to β and γ one finds all the local minima of which the global one we denote by (β_0, γ_0). For the IBM Hamiltnian of eq. (1) the energy surface takes the form

$$E_N(\beta, \gamma) = N\epsilon_s + N(N-1)u_0 + N(N-1)(1+\beta^2)^{-1}\beta^2 \left[a - b\beta\cos 3\gamma + c\beta^2 \right] \quad (4)$$

The β-γ shape of the energy surface is dictated by three combinations of parameters of the Hamiltonian: $a = u_2 + 2v_0 - 2u_0 + (\epsilon_d - \epsilon_s)/(N-1)$, $b = 2\sqrt{2/7}v_2$, $c = c_0/5 + 2c_2/7 + 18c_4/35 - u_0 + (\epsilon_d - \epsilon_s)/(N-1)$. For $\beta_0 = 0$ ($\beta_0 > 0$) the state (2) describes a spherical (deformed) shape. At a deformed minimum $\gamma_0 = 0$ (or $\pi/3$) corresponding to a prolate (oblate) shape with the z (y) axis as the symmetry axis. If the coefficient b=0, the energy surface becomes independent of γ.

INTRINSIC-COLLECTIVE RESOLUTION OF THE HAMILTONIAN

Given an IBM Hamiltonian, one calculates an energy surface through eqs. (3)-(4), finds the global minimum (β_0, γ_0), substitutes these equilibrium deformations into the state (2) and obtains the ground-state intrinsic wavefunction $|N;\beta_0,\gamma_0>$. Since the equilbrium condensate $|N;\beta_0,\gamma_0>$ is just a solution of a Hartree-Bose variational calculation, it is in general, not an eigenstate of the Hamiltonian H. However, for any given H we can construct another Hamiltonian for which which $|N;\beta_0,\gamma_0>$ is an exact zero-energy eigenstate and whose energy surface is equal to that of the original Hamiltonan H up to a constant independent of β and γ. These two requirements define the new Hamiltonian in essentially a unique way and, for reasons to be explained below, we refer to it as the intrinsic part (H_{int}) of the full Hamiltonian H. Once H_{int} is known, we define the collective part of the Hamiltonian (H_c) as the difference between H and H_{int}. In this way one obtains without any expansion or truncation procedure an **exact** resolution of the Hamiltonian into intrinsic and collective parts (Kirson *et al.*, 1985; Leviatan, 1987)

$$H \; = \; H_{int} + H_c \quad (5)$$

The intrinsic part of the Hamiltonian takes the form

$$H_{int}(\beta_0 = 0) \; = \eta_0 d^\dagger \cdot \tilde{d}(d^\dagger \cdot \tilde{d} - 1) + (1/\eta)\Big[\eta s^\dagger d^\dagger + \eta_2 (d^\dagger d^\dagger)^{(2)}\Big] \cdot \Big[h.c.\Big] \quad (6a)$$

$$\eta_0 \; = c - b^2/4a, \quad \eta_2 = v_2, \quad \eta = a \quad (6b)$$

$$H_{int}(\beta_0 > 0) \; = w_0 \Big[d^\dagger \cdot d^\dagger - \beta_0^2 (s^\dagger)^2\Big]\Big[h.c.\Big] + w_2\Big[\beta_0 s^\dagger d^\dagger \pm \sqrt{\frac{7}{2}}(d^\dagger d^\dagger)^{(2)}\Big] \cdot \Big[h.c.\Big] \quad (6c)$$

$$w_0 \; = (|b|\beta_0 - 2a)/4\beta_0^2(1+\beta_0^2), \quad w_2 = |b|/2\beta_0 \quad (6d)$$

n (6c) the upper (lower) sign is taken for $\gamma_0 = 0$ ($=\pi/3$). By construction H_{int} has the same shape or the energy surface as that of the full H. H_{int} becomes positive definite if and only if the coefficients n front of its interaction terms are non-negative. This occurs when the corresponding (β_0, γ_0) are at a lobal minimum of the energy surface. As a zero energy eigenstate of a positive definite Hamiltonian, $N;\beta_0,\gamma_0>$ is an exact ground state of H_{int}. For $\beta_0 > 0$ the state $|N;\beta_0,\gamma_0>$ is deformed and contains everal components of different angular momentum. Since H_{int} is rotational invariant all states of ood angular momentum projected from $|N;\beta_0,\gamma_0>$ form an exactly degenerate ground state band. rom numerical studies it is found that also excited states of H_{int} tend to cluster into bands from onsiderations of both energy and E2 transitions (Leviatan, 1985; Kirson *et al.*, 1985). This is seen in igures (1b) and (1d) for ^{148}Sm and ^{168}Er. In the spherical case of Fig. (1b) the spectrum of H_{int} is at of a near harmonic quadruple oscillator. In the deformed case of Fig. (1d) the spectrum of H_{int} is at of a vibrating rotor with the $J(J+1)$ rotational energy removed. The latter splitting comes from e collective part of the Hamiltonian (H_c). By construction H_c has a flat energy surface and apart om N dependent terms of no significance to excitation energy, it can be transcribed in terms of the ve body parts of the Casimir operators of the orthogonal groups in the chain

$$\overline{O(6)} \supset O(5) \supset O(3) \quad (7)$$

ere $\overline{O(6)}$ is a particular $O(6)$ group generated by $(d^\dagger \tilde{d})^{(L)}$ $L = 1, 3$ and $i(d^\dagger s - s^\dagger \tilde{d})$. The $L = 1,$ operators are the generators of an $O(5)$ group while $O(3)$ is the angular momentum group (with generators $J^{(1)} = \sqrt{10}(d^\dagger \tilde{d})^{(1)}$). The classical limit of H_c contains only momenta vanishing in the static limit and so it may be interpreted as the kinetic energy of collective rotation of the static intrinsic structure determined by H_{int}. The collective motion is related to the usual $O(3)$ rotations and also to more generalized $O(5)$ and $\overline{O(6)}$ rotations associated with the γ and β degrees of freedom respectively. In general H_c and H_{int} do not commute so that H_c can mix as well as shift and split the bands generated by H_{int}.

EXCITATION MODES

In going from a spherical to a deformed ground-state shape the $s^\dagger$ boson was replaced by the condendsate boson $b_c^\dagger(\beta, \gamma)$ (2b). Similarly, we may expect the $d_\mu^\dagger$ bosons which represent excitations of the spherical shape to be replaced by other combinations of bosons which are more suitable for describing excitations of a deformed shape. This motivates the introduction of the following non-spherical bosons (Bohr *et al.*, 1982; Leviatan 1985, 1987)

$$b_c^\dagger = (1 + \beta^2)^{-1/2}[\beta \cos\gamma \cdot d_0^\dagger + \beta \sin\gamma \cdot \frac{1}{\sqrt{2}}(d_2^\dagger + d_{-2}^\dagger) + s^\dagger] \tag{8a}$$

$$b_\beta^\dagger = (1 + \beta^2)^{-1/2}[\cos\gamma \cdot d_0^\dagger + \sin\gamma \cdot \frac{1}{\sqrt{2}}(d_2^\dagger + d_{-2}^\dagger) - \beta s^\dagger] \tag{8b}$$

$$b_\gamma^\dagger = \cos\gamma \cdot \frac{1}{\sqrt{2}}(d_2^\dagger + d_{-2}^\dagger) - \sin\gamma \cdot d_0^\dagger \tag{8c}$$

$$b_x^\dagger = \frac{1}{\sqrt{2}}(d_1^\dagger + d_{-1}^\dagger) \tag{8d}$$

$$b_y^\dagger = \frac{1}{\sqrt{2}}(d_1^\dagger - d_{-1}^\dagger) \tag{8e}$$

$$b_z^\dagger = \frac{1}{\sqrt{2}}(d_2^\dagger - d_{-2}^\dagger) \tag{8f}$$

For any choice of β, γ the six bosons in (8) form an orthogonal and complete basis which contains the usual s-d basis as a special case. Any operator written in terms of s-d bosons can be rewritten in terms of the above deformed basis. In particular the total boson number operator is diagonal in the above basis

$$\hat{N} = s^\dagger s + \sum_\mu d_\mu^\dagger d_\mu = \sum_i b_i^\dagger(\beta, \gamma) b_i(\beta, \gamma) \tag{9}$$

The "appropriate" basis for a given Hamiltonian is obtained from dynamic considerations by seleting in (8) the equilibrium deformation parameters (β_0, γ_0) of H. Each member of the basis acquires a physical interpretation through its connection with a particular degree of freedom. The condensate boson (8a) determines the exact ground state $|N; \beta_0, \gamma_0\rangle$ (2) of H_{int}. Other members of the basis represent excitations of the condensate which involve β, γ vibrations and x, y, z rotations. For large boson number these excitations are obtained by replacing a condensate boson in the equilibrium condensate by orthogonal members of the "appropriate" basis: $b_i^\dagger b_c |N; \beta_0, \gamma_0\rangle$, $i \neq c$. Some care is required in the deformed case. Consider the generator of rotation about the x- axis J_x (obtained in the usual way from the spherical form of the angular momentum operator $J_\mu^{(1)}$). Since H_{int} and J_x commute, it follows that $J_x|N; \beta_0 \gamma_0\rangle$ produces a state different but degenerate with the exact ground state $|N; \beta_0, \gamma_0\rangle$ of H_{int}. But $J_x|N; \beta_0, \gamma_0\rangle \sim \beta_0 \sin(\gamma_0 - 2\pi/3) \cdot b_x^\dagger b_c |N, \beta_0, \gamma_0\rangle$ and thus for $\beta_0 > 0$ the replacement of a condensate boson in $|N; \beta_0, \gamma_0\rangle$ by an x boson results in a spurious rather than an intrinsic excitation. This is a signature of a spontaneously broken symmetry which occurs when the ground state has a lower symmetry than the Hamiltonian. Here H_{int} is rotational invariant, yet when $\beta_0 > 0$ its exact ground state $|N; \beta_0, \gamma_0\rangle$ is deformed and does not have a well defined angular momentum. Associated with a spontaneously broken symmetry is the appearance of Goldstone bosons. In the above example the Goldstone boson is the $x-$ boson (Sd) which is associated with the broken symmetry of rotations about the x- axis. We thus see that in some circumstances governed by symmetries of the Hamiltonian and by the location of its global minimum in the energy surface, some members of the geneal basis (8) represent physical genuine modes while other represent spurious Goldstone modes. The distinction between these two types of modes is most clearly seen in the normal-mode analysis (Leviatan, 1987) to be discussed below.

Normal modes of the boson system are extracted, for large N, from H_{int} by applying to it the Bogoliubov treatment (applicable here due to the diagonality (9) of the number operator in the general basis). To leading order in N this amounts to rewritting H_{int} in terms of the "appropriate" basis $b_i^\dagger(\beta_0, \gamma_0)$ and approximating the $b_c^\dagger$ and b_c operators by the c-number $\sqrt{N}$. The resulting Bogoliubov image of H_{int} (denoted H_{int}^B) is

$$H_{int}^B(\beta_0 = 0) = \epsilon \sum_\mu d_\mu^\dagger d_\mu \tag{10a}$$

$$H_{int}^B(\beta_0 > 0, \gamma_0 = 0) = \epsilon_\beta b_\beta^\dagger b_\beta + \epsilon_\gamma (b_\gamma^\dagger b_\gamma + b_z^\dagger b_z) \tag{10b}$$

$$H_{int}^B(\beta_0 > 0, \gamma_0 = \frac{1}{3}\pi) = \epsilon_\beta b_\beta^\dagger b_\beta + \epsilon_\gamma (b_\gamma^\dagger b_\gamma + b_y^\dagger b_y) \tag{10c}$$

In each case $b_i^\dagger = b_i^\dagger(\beta_0, \gamma_0)$. The normal-mode frequencies are given in terms of the parameters of the full IBM Hamiltonian and are positive for (β_0, γ_0) at a local minimum

$$\epsilon = Na, \quad \epsilon_\beta = N\beta_0^2(4w_0 + w_2), \quad \epsilon_\gamma = 9Nw_2\beta_0^2(1+\beta_0^2)^{-1} \tag{10d}$$

The Bogoliubov image of H_{int} displays harmonic oscillators only in those members of the "appropriate" basis which represent physical intrinsic excitations. The spurious modes are absent and therefore decoupled from the intrinsic ones. For $\beta_0 = 0$ no symmetry is broken in the spherical condensate and therfore all components of the quadrupole boson appear as intrinsic modes in the form of a five dimensional harmonic oscillator (10a). In the deformed case the intrinsic modes are β and γ vibrations. The spurious modes (x, y for $\gamma_0 = 0$ and x, z for $\gamma_0 = \pi/3$) connected with rotations about directions perpendicular to the symmetry axis are absent from $H_{int}^B(\beta_0 > 0)$. The boson linked with the rotation about the symmetry axis (z for $\gamma_0 = 0$ and y for $\gamma_0 = \pi/3$) about which the rotation-symmetry is not broken, joins the γ boson to form a two-dimensional oscillator. The latter can be rewritten in terms of modes with projection $K = \pm 2$ along the symmetry axis. This is just a manifestation of the axial symmetry for prolate or oblate shapes. In case of a γ- independent energy surface ($b = 0$ in (4)), ϵ_γ (10d) vanishes and all x, y, z and γ bosons are missing from $H_{int}^B(\beta_0 > 0)$. They are Goldstone modes associated with a spontaneous broken O(5) symmetry in the condensate. Only the β- mode survives in this case as an intrinsic mode.

The contributions of terms in the collective part of the Hamiltonian to shifts in the normal-mode frequencies (10d) are given in eq. (5.31) of (Leviatan, 1987). Together with expressions (10d) they provide a large N estimate for the position of the bands. Their explicit dependence on parameters of the Hamiltonian illuminates the physical content of otherwise abstract terms and provides a criterion for selecting suitable algebraic interactions based on their spectral significance.

Once the intrinsic modes have been identified one constructs intrinsic states representing the various bands (Bohr *et al.*, 1982). For a prolate deformed shape the intrinsic states for the ground state (g), β and γ bands take the following form for large N

$$|g, K = 0\rangle = (N!)^{-1/2}(b_c^\dagger)^N|0\rangle \tag{11a}$$

$$|\beta, K = 0\rangle = b_\beta^\dagger b_c |g, K = 0\rangle \tag{11b}$$

$$|\gamma, K = \pm 2\rangle = d_{\pm 2}^\dagger b_c |g, K = 0\rangle \tag{11c}$$

In (11) $b_i^\dagger \equiv b_i^\dagger(\beta_0, \gamma_0 = 0)$. These intrinsic states have been used (Bijker *et al.*, 1982; Warner *et al.*, 1982) for calculating intrinsic transition matrix elements and have revealed the origin of characteristic IBM predictions for electromagnetic transitions in deformed nuclei (e.g. the dominance of $\beta \longrightarrow \gamma$ and $\gamma \longrightarrow g$ over $\beta \longrightarrow g$ for E2 transitions).

PROTON NEUTRON BOSON SYSTEMS

Attempts to give the boson model a microscopic iterpretation has lead to the introduction of proton (π) and neutron (ν) degrees of freedom into the model (Arima *et al.*, 1977; Otsuka *et al.*, 1978). This extended version of the interacting boson model (IBM-2) involves two commuting sets of bosons $s_\pi^\dagger, d_\pi^\dagger$ and $s_\nu^\dagger, d_\nu^\dagger$. The Hamiltonian takes the form $H = H_\pi + H_\nu + V_{\pi\nu}$ where H_ρ ($\rho = \pi, \nu$) is given as in eq. (1) and $V_{\pi\nu}$ is a proton-neutron boson interaction.

To discuss the IBM-2 in geometric terms, one introduces two sets of non-spherical bosons (Leviatan *et al.*, 1989). The proton basis is obtained from eqs. (8) by inserting β_π, γ_ν and using proton boson operators. Similarly, the neutron basis is obtained by using neutron deformations β_ν, γ_ν and neutron operators. The neutron basis is also rotated with respect to the proton basis by three Euler angles $\Omega \equiv (\phi, \theta, \psi)$

$$b^\dagger_{i,\nu}(\beta_\nu, \gamma_\nu, \Omega) = R(\Omega) b^\dagger_{i,\nu}(\beta_\nu, \gamma_\nu) R^{-1}(\Omega) \qquad i = c, \beta, \gamma, x, y, z \tag{12}$$

Here $R(\Omega)$ denotes an $O(3)$ rotation. The condensate wave function is now a product of a proton condensate and a rotated neutron condensate

$$|N_\pi, \beta_\pi, \gamma_\pi; N_\nu, \beta_\nu, \gamma_\nu, \Omega> = (N_\pi! N_\nu!)^{-1/2} \left[b^\dagger_{c,\nu}(\Gamma_\nu, \Omega)\right]^{N_\nu} \left[b^\dagger_{c,\pi}(\Gamma_\pi)\right]^{N_\pi} |0> \tag{13}$$

with $\Gamma_\rho = (\beta_\rho, \gamma_\rho)$ $\rho = \pi, \nu$. The expectation value of the Hamiltonian in the state (13) produces an energy surface which depends on the π-ν deformations as well as on the relative orientation Ω between the two quadrupole shapes. The global minimum of the energy surface denoted by $\Gamma_\rho^{(0)} \equiv (\beta_\rho^{(0)}, \gamma_\rho^{(0)})$ and $\Omega^{(0)}$ determines the form of the intrinsic wave function for the ground state band $|N_\pi, \Gamma_\pi^{(0)}; N_\nu, \Gamma_\nu^{(0)}, \Omega^{(0)}>$. As before, the resolution (5) of the Hamiltonian into intrinsic and collective parts exists. The Bogoilubov image of the intrinsic part of the Hamiltonian (H^B_{int}) is obtained by rewritting H_{int} in terms of the π-ν bases with the equilbrium deformations and approximating the π and ν condensate- boson operators by $\sqrt{N_\pi}$ and $\sqrt{N_\nu}$ respectively. A subsequent diagonalization of H^B_{int} produces the normal modes of the combined π-ν system. The normal modes are in general a mixture of particular members (other than condensate) of the separate π ν bases.

For two spherical shapes ($\beta_\pi = \beta_\nu = 0$) the normal modes take the form

$$H^B_{int}(\beta_\pi = \beta_\nu = 0) = \Lambda_s \sum_\mu \omega^\dagger_{s,\mu}\omega_{s,\mu} + \Lambda_a \sum_\mu \omega^\dagger_{a,\mu}\omega_{a,\mu} \tag{14a}$$

$$\omega_s^\dagger = (1+\eta^2)^{-1/2}(d_\pi^\dagger + \eta d_\nu^\dagger); \quad \omega_a^\dagger = (1+\eta^2)^{-1/2}(d_\nu^\dagger - \eta d_\pi^\dagger); \tag{14b}$$

The normal modes involve involve two (five dimensional) quadrupole harmonic oscillators, one symmetric (s) the other antisymmetric (a) between π ν bosons (Hamilton *et al.*, 1984). Since no symmetry is broken in the spherical condensate, all ten components of the two quadrupoles appear as intrinsic modes and there are no Goldstone modes.

For two aligned prolate shapes at equilibrium ($\gamma_\pi = \gamma_\nu = 0$, $\Omega = 0$) the normal modes take the form

$$\begin{aligned} H^B_{int}(\beta_\pi > 0, \beta_\nu > 0) = {} & \Lambda_s^{(\beta)} b^\dagger_{\beta,s} b_{\beta,s} + \Lambda_a^{(\beta)} b^\dagger_{\beta,a} b_{\beta,a} + \Lambda_s^{(\gamma)} [b^\dagger_{\gamma,s} b_{\gamma,s} + b^\dagger_{z,s} b_{z,s}] \\ & + \Lambda_a^{(\gamma)} [b^\dagger_{\gamma,a} b_{\gamma,a} + b^\dagger_{z,a} b_{z,a}] + \Lambda^{(xy)} [b^\dagger_{x,a} b_{x,a} + b^\dagger_{y,a} b_{y,a}] \end{aligned} \tag{15a}$$

$$
\begin{aligned}
b^\dagger_{i,s} &= (1+\eta_i^2)^{-1/2}[\,b^\dagger_{i,\pi} + \eta_i b^\dagger_{i,\nu}\,] \\
& \qquad\qquad\qquad\qquad\qquad\qquad i = \beta, \gamma, x, y, z \\
b^\dagger_{i,a} &= (1+\eta_i^2)^{-1/2}[\,b^\dagger_{i,\nu} - \eta_i b^\dagger_{i,\pi}\,]
\end{aligned}
\qquad (15b)
$$

with $\eta_\gamma = \eta_z$ and $\eta_x = \eta_y$. Here the normal modes consist of one-dimensional symmetric-β (β, s) and antisymmetric-β (β, a) modes, two dimensional symmetric-γ (γ, s) and antisymmetric-γ (γ, a) modes and a two dimensional scissors (sc) mode (Bohle *et al.*, 1984) composed of antisymmetric combinations of x and of y bosons. The 1^+ bandhead member of the scissors band has been recently observed in a wide range of nuclei by (e, e') (Richter, 1989) and (γ, γ') (Kneissl, 1989) reactions. The two Goldstone modes which are missing from (15) are the two symmetric combinations of x and of y bosons associated with rotations about directions perpendicular to the common symmetry z- axis. Explicit expressions for the mixing parameters and normal- mode frequencies in (14)-(15) in terms of equilibrium deformations, boson-numbers and parameters of the IBM-2 Hamiltonian can be found in (Leviatan *et al.*, 1989). The mixing parameter for the scissors mode η_{sc} ($=\eta_x = \eta_y$ in (15b)) has the following simple form

$$
\eta_{sc} = \beta_\nu \sqrt{N_\nu(1+\beta_\pi^2)} / \beta_\pi \sqrt{N_\pi(1+\beta_\nu^2)} \qquad (15c)
$$

Having identified the intrinsic modes they now serve to construct intrinsic states representing the various bands. For the aligned prolate shapes cosidered in (15) the intrinsic states for the ground (g) and excited bands take the form

$$
|(g), K = 0> = (N_\pi! N_\nu!)^{-1/2} (b^\dagger_{c,\pi})^{N_\pi} (b^\dagger_{c,\nu})^{N_\nu} |0> \equiv |N_\pi, N_\nu> \qquad (16a)
$$

$$
|(\beta, s), K = 0> = (1+\eta_\beta^2)^{-1/2} \left[b^\dagger_{\beta,\pi} |N_\pi - 1, N_\nu> + \eta_\beta b^\dagger_{\beta,\nu} |N_\pi, N_\nu - 1> \right] \qquad (16b)
$$

$$
|(\beta, a), K = 0> = (1+\eta_\beta^2)^{-1/2} \left[b^\dagger_{\beta,\nu} |N_\pi, N_\nu - 1> - \eta_\beta b^\dagger_{\beta,\pi} |N_\pi - 1, N_\nu> \right] \qquad (16c)
$$

$$
|(\gamma, s), K = \pm 2> = (1+\eta_\gamma^2)^{-1/2} \left[d^\dagger_{\pi,\pm 2} |N_\pi - 1, N_\nu> + \eta_\gamma d^\dagger_{\nu,\pm 2} |N_\pi, N_\nu - 1> \right] \qquad (16d)
$$

$$
|(\gamma, a), K = \pm 2> = (1+\eta_\gamma^2)^{-1/2} \left[d^\dagger_{\nu,\pm 2} |N_\pi, N_\nu - 1> - \eta_\gamma d^\dagger_{\pi,\pm 2} |N_\pi - 1, N_\nu> \right] \qquad (16e)
$$

$$
|(sc), K = \pm 1> = (1+\eta_{sc}^2)^{-1/2} \left[d^\dagger_{\nu,\pm 1} |N_\pi, N_\nu - 1> - \eta_{sc} d^\dagger_{\pi,\pm 1} |N_\pi - 1, N_\nu> \right] \qquad (16f)
$$

Using expressions (16) one can evaluate intrinsic matrix elements of transition operators. As an example consider the M1 operator $T(M1) = \sqrt{3/4\pi}[g_\pi J_\pi^{(1)} + g_\nu J_\nu^{(1)}]$ where $J_\rho^{(1)}$ are the π and ν angular momentum operators. For large N_π and N_ν one obtains the following estimate for the reduced transition probability of M1 transitions from the $J = 0_g^+$ state in the ground state band to the $J = 1_{sc}^+$ state in the scissors band

$$
B(M1; J = 0_g^+ \to J = 1_{sc}^+) = \frac{3}{4\pi} \cdot \frac{6 N_\pi N_\nu \beta_\pi^2 \beta_\nu^2 (g_\pi - g_\nu)^2}{N_\pi \beta_\pi^2 (1+\beta_\nu^2) + N_\nu \beta_\nu^2 (1+\beta_\pi^2)} \qquad (17a)
$$

For equal proton neutron deformations $\beta_\pi = \beta_\nu = \beta$ eq. (17a) simplifies to

$$
B(M1; J = 0_g^+ \to J = 1_{sc}^+) = \frac{3}{4\pi} 6 (g_\pi - g_\nu)^2 \beta^2 (1+\beta^2)^{-1} \frac{N_\pi N_\nu}{N_\pi + N_\nu} \qquad (17b)
$$

It is seen that the magnitude of the above B(M1) is governed by three factors: the π-ν deformations, the boson g - factors and the number of bosons N_ρ.

SELECTED APPLICATION

It is instructive to illustrate the formalism presented so far on a specific IBM-2 Hamiltonian which has often been used in IBM-2 calculations

$$H' = \sum_{\rho=\pi,\nu} \epsilon_\rho \hat{n}_{d,\rho} + \kappa Q_\pi \cdot Q_\nu + \sum_{L=1,3} \alpha_L (d^\dagger_\pi d^\dagger_\nu)^{(L)} \cdot (\tilde{d}_\nu \tilde{d}_\pi)^{(L)} + \alpha_2 [s^\dagger_\pi d^\dagger_\nu - d^\dagger_\pi s^\dagger_\nu] \cdot [h.c.] + \kappa' J^{(1)}_\pi \cdot J^{(1)}_\nu \quad (18a)$$

$$Q_\rho = d^\dagger_\rho s_\rho + s^\dagger_\rho \tilde{d}_\rho + \chi_\rho (d^\dagger_\rho \tilde{d}_\rho)^{(2)}, \quad \hat{n}_{d,\rho} = d^\dagger_\rho \cdot \tilde{d}_\rho, \quad (\rho = \pi,\nu) \quad (18b)$$

We shall focus the discussion to the case of two aligned prolate shapes ($\beta_\pi > 0$, $\beta_\nu > 0$, $\gamma_\pi = \gamma_\nu = 0$, $\Omega = 0$). The Hamiltonian H' (18) can be resolved into intrinsic and collective parts according to (5). The intrinsic Hamiltonian takes the form

$$H_{int} = \xi_0 \left[\beta_\pi \beta_\nu s^\dagger_\pi s^\dagger_\nu - d^\dagger_\pi \cdot d^\dagger_\nu\right]\left[h.c.\right] + \sum_{L=1,3} \xi_L (d^\dagger_\pi d^\dagger_\nu)^{(L)} \cdot (\tilde{d}_\nu \tilde{d}_\pi)^{(L)} + \xi_2 \left[\beta_\pi s^\dagger_\pi d^\dagger_\nu - \beta_\nu s^\dagger_\nu d^\dagger_\pi\right] \cdot \left[h.c.\right]$$
$$+ \xi_2' \left[\beta_\pi s^\dagger_\pi d^\dagger_\nu + \sqrt{\frac{7}{2}} (d^\dagger_\pi d^\dagger_\nu)^{(2)}\right] \cdot \left[h.c.\right] + \xi_2'' \left[\beta_\nu s^\dagger_\nu d^\dagger_\pi + \sqrt{\frac{7}{2}} (d^\dagger_\pi d^\dagger_\nu)^{(2)}\right] \cdot \left[h.c.\right] \quad (19)$$

The coefficients of the various terms in (19) are given by $\xi_0 = [-2\alpha_2 + (\alpha_2 - 2\kappa)(\beta^2_\pi + \beta^2_\nu)/\beta_\pi\beta_\nu + \kappa\bar{\chi}_\pi\bar{\chi}_\nu - \kappa\bar{\chi}_\pi(1-\beta^2_\nu)/\beta_\nu - \kappa\bar{\chi}_\nu(1-\beta^2_\pi)/\beta_\pi]/(1+\beta^2_\pi)(1+\beta^2_\nu)$, $\xi_1 = 2\xi_0 + \kappa\bar{\chi}_\pi\bar{\chi}_\nu + 5\kappa(\bar{\chi}_\pi/\beta_\nu + \bar{\chi}_\nu/\beta_\pi)/2 + \alpha_1$, $\xi_2 = -\xi_0 + (\alpha_2 - 2\kappa)/\beta_\pi\beta_\nu$, $\xi_2' = \kappa\bar{\chi}_\nu/\beta_\pi$, $\xi_2'' = \kappa\bar{\chi}_\pi/\beta_\nu$, $\xi_3 = 2\xi_0 - 4\kappa\bar{\chi}_\pi\bar{\chi}_\nu + \alpha_3$ where $\bar{\chi}_\rho = \sqrt{2/7}\chi_\rho$.

The deformation parameters β_π, β_ν are solutions of the follwing two extremum equations

$$\epsilon_\pi \beta_\pi (1+\beta^2_\nu)/N_\nu + \alpha_2(\beta_\pi - \beta_\nu)(1+\beta_\pi\beta_\nu) + \kappa\beta_\nu(1 - \bar{\chi}_\pi\beta_\pi - \beta^2_\pi)(2 - \bar{\chi}_\nu\beta_\nu) = 0 \quad (20)$$

The second equation for β_π, β_ν is obtained from eq. (20) by interchanging π-ν labels. For the aligned prolate shapes considered here, the Majorana parameters α_1 and α_3 as well as κ' do not appear in the extremum equations.

The normal modes and intrinsic states have the form given in eqs. (15) and (16) respectively. The β modes and frequencies are found by diagonalizing the following 2×2 matrix in the basis $b^\dagger_{\beta,\pi}$ and $b^\dagger_{\beta,\nu}$. $M^{(\beta)}_{\pi,\pi} = (\xi_0 + \xi_2 + \xi_2')N_\nu\beta^2_\nu(1+\beta^2_\pi)/(1+\beta^2_\nu)$, $M^{(\beta)}_{\pi,\nu} = (\xi_0 - \xi_2)\beta_\pi\beta_\nu\sqrt{N_\pi N_\nu}$, $M^{(\beta)}_{\nu,\nu} = (\xi_0 + \xi_2 + \xi_2'')N_\pi\beta^2_\pi(1+\beta^2_\nu)/(1+\beta^2_\pi)$. The γ modes and frequencies are obtained by diagonalizing the following 2×2 matrix given in the basis $d^\dagger_{\pi,\pm 2}$ and $d^\dagger_{\nu,\pm 2}$. $M^{(\gamma)}_{\pi,\pi} = (\xi_2 + \xi_2' + 4\xi_2'' + \xi_3/2)N_\nu\beta^2_\nu/(1+\beta^2_\nu)$, $M^{(\gamma)}_{\pi,\nu} = (-\xi_2 + 2\xi_2' + 2\xi_2'' - \xi_3/2)\beta_\pi\beta_\nu\sqrt{N_\pi N_\nu/(1+\beta^2_\pi)(1+\beta^2_\nu)}$, $M^{(\gamma)}_{\nu,\nu} = (\xi_2 + 4\xi_2' + \xi_2'' + \xi_3/2)N_\pi\beta^2_\pi/(1+\beta^2_\pi)$. The scissors modes were already given in eq. (15). Their frequency is given by

$$\lambda^{(sc)} = \left[\frac{3}{10}\alpha_1 + \frac{1}{5}\alpha_3 - \frac{1}{2}\kappa\bar{\chi}_\pi\bar{\chi}_\nu + [\alpha_2 - 2\kappa + \kappa(\bar{\chi}_\pi\beta_\pi + \bar{\chi}_\nu\beta_\nu)]/\beta_\pi\beta_\nu\right] \times \left[N_\pi\beta^2_\pi(1+\beta^2_\pi)^{-1} + N_\nu\beta^2_\nu(1+\beta^2_\nu)^{-1}\right] \quad (21)$$

The Majorana parameter α_1 affects only the frequency of the scissors mode, α_3 affects the scissors and the γ frequencies while α_2 affects the frequencies of all normal modes. The κ' interaction does not influence the normal modes and as will be shown below, it belongs to the collective part of the Hamiltonian.

The collective part of the hamiltonian H' (18) has the form

$$H_c = \epsilon_d^{(\pi)}(1 - \hat{N}_\nu/N_\nu)\hat{n}_{d,\pi} + \epsilon_d^{(\nu)}(1 - \hat{N}_\pi/N_\pi)\hat{n}_{d,\nu} + a_0\hat{N}_\pi\hat{N}_\nu$$
$$+ t_6\Big[\hat{C}_{\overline{O(6)}} - \sum_{\rho=\pi,\nu} \hat{C}^{(\rho)}_{\overline{O(6)}}\Big] + t_5\Big[\hat{C}_{O(5)} - \sum_{\rho=\pi,\nu} \hat{C}^{(\rho)}_{O(5)}\Big] + t_3\Big[\hat{C}_{O(3)} - \sum_{\rho=\pi,\nu} \hat{C}^{(\rho)}_{O(3)}\Big] \qquad (22)$$

where $a_0 = -\xi_0\beta_\pi^2\beta_\nu^2$, $t_6 = -[\kappa + \xi_0\beta_\pi\beta_\nu]/2$, $t_5 = [\kappa + \xi_0(1 + \beta_\pi\beta_\nu) - \kappa(\gamma_\pi/\beta_\nu + \gamma_\nu/\beta_\pi) - \kappa\gamma_\pi\gamma_\nu]/2$, $t_3 = [\kappa(\gamma_\pi/\beta_\nu + \gamma_\nu/\beta_\pi)/2 + \kappa\gamma_\pi\gamma_\nu/4 + \kappa']/2$. Apart from $\hat{N}_\rho$- dependent terms of no significance to excitation energies, H_c is composed of the π-ν part of the Casimir operators of the diagonal $\overline{O(6)}$, $O(5)$ and $O(3)$ groups (i.e. the groups generated by a direct sum of the generators of the separate π and ν groups discussed in (7)).

To leading order in N_ρ the Bogoliubov image of H_c contribiutes to the frequencies of the physical modes in H^B_{int} (15). For H_c of eq. (22) the resulting shifts of frequencies (denoted by $\delta\lambda$) are

$$\delta\lambda_s^{(\beta)} = 4t_6\sqrt{N_\pi N_\nu}\,\eta_\beta(1 + \eta_\beta^2)^{-1} \qquad (23a)$$

$$\delta\lambda_a^{(\beta)} = -\delta\lambda_s^{(\beta)} \qquad (23b)$$

$$\delta\lambda_s^{(\gamma)} = 4\sqrt{N_\pi N_\nu}\,\eta_\gamma(1 + \eta_\gamma^2)^{-1}(1 + \beta_\pi^2)^{-1/2}(1 + \beta_\nu^2)^{-1/2}\,[t_5\beta_\pi\beta_\nu + t_6(1 + \beta_\pi\beta_\nu)] \qquad (23c)$$

$$\delta\lambda_a^{(\gamma)} = -\delta\lambda_s^{(\gamma)} \qquad (23d)$$

$$\delta\lambda^{(sc)} = -4N_\pi N_\nu\beta_\pi\beta_\nu\left[N_\pi\beta_\pi^2(1 + \beta_\nu^2) + N_\nu\beta_\nu^2(1 + \beta_\pi^2)\right]^{-1} \times$$
$$[(3t_3 + t_5)\beta_\pi\beta_\nu + t_6(1 + \beta_\pi\beta_\nu)] \qquad (23e)$$

The expressions for the normal mode frequencies together with the shifts (23) due to H_c, can be used to obtain estimates for large boson numbers to the position of the bands.

SUMMARY

We have presented a procedure for identifying the normal modes in number- conserving algebaric bosonic systems relevant for collective states in nuclei. The procedure is part of a framework aimed at revealing the underlying intrinsic and collective structure in such systems. The main ingredients of the framework are (a) An exact resolution of the Hamlitonain into intrinsic (bandhead related) and collective (in-band related) parts (b) Introduction of shape parameters through non-spherical bases (c) Extraction of normal modes by applying the Bogoliubov treatment for large boson-number to the intrinsic part of the Hamiltonian (d) Construction of intrinsic states and evaluation of transition matrix elements.

There have been previous attempts for a geometric interpretation of the interacting boson model. The approach of (Hatch *et al.*, 1982) is based on a classical limit for the boson Hamiltonian. Several works have used this approach to determine the frequencies of preselected excitation modes (Balantekin *et al.*, 1983, 1985, 1987, 1988; Bijker, 1985; Walet *et al.*, 1985). The approach of (Dukelsky *et al.*, 1984; Pittel *et al.*, 1985) is based on a Hartree Bose procedure together with Tamm Dancoff (TDA) and (RPA) treatments. The approach of (Van Egmond *et al.*, 1985) combines a mean field approximation with a generator coordinate method. The formalism presented in this work is fully quantal and can accomodate arbitrary π-ν shapes. All normal modes are derived simultaneously and with equal ease.

The normal modes are extracted from the intrinsic part of the Hamiltonian rather than from the full Hamiltonian. This has the following virtues (i) The spurious modes are decoupled from the intrinsic ones (there is no need for an RPA treatment to restore the broken symmetries at the expense of a more complicated form for the wave functions). (ii) The derived normal modes are determined from dynamic considerations and are composed of creation operators only (not a mixture of creation and destruction operators). As a consequence an inherent feature of the IBM-2 conservation of boson number, is preserved in the structure of intrinsic states.

The formalism presented in this work illuminates the spectral significance of arbitrary algebraic interactions and identifies the intrinsic and collective modes associated with any π-ν shapes described within the IBM-2. By relating parameters of the model to observables such as bandhead energies and moments of inertia, the method provides a criterion for selecting suitable algebraic Hamiltonains. For practical applications, the explicit expressions for the approximate position of the bandhead energies may be used to provide a physical insight and guidance in numerical calculations and least square fitting procedures

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