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e^+-e^- HADRONIC MULTIPLICITY DISTRIBUTIONS

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ABSTRACT

We have have analyzed the 29 GeV multiplicity data for $e^+ - e^- \to$ hadrons using the partially coherent laser distribution (PCLD). The latter interpolates between the negative binomial and Poisson distributions as the ratio S/N of coherent/incoherent multiplicity varies from zero to infinity. The negative binomial gives an excellent fit for rather large values of the cell parameter k. Equally good fits (for full and partial rapidity range, and for the forward/backward 2 jet correlation) are obtained for the mostly coherent (almost Poissonian) PCLD with small values of k (equal to the number of jets). The reasons for the existence of this tradeoff are explained in detail. The existence of the resulting ambiguity is traced to the insensitivity of the probability distribution to phase information in the hadronic density matrix. We recommend the study of higher order correlations (intensity interferometry) among like sign-particles to resolve this question.

I. INTRODUCTION

Recently new data on multiparticle production in 29 GeV e^+-e^- collisions at the SLAC HRS have been presented[1] by Derrick et al. In addition to providing information on the two-jet and single-jet events, these authors present data for restricted rapidity windows and the forward-backward (F/B) correlation with respect to the two-jet axis. In regions of overlap this experiment basically confirms earlier results from the JADE[2], PLUTO[3] and TASSO[4] experiments: the multiplicity distributions are narrow (compared with those in hadron-hadron collisions) and obey approximate Koba-Nielson-Olesen (KNO) scaling (i.e. $\bar{n}P_n$ is energy independent when plotted as a function of the scaled variable $n/\bar{n}$.) In addition the F/B correlation is essentially zero, in contrast to the observed strong correlation observed in ISR (pp) and CERN collider ($p\bar{p}$) experiments.

In ref. 1 Derrick et al have shown that the charged multiplicity distribution P_n can be well described by the negative binomial distribution (N is the average multiplicity)

$$P_n^{NB} = \frac{(n+k-1)!}{n!(k-1)!} \frac{(N/k)^n}{(1+N/k)^{n+k}} \tag{1.1}$$

where k is typically rather large, ranging from 57.2 ($|y|<2.5$) down to 4.85 for $|y|<0.1$. Since (1.1) approaches a Poisson distribution as $k \to \infty$, one can wonder (as many have done) whether a simple Poisson is appropriate for e^+-e^- annihilations[6,7]. In fact some time ago we argued[7] that the almost-Poissonian shapes observed should be described by a k-cell partially-coherent laser distribution

$$P_n^{pc} = \frac{(N/k)^n}{(1+N/k)^{n+k}} \exp\left(\frac{-S/N}{1+N/k}\right) L_n^{k-1}\left(\frac{-kS/N}{1+N/k}\right) \quad . \tag{1.2}$$

This formula gives the photocount distribution[8] for a radiation field ensemble of k equal strength emitters with a signal intensity S and (Gaussian) noise intensity N: the average multiplicity is $\langle n \rangle = N+S$. (For early applications to particle physics see ref. 9.) As $S \to 0$ (1.2) goes over to the negative binomial (1.1) and for $N \to 0$ it gives the Poisson

$$P_n = \frac{S^n e^{-S}}{n!} \quad . \tag{1.3}$$

In our earlier paper[7] we argued that k should be literally identified with the number of jets (e.g. about two) and that the narrow distribution therefore implied a small N/S. (Indeed a few percent noise makes a large visual impact on the shape of the wings of the distribution).

As for hadronic data, it has been emphasized[10-12] that if the more general distribution (1.2) is allowed, it is very difficult to uniquely determine k and N/S from multiplicity fits. In fact one can trade off a large k in favor of a large coherence parameter S. A substantial region of parameter space gives equally good χ^2 fits. As an example, one can dispense with the large values of k found by the UA5 group in their negative binomial fits non-single diffractive data at lower energies[10,12].

The purpose of the present paper is to analyze the new high precision e^+-e^- HRS data[1] in the context of (1.2) to determine whether one can in fact distinguish it from the negative binomial (1.1). For this purpose we have paid special attention to the F/B correlation which can be much more sensitive to the S/N ratio than the KNO plot.[14] A condensed version of this work has been presented elsewhere[15].

II. ANALYSIS OF THE MULTIPLICITY DISTRIBUTIONS

In this section we consider the nature of inclusion (whole event, averaged), two-jet and single-jet events. We shall emphasize the normalized cumulant moments, defined by[16]

$$\gamma_2 = \frac{\langle (n-\langle n\rangle)^2\rangle}{\langle n\rangle^2} \equiv D^2/\langle n\rangle^2$$

$$\gamma_3 = \frac{\langle (n-\langle n\rangle)^3\rangle}{\langle n\rangle^3} \tag{2.1}$$

$$\gamma_4 = \frac{\langle (n-\langle n\rangle)^4\rangle}{\langle n\rangle^4} - \frac{3\langle (n-\langle n\rangle)^2\rangle^2}{\langle n\rangle^4} \quad .$$

As is well known, in the case of the negative binomial distribution the expressions (2.1) have the explicit form:

$$\gamma_2^{NB} = \frac{1}{\langle n\rangle} + \frac{1}{k} \tag{2.2}$$

$$\gamma_3^{NB} = \frac{1}{\langle n\rangle^2} + \frac{3}{\langle n\rangle k} + \frac{2}{k^2}$$

$$\gamma_4^{NB} = \frac{1}{\langle n\rangle^3} + \frac{7}{\langle n\rangle^2 k} + \frac{12}{\langle n\rangle k^2} + \frac{6}{k^3} \quad ,$$

etc. These results are a special case of the corresponding moments of the partially coherent distribution[7].

$$\gamma_2^{pc} = \frac{1}{\langle n\rangle} + \frac{a(1+b)}{k}$$

$$\gamma_3^{pc} = \frac{1}{\langle n\rangle^2} + \frac{3a(1+b)}{k\langle n\rangle} + \frac{2a^2(1+2b)}{k^2} \quad , \tag{2.3}$$

$$\gamma_4^{pc} = \frac{1}{\langle n\rangle^3} + \frac{7a(1+b)}{\langle n\rangle^2 k} + \frac{12a^2(1+2b)}{\langle n\rangle k^2} + \frac{6a^3(1+3b)}{k^3}$$

$$a \equiv \frac{N}{S+N} \quad , \; b \equiv \frac{S}{S+N}$$

where S is the coherent component of $\langle n\rangle$. The fraction of Gaussian noise strength is a, and b is the fraction of coherent signal strength. In the scaling limit, the negative binomial (1.1) leads to a $\langle n\rangle P_n$ of the form

$$\psi^{NB}(z) = \frac{k^{k-1}}{(k-1)!} z^{k-1} e^{-kz} \quad , \tag{2.4}$$

while the partially coherent distribution leads to the scaling form

$$\psi^{pc}(z) = k\frac{M+1}{M} \left[(M+1)z\right]^{\frac{k-1}{2}} \exp\left(-zk \frac{M+1}{M} + \frac{k}{M}\right) I_{k-1}\left(\frac{2k}{M} \sqrt{(M+1)z}\right) \tag{2.5}$$

where the parameter $M \equiv N/S = a/b$. We shall also use the variable $m = (N/S)^{\frac{1}{2}}$ to measure the strength of the noise/signal amplitude.

Notice that in the limit of large k, Eq. (1) is almost Poisson, and Eq. (2.5) is almost a singular delta function $\delta(z)$. The moment formulas (2.3) are better behaved. The distribution can be almost Poisson even for a small k, as long as N/S is small.

Using the moment formulas it is to understand our claim that it is difficult to distinguish between distributions (1.1) and (1.2). Suppose we choose a value of k (called k_{eff} for effective) in the negative binomial distribution which gives exactly the same second moment γ_2^{NB} as for the partially coherent distribution (2.2):

$$\gamma_2^{NB}(k_{eff}) = \gamma_2^{pc}(k,a,b) \quad , \tag{2.6}$$

which from (2.2)-(2.3) leads to

$$k_{eff} \equiv \frac{k}{a(1+b)} \quad . \tag{2.7}$$

It now occurs automatically, from the structure of (2.2)-(2.3) that γ_3^{pc} is very close in value to $\gamma_3^{NB}(k_{eff})$ in the domain of small noise (b<<1)

$$\gamma_3^{pc}(k) = \gamma_3^{NB}(k_{eff}) - \frac{2b^2}{k_{eff}^{\ 2}} \cong \gamma_3^{NB}(k_{eff}) \quad . \tag{2.8}$$

All the higher moments are well approximated to leading order in $1/k_{eff}$ between the two distributions (except that the higher order moments become progressively more important: This reflects the fact that the difference is more sensitive to the behavior of $\langle n\rangle P_n$ in the wings of the distribution). A very detailed examination of the fits to high precision data is needed to distinguish between the two representations.

In references 10, 11, 15, and 16, we have used the exact formulas 1-2 and standard chi-squared criteria to assess the quality of data fits.

Figs. 1-4 illustrate the application of formulas (1.1)-(1.2) to the inclusive two-jet and single jet data. In these figures we use the notations $\psi_n = \langle n\rangle P_n$, and $m = M^{1/2} = (N/S)^{1/2}$ for the noise/signal amplitude ratio.

III. MULTIPLICITY DISTRIBUTIONS FOR LIMITED RAPIDITY WINDOWS

Derrick et al have shown[1] that the multiplicity distributions for limited rapidity windows are well fit by a negative binomial whose k value decreases with decreasing acceptance in Δy. Although the qualitative trend of this effect is easy to anticipate[17], there exists no quantitative or fundamental understanding of this result. Using the same phenomenological spirit as in ref. 1 we have instead varied the noise/signal amplitude $m = (N/S)^{\frac{1}{2}}$ with k fixed at 2 for the two-jet data, as a function of $|y|_{max} \equiv y_c$. Fig. 5 shows that the two-jet data can be equally well accounted for by the negative binomial with a certain $k = k(y_c)$, or by a variable noise parameter $m(y_c)$, (k fixed at 2). The χ^2 per degree of freedom in the two cases is scarcely distinguishable (290/114 vs. 313/114).

In our alternative interpretation the fact that the entropy (noise) increases for small y_c is naturally associated with the decrease in information about the measurement which occurs as y_c decreases. In Fig. 6 we have made another comparison of the best negative binomial and partially coherent fits to the limited rapidity interval data. The solid curves of Fig. 6 giving the function $\Delta_n \equiv (\psi(pc) - \psi(NB))/\psi(NB)$ are compared with the data of ref. 1. Although $\Delta_n(z)$ is a very small number, it appears that with the stated errors the partially coherent distributi[illegible] to be preferred.

IV. THE FORWARD-BACKWARD CORRELATION

Consider the subset of all two-jet events (presumably quark jets). It is known that there is essentially no correlation between the number of hadrons in the "forward" jet and that in the "backward" jet.

Writing the joint probability $P(n_F, n_B)$ in the form

$$P(n_F,n_B) = P_{n_s} P(n_F|n_s) \tag{4.1}$$

where $n_s = n_F + n_B$ is the total population and $P(n_F|n_s)$ the conditional probability, to be measured by examining the subset with fixed n_s. The simplest assumption for the conditional probability is binomial[7,14,18]:

$$P(n_F|n_s) = \frac{n_s!}{n_F!n_B!} p^{n_F} q^{n_B} \tag{4.2}$$

with $p+q = 1$. In the present reaction symmetry further requires $\_ = q = \frac{1}{2}$:

$$P(n_F|n_s) = \frac{n_s!}{n_F!(n_s-n_F)!} (\tfrac{1}{2})^{n_s} \quad .$$

The analogous analysis of the charged neutral correlation requires the more general Eq. (4.2). This result is fairly well confirmed in ref. 1 (In contrast to the UA5 result[13] for $p\bar{p}$, that $n_s/2$ is the total number parameter in a binomial distribution). Note that if the decomposition (4.1) is to be applied to a component jet (in general, a moving source) then we can have $p \neq q$. The distribution for the total event is then composite (see Eq. 4.10 below).

In ref. 14 we developed techniques for deriving joint distributions from assumptions (4.1)-(4.2) for all P_{n_s} belonging to the class of Poisson transforms. As a particular case, the negative binomial gives

$$\langle n_B(n_F)\rangle = \langle n\rangle \frac{(n_F + k)}{\langle n\rangle + 2k} \tag{4.4}$$

for $p = q = \frac{1}{2}$. The slope parameter b usually presented by experimentalists is the best linear fit parametrized as

$$\langle n_B(n_F)\rangle = a + bn_F \quad . \tag{4.5}$$

In the case of the negative binomial distribution linearity is exact and

$$b^{NB} = \frac{\langle n\rangle}{\langle n\rangle + 2k} \quad . \tag{4.6}$$

For the partially coherent distribution (1.2) one gets instead

$$\langle n_B(n_F)\rangle = \frac{N}{2(1+N/2k)}[L^k_{n_F}(-x) + \frac{M}{1+N/2k}L^{k+1}_{n_F}(-x)]/L^{k-1}_{n_F}(-x)$$

$$x = \frac{k}{M}\frac{1}{1+M/k} \quad . \tag{4.7}$$

Here L^a_b is the associated Laguerre polynomial (positive definite for negative argument.) In general the function (4.7) is not a straight line. However, for large k <u>or</u> small noise N (or M), we find the approximate form

$$\langle n_B(n_F)\rangle \cong \langle n\rangle\ (1 - \frac{M\langle n\rangle}{2k} + \frac{M}{k}\ n_F) \quad . \tag{4.8}$$

In these limits the slope parameter is

$$b^{pc} = \tfrac{1}{2}\langle n\rangle/k_{eff} \tag{4.9}$$

where k_{eff} is given in (2.7). For small noise, $k_{eff} \cong kS/N >> 1$ and Eq. 2 is nearly Poisson. As $N \to 0$ the slope goes to zero, as it must from the exact calculation[14].

Although (4.3) is reasonably well confirmed experimentally, consideration of the underlying dynamics leads to the realization that

$P(n_F, n_B)$ should be considered as the result of a summation over the contributions of individual sources[14,19-21]. The sources can be discrete or represented by a continuum, with suitable weights in either case. Here we shall restrict our attention to the simple case of two (equal weight) jets. Each source is represented as (4.1) but with $p \neq q$ in (4.2) to represent the effect of source motion. The quantities p, q are clearly theoretical constructs since the experimentalist cannot tell from which source, a backwards particle came, for example. (We also note that this classical composition of probabilities could be suspect, a point worthy of investigation.)

In our previous work on the F/B correlation[14] we studied the sensitivity to the emission probabilities p_j, q_j of the individual clusters j with respect to the chosen axis. (A condensed discussion was given in Eqs. (29) ff of ref. 14. Thus instead of (4.1) we should write

$$P(n_F,n_B) = \prod_{j=1}^{k} P_j(n_{Fj},n_{Bj})\delta(\Sigma n_{Fj}-n_F)\delta(\Sigma n_{Bj}-n_B) \quad . \tag{4.10}$$

The individual P_j of (4.10) are now assumed to obey (4.1) with p_j, q_j chosen by models, theories or instinct. $p_j - q_j > 0$ clearly means that source j is emitting predominantly to the right (hence presumably moving to the right). For $p_j = q_j = \frac{1}{2}$, all j, Eq. (4.10) reduces again to (4.1).

Figs. 7-12 present 2 cell (k = 2) results calculated using the methods of ref. 14. The top half of each curve shows $\langle n_B \rangle_F$ as a function of n_F for p_R = 0.5, 0.80 and 0.95 (implying the symmetrical choices p_L = 0.5, 0.20 and 0.05). The lower curves give the ratio of $P_s(n_F)$

(i.e. n_s fixed) compared with the simplest case of (4.3). This ratio is unity for $p = \frac{1}{2}$ but can and does deviate for the other choices.

The above deviation can be analytically calculated for the important case of the Poisson distribution. Consider the individual P_j's of (4.10) with binomial distribution in (n_F, n_B), Poisson distribution in $N_s = n_F + n_B$, and with arbitrary p_j, i.e.,

$$P_j(n_{F_j}, n_{B_j}) = e^{-\bar{n}_{sj}} \bar{n}_{sj}^{\;n_{sj}} \frac{(p_j)^{n_{F_j}} (q_j)^{n_{B_j}}}{n_{F_j}! \; n_{B_j}!} \quad . \tag{4.11}$$

Equation (4.10) can be evaluated explicitly by using the relationship

$$\prod_j (\bar{n}_j)^{n_j} p_j^{\;n_j}/n_j! = \Sigma(\bar{n}_j \; p_j)^n/n! \quad , \tag{4.12}$$

where

$$n = \sum_j n_j \quad .$$

Defining

$$\langle p \rangle \equiv (\Sigma \; \bar{n}_{s_j} p_j)/\Sigma \bar{n}_{s_j} \quad , \tag{4.13}$$

$$\langle q \rangle = (\Sigma \; \bar{n}_{s_j} q_j)/\Sigma \bar{n}_{s_j} \quad , \tag{4.13}$$

we get

$$P(n_F, n_B) = e^{-\langle n_s \rangle} \langle n_s \rangle^{n_s} \frac{\langle p \rangle^{n_F} \langle q \rangle^{n_B}}{n_F! \; n_B!} \quad . \tag{4.14}$$

Notice that (4.14) is of exactly the same form as the individual P_j of (4.11); it is therefore form invariant under the convolutional procedure of (4.10). This is a reflection of the underlying assumption of independence in both the Poisson and the binomial distribution. Applying Eq. (4.14) to the extreme case of symmetrical jets, we get $\langle p\rangle=\langle q\rangle$, and the forward/backward correlation b = 0. When n_{sj} distributions are highly coherent, we also expect the compound distribution of (4.10) to be nearly binomial in (n_F, n_B) with b nearly zero.

Fig. 7 gives numerical results for the partially coherent distribution with k = 2, $\langle n\rangle = 10.86$ and $m = (N/S)^{\frac{1}{2}} = 0.13$. The resulting slope for $p = \frac{1}{2}$ (b = 0.041) is small, but differs from the observed value (0.001) by an amount greater than the quoted error. Biasing the jet decay by choosing p = 0.80 or 0.95 produces an acceptable slope value. At first sight the lower curve might seem incompatible with our statement that (4.3) is approximately verified. Visual inspection of the data shows that the modification is entirely acceptable.

In Figs. 11-12 we compare 2 jet, full rapidity range data with F/B correlation predictions for the negative binomial and for the partially coherent distribution. The parameters are determined by fits to the KNO data plot. For a given parameter p the negative binomial slope is about 50% greater than that for the partially coherent distribution. However, in each case a plausible value of p (p>.8) can be found for which the experimental slope of $b \cong 0.006$ can be matched.

V. CONCLUSIONS AND DISCUSSIONS

We have shown the impossibility of distinguishing between the negative binomial and the partially coherent distributions by means of a phenomenological fit to multiplicity distributions for varying rapidity intervals as well as for the forward-backward correlation. Indeed the deviation of the HRS data from Poisson is negligibly small, in fact a smaller deviation than for JADE and TASSO data in the same energy range[4]. It seems likely that systematic errors are responsible for the (small) differences among these experiments.

What is clearly missing is a persuasive dynamical underpinning to the mysteriously successful statistical formulas Eq. 1 and Eq. 2. In Ref. 7 we suggested on the basis of an analogy to QED (i.e., that the radiation from a classical current source produces a coherent state) that the hadronization might be highly coherent too. Indeed, QCD calculations show the importance of coherence at the quark-gluon level. In particular Catani, Ciafaloni, and Marchesini[21] have exhibited[22] a very suggestive form for the S-matrix in which the typical exponentiation characteristic of coherent states is seen. Unfortunately, no one presently knows how to bridge the gap to real hadrons, or can connect coherence in the quark-gluon sector to that in the hadronic density matrix.

For clarity we review briefly some well-known results relating coherent states to the (classically) forced harmonic oscillator[23]. These states arise naturally[24] in the QED infrared problem in which the emitting charge current suffers negligible recoil fluctuations in the emission of long wavelength photons. In this case each momentum state of the radiation couples linearly to the p^{th} Fourier component of the current. The p^{th} oscillator has the forced oscillator Hamiltonian $\omega(a^+a+\frac{1}{2}) - x_o(a+a^+)F(t)$, when x_o measures the zero point fluctuation. To solve the scattering problem posed by this Hamiltonian we introduce[23,25] the coherent state $|\alpha\rangle$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad . \tag{5.1}$$

These states are eigenstates of the destruction operator: $a|\alpha\rangle = \alpha|\alpha\rangle$ for any complex α (hence not orthogonal though (over) complete). Hence,

$$\bar{n} = \langle\alpha|a^{+}a|\alpha\rangle = |\alpha|^2$$

$$\langle\alpha|x(t)|\alpha\rangle = x_o(\alpha\, e^{-i\omega t} + \alpha^{*}\, e^{i\omega t}) \tag{5.2}$$

$$= \alpha x_o \langle n\rangle^{\frac{1}{2}} \cos(\phi - \omega t)$$

where $x(t)$ is the (Heisenberg) displacement operator and ϕ is the phase of $\alpha = \langle n\rangle^{\frac{1}{2}} \exp(i\phi)$. The counting distribution of 5.1 is Poisson:

$$P_n = |\langle n|\alpha\rangle|^2 = \langle n\rangle^n e^{\langle n\rangle}/n! \quad .$$

The connection of the states $|\alpha\rangle$ with classical-like motion has been much studied[26]. Here we note that these are the states created by the action of a linearly coupled classical driving force. To express this most succinctly note that (5.1) can be written as

$$|\alpha\rangle = D(\alpha)|0\rangle \tag{5.4}$$

where $|0\rangle$ is the ground state and $D(\alpha)$ the unitary displacement operator

$$D(\alpha) = \exp(\alpha a^{+} - \alpha^{*} a) \tag{5.5}$$

$$[a, D(\alpha)] = \alpha\, D(\alpha) \quad .$$

As shown in Ref. 23, the in and out operators are connected by

$$a_{out} = a_{in} + if_o$$

$$= S^+ a_{in} S$$

$$S = D(if_o) = \exp i\left[\frac{a_{in}^+ F(\omega) + a_{in} F^*(\omega)}{(2\Pi \hbar \omega)^{1/2}}\right] \tag{5.6}$$

where $F(\omega) = \int dt\, e^{i\omega t} F(t)$. Hence the S matrix creates coherent states when a force F(t) acts on the ground state.

Note that the observation of a Poisson distribution gives no information on the phases in the above example. Although (5.1) leads to Poisson, the inverse deduction cannot be made. Put in another way the (pure) density matrix

$$\rho = |\alpha\rangle\langle\alpha| \tag{5.7}$$

corresponding to classical-like coherent motion has rich phase information in the off diagonal elements not visible in the diagonal element $P_n = \langle n|\rho|n\rangle$ (see Eq. (5.3). It is exactly this off-diagonal structure responsible for the occurrence of coherent motion, Eq. (5.2).

The density matrix corresponding to a Gaussian mixture,

$$\rho = \int d^2\alpha \frac{\exp(-|\alpha|^2/N)}{\pi N} |\alpha\rangle\langle\alpha| \quad , \tag{5.8}$$

leads to the Bose-Einstein distribution. (For k equal strength oscillator modes one gets the negative binomial.) This density matrix has no off diagonal elements. The displaced Gaussian weight

$$\rho = \int d^2\alpha \, \frac{\exp(-|\alpha-\beta|^2/N}{\pi N} \, |\alpha\rangle\langle\alpha| \tag{5.9}$$

leads to (5.2) for k = 1. (Again k equally weighted modes leads to the general formula (5.2).)

These techniques are explained in detail in Ref. 16. Here we simply stress that what is needed to discriminate between physical models having the same (to within experimental error) counting distribution $P_n = \langle n|\rho|n\rangle$ is information on the off-diagonal elements of the density matrix. As is well known in quantum optics[27], the requisite phase information is contained in intensity correlation functions; the first example being the Hanbury-Brown Twiss effect (corresponding to a Gaussian field ensemble for starlight). As has been noticed, the Bose-Einstein correlation is an analogue to this effect, except that mixed coherence is possible. Indeed Fowler and Weiner[28] have proposed transplanting formulas like (5.8)-(5.9) into rapidity space, in which case the ($Q^2 = 0$) intercept of the ratio R of like sign particles to all directly measures N/S. Pure Gaussian noise corresponds to R = 2 and pure coherence to R = 1. Experimental uncertainties abound but it appears that R is larger for hadron-hadron data than for $e^+ - e^-$, indicating greater coherence in the latter. We terminate the discussion of this point because further work is needed to clean up this idea.

Malaza and Webber[29] have computed quark and gluon jet multiplicity moments in order α_s. It is interesting to compare their results to those from the statistical formula (1.2), even though the latter was of necessity fit to hadronic data. A convenient measure of the moment prediction is given by the quantity

$$K_{L,i} \equiv \frac{1}{L} (\xi_{L+1}/\xi_{L-1}) \tag{5.10}$$

where i denotes quark or gluon, and the ξ_L^i are the normalized L^{th} order factorial moments

$$\xi_L^i \equiv \frac{\langle n(n-1)\ldots(n-L+1)\rangle_i}{\langle n\rangle_i^L} \quad . \tag{5.11}$$

In Ref. 7 the ξ_L were given for the partially coherent distribution (1.2):

$$\xi_L = \frac{L!}{k^L} L_L^{k-1} (-kS/N) a^L \quad . \tag{5.12}$$

In the limt $S \to 0$, $a \to 1$ one easily finds

$$K_L = 1/k \tag{5.13}$$

for the negative binomial, in agreement with Ref. 28.

Malaza and Webber[29] find on evaluating the left hand side of (5.10) from their QCD calculation effective values of k (i.e., in 5.13) between 6 and 12. As we have seen such values do not give sufficiently narrow negative binomial fits to the data.

If we use (1.2) and its consequence (5.12), the evaluation of (5.10) leads to a modification of (5.13). For simplicity consider the limit of dominant coherence, defined by

$$x \equiv kS/N >> 1 \quad . \tag{5.14}$$

Using the asymptotic expansion

$$L_L^k(-x) \simeq \frac{x^L}{L!} (1 + L(L + k-1)/x + \ldots) \tag{5.15}$$

we find for (5.10), independent of L (as indicated by the theoretical calculation):

$$K_L \sim 2N/kS = 2a/k = 2/k_{eff} \tag{5.16}$$

where k_{eff} is the "effective" k already defined in (Eq. (2.7). As before the actual k in (1.2) can be small while k_{eff} is large due to dominance of coherence.

A numerical illustration is provided by our m = 0.06, k = 2 fit to the total inclusive data. $a \cong m^2 = 3.6 \times 10^{-3}$. To compare with the experimental dispersion we take L = 1 and rewrite

$$\begin{aligned} K_1 &= \xi_2/\xi_1 - 1 = \langle n(n-1)\rangle/\langle n\rangle^2 - 1 \\ &= D^2/\langle n\rangle^2 - 1/\langle n\rangle \quad . \end{aligned} \tag{5.17}$$

Derrick <u>et</u> <u>al</u>. give[1] $R_1^{expt.} = 3.9 \times 10^{-3}$ (which corresponds to their large negative binomial $k_{NB}^{-1} = 3.9 \times 10^{-3}$. Using k = 2 and our fit $a = m^2 = 3.6 \times 10^{-3}$ we get 3.6×10^{-3} for the evaluation of K_1, almost too good to be true. Ignoring the lack of justification for the comparison of (1 2) with QCD predictions we see that the seeming discrepancy between (5.13) and the HRS data are completely removed if coherence dominates.

A few remarks on KNO scaling can be made in the context of the present work. Although currently available data are compatible with scaling, no theory we know of can make a decisive statement on this topic. For (1.1) KNO scaling obtains for fixed k, and observed deviations in hadron-hadron collision can be parameterized by a energy-dependent k. In Ref. 7 we noted that the existence of scaling at different energy requires a careful tuning of the noise parameter, not predicted by our theory. Chou and Yang remark that if the $e^+ - e^-$ multiplicity distributions are indeed Poisson then no KNO scaling is to be expected[6,30].

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Figure Captions

Fig. 1. The inclusive whole event KNO data $\psi_n = \bar{n}P_n$ (vertical lines) are compared with the partially coherent distribution Eq. (1.2) for the parameter choice $k = 2$, $m = (N/S)^{\frac{1}{2}} = 0.0245$ (upper curve). The χ^2/DF is 11.1/14 in this case. In the lower curve the deviation of the data from Poisson is measured by $\Delta_n = (\psi_n - \psi_n(P))/\psi_n(P)$ in the vertical lines. The deviation of the negative binomial (dashed line) is indicated for the case $k = 200$.

Fig. 2. The inclusive single-jet KNO data $\psi_n = \bar{n}P_n$ (vertical lines) are compared with the partially coherent distribution Eq. (1.2) for the parameter choice $k = 1$, $m = (N/S)^{\frac{1}{2}} = 0.014$ (upper curve). The χ^2/DF is 11.4/18 in this case. In the lower curve the deviation of the data from Poisson is measured by $\Delta_n = (\psi_n - \psi_n(P))/\psi_n(p)$ in the vertical lines. The deviation of the negative binomial (dashed line) is indicated for $k = 100$.

Fig. 3. The two-jet KNO data $\psi_n = \bar{n}P_n$ (vertical lines) are compared with the partially coherent distribution (1.2) for the parameter choices $k = 1$, $m = 0.0245$ (upper curve) the χ^2/DF is 7.6/14. In the lower curve the deviation of the data from Poisson is measured by $\Delta_n = (\psi_n - \psi_n(P))/\psi_n(P)$ in the vertical lines. The deviation of the negative binomial (dashed line) is indicated for the case $k = 200$.

Fig. 4. The single jet data (taken from the two jet sample) KNO function $\psi_n = \bar{n}P_n$ (vertical lines) are compared with the partially coherent distribution Eq. (1.2) for the parameter choices $k = 1$, $m = 0.014$ (upper curve). The χ^2/DF is 25.5/18. In the lower curve the deviation of the data from Poisson is measured by $\Delta_n = (\psi_n - \psi_n(P))/\psi_n(P)$ in the vertical lines. The deviation of the negative binomial (dashed line) is indicated for $k = 100$.

Fig. 5. The best-fit parameters to the limited-rapidity multiplicity distributions $-y_c<0<y_c$ are shown for the partially coherent formula (1.2) (i.e., the left hand ordinate gives $m = (N/S)^{\frac{1}{2}}$ as a function of y_c) and for the negative binomial formula (1.1) (i.e., the right hand ordinate gives k_{NB} (and $\langle n\rangle$) as a function of y_c).

Fig. 6. The (very small) discrepancy between the partially coherent and (1) the negative binomial fit, and (2) the data are shown here, by the solid curve. The data points show the typical fractional discrepancies of the data from the negative binomial, i.e., $\Delta_n^{exp} = \psi_n^{exp}/\psi_n^{NB} - 1$.

Fig. 7. The upper curve shows the theoretical prediction for $\langle n_B\rangle_F$ as a function of n_F for $\langle n\rangle = 10.86$, $k = 2$ and $m = (N/S)^{\frac{1}{2}} = 0.13$ in the partially coherent distribution. The three curves are for an intrinsic probability of $p = 0.5$, 0.8 and 0.95 per cell for emission along the direction of motion. The slope parameter b is indicated in each case. In the lower curve the ratio of $P_s(n_F)$ to $P_s(n_F$, Binomial) is shown for the same choices of p, with n_s fixed at 11.

Fig. 8. The upper curve shows the theoretical prediction for the negative binomial distribution for $\langle n\rangle = 10.86$ $k = 58$ and $m = (N/S)^{\frac{1}{2}} = \infty$. The three curves correspond to an intrinsic probability of $p = 0.5$, 0.8 and 0.95 per cell for emission along the direction of motion. The slope parameters b are roughly twice as big as for the pc distribution of Fig. 7 with parameters chosen to fit the total multiplicity distribution.

Fig. 9. These curves differ from those of Fig. 7 only in the choice of parameters $\langle n\rangle = 4.27$, $k = 2$, $m = 0.32$ (upper) and $n_s = 5$ (lower).

Fig. 10. These curves differ from those of Fig. 8 only in the choice of parameters $\langle n \rangle = 4.27$, $k = 9$ (upper) and $n_s = 5$ (lower).

Fig. 11. The full rapidity interval 2 jet data are compared with negative binomial assumptions in the appropriate (large) k values. For these parameters determined by the fit to the KNO plot, one needs a forward hadron emission probability $p > 0.8$ to approach the experimental value of $b \cong 0.006$.

Fig. 12. As in Fig. 11 except that k is fixed at two and the noise/signal amplitude is $m = 0.06$ as in Fig. 1.