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PREPRINT UCRL-80009

Lawrence Livermore Laboratory

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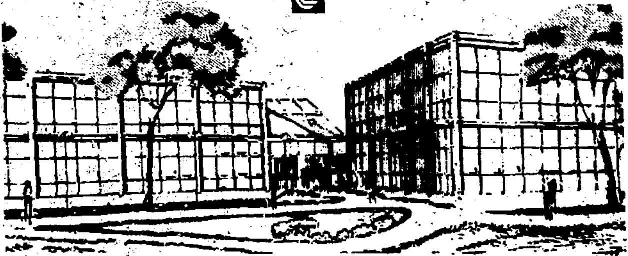
August 25, 1977

This paper was prepared for submission to
Proceedings of the College on Theoretical & Computational Plasma Physics,
Trieste, Italy, 3/22 - 4/9/77



For Trieste Proceedings

HYBRID SIMULATIONS OF QUASINEUTRAL PHENOMENA IN MAGNETIZED PLASMA

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We present a new class of numerical algorithms for computer simulation of low frequency ($\omega << \omega_{ce}, \omega_{pe}$) electromagnetic and electrostatic phenomena in magnetized plasma. Maxwell's equations are solved in the limits of quasineutrality and negligible transverse displacement current (Darwin's model): $\nabla \cdot \underline{J} = 0$, $\nabla \times \underline{B} = 4\pi c^{-1}\underline{J}$, and $-\partial \underline{B}/\partial t = c\nabla \times \underline{E}$. Electrons are modeled as a fluid with polarization effects ignored: $0 \approx -n_e e\underline{E} + \underline{J}_e \times \underline{B}c^{-1} - m_e \nu_{ei}\underline{J}e^{-1} - \nabla \cdot \underline{P}_e$, $n_e \approx n_i$. Ions are described as particles. A novel feature of these algorithms is the use of the electron fluid equation of motion to determine the electric field, which renders these numerical schemes remarkably simple and direct. The simulation plasma is either periodic, or bounded by particle reflecting conducting walls. Both fully nonlinear codes with spatial grids and linearized gridless codes have been implemented. We have implemented several variations of the basic algorithm which utilize the conservation of canonical momentum or which relax this constraint for more general situations. The numerical dispersion and stability of the various algorithms are investigated analytically and verified by computer experiments. Stability generally requires that $(kV_A)_{eff}\Delta t \lesssim 1$ and $\omega_{ci}\Delta t \lesssim 1$, where the effective Alfvén frequency $(kV_A)_{eff}$ includes space-time grid effects. The goal is to simulate low frequency ($\omega \sim \omega_{ci} << \omega_{pi}$) electrostatic and magnetostatic plasma collective behavior. Special emphasis is given to mirror physics applications, e.g. low frequency microinstabilities and build-up to and stability of field-reversed configurations. Maxwell's equations are solved in the _Darwin_ approximation (no transverse

Basic algorithm:

$$\underline{E} = \underbrace{-\frac{\nabla^2}{\omega_{pi}^2}\underline{A}^t c \times \underline{\omega}_{ci} - \frac{4\pi}{\omega_{pi}^2}\underline{J}_i \times \underline{\omega}_{ci}}_{\underline{E} \times \underline{B}} - (n_i q_i)^{-1} \underset{\substack{\uparrow\\ \text{diamagnetic}\\ \text{effects}}}{\nabla\cdot\underline{\underline{P}}_e} + \underset{\substack{\uparrow\\ \text{resistivity}}}{\frac{\nu_{ei}}{\omega_{pe}^2}\nabla^2\underline{A}^t c}$$

$$-\frac{1}{c}\frac{\partial\underline{A}^t}{\partial t} = \underline{E}^t \qquad \underline{E} = \underline{E}^t + \underline{E}^\ell \qquad \underline{B} = \nabla \times \underline{A}^t + \underline{B}_o \qquad \underline{\underline{P}}_e = \underline{\underline{P}}_e(\underset{\substack{\shortparallel\\ n_i}}{n_e}, T_e)$$

$$d\underline{V}_i/dt = (q_i/m_i)\left(\underline{E} + \underline{V}_i \times \underline{B}c^{-1}\right)$$

$$d\underline{X}_i/dt = \underline{V}_i$$

$$\underline{X}_i, \underline{V}_i \rightarrow n_i, \underline{J}_i$$

Note that the vector potential is propagated ahead in time and that the electric field appears as part of the electron current, $\underline{J}_e = n\frac{\underline{E}\times\underline{B}}{B^2}$, and is obtained by solving for $\underline{J}_e = -\nabla^2\underline{A} - \underline{J}_i$. It is possible to construct algorithms that are perfectly time centered and that require no iteration. In one dimension the algorithm is particularly straightforward. There is a longitudinal electric field E_x, a transverse electric field E_y, a vector potential A_y, and a magnetic field B_z. It is the geometric decoupling of transverse and longitudinal vector fields that makes one-dimensional versions of Darwin models especially simple. Another simplification possible in one dimension is the use of conservation of y momentum.

Compressional Alfvén Waves

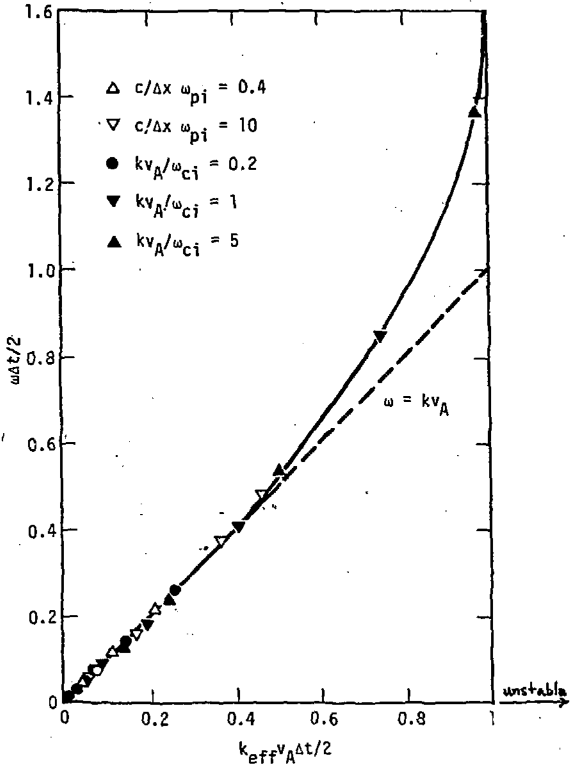

Fig 1.

(A) UNSTABLE DCLC MODE

$F(v) = \exp(-v^2) - \exp(-Rv^2)$

$R = 10$ $k_x\rho_i = 7$ $\beta = 0.1$ $\rho_i/R_p = 0.1$

Result shows initial high frequency transient and then transitions to an instability with $\frac{\omega}{\omega_{ci}} = 0.65$ $\frac{\gamma}{\omega_{ci}} = 0.45$

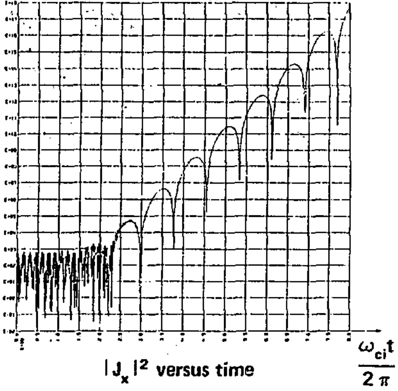

Fig 2.

$|J_x|^2$ versus time

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displacement current) with quasineutrality additionally imposed. The plasma is a nonrelativistic ensemble of particle ions and an electron fluid.

Quasineutrality:

$$\frac{\partial}{\partial t} e(n_i - n_e) + \nabla\cdot(\underline{J}_i + \underline{J}_e) = 0$$

Boundary conditions: $\left.(\underline{J}_i + \underline{J}_e)^{\ell}\cdot\hat{n}\right|_{\text{surface}} = 0$

$$\therefore \quad (\underline{J}_i + \underline{J}_e)^{\ell} = 0$$

Therefore, quasineutrality + Darwin approximation $\frac{1}{c}\ \frac{\partial}{\partial t}\underline{E}^t \rightarrow 0$ reduces Ampere's law to

$$\nabla \times \underline{B} = \frac{4\pi}{c}(\underline{J}_i + \underline{J}_e)^t = \frac{4\pi}{c}(\underline{J}_i + \underline{J}_e)$$

To this add

$$\nabla \times \underline{E} = -\frac{1}{c}\ \frac{\partial}{\partial t}\underline{B} \qquad \text{Faraday's law}$$

and

$$n_e m_e\left(\frac{\partial}{\partial t} + \underline{V}_e\cdot\nabla\right)\underline{V}_e \approx 0 \approx n_e q_e\left(\underline{E} + \underline{V}_e \frac{\times}{c}\underline{B}\right) - n_e m_e(\underline{V}_i - \underline{V}_e)\nu_{ei} - \nabla\cdot\underline{\underline{P}}_e$$

We neglect the electron inertia, use $n_e \approx n_i$, and solve the electron fluid equation for $\underline{E}$.

To verify that our algorithm is accurate and stable we have analyzed the linear behavior of the one-dimensional finite difference algorithm. We compare in Fig. 1 the analysis and measurements from the code for compressional Alfvén waves.

The extension to higher dimensions is not straightforward. Difficulties arise in trying to time center all of the equations without introducing spurious effects at a frequency related to the sampling rate. We also wish to relax the constraint that canonical momentum be conserved in any direction. A general predictor-corrector scheme has been developed. The use of the predictor alone has the usual difficulties associated with leapfrog schemes. The corrector damp out the spurious high frequency modes but also can severely damp desired low frequency modes unless the time step is sufficiently small. For example with $\omega\Delta t = kV_A\Delta t = 0.125$ we observe a reduction in wave amplitude by 7% in a time $\omega_{ci}t = 50$. This may or may not be acceptable depending on the application. The point is that the algorithm is stable without extra spurious branches and with some care will not seriously alter the significant physics.

We have successfully simulated with linearized codes three examples of microinstability: two essentially electrostatic modes, the Dory-Guest-Harris instability[1,2,3] and the drift-cyclotron-loss-cone mode[4]; and the Alfvén-ion-cyclotron mode[5] which is electromagnetic. The simulation results agree well with linear analytical theory, and the generally stabilizing influence of the electromagnetic modifications of the dominantly electrostatic modes is demonstrated. We have also applied these simulation models to the study of field-reversed magnetic-mirror systems. Our simulations verify that electron return currents cancel an embedded, linearly rising external current which is perpendicular to the vacuum magnetic field, only for times up to the Alfvén transit time of a plasma bounded by conducting walls.[6] After this, there is a growth of net current and concomitant magnetic field modification.

Examples of results from a linearized version of the code for a drift-cyclotron-loss-cone mode are shown in Figs. 2, 3. For the case $k_x\rho_i = 7$, we see an instability with both growth rate and frequency agreeing with the theory. For $k_x\rho_i = 1$, the mode is observed to be stable, again in agreement with the theory.

Similar agreement is obtained for the Dory-Guest-Harris instability the Alfvén ion cyclotron mode.

These algorithms are presently being extended to 2D nonlinear versions for inhomogeneous plasmas. They are expected to be important for a variety of mirror-confined plasma problems.

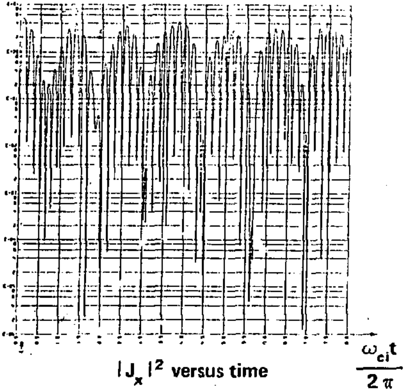
Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

(B) STABLE DCLC MODE

$k_x \rho_i = 1$ other parameters same as in (a)

NO growth is observed; the primary frequency is close to $\frac{\omega}{\omega_{ci}} = \frac{kv_A}{\omega_{ci}} = 3$

Fig 3.



$|J_x|^2$ versus time