RIPPLED-BEAM FREE-ELECTRON LASER

B. E. Carlsten, Los Alamos National Laboratory, Los Alamos, NM 87545 USA

Abstract

We describe a new microwave generation mechanism involving a scalloping annular electron beam. The beam interacts with the axial electric field of a TM_{0n} mode in a smooth circular waveguide through the axial free-electron laser interaction, in which the beam ripple period is synchronous with the phase slippage of the rf mode relative to the electron beam. Due to nonlinearities in the orbit equation, the interaction can be made autoresonant, where the phase and amplitude of the gain is independent of the beam energy.

1 INTRODUCTION

In this paper, we will introduce a new amplifier interaction mechanism between the electron beam and a fast-wave rf mode in a smooth waveguide, in which the annular beam is radially rippled to establish synchronism. There is a uniform, axial magnetic field, and the beam ripple is caused by injecting the beam either displaced from its equilibrium position or with some initial radial velocity. The rf mode profile is unperturbed in the interaction region, leading to a smooth transition between the input signal and the growing mode and also between the growing mode and the output signal. The beam interacts with the axial electric field of a mode with a phase velocity greater than the speed of light and is bunched axially, and thus this interaction is in the class of the axial free-electron laser. We demonstrate this interaction in Fig. 1, where a TM_{02} mode is used and the beam oscillates about the radius where there is a null in the axial electric field. Assume that the axial electric field opposes the electronic motion at position A, where the electron is at a radius greater than the null of the axial field. As the electron travels to the right, the rf phase slips by because the phase velocity is greater than the speed of light. At position B, the electron is now at a radius less than the null of the axial field. If exactly one-half of an rf wavelength has slipped by the electron, the axial field at this location will also oppose the electronic motion, and a synchronous interaction between the electron and the rf will be established. The single-particle synchronism equation,

$$\frac{\omega}{c\beta_z} = \frac{\omega}{v_p} + \frac{2\pi}{\lambda_r} \quad , \tag{1}$$

is exactly the same as for all FEL interactions (where ω is the radian mode frequency, c is the speed of light, β_z is the electron's axial velocity normalized to the speed of light, v_p is the phase velocity of the rf mode, and λ_r is the period between ripples). The resonant axial electric field seen by an electron at an arbitrary phase is given by

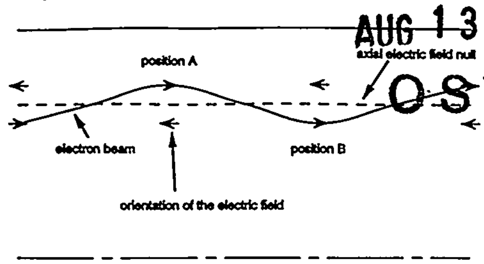

Figure 1. Rippled beam interaction, showing single electron synchronism.

$$E_z = Ak_c\kappa J_1(k_c r_o)\cos(k_r z)\cos(k_r z + \phi) \quad , \tag{2}$$

where the mode has a maximum on-axis axial field strength of A, k_c is the mode's radial wavenumber, κ is the maximum ripple amplitude, r_o is the equilibrium annulus radius, k_r is the ripple wavenumber ($k_r = 2\pi / \lambda_r$), and ϕ is the relative phase between the rf and the electron. For an intense electron beam, Eqn. (1) is somewhat modified by the effects of the axial space-charge forces between particles; we will derive the more general dispersion relation in Section 3, which we will then use to numerically calculate the growth rate of the mode in Section 4. Note that this interaction vanishes on axis, and is only suitable for an annular electron beam. This interaction shares some features with the cyclotron autoresonance maser (CARM) interaction [1]. In particular, we will see that due to the inherent nonlinearities in the orbit equation, the rippled beam interaction can stay in resonance as either the axial magnetic field or the beam energy is changed, demonstrating autoresonance characteristics.

2 ORBIT EQUATIONS

The radial equation of motion of the center of an annular beam in a uniform axial magnetic field is

$$m\frac{d}{dt}\gamma\dot{r} = e\frac{E_r}{\gamma^{*2}} - ev_\theta B + \gamma m\frac{v_\theta^2}{r} \quad , \tag{3}$$

where a dot refers to a time derivative, γ is the relativistic mass factor, m and e are the electronic mass and charge, respectively, E_r is the radial space-charge force at that point, γ^* is the effective relativistic mass factor from the beam's axial velocity only

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$\left(\gamma^* = \left(1-\beta_z^2\right)^{-1/2}\right)$, v_θ is the azimuthal velocity of the center of the annulus, B is the total axial magnetic field. We assume that the radius of the annulus center is very close to an equilibrium radius r_o, and can be written as $r = r_o + \delta r$.

After reducing Eqn. (3) [2], the equation for the equilibrium radius becomes

$$r_o^2 = \frac{b+\sqrt{b^2+4\hat{r}^4}}{2} \quad , \qquad (4)$$

where $I_A = 4\pi\varepsilon mc^3/e$, $\hat{r}$ is the injection radius, and $b = \left(\gamma cm/eB\right)^2 4I/I_A\gamma\gamma^{*2}\beta_z$. The radial displacement from the equilibrium orbit is then to lowest order

$$\delta r = \kappa e^{j\left(\frac{eB\sqrt{\left(r_o^4+\hat{r}^4\right)/2}}{\gamma m r_o^2}\right)t+j\phi} \qquad (5)$$

where κ is the maximum orbit displacement and ϕ is the phase of the oscillation. The ripple period is then $\lambda_r = 2\pi c\beta_z\gamma m r_o^2 / eB\sqrt{\left(r_o^4+\hat{r}^4\right)/2}$. The average axial velocity is given by

$$\beta_z^2 = 1 - \frac{1}{\gamma^2} - \overline{\beta}_r^2 - \overline{\beta}_\theta^2 \qquad (6)$$

where the rms azimuthal velocity is

$$\overline{\beta}_\theta = \frac{eB}{cm\gamma}\kappa\left(\frac{1}{2}+\cos^2(\phi)\right)^{1/2} \quad , \qquad (7)$$

and the rms radial velocity is

$$\overline{\beta}_r = \frac{eB}{cm\gamma}\kappa\frac{1}{\sqrt{2}} \quad . \qquad (8)$$

3 SINGLE MODE DISPERSION RELATION

We can use standard techniques to derive the single mode dispersion relation [2,3,4], which becomes

$$\left(\left(\beta_e + j\Gamma\right)^2 + \left(\Gamma^2+k^2\right)\hat{\beta}_q^2\right)\left(\left(\Gamma - jk_r\right)^2 + \beta_1^2\right) + 2\beta_1^4 C^3 \frac{\left(k^2+\Gamma^2\right)}{k_c^2} = 0 \quad , \qquad (9)$$

where C is Pierce's gain parameter, defined by $C^3 = K\beta_e/2R_o\beta_1$, Γ is the exponential growth rate of the mode (all rf terms vary as $e^{-\Gamma z}$), $\beta_e = \omega/c\beta_z$, $k = \omega/c$, $\beta_1 = \omega/v_p$, k_c is the mode's cutoff wavenumber, $R_o = 2\gamma^3 m(c\beta_z)^2/eI$, K is the peak magnitude of the synchronous electric field (Eqn. (2)) squared divided by the product of four times the power in the waveguide times β_1^2, and $\hat{\beta}_q^2$ is the square of the reduced plasma wavenumber, given by

$$\hat{\beta}_q^2 = 2\chi_o \frac{I}{I_A\gamma^3\beta_z^3}\ln\frac{r_w}{r_o} \quad , \qquad (10)$$

where r_w is the wall radius, χ_o is the geometrical factor $I_o(hr_o)\left(K_o(hr_o)I_o(hr_w) - K_o(hr_w)I_o(hr_o)\right)/I_o(hr_w)\ln\frac{r_w}{r_o}$ and $h^2 = (j\Gamma)^2 - k^2$.

4 NUMERICAL EXAMPLES

In this section, we present some examples in which we numerically solve Eqn. (9) along with the orbit equations. For our nominal case, we will assume a 4.5-kA, 650-keV electron beam with an annulus radius of 2.9 cm in a waveguide with radius 3.37 cm. A 17-GHz rf signal in the TM_{02} mode will have a cutoff wavenumber of $k_c = 163.8\ \text{m}^{-1}$ in this waveguide, with an axial wavenumber of $\beta_1 = 316.4\ \text{m}^{-1}$. We will additionally assume that there is an inner conductor in the waveguide, located at the radius of the first axial electric field null of the TM_{02} mode (at 1.47 cm), which will not affect the mode pattern, but will reduce the power required for a given field strength. For a TM_{02} mode with on-axis amplitude A, the power required in the mode is $P_{req} = A^2 6.43\left(10^{-7}\right)$ watts. The space charge term used in the dispersion relation is $\hat{\beta}_q^2 = 0.00358/\beta_z^3$.

Our nominal case will be a 10% ripple with a period of 5.19 cm (an axial field of 0.40 T). This ripple period and ripple amplitude leads to a gain of about 26 dB per meter of interaction. We plot the normalized electronic admittance $\left(2\beta_1^4 C^3/\left(\left(\beta_e + j\Gamma\right)^2 + \left(\Gamma^2+k^2\right)\hat{\beta}_q^2\right)\right)$ in Fig. 2, along with the normalized circuit admittance $\left(-\left(\left(\Gamma - jk_r\right)^2 + \beta_1^2\right)k_c^2/\left(k^2+\Gamma^2\right)\right)$, as a function of $j\Gamma$ (the dashed line is the circuit admittance and the solid line is the electronic admittance). The dispersion relation is satisfied where these two curves meet, in which case Γ is purely imaginary (there is no gain). Since the dispersion relation is quartic, there will always be four roots, and in order to have gain, there must be only two intersections between these two curves (as in Fig. 2). In order to have a root with gain, as the circuit admittance crosses the $j\Gamma$ axis, it must pass through the left gap where the electronic admittance separates due to the slow space-charge wave [3]. With small ripple amplitudes, we can change the detuning by varying the axial magnetic field; by increasing the magnetic field, the ripple period

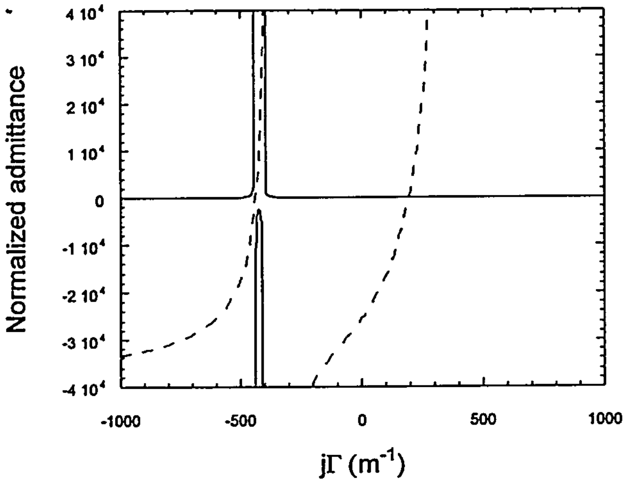

Figure 2. Normalized electronic admittance (solid line) and normalized circuit admittance (dashed line) as a function of $j\Gamma$.

decreases, and the ripple wavenumber k_r increases, making the detuning more negative and shifting the circuit admittance curve more to the left. As the magnetic field is increased, the average axial velocity also drops, making both β_e and $\hat{\beta}_q$ larger, and moving the slow space-charge wave gap in the electronic admittance also to the left. For large ripple amplitudes, the movement in the electronic admittance is larger than the movement in the circuit admittance, and for sufficiently large amplitudes, the slow space-charge wave gap is always further to the left than the lowest zero-crossing of the circuit admittance, and there is no resonance. Because the amount of space-charge determines the separation between the slow and fast space-charge wave gaps, this effect increases as the beam current is increased.

There is a peculiar regime in between these extremes where the movements are matched, and there is resonance for a very large range of axial magnetic fields and resulting ripple periods, and where a beam will stay in resonance as it loses energy to the rf field.

In Fig. 3, we plot the gain per meter of interaction length as a function of the applied axial magnetic field, for a 10% ripple amplitude. There are two regions of gain - one with a field ranging from 0.38-0.42 T, and a second, more narrow region at a field of about 1.05 T. Resonance is established in the first region with a fairly high β_z, where the axial field effectively only modifies the ripple period. Resonance is established in the second region with a relatively low β_z (< 0.5), where the increasing axial field mostly increases the space-charge wavenumber $\hat{\beta}_q^2$.

At 22 kA (Fig. 4), the zero-crossing of the electronic admittance and the gap in the electronic admittance due to the slow space-charge wave are cotangent at a magnetic field of about 0.7 T. This is the condition of auto-stable resonance, and the first derivative of the phase and of the amplitude of the gain with respect to energy vanish here.

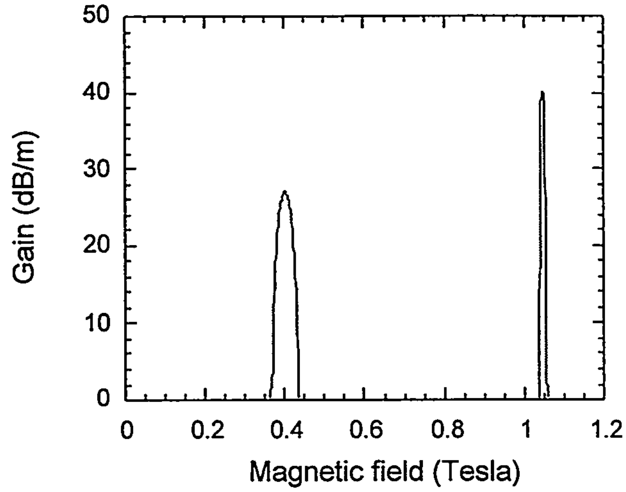

Figure 3. Gain of nominal case as a function of axial magnetic field, 10% beam ripple.

5 ACKNOWLEDGEMENTS

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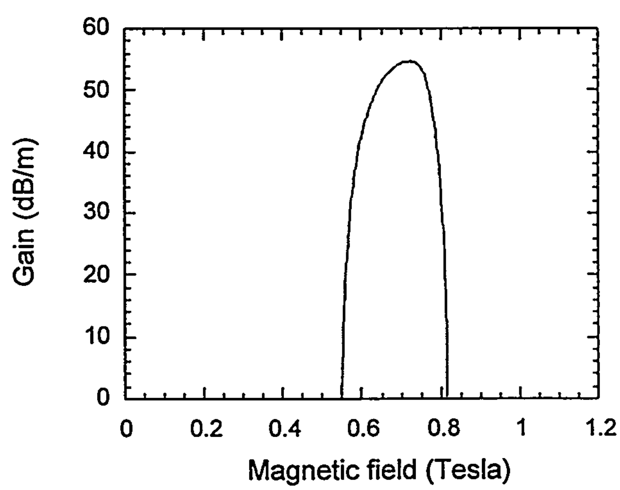

Figure 4. Gain curve for 22 kA (auto-stable resonance condition is at about 0.7 T)