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DENSITY FLUCTUATIONS FROM THE QUARK-HADRON EPOCH AND PRIMORDIAL NUCLEOSYNTHESIS

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ABSTRACT

We present a simple thermodynamic model of the quark-hadron transition in the early universe and use this model to estimate how the size of isothermal baryon number fluctuations which emerge from this epoch depend on the temperature of the transition and other uncertain quantities of the underlying QCD physics. We calculate primordial nucleosynthesis in the presence of these fluctuations and find that $\Omega = 1$ in baryons is possible only if the measured abundances of ^7Li and ^2H reflect substantial destruction during the evolution of the galaxy.

I. INTRODUCTION

A transition from quark-gluon plasma to confined hadronic matter must have occured at some point in the evolution of the early universe. There may be a first order phase transition at this epoch associated with either the color confinement transition[1] or the chiral symmetry breaking transition.[2] It is of interest to explore whether there might be a relic signature of the quark-hadron transition in primordial light element abundances or elsewhere.

There have been several recent papers[3,4,5] based on the possibility that the quark-hadron phase transition in the early universe may have led to the formation of isothermal baryon number fluctuations. The basis for the production of isothermal baryon number fluctuations from a phase transition in the early universe lies in the separation of cosmic phases scenario first discussed by Witten[6] and later by Applegate and Hogan.[7] This cosmic separation of phases scenario has been discussed in some detail by Kajantie and Kurki-Suonio.[9] In this scenario isothermal density fluctuations could arise because when macroscopic regions of different phase are in thermal, pressure and possibly, chemical equilibrium the quark-gluon plasma phase has a higher baryon concentration than does the hadron gas phase.

We will discuss the microphysics of the fluctuation generation mechanisms and we identify two scenarios. In the limit of chemical equilibrium between the two phases in coexistence there will be a different baryon concentration in each phase. In the limit where exchange of baryon number across the boundary between the two phases is not rapid enough to achieve chemical equilibrium, the fluctuations generated are always larger than in the equilibrium case, since nucleated confined vacuum contains no net initial baryon number. These scenarios for the generation of fluctuations depend on the phase transition being first-order so that latent heat is released.

The isothermal density fluctuations from the quark-hadron phase transition could be further modified at a later epoch by a diffusive separation of neutrons and protons, as pointed out by Applegate, Hogan, and Scherrer,[3] resulting in low density neutron rich regions and high density proton rich regions at the time of nucleosynthesis. The primordial nucleosynthesis resulting from a universe which is inhomogeneous in density and neutron-to-proton ratio is very different from that in the standard big bang and, in fact, may give acceptable light element abundances even with $\Omega = 1$ in baryons. This result is in contrast to previous studies of primordial nucleosynthesis with $\Omega = 1$ baryons in which not enough deuterium is produced.[10-14]

II. THERMODYNAMICS OF THE QUARK-HADRON PHASE TRANSITION

It will be most convenient for our purposes to compute the thermodynamic potential, Ω, for both the quark-gluon plasma phase and the hadron phase. We caution that we have used Ω for both the thermodynamic potential and the critical parameter for the universe. Our definition of Ω conforms with that in Landau and Lifshitz[15], so that

$$\Omega = - T \ln(Z), \tag{1}$$

where Z is the grand partition function.

It is straightforward to compute the grand partition function, and thus Ω, if we assume that the particles are non-interacting except for an overall QCD vacuum energy in the unconfined phase. The background relativistic particles which are not strongly interacting and are in thermal equilibrium with both the confined and unconfined phases contribute (for $\mu = 0$)

$$\Omega = -V \left(g_b + \frac{7}{8} g_f\right) \frac{\pi^2}{90} T^4, \tag{2a}$$

where g_b and g_f are the statistical weights of bosons and fermions, respectively. At the epoch of the quark-hadron phase transition photons ($g_b = 2$), electrons ($g_f = 4$), muons ($g_f = 4$), and neutrinos ($g_f = 6$), yield $g = g_b + \frac{7}{8}g_f = 14.25$ so that the pressure contribution from these particles common to both phases is

$$P \approx 1.56\ T^4. \tag{2b}$$

We treat the unconfined quark-gluon plasma as a gas of noninteracting relativistic particles plus an overall vacuum energy. In the limit of vanishing quark masses, there is a simple expression for Ω which is valid for any temperature and chemical potential:[16a-e]

$$\Omega_{qg} = \frac{-7\pi^2}{180} \cdot N_c N_f V\ T^4 \left[\ 1 + \frac{30}{7\pi^2} \left(\frac{\mu_q}{T}\right)^2 + \frac{15}{7\pi^4} \left(\frac{\mu_q}{T}\right)^4\ \right] - \frac{\pi^2}{45} N_g V T^4 + BV \tag{3a}$$

Here N_c is the number of colors (3), N_f is the number of relativistic quark flavors (2 at lower temperatures corresponding to the u and d quarks, and 3 at higher temperatures where the strange quark becomes relativistic), and B is the QCD vacuum energy, or bag constant. The number of gluons is $N_g = 8$. The quark chemical potential is $\mu_q = \frac{1}{3}\mu_b$, where μ_b is the baryon chemical potential.

For the early universe $(\mu_b/T) \sim 10^{-8}$, so that for 2 relativistic quark flavors the contribution to the pressure in the unconfined phase from quarks and gluons is

$$P = 4.06\ T^4 - B \tag{3b}$$

The total pressure in the early universe in the unconfined phase includes the contributions from equations (2b) and (3b).

Note that the QCD vacuum energy contributes negatively to the pressure. The value of this vacuum energy, or bag constant, is not known. It parameterizes the temperature at which the confined and unconfined phases coexist. We emphasize that in what follows it will be this coexistence temperature T_c which will be the fundamental quantity of interest. The vacuum-energy (bag-model) approach is simply an easy and concise way to calculate the essential thermodynamics of the unconfined phase in a consistent manner.

We caution that for two reasons it is difficult to connect the value of the bag constant used in various models of hadrons with the relevant value of B in equations (3ab). First, most current research on modeling hadronic properties with the bag must address the problem of treating the bag surface.[17-22] Surface effects can be appreciable for individual hadrons. However, we expect the regions of unconfined quark gluon plasma which are in coexistence with confined hadronic matter to be of macroscopic size, so that surface effects are minimal. Second, it is dangerous to use a value of B in equations (3a) and (3b) that is derived from a particular model of hadrons because we are interested in the <u>high temperature</u> QCD vacuum energy. This vacuum energy probably comes down with increasing temperature.[18,20,22]

For the confined, or hadronic, phase we will not use the bag model but rather the spectrum[23] of known masses for baryons and mesons. This is not inconsistent with the bag model treatment of the quark-gluon phase because the isothermal baryon number fluctuations we are interested in are largest at low coexistence temperature where the neutron, proton, pion, and other light hadron masses (which do not depend on B) dominate the hadronic level density. At higher temperatures there may be an inconsistency and we discuss this limit below.

Computing Ω for mesons first (note that $\mu = 0$ for the mesons), we find for each meson that

$$\Omega = \frac{-gV\,T^4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^4}\, \bar{K}_2\,(nm/T), \tag{4a}$$

where m is the mass of the meson, and g is the spin plus isospin degeneracy factor, $g = (2J + 1)\,(2I + 1)$. The $\bar{K}_2$ is related to the modified Bessel function of the second order (K_2), as in Fowler and Hoyle,[24]

$$\bar{K}_2(x) \equiv \frac{x^2}{2} K_2(x) \qquad (4b)$$

In the conditions we are interested in (T ~ 100-200 MeV) the pions will make the dominant contribution to the pressure in the hadronic phase; however, in the numerical calculations presented below we have summed the contributions to Ω in equation (4a) over all of the known mesons[23] and calculated all $\bar{K}_2$ numerically.

For baryons in the small chemical potential limit ($|\mu_b| < m$)

$$\Omega = \frac{-g T^4 V}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} \cosh\left(\frac{n\mu_b}{T}\right) \bar{K}_2(nm/T) \quad . \qquad (5a)$$

The net baryon number density corresponding to this Ω is

$$n_b = \frac{2g T^3}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \sinh\left(\frac{n\mu_b}{T}\right) \bar{K}_2(nm/T) \quad . \qquad (5b)$$

Note that n_b is the number density of baryons minus that of the antibaryons. As above, we obtain the total Ω by summing over the spectrum of known baryonic masses.[23]

It is possible to make a simple analytic model of the essential statistical mechanics of the quark-hadron system and compare this to our numerical results. We define $a \equiv \pi^2/30$ and the statistical weight for the photons, neutrinos, electrons, and muons common to both phases as $g_{\nu\gamma}$ = 14.25. We then define the intrinsic statistical weight for the quark-gluon plasma as g_Q and that for the hadron soup as g_H. The total statistical weight for each phase in coexistence is then defined as

$$g_q \equiv g_Q + g_{\nu\gamma} \qquad (6a)$$

$$g_h \equiv g_H + g_{\nu\gamma}. \qquad (6b)$$

In the model of the quark-gluon plasma discussed above g_Q = 37 and thus g_q = 51.25. At T_c ~ 100 the pions dominate the contribution to Ω in the sum of equations (4a) and (5a), so that very roughly g_H = 3 and g_h = 17.25. The essential thermodynamics is contained in this difference in statistical weight between the phases.

The pressure, P_q, energy density, E_q, and entropy density, s_q, in the quark-gluon phase are given as

$$P_q = \frac{1}{3} g_q aT^4 - B \quad (7a)$$

$$s_q = \frac{4}{3} g_q aT^3, \quad (7b)$$

while for the hadron phase the corresponding quantities are

$$P_h = \frac{1}{3} g_h aT^4 \quad (8a)$$

We define the ratio of statistical weights in the two phases to be

$$x = \frac{g_q}{g_h} = \frac{s_q}{s_h} \quad (9)$$

Note that x in this paper is the same as r in ref. 9. Inclusion of the hadronic resonances makes g_h, and hence x, a function of temperature. Also, at high temperature, interaction corrections to g_h and x may be appreciable. In Figure 1 we present x as a function of temperature for the known mass spectrum of hadrons[23] and different numbers of relativistic quark flavors. At the lower temperature when baryon number fluctuations are expected to be largest, temperature and interaction corrections to x will be small.

The latent heat can be expressed as

$$L = T_c\, s_q \left(1 - \frac{1}{x}\right) \simeq 4B, \quad (10)$$

where the latter approximation follows for constant $x = 2.971$.

Pressure equilibrium, or co-existence, between the phases, $P_h = P_q$, occurs for a temperature,

$$T_c = (g_q - g_h)^{-\frac{1}{4}} \left(\frac{3}{a}\right)^{\frac{1}{4}} B^{\frac{1}{4}} \simeq 0.72\, B^{\frac{1}{4}}, \quad (11)$$

where the latter approximations assumes $x = 2.971$. Values of $B < (300\text{ MeV})^4$ lead to a coexistence temperature, $T_c < 250$ MeV, in which the separation of

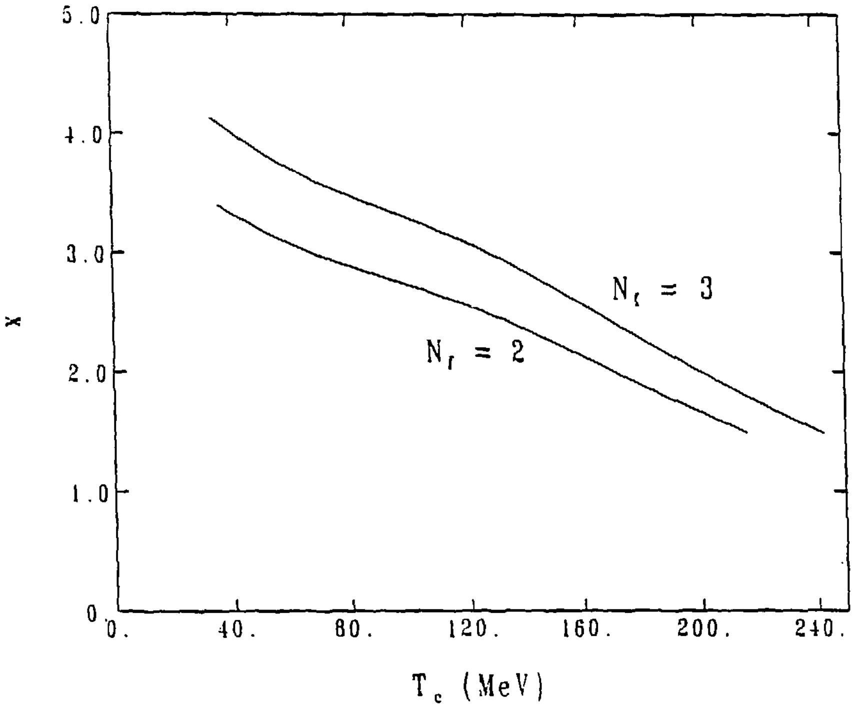

Figure 1) The ratio of the effective statistical weights of the unconfined phase to the confined phase, x, as a function of coexistence temperature, T_c. The contribution of all known hadronic resonances is included in the computation of the pressure equilibrium.

phases scenario outlined above could produce isothermal baryon number fluctuations of appreciable amplitude. If, on the other hand, the vacuum QCD energy is $B > (300\ \text{MeV})^4$ then no coexistence and separation of phases is possible because the hadronic pressure always exceeds the quark-gluon plasma pressure.

Finally, even if the QCD vacuum energy is high, $B \sim 300$ MeV, it might be possible that the quark-gluon plasma phase is able to supercool well below T_c before nucleation of the hadronic phase begins. In this extreme case the QCD vacuum energy will come to dominate the pressure of the universe and a short de-Sitter exponential expansion of the universe can ensue. This miniinflation will, however, result in only a 15% increase in the scale factor.[5]

III. NUCLEATION

When a thermodynamic system is cooled (or heated) through the coexistence temperature at which a first order phase transition occurs, the new phase does not appear immediately. Significant reorganization of fundamental degrees of freedom is needed for the creation of new phase, and the thermodynamic fluctuations which can produce such reorganization have low probability. Furthermore, it is energetically unfavorable to create small volumes of new phase because of the free energy associated with the surface separating the two phases. These effects ensure that some supercooling below the coexistence temperature occurs before stable nuclei of the new phase are produced. These nuclei grow rapidly, since the old phase is metastable, releasing latent heat and thus reheating the system back to the coexistence temperature.

The physics of nucleation in the quark-hadron phase transition is less well understood than the bulk thermodynamics. However, there are three plausible mechanisms for this process. We call these mechanisms homogeneous nucleation, heterogeneous isothermal nucleation and heterogeneous non-isothermal nucleation. In what follows we will discuss only the most likely mechanism, homogeneous nucleation.[15]

If the universe is pure and isothermal, then the new phase originates through spontaneous fluctuations in the metastable phase: The nucleation rate is determined by the probability that a spontaneous fluctuation in the metastable (quark) phase will produce a "critical nucleus" of the stable (hadron) phase. This critical nucleus has a radius, r_c, determined by

$$P_h - P_q = \frac{2\sigma}{r_c} \quad , \tag{12}$$

where σ is the free energy per unit surface area associated with the boundary of the nucleus, which is assumed to be spherical.

New nuclei with radii less than r_c will collapse and disappear, while nuclei of radii larger than r_c will expand until a macroscopic amount of new phase is produced. The probability of a fluctuation of radius r_c is $\exp(-W/T)$ where

$$W = \frac{4\pi}{3} r_c{}^3 \; (P_q - P_h) + 4\pi\sigma \; r_c{}^2. \tag{13}$$

The first term in (14) is the difference between the thermodynamic potentials of the two phases and is negative. The second term is the surface free energy of the boundary between the phases.

The rate, p(T), at which nuclei form per unit volume per unit time is given by[15]

$$p(T) = CT_c{}^4 \exp\,(-W/T), \tag{14}$$

where C is a coefficient of order unity, which turns out (see below) to have no quantitative significance when we calculate the number density of nucleation sites.

The amount of supercooling is very small. Accordingly, it is convenient to expand about the coexistence temperature, T_c. We define

$$P_h - P_q = L\eta, \tag{15a}$$

$$\eta \equiv \frac{T_c - T}{T_c}, \tag{15b}$$

where L is the latent heat per unit volume of the phase transition and η is the "supercooling parameter." It follows that:

$$p(\eta) = CT_c{}^4 \exp\,(-\frac{16\pi}{3} \frac{\sigma^3}{T_c L^2 \eta^2}). \tag{16}$$

The important characteristics of Eq. (16) are that $p(T) \to 0$ as $T \to T_c$, and that for small supercooling ($\eta \ll 1$) the argument of the expontential is large and negative, yielding a very strong increase of p with η. This means that almost all nucleation occurs at the lowest temperature achieved during the supercooling phase (i.e. just prior to reheating). Furthermore, metastable regions are reheated by the release of latent heat around individual nucleation sites, and no appreciable nucleation occurs in the reheated regions.

The important parameter for the purpose of primordial nucleosynthesis will be the mean comoving distance between nucleation sites. We estimate this distance from Eq. (16) for a given nucleation rate, using an analysis similar to that of Kajantie and Kurki-Suonio[9]. In this analysis, each expanding volume of hadronic phase acts as a piston driving a weak shock wave which expands into the quark phase at just above the sound speed $V_s \approx 3^{-\frac{1}{2}}$. These shocks reheat the quark material. No further nucleation will occur once a region has been crossed by one or more shocks. Then it follows that the fraction of the universe which is unaffected by nucleation during this period is:

$$f(t) = \exp - \int_{t_c}^{t} f(t') \, p(T') \, dt' \, \frac{4\pi}{3} V_s^3 \left(\frac{T}{T'}\right)^3 (t - t')^3 \qquad (17)$$

The total number of nucleation sites is

$$N_n = \int_{t_c}^{\infty} f(t) \, p(T) \, dt \qquad (18)$$

In these equations t is time, t_c is the time at which the universe first cools through $T = T_c$, T is the temperature at time t and T' the temperature at time t'.

An approximate (±10%) solution of for the nucleation parameter is,

$$\eta_f \approx 1.4 \frac{\sigma^{3/2}}{T_c^{1/2} L} \, . \qquad (19)$$

The number density of nucleation sites is given by

$$N_n t^3 = \pi^2 \left(\frac{8}{3} \cdot \frac{\sigma^3}{V_s T_c L^2 \eta_f^3}\right)^3 . \qquad (20)$$

The mean separation, ℓ, between nucelation sites is

$$\ell \equiv N_n^{-\frac{1}{3}} \approx 0.8 \frac{\sigma^{\frac{3}{2}} t}{T_c^{\frac{1}{2}} L} \qquad (21a)$$

$$\ell = (5.43 \times 10^{6} m) \frac{\sigma^{3/2}}{T_c^{3/2}}, \qquad (21b)$$

where the second equation follows from the thermodynamic model of the last section. This length scale should be greater than the comoving proton diffusion length in order that the fluctuations not damp out entirely before the onset of nucleosynthesis. In addition, should this length turn out to be greater than the comoving neutron diffusion length, then the approximate treatment of diffusion and nucleosynthesis that is described in section VI is inadequate; however, there will still be significant modification of the nucleosynthesis yield and our broad conclusion will be qualitatively correct.

This length scale depends on two bulk properties of the phase transition (T_c and L) that may be calculated in terms of our model equations. In addition, this length scale depends on the surface tension, σ , for which we have no model calculation. Following Farhi and Jaffe[25] and Alcock and Farhi,[26] we expect the "intrinsic" contribution to σ to be small (i.e. $\sigma_I^{1/3} \ll B^{1/4}$) and the "dynamical" contribution to σ to be $\sigma_D \leq (70\ \mathrm{MeV})^3$. If σ is near this upper limit, the length scale is long compared to the neutron diffusion length, which is ~ 30m.[3]

More important for our analysis would be a lower bound to σ since this would establish whether or not proton diffusion is important. The only strict lower bound is $\sigma > 0$, since $\sigma \leq 0$ would not result in separated phases. It is unlikely that σ is very small compared to the relative contributions of the bulk free energies of the phases, but this possibility cannot be excluded.

IV. DYNAMICS OF THE UNIVERSE DURING THE CONSTANT TEMPERATURE COEXISTENCE EPOCH

The last section described the supercooling/nucleation scenario in which the quark-gluon plasma phase is separated from the hadron phase. The duration of this nucleation epoch is short compared to the Hubble time since the fractional supercooling is small ($\eta \sim 10^{-3}$). Again, because the duration of this epoch is short, the entropy generation associated with reheating to T_c is expected to be small compared to the initial entropy. At the end of this nucleation epoch the universe is left with bubbles of hadron phase surrounded by quark gluon plasma, all in pressure equilibrium at T_c. Since the hadron bubbles were nucleated through random thermal or quantum processes they contain, on average, no net baryon number; all of the net baryon number resides in the quark-gluon plasma phase.

The subsequent evolution of this configuration of phases takes place at constant temperature, T_c.[6-9] The hadron bubbles grow with time and the fraction of the total volume of the universe which is in quark-gluon plasma, f_v, decreases from $f_v = 1.0$ to $f_v = 0$ at the end of the phase transition. The universe remains at constant temperature by trading volume from the unconfined vacuum to the confined vacuum, and thus releasing the latent heat. We summarize the dynamics and specialize our discussion to baryon-number transport in what follows.

If we denote the scale factor in the Robertson-Walker metric by A, then the Einstein equation and energy-momentum conservation can be used to solve for A(t).

$$\frac{A(t)}{A_i} = (4x)^{1/3} \left[\cos\left(\frac{3x\,(t - t_i)}{2\,(x-1)^{\frac{1}{2}}} + \arccos\frac{1}{2x^{\frac{1}{2}}}\right)\right]^{2/3} , \qquad (22a)$$

where the beginning of the constant-temperature epoch (end of the nucleation epoch) is taken at time t_i, corresponding to a scale factor $A(t_i) = A_i$. Similarly it can be shown that the volume fraction of the universe in quark-gluon plasma is

$$f_v = \frac{1}{4\,(x-1)} \left\{ \tan^2\left(\arctan\,(4x - 1)^{\frac{1}{2}} + \frac{3x\,(t_i - t)}{2\,(x - 1)^{\frac{1}{2}}}\right) - 3 \right\} \qquad (22b)$$

where we have assumed that $f_v = 1$ at the end of the nucleation phase. This approximation will be adequate for our purpose of following the baryon number transport properties.

The total proper surface area, Ξ, of the domain wall which separates the confined and unconfined phases is

$$\Xi \approx 4\pi\, r^2\, N_n \quad V_o \qquad (23a)$$

where N_n ($\sim 10^{-9}$ cm^{-3}) is the number of nucleated sites per unit volume from equation (20), V_o is the proper volume of the universe (horizon volume) at the end of the nucleation epoch and r is taken to be a typical radius of a hadron bubble. Since we have approximated $f_v = 1$ at the beginning of the constant temperature epoch we must take $r = 0$. As f_v drops from 1.0 to 0.5 the radius of a typical hadron bubble increases from 0 to some maximum value r_{max}. At this point, $f_v = 0.5$, the bubbles of hadron phase percolate with the quark gluon plasma. As f_v drops from 0.5 to 0 we interpret r as the radius of a shrinking bubble of quark-gluon plasma. The assumption of spherical bubbles is strictly correct only near the beginning and the end of the phase transition where the bubble surface energy dominates the volume energy. Nevertheless, the typical bubble radius remains a useful parameterization of the total surface area in the boundary separating the phases, even if the actual surface area is larger than that given in equation (23a) because of bubble nonsphericity. During the phase transition the proper bubble radius is

$$r(t) \approx \left(\frac{4}{3}\pi N_n\right)^{-1/3} \cdot \frac{A(t)}{A_i} \cdot \bar{f}_v \tag{23b}$$

where $\bar{f}_v$ is defined by

$$\bar{f}_v = \begin{cases} 1 - f_v & \text{if } f_v \leq 0.5 \\ f_v & \text{if } f_v \geq 0.5 \end{cases} \tag{23c}$$

The mechanism of heat transport will be discussed briefly below as regards fluctuation size. We note however, that hydrodynamic heat transport is favored when the supercooling is small[7]. The neutrino mean free path is of the order ~ 1 m, so that a neutrino can random walk ~ 1 km, or 10% of the horizon distance, in a Hubble time. On the other hand, the nucleon mean free path is ~ 10^{-11} cm (where the density of mesonic and baryonic states has been estimated using equations 4a through 5a) so that a nucleon moves only ~ 10^{-3} cm in a Hubble time. In the simple model of noninteracting quarks and gluons presented here we expect the quark mean free path to be roughly comparable to that of the neutrino.

V. BARYON NUMBER TRANSPORT ACROSS THE PHASE BOUNDARY AND CHEMICAL EQUILIBRIUM

The coexistence of the two phases across a boundary is shown schematically in figure 2. Note that the latent heat transport is by the motion of the boundary wall, converting volume from one vacuum to another. This vacuum energy difference can drive particle creation. Neutrinos or photons could carry heat or entropy across the phase boundary, but baryon number is not thermally created in the hadron phase, and thus must actually flow across the boundary if the hadron phase is to have any net baryon number

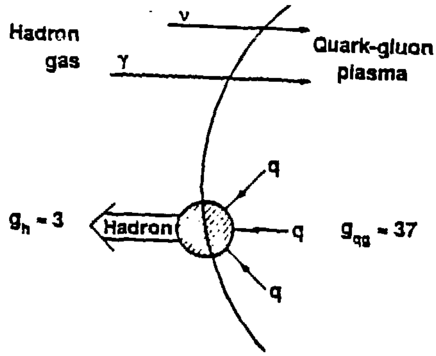

Figure 2) A schematic illustration of a phase boundary separating confined vacuum on the left from unconfined vacuum on the right. The approximate intrinsic statistical weight of relativistic degrees of freedom is shown for each phase. Entropy can be readily carried across the wall by photons, neutrinos, and other light leptons. Baryon number is transported across the boundary only by strong interaction processes. The indicated processes are reversible.

at all. Any transport of baryon number across the phase boundary must be due to strong interaction processes. We therefore consider two limits to the efficiency of baryon number transport. The first is that the baryon number is rapidly and efficiently transported across the front to achieve chemical equilibrium; and the second involves an inefficient transport process in which chemical equilibrium is not necessarily achieved on the timescale of the phase-coexistence evolution. We will show that in both of these limits isothermal baryon number fluctuations can result.

If we define,

$$R \equiv (n_b^q / n_b^h), \tag{24}$$

where the ratio of baryon number densities (n_b^q and n_b^h are the baryon densities in the quark-gluon plasma and hadron gas respectively) is to be taken immediately after decoupling of the bubbles of quark-gluon plasma, then R is a measure of the amplitude of the isothermal baryon number fluctuations. In the limit of chemical equilibrium across the front, the baryon chemical potentials

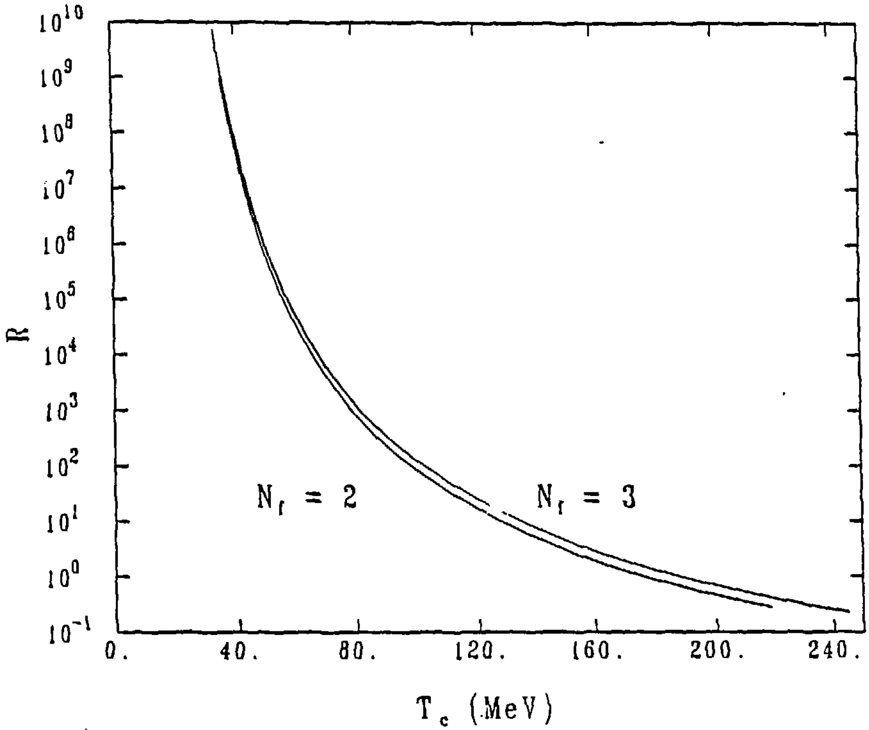

Figure 3) The equilibrium baryon number density ratio, R, as a function of coexistence temperature, T_c. Curves are given for 2 and 3 relativistic quark flavors.

are equal in the two phases and

$$R \approx \frac{2}{9} \left(\frac{\pi^3}{8}\right)^{\frac{1}{2}} \left(\frac{T_c}{m}\right)^{3/2} e^{m/T_c} \tag{25}$$

The sum over baryonic resonances expected to be in equilibrium in the hadron soup tends to decrease the value R by providing a higher statistical weight for baryon number in the hadron phase. This effect is smallest at low temperature where R will be largest. Figure 3 gives this equilibrium amplitude R as a function of T_c, including the hadronic resonances. As we showed in our previous paper, when $T_c < 125$ MeV, $R \geq 20$ and these flucuations will have a significant influence on nucleosynthesis. The difference in baryon number concentration in the two phases results from the difference in the mass of the particles which carry baryon number: nonrelativistic neutrons, protons, and other baryons on the one hand; and relativistic quarks on the other.

The amplitudes of isothermal baryon number fluctuations generated in the chemical equilibrium case represent, to some extent, lower limits when the actual baryon number transport processes are considered. The baryon content of the universe is initially in the shrinking bubbles of the high-temperature phase, so that unless the baryon transport rate is efficient, chemical equilibrium will never be reached.

The baryon number density in each phase can be followed in time as baryon number moves across the phase boundary. If n_b^h and n_b^q are the net baryon number densities in the hadron and quark-gluon plasma phases, respectively, then

$$\dot{n}_b^q = -n_b^q \lambda_q + n_b^h \lambda_h - n_b^q \left(\frac{\dot{V}}{V}\right) \tag{26a}$$

$$\dot{n}_b^h = + n_b^q \lambda_q - n_b^h \lambda_h - n_q^h \left(\frac{\dot{V}}{V}\right) \tag{26b}$$

where $\dot{n}_b^q$ and $\dot{n}_b^h$ are the time rates change of baryon number density in each phase, λ_q and λ_h are characteristic baryon number transfer rates from quark-gluon plasma to hadron phase and the reverse, respectively. The last terms in equations (26a) and (26b) are volume redshift factors. The total baryon number per comoving volume is constant at N_o,

$$\frac{N_o}{V} = n_b^q f_v + n_b^h (1 - f_v). \tag{27}$$

The total baryon number swept up by the wall in the quark-gluon phase, and pushed through to the hadron phase, divided by the total volume in the quark phase is

$$n_b^q \lambda_q = 4\pi r^2 \,.\, N_n \,.\, \frac{V_o}{f_v V} \,.\, V_f \,.\, F, \tag{28a}$$

where r is a typical bubble radius, N_n is as in equation (26a), V_o is the horizon volume at the end of the nucleation epoch, and F is the fraction of baryon number passed by the wall. It can be shown from simple phases space arguments that

$$F = (413)\ \Sigma_h \left(\frac{MeV}{T}\right) \exp\left(-\frac{938\ MeV}{T}\right) v_f^{-1} \tag{28b}$$

where v_f is the speed of the wall in units of c and Σ_h is the probability that the three quarks in a proton, neutron or other bound hadron will pass through the wall into the free quark-gluon plasma regime. Detailed balance can be used to show that probability for 3 quarks to recombine at the wall, Σ_q, is related to Σ_h by

$$\Sigma_q = (5.4 \times 10^6) \left(\frac{T}{100\ MeV}\right)^{-10} \exp\left(\frac{-938\ MeV}{T}\right) \Sigma_h. \tag{28c}$$

Using the above we can show that,

$$\lambda_q = (1.24 \times 10^{13}\ s^{-1}) \left(\frac{MeV}{T_c}\right) \exp\left(\frac{-938\ MeV}{T_c}\right) \left(\frac{cm}{r}\right) \cdot \Sigma_h. \tag{28d}$$

By reasoning similar to that employed above it can be shown that

$$n_b^h \lambda_h = \frac{1}{3} n_b^h V_b \cdot \left(\frac{4\pi r^2 \cdot N_n \cdot V_o}{f_v V}\right) \cdot \Sigma_h , \tag{29a}$$

where V_b is a typical thermal velocity for a nucleon, $V_b \approx (3T/m)^{\frac{1}{2}}$, from which we derive,

$$\lambda_h \approx (1.70 \times 10^9\ s^{-1}) \left(\frac{T_c}{MeV}\right)^{\frac{1}{2}} \left(\frac{cm}{r}\right) \cdot \Sigma_h . \tag{29b}$$

The first order differential equations for n_b^q and n_b^h are easily integrated. In the limit of a static universe, $\frac{\dot{V}}{V} = 0$, which corresponds to a static phase boundary, after a sufficiently long time $t >> \lambda_h^{-1}$ we obtain the equilibrium baryon number density ratio,

$$\left(\frac{n_b^q}{n_b^h}\right)_{eq.} = \frac{\lambda_h}{\lambda_q} , \tag{30}$$

which is the same ratio given in equation (28). In the expanding universe case, figure 4a and 4b present the ratio $R = n_b^q / n_b^h$ as a function of time from the onset of the constant temperature epoch to the end of the phase transition. Figure 4a corresponds to T_c = 100 MeV, while figures 4b corresponds to T_c = 200 MeV. The baryon number density ratio, R, is computed at each T_c for several values of the nucleon transmission probability: Σ_h = 1,

10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, and 10^{-6}. We note that for $\Sigma_h = 1$, baryon number chemical equilibrium is rapidly established, with R being driven to the equilibrium value on a time scale short compared to either the Hubble time or the duration of the constant temperature epoch. For $\Sigma_h << 1$ chemical equilibrium is not obtained and R can be much larger than R_{eq}.

The appropriate value of the baryon transmission probability, Σ_h, depends on the nature of the phase boundary separating the confined vacuum from the unconfined vacuum. Any calculation of Σ_h, therefore, will be highly model dependent.

The basis for isothermal baryon number fluctuation generation lies in the difference in baryon number concentration between the unconfined and confined phases; $R = n_b^q/n_b^h$. If equilibrium is not established across the phase boundary then R is larger than the equilibrium value. The amplitude of the isothermal baryon number fluctuations has been considered in our previous paper[5] and was shown to be order R.

VI. BARYON DENSITY FLUCTUATIONS AND PRIMORDIAL NUCLEOSYNTHESIS

The isothermal baryon number density fluctuations discussed in this paper can have interesting effects on primordial nucleosynthesis. There are two influences which must be considered. One is the fact that more than one baryon to photon ratio must be averaged to produce the final nucleosynthetic yield. Such an averaging by itself can significantly alter the yields from primordial nucleosynthesis.[4] The second intriguing effect[3] is that the neutron and proton components of these density fluctuations will diffuse differently after the weak reactions fall out of equilibrium. Essentially, the neutron collision mean free path is much larger (by more than an order of magnitude) than the proton mean free path since the neutron only interacts with the background plasma via nucleon scattering or via the small neutron dipole momment. The diffusion of protons, on the other hand, is slowed by the more dominant proton-electron scattering. This diffusion of neutrons leads to a filling of the low density voids between the high density fluctuations with neutron-rich material. The high-density regions are correspondingly proton-rich relative to standard big bang nucleosynthesis.[10]

In this paper we extend our previous work[5] to consider the effects of

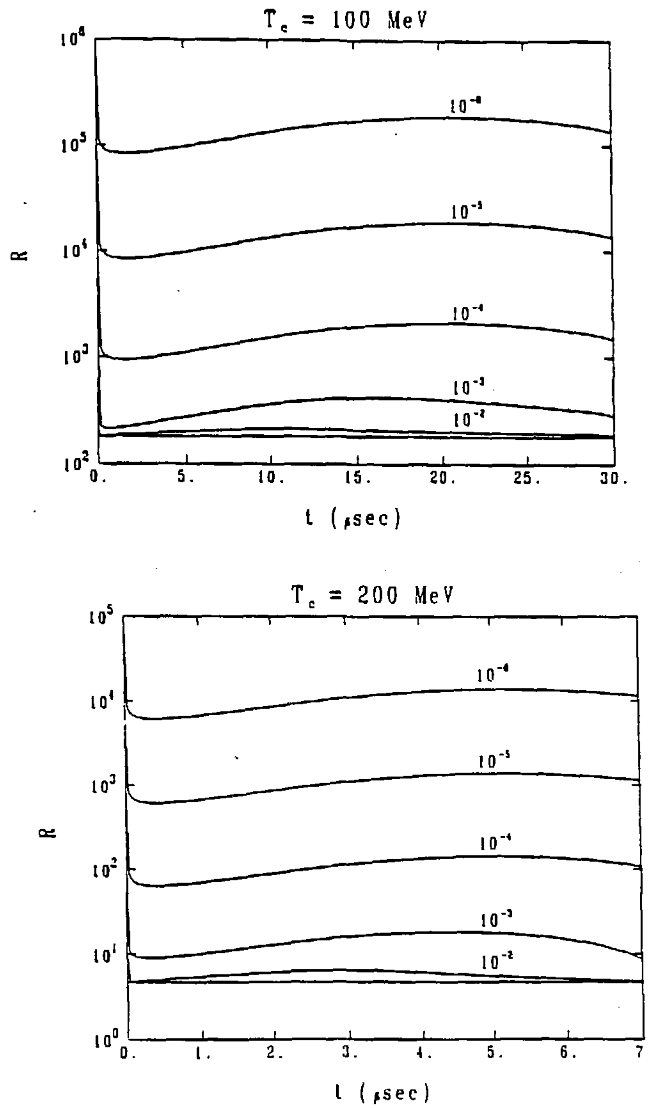

Figure 4) The baryon number ratio, R, as a function of time during the phase transition for T_c = 100 MeV and 200 MeV. Curves are labelled by the value of the nucleon transmission probability, Σ_h; The lowest curve corresponds to Σ_h = 1.

these baryon inhomogeneties in a universe with $\Omega \neq 1$, and as a function of the ratio of the baryon densities in the two phases. Our aim is to delimit the parameter space of this model which is consistent with the constraints from light element abundances.

For the purposes of this parameter study we utilize the two-zone complete diffusion scenario discussed in ref. 5 which reduces the parameter space to three quantities, the total average baryon denisty, Ω_b, the ratio of densities in the two regions, R, and the volume fraction, f_v, in the high-denisty regions just before nucleosynthesis.

We have utilized the big-bang nucleosynthesis code of Wagoner[10] with three neutrino flavors, a neutron half life of 10.6 min., and a number of nuclear reactions updated from the original version. We begin the nucleosynthesis calculations with relative baryon densities in the high density and low density regions (Ω_h or Ω_l) as a function of f_v and average Ω.

$$\Omega_h = \frac{\Omega R}{f_v (R-1) + 1} \tag{31a}$$

$$\Omega_l = \Omega_h / R \tag{31b}$$

We assume that the neutron diffusion occurs between the freezout of the weak reactions (at $T \approx 1.3 \times 10^9$ K, 0.11 MeV) and the onset of nucleosynthesis (at $T \approx 0.9 \times 10^9$ K) about 100 sec later. Note that by weak "freezeout" we mean that temperature at which free neutron decay dominates all other $n \rightarrow p$ reactions. The weak interaction drops out of equilibrium at a much higher temperature, $T \leq 1$ MeV, when typical weak rates become small compared to the universal expansion rate. As nucleosynthesis begins after neutron diffusion, the universe consists of a high baryon density proton-rich region, (1) with

$$\Omega^{(1)} = X_n + \Omega_h [1 - X_n], \tag{32}$$

$$X_n^{(1)} = X_n / \Omega^{(1)}, \tag{33}$$

where X_n is the neutron mass fraction before nucleon diffusion at $T = 1.3 \times 10^9$K, and a low baryon density neutron-rich region, 2, with

$$\Omega^{(2)} = X_n + \Omega_h [1 - X_n] \quad (34)$$

$$X_n^{(2)} = X_n/\Omega^{(2)} \quad (35)$$

The final averaged mass fractions for each nuclide are then,

$$\bar{X}_i = \frac{f_v X_i^{(1)} \Omega^{(1)} + (1 - f_v) X_i^{(2)} \Omega^{(2)}}{f_v \Omega^{(1)} + (1 - f_v) \Omega^{(2)}} \quad (36)$$

In the standard big bang, at high baryon densities, essentially all of the available neutrons are quickly converted into ^{4}He at t ~ 100 sec. In the low-density regions, the neutron mass fraction is set to close to unity when the baryon diffusion is turned on at t ~50 sec. It is then the proton mass fraction which is absorbed to make ^{4}He. Further ^{4}He production must await the decay of neutrons into protons. Thus, there is a significant neutron mass fraction until much later times. ^{2}H is therefore produced at lower densities and survives further nuclear reactions. Since ^{3}H, ^{3}He and ^{4}He are large, there is also significant production of ^{7}Li in these regions.

In figures 5a-b we show averaged nucleosynthesis yields and nucleosynthesis in the proton-rich and neutron-rich regions for Ω = 0.1 and 1.0 when R = 50. This corresponds to the ratio of baryon densities for chemical equilibrium when the coexistence temperature is 110 MeV with three relativistic quark flavors, or when T_c >110 MeV and Σ_h < 1. The intriguing feature of these results is that, for Ω_b =1, over a broad range for the parameter f_v, the main abundance constraints from primordial nucleosynthesis (i.e. ^{4}He, and ^{2}H) can be satisified. There is, perhaps, a slight under production of ^{3}He and a larger over production of ^{7}Li. This overproduction of ^{7}Li is due to the fact that ^{7}Li can be produced in significant amounts in both the neutron-rich and proton-rich regions. However, this overabundance may not be too significant due to the uncertainities[27] in the ^{7}Li abundance, the nuclear reaction rates, and stellar destruction of ^{7}Li. The overabundance appears to be a consequence of the large contribution from the ^{3}He $(\alpha, \nu)^7$ Be reaction in the high baryon-density regions and the ^{3}H$(\alpha,\gamma)^7$Li reaction in the low baryon density regions. However, diffusion during nucleosynthesis may deplete ^{7}Li.[28]

In figures 5a,b there is little dependance of the average ^{7}Li mass

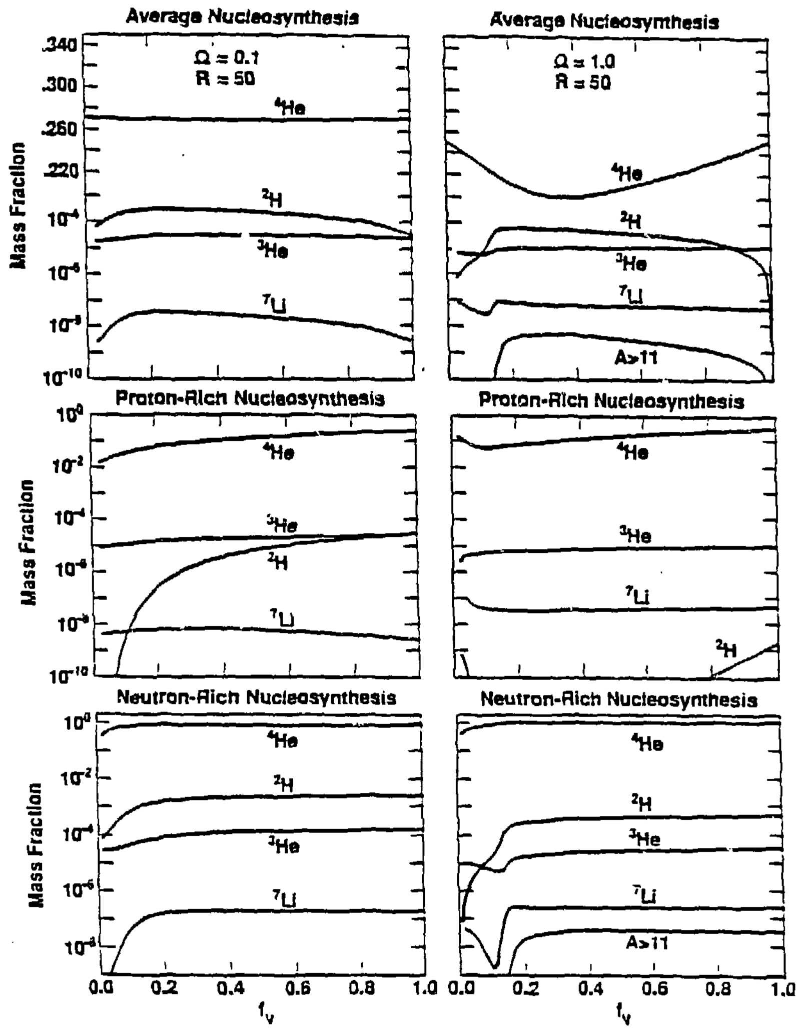

Figure 5) Primordial nucleosynthesis for the high and low density regions and their average yields as a function of volume fraction, f_v, for a fixed fluctuation amplitude, R = 50, and with: a) $\Omega_b = 0.1$; b) $\Omega_b = 1.0$. Note that the average yields for ^2H are relatively insensitive to Ω_b. Note also that the yields do not depend much on f_v for most of the range, $0.2 \le f_v \le 0.8$.

fraction with f_v, although ^2H decreases dramatically when f_v is near the extrema, f_v = 0. or 1., where the results of standard big bang nucleosynthesis are reproduced. For this reason we restrict our discussion to the f_v in the range $f_v = 0.5 \pm 0.4$ for which the abundances are largely insensitive to f_v and there is sufficient production of ^2H.

Figures 6a,b illustrate how the nucleosynthesis yields vary as a function of R for fixed f_v and Ω_b = 1.0 or 0.1. Note, however, for R > 20 (1/R < .05) the yields become essentially independent of fluctuation amplitudes. Typically the average ^7Li and ^2H mass fractions can increase by as much as an order of magnitude over the interval of R = 1. → 20. (Note that R = 1 corresponds to standard big-bang nucleosynthesis with no neutron diffusion). The slight increase for both ^7Li and ^2H with increasing R is basically due to the fact that the relative baryon density in the low-density neutron-rich region decreases with R. For Ω_b = 1, one can see the effect of the minimum in ^7Li production as a function of baryon density due to the increased ^7Li destruction before ^7Be production from the $^3He(\alpha, \gamma)^7Be$ reaction.

In figure 7 we delimit the regions of the R - Ω_b plane which are consistent with the constraints from light element abundances in the two-zone model. Contours are drawn on this figure which show the values of R and Ω_b which satisfy the constraint noted in the figure captions. Curves are drawn corresponding to both deuterium production and ^7Li production. Due to the large uncertainties in the primordial ^7Li abundance, contours are shown for a number of different upper limits to primordial ^7Li ranging from the lowest values from Pop II halo stars to the highest values corresponding to a large fraction of the primordial abundance having been destroyed in stars. Two different ^2H constraints are shown to illustrate the sensitivity to the deuterium abundance constraint.

From this figure several interesting conclusions can be drawn. One is that the deuterium constraint can only be satisfied in an Ω = 1 universe if the fluctuation amplitude, R > 20. Perhaps more useful is the observation that if the smallest upper-limits to the ^7Li abundance are correct then this constraint can only be satisfied in a universe with $R \leq 2$, which would correspond to standard big bang nucleosynthesis without fluctuations. Thus, if the Pop II or even Pop I disk ^7Li abundances are ever accepted as firm upper limits to the primordial lithium, then one could use this constraint to

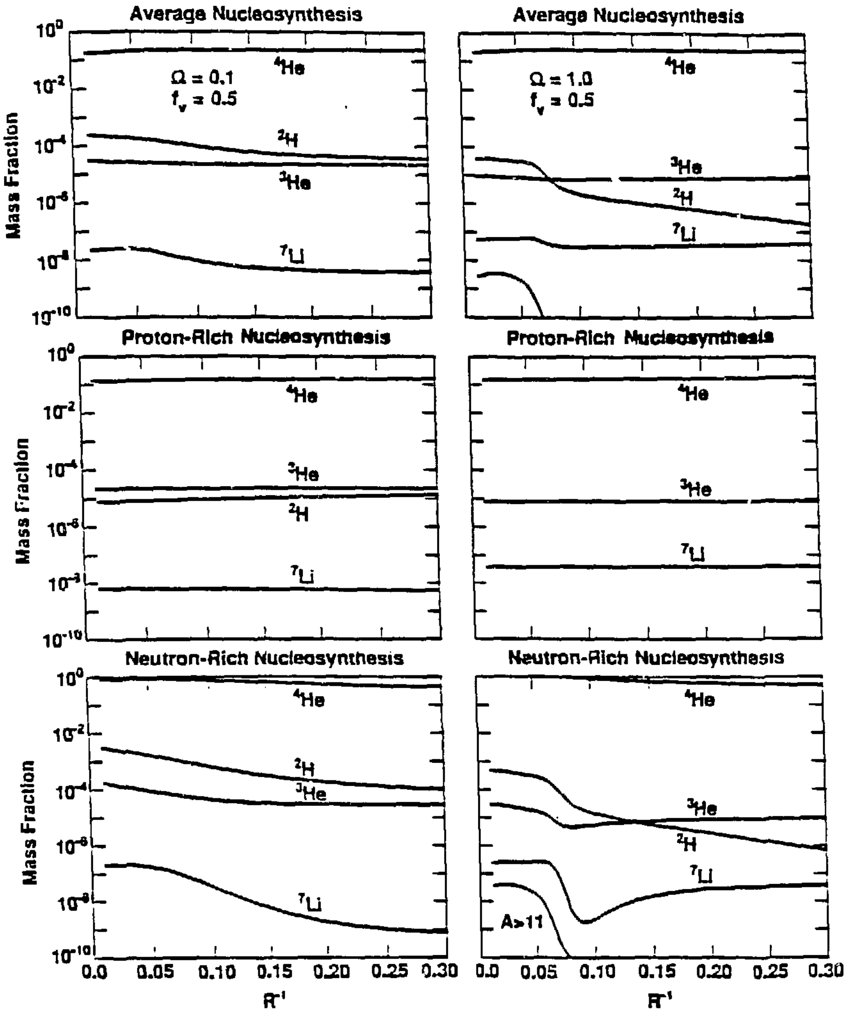

Figure 6) Primordial nucleosynthesis yields in the high and low density regions and their average as a function of inverse fluctuation amplitude, 1/R, for f_v = 0.5 and: a) Ω_b = 0.1: b) Ω_n = 1.0. Note that for R > 20, all yields become virtually independent of the fluctuation amplitude.

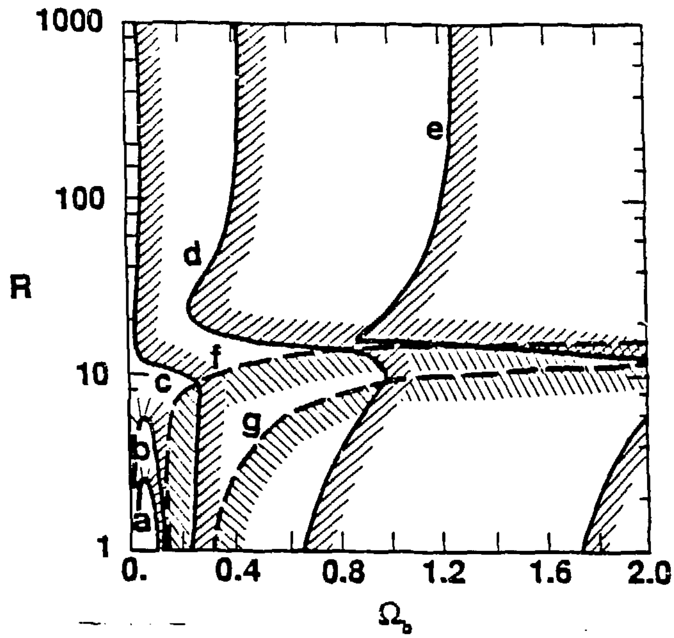

Figure 7) Regions of the R - Ω plane which are consistent with the ^{7}Li and ^{2}H abundances. The solid lines denote regions allowed by the ^{7}Li abundance corresponding to upperlimits taked from: a) $^7Li/H < 1.8 \times 10^{-10}$ (Pop II halo abundance); b) $^7Li/H < 8 \times 10^{-10}$ (Pop I disk abundance); c) $^7Li/H < 2.6 \times 10^{-9}$ (CCI metoritic abundances); d) $^7Li/H < 8 \times 10^{-9}$; e) $^7Li/H < 8 \times 10^{-9}$ (Astration factor = 10 x Pop I abundance). The dashed lines are from the lower limit to the ^{2}H abundance: f) $D/H > 10^{-5}$; g) $D/H > 10^{-6}$.

set an upper limit to the magnitude of baryon density fluctuations to emerge from the quark-hadron phase transition. On the other hand, if the primordial ^{7}Li abundance is at least 3-5 times the meteoritic value, then this constraint could be satisfied for $R \geq 15$. This suggests that if ^{7}Li destruction in stars (astration) has been significant over the history of the galaxy, then the light element abundances could be consistent with $\Omega = 1$ in baryons.

If ^{7}Li has been significantly astrated then it follows that ^{2}H must also have been astrated since this nuclide is less tightly bound than ^{7}Li. It is therefore encouraging that deuterium is overproduced along with ^{7}Li in models with $R \geq 50$. It is useful to consider the constraint based upon the ratio of $^7Li/^2H$ in which the effects of astration cancel out to first order[23] by choosing the highest present day ^7Li abundance, and the lowest deuterium abundance a firm upper limit to this ratio can be derived ($^7Li/^2H < 3 \times 10^{-4}$). We find that for sufficiently large values of R it is possible to have Ω_b very close to unity and still satisfy this constraint.

VII. CONCLUSION

We have shown in this paper that isothermal baryon number density fluctuations are a plausible and perhaps unavoidable consequence of the transition from quark-gluon plasma to confined hadronic matter. We identify the coexistence temperature, T_c, and the nucleon phase boundary transmission probability, Σ_h, as the main determinants of fluctuation size.

We have calculated the effect of isothermal baryon number fluctuations on primordial nucleosynthesis and have shown that the light element abundance yields in a high baryon content universe could reproduce accepted primordial abundances, in a relatively parameter independent way, with the exception of ^7Li. The point is that the limit on Ω_b from primordial nucleosynthesis now rises or falls on the basis of what is taken to be the primordial ^7Li abundance. The measurements of the ^7Li abundances and the $^7Li/^6Li$ isotopic ratios, when coupled with self-consistent models of galactic chemical evolution, now seem to point to a low primordial Li abundance.[27]. If one takes the view that the measured ^7Li abundance is primordial then the present study might constrain the physics of the QCD transition. However, given the uncertainities in the primordial ^7Li abundance it is premature to exclude the study of $\Omega_b = 1$ universes.

VIII. ACKNOWLEDGMENTS

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