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PROTON ACCUMULATOR RING INJECTION STUDIES*

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1. INTRODUCTION

Protons may be created in an accelerator or storage ring by stripping electrons from neutral hydrogen atoms that have been injected into the machine. Because Liouville's theorem is violated by this type of injection, particles may be continually injected into a region of phase space that is already populated, and the density in that region increases with time.

The purpose of this work is to investigate computationally the evolution of the distribution of particles in longitudinal phase space during such an injection process. We consider a storage ring operating below the transition energy. The variables considered are $u = \Delta E/E_o$ (i.e., the deviation in energy divided by the reference energy E_o) and the phase $\varphi = h(\theta - \omega_o t)$ in which h is the harmonic number, θ the azimuthal position, and ω_o the circulation frequency for particles with energy E_o. The Vlasov equation is integrated numerically to determine the function $\psi(t,\varphi,u)$ that represents the distribution of particles in φ - u space at time t. Two calculations are performed.

In the first calculation an rf cavity is present in the ring and particles are injected into the stable phase region once each revolution. The purpose of this calculation is to determine the rf voltage necessary to overcome the longitudinal self-forces and contain the particles within the region of stable phase. In the second calculation the rf is turned off, so that we are treating the spreading in azimuth of the injected particles (i.e., de-bunching). The de-bunching occurs because of the initial energy spread and the action of the self-forces. One purpose of the calculation is to determine the total energy spread after a given number of revolutions. Another purpose is to elucidate the effect of finite resistance in the vacuum tank walls. For sufficiently high current, the finite resistance can cause bunching of a beam that is initially uniform in azimuth. Therefore it might be expected that the finite resistance would inhibit or prevent de-bunching once the number of particles injected reaches some threshold, and that this threshold would depend upon the energy spread in the beam.

2. THE MODEL

Quantities appearing in the Vlasov equation include $\eta = \gamma^{-2} - \gamma_t^{-2}$, with γ_t the transition value of γ for the ring, R = circumference/2π, the peak rf voltage V, the azimuthal self-electric field $\mathcal{E}$, and the dimensionless time $\tau = \omega_o t$. In terms of these

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variables the Vlasov equation takes the form

$$\frac{\partial\psi}{\partial\tau} + \frac{h\eta u}{\beta^2}\frac{\partial\psi}{\partial\varphi} + \frac{e}{E_o}\left[R\mathcal{E}(\varphi) - \sum_{n=0}\delta(\tau - n2\pi)\frac{V}{2\pi}\sin\varphi\right]\frac{\partial\psi}{\partial u} = S(\varphi,u,t) \tag{1}$$

The right hand side of this equation is the source term that represents the injection of particles. The particular form chosen is

$$S(\varphi,u) \propto \left[1 + e^{10(Y-1)}\right]^{-1} , \tag{2}$$

with

$$Y = \left[\left(\frac{u}{u_o}\right)^2 + \left(\frac{\varphi - \alpha u}{\varphi_o}\right)^2\right]^{\frac{1}{2}} \tag{3}$$

The quantity α in Eq. (3) represents a shift from a centered distribution in φ. This shift occurs during transport of the beam to the ring.

The LAMPF 800 MeV linac is taken as the source of particles for the ring, thus we take $\gamma = 1.85$. When used as an injector for the ring, the output of LAMPF should be micropulses of ~200 ps duration separated by 5 ns and containing 5×10^8 particles. The anticipated value of $\Delta p/p$ is of the order of 10^{-3}. We choose $\varphi_o = 0.2$ and $u_o = 7 \times 10^{-4}$ in Eq. (3) The model of the ring has a circumference of 75.4 m so that the rf operates on the 60th harmoni in order to be synchronous with the incoming bunches. A value of 0.1 is used for η. The normalization of S is such that 5×10^8 particles are injected into each bucket each turn.

The model of the beam used in calculating the electric self-field assumes that the particles are confined to the surface of a cylinder of radius a = .01 m, concentric with the vacuum pipe of radius b = .05 m. (For calculation of electric field effects the curvature of the machine is neglected.) The surface charge density is related to the distribution function through the relation

$$\lambda(\varphi) = e\frac{h}{R}\int_{-\infty}^{\infty}\psi\, du \quad .$$

This charge density is Fourier analyzed as

$$\lambda(\varphi) = \sum_{-\infty}^{\infty} \ell_n e^{in\varphi} \quad .$$

In a perfectly conducting pipe this charge density gives rise to a longitudinal electric field acting back on the particles:

$$\mathcal{E}_o = \frac{-1}{\gamma^2 2\pi\varepsilon_o}\frac{h}{R}\sum_{-\infty}^{\infty} in\,\ell_n\, I_o^2\left(\frac{nha}{\gamma R}\right)\left(\frac{K_{oa}}{I_{oa}} - \frac{K_{ob}}{I_{ob}}\right)e^{in\varphi} \quad . \tag{4}$$

where I_o and K_o are modified Bessel functions with argument either $nha/\gamma R$ or $nhb/\gamma R$ as indicated by a second subscript. If the arguments of the Bessel functions can all be taken to be small, then this expression reduces to

$$\mathcal{E}_o \doteq -\frac{2\ln(b/a)}{\gamma^2}\frac{h}{4\pi\varepsilon_o R}\frac{\partial\lambda}{\partial\varphi} \quad , \tag{5}$$

which is a more familiar form.

To include the effects of finite conductivity on the particle motion, the magnetic field at r = b is calculated as though the pipe were perfectly conducting. This field is expressed as

$$B_o = \sum_{-\infty}^{\infty} b_n\, e^{in\varphi} \quad .$$

Each Fourier component of this field can be thought of as producing at the wall a component of a longitudinal electric field given by[1]

$$e_n = (1 \mp i)\,\frac{b_n}{\mu_o\sigma\delta_n} \quad ,$$

in which σ is the conductivity of the wall material ($\sigma = 1.1 \times 10^6$ mho/m), δ_n is the skin depth of the nth harmonic of the field ($\delta_n = \sqrt{2/hn\omega_o\mu_o\sigma}$), and the lower sign is taken for $n < 0$. This electric field evaluated at the beam is then given by

$$\mathcal{E}_1 = \frac{-v}{2\pi\sigma b} \sum_{-\infty}^{\infty} \ell_n\, (1 \mp i)\,\frac{1}{\delta_n}\,\frac{I_{oa}^2}{I_{ob}^2}\, e^{in\varphi} \tag{6}$$

The computer code calculates the electric fields $\mathcal{E}_o$ and $\mathcal{E}_1$ once per turn. The number of time steps n_τ per turn is chosen to satisfy the more stringent of the stability criteria for the finite difference scheme used:

$$\frac{h\eta|u|_{max}}{\beta^2}\,\frac{\Delta\tau}{\Delta\varphi} < 1 \quad ,$$

and

$$\frac{e|\mathcal{E}|_{max}R}{E_o}\,\frac{\Delta\tau}{\Delta u} < 1 .$$

The finite difference equations are those of the simple "upstream-downstream" method[2]:

$$\psi_{ij}^{n+1} = \psi_{ij}^{n} - \frac{\Delta\tau}{\Delta\varphi}\frac{h\eta}{\beta^2} u_j \begin{Bmatrix} \psi_{i,j}^{n} - \psi_{i-1,j}^{n} \\ \psi_{i+1,j}^{n} - \psi_{i,j}^{n} \end{Bmatrix}$$

$$- \frac{\Delta\tau}{\Delta u}\frac{eR}{E_o}\mathcal{E}_i \begin{Bmatrix} \psi_{i,j}^{n} - \psi_{i,j-1}^{n} \\ \psi_{i,j+1}^{n} - \psi_{i,j}^{n} \end{Bmatrix} + S_{ij}\sum_{k=0}^{\infty}\delta_{n,kn_\tau}$$

where in the bracketed expressions the upper expression is to be used if the subscripted factor in front of the brackets is positive. The Kronecker deltas indicate that the source term is added to the distribution function once per revolution. The effect of the rf voltage is taken into account once per turn, immediately after injecting a new bunch of particles. This calculation amounts to a shift in the distribution function along the lines φ = constant by the amount

$$\Delta u = \frac{-eV_{rf}\sin\varphi}{E_o} \quad .$$

3. RESULTS

Figure 1 shows the initial distribution function (= the source function) used for all calculations. The horizontal axis is φ, varying from $-\pi$ to π in 128 steps. The axis extending into the plane of the figure is the u axis, varying from $-3u_o$ to $3u_o$ in 30 steps. The rf voltage required may be estimated by assuming that after a large number of turns we have $\lambda(\varphi)$ of the form

$$\lambda(\varphi) = \lambda_o(1 + \cos\varphi) \quad ,$$

and employing the approximate relation Eq. (5). We find the maximum $\mathcal{E}(\varphi)$ to be such that $2\pi R\mathcal{E}_{max}$ ~40 kV after 200 injected turns. The total voltage V_e required to contain a distribution with half-height $\Delta E/E_o$ is given by the relation

$$\frac{\Delta E}{E_o} = \beta\left(\frac{2eV_e}{\pi h\eta E_o}\right)^{1/2} \quad .$$

For $\Delta E/E = 7 \times 10^{-4}$ and the parameters used above, we find V_e ~10 kV, so that the applied voltage should be of the order of 50 kV.

Figure 2 is the solution after 200 turns of injection with 100 kV of applied rf. This voltage appears to be not quite sufficient to contain all the particles injected, as can be seen from the finite value of the distribution function at $\pm\pi$. However, there is considerable numerical diffusion inherent in the finite differencing scheme. This diffusion (which is not physical) could possibly account for particles reaching the boundaries at $\varphi = \pm\pi$. A more sophisticated differencing scheme may be employed in future calculations.

Figures 3 and 4 show the distribution function after 100 and 200 injected turns, respec-ively, with no external voltage applied. The bunches can be seen to be well distributed in zimuth, the desired behavior, even though the injection method, which injected bunches directl n top of other bunches, was the least conducive to a rapid azimuthal de-bunching. No effects f the finite resistance are apparent. The distribution has spread in u to the extent that the ialf width at half maximum is $\sim 2u_0$. Simple analytic theory predicts that this energy spread s sufficient to suppress the longitudinal resistive wall instability in an azimuthally uniform eam with much higher circulating current than is present here after 200 turns. This calcula-ion will be continued to 1000 turns using a better differencing scheme.

References

. J. D. Jackson, Classical Electrodynamics, Wiley, 1962, p 238.

'. J. A. Byers, and J. Killeen, Finite Difference Methods for Collisionless Plasma Models, Methods Computational Physics, Vol. 9 (1970).

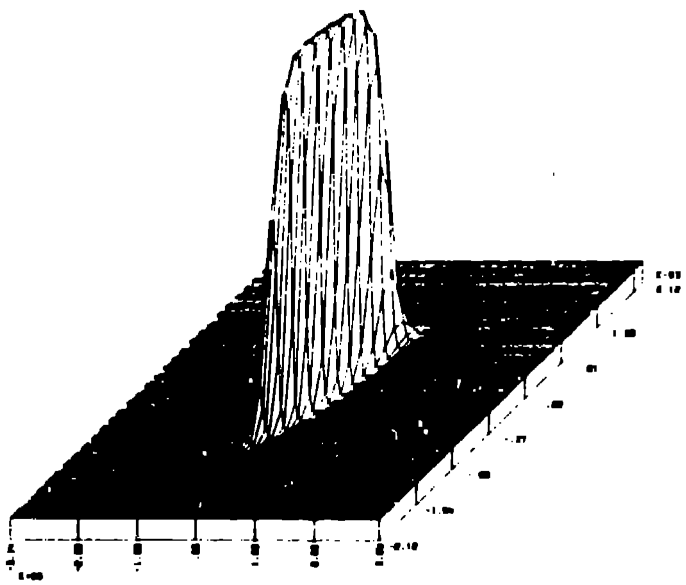

Fig. 1. The Initial Distribution Function

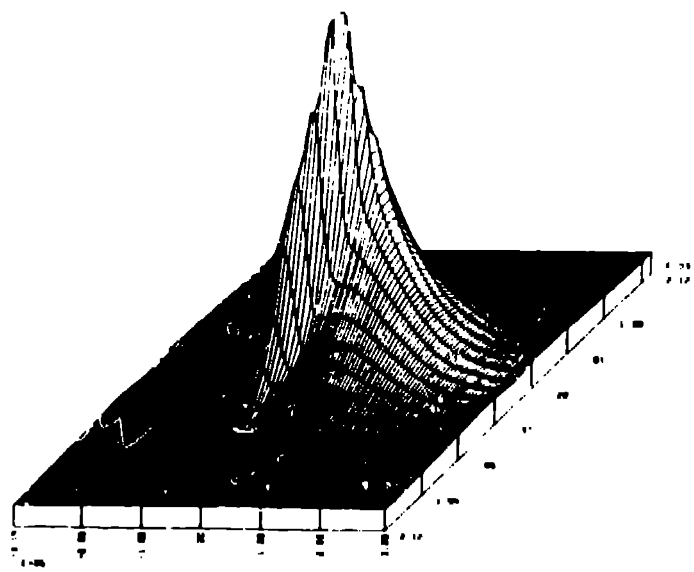

Fig. 2. The Distribution Function After 200 Turns with 100 kV RF Voltage

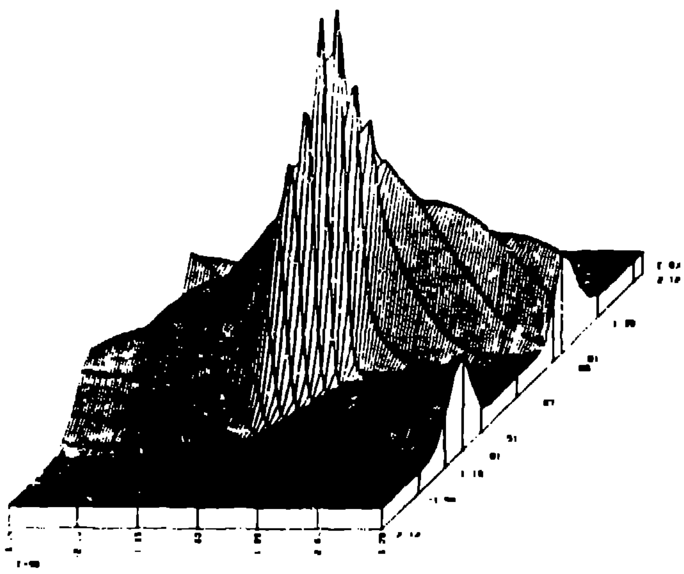

Fig. 3. The Distribution Function After 100 Turns, No RF

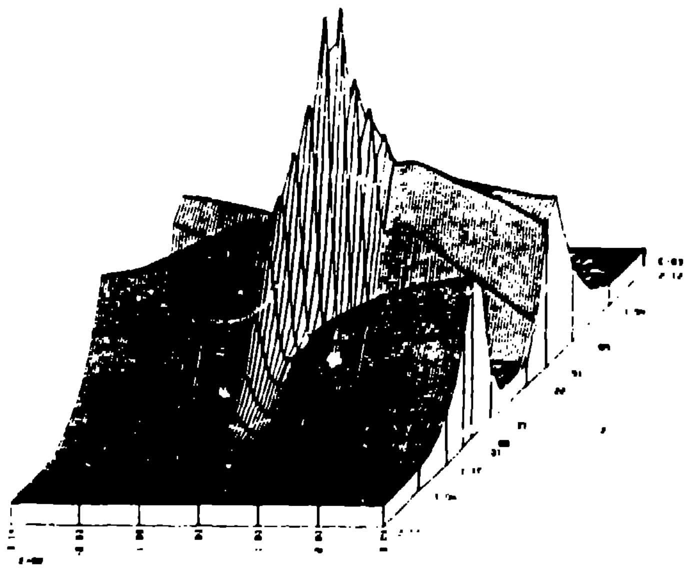

Fig. 4. The Distribution Function After 200 Turns, No RF