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IMPROVING THE PERFORMANCE OF POWER-LIMITED TRANSVERSE STOCHASTIC COOLING SYSTEMS*

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We present the formulas relevant to the behavior of (transverse) stochastic cooling systems which operate under the the not uncommon condition that performance is limited by available output power, and contrast the operation of such systems with non-power-limited ones. In particular, we show that for power-limited systems, the two most effective improvements are the use of pickups/kickers which operate in both planes simultaneously[1] and/or plunging of the cooling system electrodes, and present an example where increasing bandwidth is counter-productive. We apply our results to the proposed upgrade of the Fermilab $\overline{p}$ source.

INTRODUCTION

Conventional analyses of stochastic cooling systems assume that performance is not limited by available electronic gain, and that the latter quantity can be set to maximize the cooling rate. Under these conditions, one can expect an improvement of as much as a factor of 4 in the cooling time by doubling the midband operating frequency of the cooling system. In practical systems, cost-induced limitations on the maximum available output power may restrict the maximum attainable gain to be less that its optimal value; such is the case in the anti-proton sources at both CERN and Fermilab. We show that the criteria that one would employ in upgrading such power-limited systems (we limit our treatment throughout to the case of systems which cool the transverse phase space of the beam) are rather different from those for systems for which one can optimize the gain; in particular, the maximum expected improvement resulting from doubling the operating frequency of such a power-limited system is less than a factor of 2. We apply our results to the specific case of improving the performance of the Fermilab debuncher ring.

FORMULARY FOR POWER-LIMITED SYSTEMS

The cooling rate of a stochastic cooling system is given by[2]

$$\frac{1}{\tau} = \frac{W}{N}\left[2g - g^2(M+U)\right] \qquad (1)$$

where W is the signal bandwidth of the cooling system, M is the so-called mixing factor, U is the noise-to-signal ratio, and g is usually referred to as the system gain; in a transverse cooling system, it represents the fraction of the beam-sample centroid error corrected in a single pass through the pickup and kicker. For *non*-power-limited systems, one minimizes τ by setting $g = 1/(M+U) \equiv g_{opt}$, its optimum value, thereby yielding the familiar result

$$\frac{1}{\tau_{opt}} = \frac{W}{N(M+U)} \qquad (2)$$

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We now consider the results for power-*limited* systems.[a] If we define g_{lim} as the maximum available (i.e. power-limited) gain, we can write

$$\frac{1}{\tau_{lim}} = \frac{1}{\tau_p}\left[2 - \frac{g_{lim}}{g_{opt}}\right] \tag{3}$$

where, for analyzing power-limited systems, it is convenient to introduce the quantity

$$\frac{1}{\tau_p} \equiv \frac{1}{\tau_{opt}}\,\frac{g_{lim}}{g_{opt}} \tag{4}$$

The quantity τ_p^{-1} is given by

$$\frac{1}{\tau_p} = \frac{e c f_o \overline{\beta} \alpha n_L (Z'_L)^2}{\sqrt{2}\pi E/e \; f_B Z_c}\sqrt{\frac{P_{out} W}{(1+\frac{1}{U})\, k(T_R+T_A)}} \tag{5}$$

where

N = total number of particles
f_o = particle revolution frequency
E = total proton energy (rest + kinetic)
e = proton charge
f_B = mid-band beam (signal) frequency
c = velocity of light
$\overline{\beta}$ = (geometric) mean of beta functions at pickup and kicker
n_L = number of kicker/pickup loop pairs
Z_L' = single loop-pair (transverse) transfer impedance
K_L = single loop-pair (transverse) kicker constant = $cZ_L'/\pi f_B Z_C$
Z_C = characteristic impedance of external signal lines
α = voltage attenuation in the pickup and kicker circuitry located between the electrodes and the amplifier circuits

Calculating τ_{opt} from Eq. 2, one can now use Eq. 4 to obtain

$$\frac{g_{lim}}{g_{opt}} = \frac{\tau_{opt}}{\tau_p} \tag{6}$$

Finally, to evaluate the ability of a system to cool a beam from an initial emittance ε_i to a final emittance ε_f, one substitutes Eqs. 5 and 6 into Eq. 3, and integrates to obtain the total cooling time T_{tot}.

$$T_{tot} = -\int_{\varepsilon_i}^{\varepsilon_f} \frac{\tau(\varepsilon)\,d\varepsilon}{\varepsilon} \tag{7}$$

GENERAL CONCLUSIONS

We begin by reviewing the situation for systems which are *not* power limited. Let us assume for definiteness that we have a cooling system which operates over a one-octave frequency range. Eq. 2 shows that doubling the mid-band frequency doubles the cooling rate by doubling W. If the system is *mixing*-limited, an additional factor of two results from halving M. A similar additional factor of 2 is usually obtained for *noise-limited* systems as well: Under the combined assumptions that the length of individual pickup elements is proportional to the operating frequency, that it is possible to preserve the same pickup

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[a] For a derivation of the formulas, the reader is referred to Reference 3.

impedance for the higher frequency electrodes, and that the total space available for electrodes remains unchanged, doubling the operating frequency permits a doubling of the number of electrodes, and hence a halving of U and a doubling of the cooling rate. In practice, this gain is partially offset by the increases in the preamplifier noise temperature and external circuit attenuation which accompany an increase in operating frequency. Hence overall, the cooling rate increases proportional to something between the first and second power of f_B.

Let us now consider the power-limited system. From Eq. 3, we see that the quantity which best characterizes the performance of such a system is τ_p, which is defined by Eq. 4. For $g_{lim}/g_{opt} << 1$, the power-limited cooling rate τ_{lim}^{-1} is simply given by $2 \cdot \tau_p^{-1}$; as the beam cools, the gain ratio approaches unity, and the cooling rate falls by a factor of 2 to τ_p^{-1}, while at the same time τ_{opt} approaches τ_p. As the ratio exceeds unity, the system is of course no longer power limited, and the maximum cooling rate is determined by τ_{opt} from Eq. 2. The situation is illustrated in Fig. 1, where we have replaced τ_{lim} by τ_{opt} in the region where the gain ratio would exceed unity.

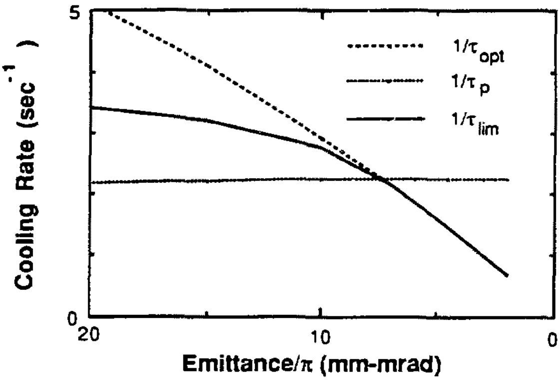

FIGURE 1 Transition from power-limited to non-power-limited operation as beam is cooled

Using τ_p^{-1} as our basic figure of merit, we see from Eq. 5 that most, if not all, the advantage in going to higher frequency is lost when the system is power-limited. The doubling of n_L made possible by the reduced electrode length is offset by the factor of f_B in the denominator, which arises from the 1/f dependence of the kicker constant (this is based on the reasonable assumption that it is the transfer impedance, rather than the kicker constant, which one can preserve when raising the frequency). Also, because g_{lim} decreases as $W^{-1/2}$ due to the increased noise bandwidth at higher frequency, the explicit W-dependence of τ_p^{-1} is as the one-half power, rather than the usual linear one. Moreover, this improvement is likely to be at least partly offset (possibly even *more than offset)* by increases in attenuation and amplifier noise which usually characterize a frequency increase.

As shown in Fig. 1, cooling of the beam may cause the system's operating range to span both the power-limited and noise-limited regimes. As might be surmised, (and as is shown explicitly in Ref. 3), such a system exhibits a greater than $\sqrt{2}$ improvement with a doubling of the operating frequency even prior to emerging from its power-limited condition.

An additional distinction between power-limited and non-power-limited systems concerns their scaling with beam aperture. Assuming that the pickup impedance Z_L' scales as the reciprocal of the gap width, it is straightforward to show that for the latter type of system, the time to cool to a given *fraction* of the initial emittance is independent of the initial gap. However, as shown in Ref. 3, for a power-limited system, that time *increases* as the gap increases.

TABLE Xa. 0-0 Transition Energies of MS-C

Matrix	T_1		S_1		S_2		S_3	
	nm	cm^{-1}	nm	cm^{-1}	nm	cm^{-1}	nm	cm^{-1}
Argon	n.a.		335*	29850*	n.a.		n.d.	
Xenon	n.a.		337*	29670*	n.a.		252	39680
SF_6	n.a.		332*	30120*	n.a.		n.d.	

TABLE Xb. 0-0 Transition Energies of MS-E

Matrix	T_1		S_1		S_2		S_3	
	nm	cm^{-1}	nm	cm^{-1}	nm	cm^{-1}	nm	cm^{-1}
Xenon	370.0	27030	317.3	31510	n.a.		246	40650
SF_6	367.6	27200	311.7*	32080	n.a.		n.d.	

* Values marked with asterisks represent crossing point between excitation and emission spectra, all others are the wavelength or frequency at half height of the leading peak in structured excitation, emission, or absorption spectra.

and 5 kW; the effects of the notch filter are included for all systems. We consider each system at intensities of N = 4, 8, and 16 x10^7. The remaining system parameters used in our calculations are listed in Table I.

We made two sets of calculations, one for fixed electrodes and one for so-called plunged electrodes, where the electrodes are moved inward to follow the envelope of the beam as the beam cools. *For these calculations, we made the conservative assumption* that the pickup impedance increased as the reciprocal of the electrode gap.

TABLE I Assumed Parameters for Various Choices of Electrodes

System Parameter	Upgraded 2-4 GHz	Bi-Planar 2-4 GHz	Uni-Planar 4-8 GHz	Bi-Planar 4-8 GHz
f_B (GHz)	3	3	6	6
W (GHz)	2	2	4	4
$T_R + T_A$ (°K)	140	140	180	180
α, α_p	0.64	0.64	0.5	0.5
M	10	10	5	5
Electronic Gain* (dB)	151	151	147	147
n_L	128	256	256	512

*Gain figures for 4-8 GHz are for P_{out} = 2.5 kW (per plane); for 5 kW system, values are 3 dB greater

Parameters in common :

$Z_L' = 16.3\ \Omega/\text{cm}$
$Z_C = 50\ \Omega$
$\beta = 10$ m
E/e = 8.938 GV (K.E. = 8 GeV)
f_0 = .590 MHz

A summary of the results of the calculations is presented in Table II; for all of the cooling scenarios we list the cooling times from an initial (full) emittance of 30π to several final emittances, including the present goal of 7π (underscored for ease of identification), and a value as low as 3π (to illustrate the effects of such small emittances). Bold-face entries are used to show the points at which the cooling system is no longer power-limited. More detailed results, showing all of the calculated quantities at a number of intermediate emittances, are presented in Ref. 3. Because the 2.5 kW 4-8 GHz systems remain power-limited down to nearly the smallest emittance, we felt it reasonable to calculate the effect on their performance of an additional doubling of the output power to 5 kW.

As anticipated, the bi-planar 4-8 GHz system outperforms the uni-planar system by roughly a factor of two throughout, by virtue of having twice as many electrodes (which, as noted above, doubles its performance in both the power-limited and non-power limited regimes). What is perhaps more surprising, is that for all but the highest intensity and lowest emittances (i.e.those smaller than presently required), not only does this advantage enable the *2-4 GHz* bi-planar system to perform adequately to meet system requirements, but actually to yield cooling times comparable to those obtained with a 4-8 GHz uni-planar system having the same total output power!

We also note that, as expected, for the 4-8 GHz system doubling the available output power is less efficacious than either bi-planarity or plunging, and its efficacy decreases in precisely those regimes, i.e. high intensity and low emittance, where the demands on the cooling system are greatest. Furthermore, it would be of even less benefit at 2-4 GHz, where (as one can see from the table) one is less severely power-limited.

In conclusion, bi-planarity and plunging offer comparable improvements in perform-

TABLE II Cooling Times for Uni- and Bi-Planar Arrays

		2 to 4 GHz				4 to 8 GHz							
		2.5 kW				2.5 kW				5 kW			
No. of Part.	Envelope Emittance	Fixed		Plunged		Fixed		Plunged		Fixed		Plunged	
		Uni	Bi	Uni	Bi	Uni	Bi	Uni	Bi	Uni	Bi	Uni	Bi
4×10^7	30π	----	----	----	----	----	----	----	----	----	----	----	----
	15π	*0.83*	*0.47*	*0.62*	*0.37*	*0.72*	*0.39*	*0.53*	*0.29*	*0.53*	*0.28*	*0.39*	*0.21*
	7π	1.75	0.99	1.01	0.64	*1.37*	*0.72*	*0.77*	*0.42*	*1.03*	*0.55*	*0.57*	*0.32*
	4π	2.86	1.61	1.29	0.84	*1.93*	*1.01*	*0.88*	0.49	*1.50*	0.79	0.67	0.38
	3π	3.68	2.05	1.43	0.94	*2.27*	*1.19*	0.93	0.52	1.82	0.96	0.72	0.42
8×10^7	30π	----	----	----	----	----	----	----	----	----	----	----	----
	15π	*0.93*	*0.59*	*0.74*	0.51	*0.77*	*0.43*	*0.58*	*0.34*	*0.57*	*0.32*	*0.43*	*0.25*
	7π	1.98	1.26	1.28	0.94	*1.45*	*0.80*	*0.85*	*0.50*	*1.09*	*0.61*	*0.64*	0.39
	4π	3.21	1.99	1.67	1.25	*2.02*	*1.11*	0.98	0.59	1.59	0.88	0.77	0.48
	3π	4.09	2.49	1.87	1.41	*2.38*	*1.30*	1.04	0.64	1.93	1.07	0.83	0.53
1.6×10^8	30π	----	----	----	----	----	----	----	----	----	----	----	----
	15π	*1.19*	0.92	1.03	0.86	*0.87*	*0.52*	*0.67*	*0.42*	*0.65*	*0.40*	*0.50*	*0.33*
	7π	2.53	1.91	1.88	1.60	*1.60*	*0.94*	*1.00*	0.65	*1.22*	*0.74*	0.78	0.53
	4π	3.99	2.86	2.51	2.14	*2.21*	*1.29*	1.18	0.80	1.77	1.06	0.96	0.68
	3π	4.98	3.48	2.83	2.42	*2.59*	1.51	1.27	0.87	2.14	1.28	1.06	0.76

ance, and for power-limited situations, such as that which exists at the Fermilab debuncher ring, offer performance improvements greater than those resulting from an increase in operating frequency. Moreover, the first two approaches permit one to utilize the electronics associated with the existing cooling system, thereby giving them a decided advantage in both time and cost. (As of this writing, based on the above analysis, the Fermilab beam-cooling group has decided to abandon any 4-8 GHz scenario for upgrading the debuncher.) If one can arrive at a bi-planar electrode design which admits of plunging, it would seem to be worthwhile to implement it. Otherwise, it is not clear that the mechanical complexity and expense involved in a plunged system is warranted.

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