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Effective Theories and Thresholds in Particle Physics

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Abstract
The role of effective theories in probing a more fundamental underlying theory and in indicating new physics thresholds is discussed, with examples from the standard model and more speculative applications to superstring theory.

1. Introduction

Effective field theories have proven to be a useful phenomenological tool in elementary particle physics. They serve as probes of the underlying symmetries and structure of more fundamental theories, and the very limitations on their domain of validity can point to thresholds for new physics. I will first illustrate these points with examples from the Standard Model. The large Higgs mass limit of the Standard Model provides both a theoretical laboratory for checking the validity of an effective theory and also as a model for possible physics scenarios at the SSC/LHC.

Finally I will consider the Standard Model itself as an effective four-dimensional field theory that is the low energy limit of ten-dimensional superstring theory. This entails the study of four-dimensional effective supergravity theories that emerge as limits of the string theory at scales μ just below the string and/or compactification scales, and that should reduce to the Standard Model at still lower scales: $\mu \ll M_{Pl}$, where $M_{Pl} = (8\pi G_N)^{-\frac{1}{2}} \simeq 1.8 \times 10^{18} GeV$ is the reduced Planck mass. In addition, attempts to make the connection between superstrings and observed particle physics must be able to account for the origin of supersymmetry (SUSY) breaking; this motivates the study of effective lagrangians for gaugino condensation. The hope is to find a formulation that generates the observed large hierarchy of scales.

2. Effective Theories in the Standard Model

2.1. Fermi Theory

Low energy weak interactions are well described by the Fermi lagrangian:

$$\mathcal{L}_{tree} = 2\sqrt{2}G_F(\bar{\psi}_L\gamma^\mu\psi_L)(\bar{\psi}_L\gamma_\mu\psi_L), \tag{1}$$

which is now understood as the low energy limit of intermediate boson (W) exchange, with the identification

$$\sqrt{2}G_F = \frac{g^2}{4m_W^2} = \frac{\pi\alpha_2}{m_W^2}, \tag{2}$$

where $\alpha_2 = g^2/4\pi$ is the $SU(2)_L$ coupling constant. If we use (1) to calculate the one loop contribution to the four fermion coupling we get a quadratically divergent contribution

$$\mathcal{L}_{1-loop} \sim \frac{\sqrt{2}G_F\Lambda^2}{8\pi^2}\mathcal{L}_{tree}. \tag{3}$$

Evaluating instead the same coupling at the one-loop level of the renormalizable Yang-Mills theory, and then taking the limit of low external momenta $|p|^2 \ll m_W^2$, gives

$$\mathcal{L}_{1-loop} \sim \frac{\alpha_2}{8\pi}\mathcal{L}_{tree}, \tag{4}$$

which, using (2), is the same as (3), provided we make the identification

$$\Lambda \to m_W. \tag{5}$$

Similarly, one-loop corrections in the effective theory (1) include logarithmically divergent terms, for example, an 8-fermion coupling, that agree, after the substitution (5), with those calculated in the low energy limit of the Standard Model.

The effective theory defined by (1) provides a good description of weak interactions for energies $E^2G_F \ll 1$, and the loop expansion converges if the cut-off satisfies $\Lambda^2G_F < 1$. Before it was understood that the underlying physics of the Fermi theory was a Yang-Mills theory, these observations pointed to a new physics threshold $\Lambda < G_F^{-\frac{1}{2}} \approx 300 GeV$, a threshold that we now associate with the mass m_W of the intermediate boson. Veltman was one of the first people to recognize[1] the importance of Yang-Mills theories in this context.

When flavor changing four-fermion couplings are included in $\mathcal{L}_{tree}$, a cut-off less than a few GeV is required for consistency with observation; this led to the prediction[2], later made more precise by calculations[3] within the renormalizable Yang-Mills theory, of the charmed quark mass.

2.2. Pion Chiral Dynamics

As another example, the low energy physics of pions is described by an effective lagrangian where the pion field can be viewed as an interpolating field for the quark bilinear field operator:

$$\bar{q}\frac{\vec{\tau}}{2}\gamma_5 q \Rightarrow \vec{\pi}. \tag{6}$$

In this case we do not know how to take an analytic limit of the underlying QCD theory to obtain the effective pion theory. Rather the low energy limit of the latter is dictated by symmetries and their quantum anomalies:

$$\mathcal{L}_{eff} = \frac{1}{2}\partial_\mu \pi^i \partial^\mu \pi^j \left(\delta_{ij} + \frac{\pi_i \pi_j}{f_\pi^2 - \pi^2}\right) + \frac{\alpha}{6\pi}\frac{\pi^0}{f_\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} + \cdots. \tag{7}$$

The first term in (7) is the unique two-derivative term that respects the chiral $SU(2)_L \otimes SU(2)_R$ symmetry of QCD with two massless quarks. It defines a nonrenormalizable effective theory for which the loop expansion series converges if the cut-off, $\Lambda < 4\pi f_\pi \sim GeV$, lies below the observed resonance mass scale. In fact, for a suitable choice of cut-off the one-loop corrections in this effective theory reproduce, for example, the low energy tail of the ρ resonance. The second term in (7), which induces the neutral pion decay $\pi^0 \to \gamma\gamma$, arises from the chiral anomaly[4] present in quark QED. Both terms will have analogues in the effective theory for gaugino condensation to be discussed below.

3. Is the Standard Model an Effective Theory?

In the above examples, the notion that the cut-off should indicate a scale of new physics is related to the unacceptability of fine tuning. For example, one could absorb the correction (3) (along with the leading divergent corrections in higher loop order) into the definition of the Fermi constant G_F. Since in the effective nonrenormalizable theory new (*e.g.*, the 8-fermion coupling) terms are only log divergent, the limit on the cut-off would be much less stringent. However this would require arranging cancellations among large corrections to produce a very small number. In the spirit of avoiding fine tuning, we can point to two fine tuning problems in the standard model that might suggest new physics thresholds.

3.1. The Strong CP Problem

The QCD lagrangian in the Standard Model takes the form

$$\mathcal{L}_{QCD} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_3}{8\pi}\theta\tilde{F}_{\mu\nu}F^{\mu\nu} + \bar{q}i\not{D}q + (q_L M q_R + h.c.), \qquad (8)$$

where α_3 is the QCD coupling constant, M is the quark mass matrix and the parameter θ violates P and CP. As discussed at this meeting by Eduardo de Raphael, the experimental limit on the neutron electric dipole moment sets a stringent bound: $\theta < 10^{-9}$, if we work in a basis where the quark mass matrix is hermitian $M = M^{\dagger}$. When radiative corrections from the CP violating weak sector are included, the quark mass matrix acquires a logarithmically divergent, nonhermitian correction. Rediagonalization of M then induces a correction[5] $\delta\theta$ to θ, that in the standard model is divergent first in 7-loop order:

$$\delta\theta_{SM} = \delta_{ud} + \delta_{\text{finite}},$$

$$\delta_{\text{finite}} \sim \left(\frac{\alpha_3}{\pi}\right)^4 \left(\frac{\alpha_2}{\pi}\right)^2 \frac{m_c^2 m_s^2}{m_W^4}\theta_{CKM} \sim 10^{-16},$$

$$\delta_{ud} = \frac{\alpha_1}{\pi}\left(\frac{\alpha_2}{\pi}\right)^6 \frac{m_t^4 m_b^4 m_c^2 m_s^2}{m_W^{12}}\theta_{CKM}\ln(\Lambda/m_t) < 2\times 10^{-28} \qquad (9)$$

assuming $m_t < 200GeV$ and $\Lambda < M_{Pl}$. Here $\theta_{CKM} = s_1^2 s_2 s_3 \sin\delta$ is the usual CP violating parameter of the Cabibbo-Kobayashi-Maskawa matrix.[6] Although the contribution (9) increases when one includes additional couplings, such as in $SU(5)$ grand unification, it is clear that the strong CP "problem" does not provide useful information on possible new thresholds.

3.2. The Gauge Hierarchy Problem

In the standard model the renormalized Higgs mass is determined as

$$m_H^2 = \frac{\lambda}{8}(TeV)^2 = m_H^2(\text{tree}) + a\frac{g^2}{16\pi^2}\Lambda^2 + \cdots, \qquad (10)$$

where λ is the renormalized Higgs self-coupling constant, and a is a numerical coefficient of order unity. If the Higgs sector is weakly coupled, $\lambda < 1$, absence of fine tuning suggests a new threshold at a scale $\Lambda < 3TeV$, which is the well-known "second threshold", first emphasized by Veltman.[7]

A priori, there is nothing sacred about weak coupling. If we allow λ, and hence m_H, to become arbitrarily large, the Higgs sector becomes strongly coupled. At scales well below the Higgs mass m_H, the strong self-couplings of the three eaten Goldstone bosons φ^i of the Higgs sector manifest themselves as strong self-couplings among the longitudinally polarized intermediate bosons, $W^{\pm}, Z$. More precisely the S-matrix for the eaten Goldstones is equivalent,[8] up to corrections of order m_W^2/E_W^2, to that for the longitudinally polarized bosons. A recent alternative proof[9] by Hélène Veltman of this "equivalence theorem"[10] has resolved questions[11] that had been raised about its validity. To the extent that the linear σ-model is equivalent to the linear one (a possible discrepancy at the two loop level has been pointed out by van der Bij and M. Veltman[12]), the effective lagrangian for this system is identical to the QCD lagrangian for low energy pions, Eq.(7), with the substitutions $\pi \to \varphi$ and $f_\pi \to v$, where $v \sim \frac{1}{4}TeV$ is the vacuum expectation value of the Higgs field:

$$\mathcal{L}_{eff} = \frac{1}{2}\partial_\mu\varphi^i\partial^\mu\varphi^j\left(\delta_{ij} + \frac{\varphi_i\varphi_j}{v^2-\varphi^2}\right)\left(1 - \frac{\Lambda^2}{8\pi^2v^2}\right) + \cdots. \tag{11}$$

Eq.(11) gives a valid description of strong W, Z interactions over an energy range $m_W^2 \ll E^2 \ll \Lambda^2$, where Λ is the ultraviolet cut-off for the effective theory, and I have displayed the quadratically divergent part of the one-loop correction, which could be reabsorbed as a renormalization:

$$\varphi_{Ren} = Z^{\frac{1}{2}}\varphi, \quad v_{Ren} = Z^{\frac{1}{2}}v \approx 250GeV, \quad Z = 1 - \frac{\Lambda^2}{16\pi^2v^2} + \cdots.$$

This shows that, in order to avoid fine tuning, a new physics threshold $\Lambda < 4\pi v \sim 3TeV$ is indicated even in the strongly interacting limit of the electroweak sector.

One popular scenario for this new physics is technicolor;[13] in this case the strongly coupled part of the electroweak sector closely resembles the pion sector of QCD, including the resonance region, with a scaling in energy by a factor $v/f_\pi \approx 2800$. However there is no explicit realization of this scenario without phenomenological problems.

Veltman realized some time ago[14] that the cancellations among fermions and bosons in a supersymmetric theory[15] would damp the quadratic divergence in (10), provided the fermion-boson mass gap is not too large. It has proven difficult to construct a phenomenologically viable renormalizable theory with spontaneously broken supersymmetry (SUSY), but quite easy to accommodate explicit soft SUSY breaking, in the form of scalar and gaugino masses and trilinear scalar self-couplings. The scale parameter that determines the size of these effects plays the role of the cut-off in (10). In fact, the most recent measurements of the Standard Model gauge constants indicate that their unification is ruled out in the minimal Standard Model, while unification within the minimal supersymmetric extension of the Standard Model fits the data well, with a SUSY mass gap of about a TeV.[16]

In this context one must still understand why the mass gap is as small as it must be to conform to the data with no fine tuning. A favorite hypothesis is that local supersymmetry, in the form of a nonvanishing gravitino mass $m_{\tilde{G}}$, is broken spontaneously in a "hidden sector" of a (nonrenormalizable) supergravity theory, which may in turn be the low energy limit of a (finite) superstring theory. It is then the task of the superstring theorists to predict the correct scale for the SUSY mass gap.

4. The Heterotic Superstring

In the superstring scenario, one starts from a string theory[17] in ten dimensions with an $E_8' \otimes E_8$ gauge group, and ends up[18] in four dimensions with an effective $N = 1$ superstring theory with gauge group $(G' \subset E_8') \otimes (SU(3) \otimes SU(2) \otimes U(1) \subset G \subset E_8)$. G is the gauge group of a SUSY Yang-Mills theory coupled to matter, including the quarks, leptons and Higgs particles of the Standard Model and their superpartners. G' is the gauge group of a "hidden" SUSY Yang-Mills sector, that has only gravitational strength couplings to observed matter. A popular candidate mechanism for SUSY breaking is gaugino condensation[19] in the hidden sector, which is assumed to be asymptotically free and infrared enslaved, so that the SUSY Yang-Mills theory becomes confined at some scale Λ_c where gaugino condensation occurs:

$$< \bar{\lambda}\lambda >_{hid} \sim \Lambda_c^3. \tag{12}$$

An additional source of SUSY breaking could be[20] the (quantized) vev of the field strength H_{lmn} of ten-dimensional supergravity.

$$\int dV^{lmn} < H_{lmn} >= 2\pi n \neq 0, \;\; l, m, n = 4, \ldots, 9,$$
$$H_{LMN} = \nabla_L B_{MN}, \;\; L, M, N = 0, \ldots, 9. \tag{13}$$

When both sources of SUSY breaking are present, it is possible[20] to have "local" SUSY breaking, in the sense that the gravitino acquires a mass: $m_{\tilde{G}} \neq 0$, with a vanishing cosmological constant at the classical level of the effective theory. Supersymmetry breaking should be communicated by radiative corrections to the observable sector, resulting in a SUSY mass gap, *i.e.*, "global" SUSY breaking.

4.1. The Effective Supergravity Theory

The particle spectrum of the effective four dimensional field theory includes the gauge supermultiplets W^a, the matter chiral multiplets Φ^i and the supergravity multiplet. In addition there is a gauge singlet chiral multiplet S, with a scalar component Res that is the "dilaton" field whose *vev* determines the value of the gauge coupling constant at the unification scale:

$$< \mathrm{Re} s >= g^{-2}, \tag{14}$$

as well as singlet chiral multiplets T_α, called "moduli", whose scalar components t_α are related to the structure of the compact manifold. In the case of a single modulus[21] the unification (or compactification) scale is determined as

$$\Lambda^2_{GUT} = \frac{M^2_{Pl}}{< \mathrm{Re} s \mathrm{Re} t >} + O(< |\varphi^i|^2 >) \tag{15}$$

where φ^i is the (complex) scalar component of the superfield Φ^i.

The effective theory just below the compactification scale is an $N = 1$ supergravity theory. In the Kähler covariant superfield formulation[22] of supergravity, the lagrangian takes the simple form

$$\mathcal{L} = \mathcal{L}_E + \mathcal{L}_{pot} + \mathcal{L}_{YM}. \tag{16}$$

The first term

$$\mathcal{L}_E = -3 \int d^4\Theta \mathcal{E} \mathcal{R} + h.c. \tag{17}$$

is the generalized Einstein term. It contains the pure supergravity part as well as the kinetic energy terms for the chiral supermultiplets. The second term:

$$\mathcal{L}_{pot} = \int d^2\Theta \mathcal{E} e^{K(Z,\bar{Z})/2} W(Z) + h.c., \tag{18}$$

contains the Yukawa couplings and the scalar potential, and the third term

$$\mathcal{L}_{YM} = \frac{1}{4}\int d^2\Theta \mathcal{E} f_b^a(Z) W_a^b W_a^a + h.c. \qquad (19)$$

is the Yang-Mills lagrangian. The above lagrangian is invariant under a Kähler transformation, which is a redefinition of the Kähler potential $K(Z,\bar{Z}) = K(Z,\bar{Z})^\dagger$ and of the superpotential $W(Z) = \overline{W}(\bar{Z})^\dagger$ by a holomorphic function $F(Z) = \bar{F}(\bar{Z})^\dagger$ of the chiral supermultiplets $Z = \Phi^i, S, T_\alpha$:

$$K \to K' = K + F + \overline{F}, \quad W \to W' = e^{-F}W. \qquad (20)$$

Since this transformation changes $e^{K/2}W$ by a phase that can be compensated by a phase transformation of the integration variable Θ, the theory defined above is classically invariant[23,22] under Kähler transformations provided one transforms the superfields $\mathcal{R}$ and W_α^a by a compensating phase; for example the Yang-Mills superfield transforms as:

$$W_\alpha^a \to e^{-i\text{Im}F/2} W_\alpha^a. \qquad (21)$$

This last transformation, which implies a chiral rotation on the left-handed gaugino field λ_L^a:

$$\lambda_\alpha^a \to e^{-i\text{Im}F/2} \lambda_\alpha^a, \qquad (22)$$

is anomalous at the quantum level, a point that will be important in the discussion below. (Here a is a gauge index and α is a Dirac index.).

The theory is completely specified by the field content, the gauge group and the three functions K, W and f of the chiral superfields. In theories from superstrings one has $f_b^a(Z) = \delta_b^a S$, resulting in the identification (14). The Kähler potential depends on the dilaton and the moduli fields in such a way that the compactification scale (15) is determined by the vev :

$$\frac{\Lambda_{GUT}}{M_{Pl}} = (2g)^{\frac{1}{3}} < e^{K/6} > . \qquad (23)$$

4.2. The Effective Lagrangian for Gaugino Condensation

In order to incorporate supersymmetry breaking, we include an effective potential for gaugino condensation that is constructed by the introduction of a composite superfield operator[24] U as an interpolating field for the Yang-Mills composite operator:

$$\frac{1}{4}W_\alpha^a W_a^\alpha \Rightarrow U = e^{K/2}\widetilde{W}(H). \tag{24}$$

Here H is a chiral supermultiplet that represents the lightest bound state of the confined SUSY Yang-Mills sector, in the same way that the pion is an interpolating field for the composite quark operator, Eq.(6), in low energy QCD. Kähler invariance requires

$$\widetilde{W}(H) \to e^{-F}\widetilde{W}(H) \tag{25}$$

under (20).

Just as the symmetries of the Standard Model and their quantum anomalies uniquely determine the low energy pion lagrangian (7), the symmetries of the effective supergravity theory determine the effective supergravity lagrangian for the bound state supermultiplet H. In addition to the chiral anomaly related to the transformation (21), (22) under Kähler transformations, there is a conformal anomaly associated with a rescaling of the cut-off (15), (23) under (20):

$$\Lambda_{GUT} \to e^{\mathrm{Re}F/3}\Lambda_{GUT}. \tag{26}$$

The effective lagrangian for gaugino condensation is defined by[25,26]

$$\mathcal{L}_{pot}^{eff} \equiv \int d^4\Theta\mathcal{E}e^{K/2}W(H,S) = \int d^4\Theta\mathcal{E}e^{K/2}\widetilde{W}(H)2b_0\lambda\ln(H/\mu) + h.c.$$

$$= \int d^4\Theta\mathcal{E}e^{K/2}2b_0\lambda e^{-3S/2b_0}H^3\ln(H/\mu) + h.c., \tag{27}$$

where b_0 determines the β-function for the confined Yang-Mills theory:

$$\frac{\partial g}{\partial\ln\mu} = -b_0 g^3,$$

and λ and μ are constants of order unity. The H-superfield kinetic energy term is determined by the Kähler potential[26,27]:

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}-|\Phi|^2-|H|^2). \tag{28}$$

Then under a Kähler transformation (20), (25), with $H \to e^{-F/3}H$, the lagrangian (27) undergoes the shift

$$\delta\mathcal{L}_{pot}^{eff} = -\frac{2b_0}{3}\int d^4\Theta\mathcal{E}F(Z)U + h.c.$$

$$= \sqrt{-\det g}\frac{b_0}{3}(\mathrm{Re}F(z)F^{\mu\nu}F_{\mu\nu} + \mathrm{Im}F(z)\tilde{F}^{\mu\nu}F_{\mu\nu} + \cdots), \tag{29}$$

which correctly reproduces the known variations under the trace and conformal anomalies.[24] Note that in this formalism the anomaly is reflected in the interaction term (27), but the H kinetic energy term, defined by (28), respects the classical symmetry of the theory.

If we now solve for the vacuum value of the scalar component h of the superfield H:

$$< h >= \mu e^{-\frac{1}{3}}, \tag{30}$$

and integrate out the H-supermultiplet, which at the classical level of the theory defined by (27) and (28), amounts to fixing H at its ground state value (30), we obtain an effective theory for Φ^i, S, T and the observable-sector Yang-Mills fields that is defined by (28) with $H =< h >$ and by the superpotential

$$W(Z) = c_{ijk}\Phi^i\Phi^j\Phi^k + \tilde{c} + \tilde{h}e^{-3S/2b_0}, \quad \tilde{h} = -\frac{2b_0}{3}\lambda\mu^3 e^{-1}, \tag{31}$$

where the constant $\tilde{c}$ is proportional to the vev (13). This is precisely the effective theory obtained earlier by Dine *et al.*[20] using arguments based on a nonanomalous chiral $U(1)$ symmetry:

$$\lambda_\alpha^a \to e^{3i\beta/2}\lambda_\alpha^a, \quad s \to s + 2ib_0\beta. \tag{32}$$

The effective theory defined by (31) has a positive semi-definite potential which vanishes at the minimum. If $\tilde{c} = 0$, the vacuum energy is minimized for $\tilde{h} = 0$ ($< H >= 0$) or $< s >\to \infty$ ($g = 0$), that is, condensation does not occur and supersymmetry remains unbroken. For $\tilde{c} \neq 0$ the effective theory has the following properties at the classical level[20] and at the one-loop[28] level: the cosmological constant vanishes, the gravitino mass $m_{\tilde{G}}$ can be nonvanishing, so that local supersymmetry is broken, in which case the vacuum is degenerate, and there is no manifestation of SUSY breaking in the observable sector. Nonrenormalization theorems for supergravity, together with a classical $SL(2,\mathcal{R})\otimes U(1)$ symmetry of this effective theory, indicate[27] that these results will persist to all orders of the effective theory defined by (31).

The $SL(2,\mathcal{R})\otimes U(1)$ symmetry effects a Kähler transformation (20), and hence is broken by anomalies at the quantum level. The effects of anomalies can be made manifest in the effective low energy theory if we first integrate out the H supermultiplet at the one-loop level. Then soft SUSY breaking in the form of gaugino masses appears at the one loop level of the effective low energy theory; more precisely these terms arise from diagrams with one H loop and one "light"particle loop. Evaluating this contribution requires first fully determining the one-H-loop effective lagrangian and performing the appropriate wave function renormalizations and the Weyl transformation needed to recast the renormalized Einstein curvature term[29] in canonical form.

Including loop corrections from the H-sector, one finds[26] that masses are generated for the gauginos of the observable sector that are of order

$$m_{\tilde{g}} \sim \frac{m_{\tilde{G}} m_H^2 \Lambda_c^2}{(4\pi M_{Pl})^4} < 4 \times 10^{-15} M_{Pl} \sim 7TeV,$$

$$\text{for} \;\; m_{\tilde{G}} < m_H \sim \Lambda_c < 10^{-2} M_{Pl}, \tag{33}$$

where m_H is the mass of the H-supermultiplet. The factor $(4\pi)^{-4}$ appears in (33) because the effect arises first at two-loop order in the effective theory, the factor $m_{\tilde{G}}$ is the necessary signal of SUSY breaking, the factor m_H^2 is the signal of the anomalous breaking of $SU(2,\mathcal{R}) \otimes U(1)$, and Λ_c^2 is the effective cut-off. This last factor arises essentially for dimensional reasons: the couplings responsible for transmitting the knowledge of symmetry breaking to the observable sector are nonrenormalizable interactions with dimensionful coupling constants proportional to M_{Pl}^{-2}. Note that the ground state equations give

$$m_{\tilde{G}} =< e^{K/2} W > \approx \frac{\lambda \mu^3 \Lambda_c^3}{2 c g^4 M_{Pl}^2}, \quad \mu \Lambda_c \sim \left(\frac{\tilde{c} c}{2\lambda}\right)^{\frac{1}{3}} \Lambda_{GUT}, \tag{34}$$

so it is not possible to generate a hierarchy of more than a few orders of magnitude between $m_{\tilde{G}}$ and Λ_{GUT} if $\tilde{c}$ is quantized as in (13). However this initial small hierarchy is enough to generate a viable gauge hierarchy if observable SUSY breaking is sufficiently suppressed, as in (33), relative to local SUSY breaking. For example, recent LEP data[18] suggest $\Lambda_{GUT} \sim 10^{16} GeV$, $g^{-2} \sim 2$, so for a hidden E_8 gauge group ($b_0 = .56$) we get $\Lambda_c \sim .6 \Lambda_{GUT} \sim 3 \times 10^{-3} M_{Pl}$.

4.3. Restoration of Space-Time Duality

In the formalism presented above, the classical $SL(2,\mathcal{R}) \otimes U(1)$ symmetry is broken by anomalies to a Peccei-Quinn type $U(1)$ symmetry: $T \to T + i\gamma$. However the discrete subgroup $SL(2,Z)$ of $SL(2,\mathcal{R})$ is known[30] to be an exact symmetry to all orders in string perturbation theory. Similar symmetries are present in more general string compactifications.

This so-called "modular invariance", which includes the "duality" inversion $R \to R^{-1}$ of the radius of compactification, is restored by adopting, instead of (27), the effective lagrangian[31]

$$\mathcal{L}_{pot}^{eff} = \int d^4\Theta \mathcal{E} e^{K/3} 2b_0 \lambda e^{-3S/2b_0} H^3 \ln(H\eta^2(T)/\mu) + h.c., \qquad (35)$$

where $\eta(T)$ is the Dedekind η-function. This is the unique function of the chiral superfields that has the required analyticity and $SL(2, Z)$ transformation properties.[32,31] For different compactifications it will be replaced by different moduli-dependent functions.[33,34] This additional contribution to the Yang-Mills wave function renormalization can be understood[35] as arising from finite threshold corrections[30,33] to the leading log approximation that arise from heavy string mode loops, and is closely related to the anomalous quantum correction due to the (nonrenormalizable) coupling of the Kähler connection,

$$\Gamma_\mu = K_i \partial_\mu z^i - K_{\bar{i}} \partial_\mu \bar{z}^{\bar{i}} = \frac{i}{4}\left[\frac{\partial_\mu (s - \bar{s})}{s + \bar{s}} + \frac{\partial_\mu (t - \bar{t}) + \varphi^i \partial_\mu \bar{\varphi}^{\bar{i}} - \bar{\varphi}^{\bar{i}} \partial_\mu \varphi^i}{t + \bar{t} - |\varphi|^2}\right],$$

to the axial $U(1)$ current.[27,37,35] This ABBJ-type anomaly[4] induces a coupling of the moduli to the fermion axial current and hence to the gauge field strength $F_{\mu\nu}\tilde{F}^{\mu\nu}$, in analogy with the $\pi\gamma\gamma$ coupling in (7).

Whether or not this "corrected" effective lagrangian, or its generalization to more realistic compactifications, can produce as promising a result for phenomenology as the one in (33) remains to be seen.[36]

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