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Report

UNIFIED FLUID/KINETIC DESCRIPTION OF MAGNETIZED PLASMAS. PART II: APPLICATIONS

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The unified fluid/kinetic equations developed in part I of this work are used to study plasma drift type microinstabilities. A generalized perturbed Ohm's law is derived (for a sheared slab magnetic field model) which is uniformly valid for arbitrary collisionality ω/ν and adiabaticity $\omega/k_{\parallel}v_t$. For electron drift waves we demonstrate that the collisional and collisionless drift wave instabilities can be uniformly described by replacing the Spitzer resistivity with a generalized pseudo-resistivity. Similarly, for ion drift type modes we obtain a uniformly valid dispersion relation for the ion temperature gradient modes (η_i modes). The η_i threshold as a function of ion-ion collisionality and ion Landau damping strength is addressed. Applying the unified equations to electron electromagnetic modes leads to generalized coupled equations for $\tilde{\phi}$ and $\tilde{A}_{\parallel}$ which include electron Landau damping effects and are valid for any ω/ν. It is shown that the semi-collisional micro-tearing and drift-tearing modes of Drake and Lee [Phys. Fluids **20**, 1341 (1977)] can be easily reproduced in the appropriate limit. Generalization of the two-field Hasegawa-Wakatani turbulent equations [Phys. Rev. Lett. **50**, 682 (1983)] to include electron temperature fluctuations and linear Landau damping effects is also discussed. Finally, a new method is presented to facilitate the study of magnetic trapped particle modes using our kinetic closure procedure. It is found that by including the trapped particle effects in the closure relations, the usual separation of the fluid equations into trapped and untrapped components becomes unnecessary.

I. INTRODUCTION

In part I of this work,[1] we developed a set of closed fluid moment equations for perturbed density ($\tilde{n}$), parallel flow ($\tilde{u}_{\parallel}$) and temperature ($\tilde{T}$) that are uniformly valid for arbitrary collisionality ω/ν, where ω is the fluctuation mode frequency, ν is the collision frequency, and adiabaticity $\omega/k_{\parallel}v_t$, where $k_{\parallel}$ is the parallel mode number and v_t is the species thermal velocity. Due to the usage of a Chapman-Enskog-like procedure and a moment approach (see part I) these equations exhibit the same structure as the classical Braginskii fluid equations[2] except that the transport coefficients (viscosity μ, thermal diffusivity χ, *etc.*) have been self-consistently generalized to the low collisionality (or even collisionless) regime, and the important linear wave-particle Landau damping effects are included. The establishment of this unified description of plasmas is mostly suitable for drift type microinstability studies. It not only greatly simplifies the theoretical analyses of plasma microinstabilities, but also enlarges the window of validity for present microturbulence theory models and facilitates the numerical simulation of plasma turbulence that is more realistic with respect to present tokamak operational regimes.

The main goal of this paper is to show how to use the unified equations to study various drift wave problems and how the resultant unified theories recover the previous results from collisional fluid analysis and collisionless kinetic analysis in the corresponding limits. In Section II we first review the fluid/kinetic equations developed in part I. A generalized perturbed Ohm's law which generalizes the widely used Hazeltine, Dobrott and Wang's result[3] to include the electron Landau damping effects will be derived in Section III. In the fluid limit ($\nu, \omega \gg k_{\parallel}v_t$) we recover Hazeltine *et al.*'s result; in the adiabatic limit we show that electron Landau damping replaces the Coulomb collision dissipative effect and resists the current flow. Section IV through Section VII discuss applications of this work to many existing branches of plasma microinstability theory. In Section IV the linear electron drift wave theory is developed using our unified equations. It is shown that the resultant dispersion equations (local and nonlocal) have exactly the same structure as the usual fluid ones except for the replacement of the Spitzer resistivity by the generalized pseudo-resistivity from Section III. These dispersion equations describe not only the dissipative electron drift instability, which is driven by the collisional drag force, but also the collisionless instability (universal instability) driven by the electron Landau damping mechanism. A similar situation is shown to happen in the ion drift wave analysis in Section V where the ion temperature gradient modes (η_i modes) are addressed. The well-known instability threshold values (η_c) for the fluid η_i branch ($\eta_c = 2/3$) and kinetic η_i branch ($\eta_c = 2$) are again recovered. Using the new uniformly valid dispersion relation we obtain the η_c as a function of the ion-ion collisionality and ion Landau damping strength. The electron electromagnetic modes are studied in Section VI where the generalized coupled eigenvalue equations (for $\tilde{\phi}$ the electrostatic potential and $\tilde{A}_{\|}$ the parallel vector potential) for micro-tearing type modes are derived. When the mode is purely magnetic, our dispersion relation easily reproduces the previous semi-collisional micro-tearing[4] and drift-tearing[3, 4] instabilities in the appropriate limits.

Since all nonlinear terms are retained in the three moment equations (though the closures are linearized), we can use them to study plasma microturbulence problems. In Section VII, we discuss the generalization of the Hasegawa-Wakatani two field ($\tilde{\phi}$ and $\tilde{n}$) equations[5] which have been widely used for electron dissipative drift wave turbulence studies.[6] Due to the complicated ω dependence in the closure coefficients, the direct application of the generalized three-field ($\tilde{\phi}$, $\tilde{n}$ and $\tilde{T}$) equations for arbitrary ω/ν and $\omega/k_{\|}v_t$ becomes difficult. A suggestion for simplification arising from one-pole approximations to the plasma dispersion functions[1] (Z and Z') is proposed. The net result is an extended plasma turbulence description which requires the inclusion of more time evolution equations (for the closure terms).

Although the entire work in this paper and its corollary part I have been developed for a simple sheared slab magnetic geometry, the idea is equally applicable to a toroidal geometry where magnetic trapped particle effects become another important means of accessing the plasma free energy. In Section VIII, the usual drift-kinetic equation is used to calculate the closure relations for a toroidal geometry. It is demonstrated that by including the trapped particle effects in the closure relations we can dramatically simplify the description of trapped particle modes. In the new description the usual separate fluid equations for trapped and untrapped components of a single plasma species can be avoided. The trapped particle effects are included via the closure relations. A simple example is given for the dissipative trapped electron mode. The summary and conclusions are given in Section IX.

II. REVIEW OF THE UNIFIED FLUID/KINETIC EQUATIONS

The closed set of the fluid/kinetic equations for a sheared slab magnetic geometry developed in part I of this work[1] can be summarized as follows. The three basic fluid moment equations for perturbed density ($\tilde{n}$), parallel flow ($\tilde{u}_{\parallel}$) and temperature ($\tilde{T}$) are (for both electrons and ions)

$$\left(\frac{\partial}{\partial t} + \tilde{\mathbf{V}}_E \cdot \nabla_\perp\right) \tilde{n} = -n_0 \nabla_\parallel \tilde{u}_\parallel - n_0 \nabla_\perp \cdot \tilde{\mathbf{V}}_p - \tilde{\mathbf{V}}_E \cdot \nabla n_0, \tag{1}$$

$$\begin{aligned} n_0 m \left(\frac{\partial}{\partial t} + \tilde{\mathbf{V}}_E \cdot \nabla_\perp\right) \tilde{u}_\parallel &= e n_0 \tilde{E}_\parallel - \nabla_\parallel \left[p - \frac{p_0}{2\Omega}\mathbf{b} \cdot (\nabla \times \tilde{\mathbf{u}}_\perp)\right] \\ &\quad -\mathbf{b} \cdot \nabla \cdot \tilde{\mathbf{\Pi}}_\parallel + \tilde{R}_\parallel, \end{aligned} \tag{2}$$

$$\begin{aligned} \frac{3}{2} n_0 \left(\frac{\partial}{\partial t} + \tilde{\mathbf{V}}_E \cdot \nabla_\perp\right) \tilde{T} &= -\frac{3}{2} n_0 \tilde{\mathbf{V}}_E \cdot \nabla T_0 - p_0 \nabla_\parallel \tilde{u}_\parallel - \nabla_\parallel \tilde{q}_\parallel \\ &\quad -p_0 \nabla_\perp \cdot \tilde{\mathbf{V}}_p, \end{aligned} \tag{3}$$

where $\tilde{\mathbf{V}}_E \equiv c\tilde{\mathbf{E}} \times \mathbf{B}/B^2$ is the usual $\tilde{\mathbf{E}} \times \mathbf{B}$ flow velocity and $\mathbf{b} \equiv \mathbf{B}/B$ is a unit vector along the magnetic field. The divergence of the polarization flow is given by

$$\nabla \cdot \tilde{\mathbf{V}}_p = \nabla_\perp \cdot \frac{\mathbf{b}}{\Omega} \times \left(\frac{\partial}{\partial t} + \tilde{\mathbf{V}}_E \cdot \nabla_\perp\right) \tilde{\mathbf{u}}_\perp - \frac{p_0}{n_0 m \Omega^2} \nabla_\perp^2 \nabla_\parallel \tilde{u}_\parallel. \tag{4}$$

The pressure gradient term in Eq. (2) should be understood as

$$\nabla_\parallel p = i k_\parallel \tilde{p} + \tilde{\mathbf{b}} \cdot \nabla (p_0 + \tilde{p}). \tag{5}$$

The extra part in the second term of the right side of Eq. (2) is a parallel vorticity induced pressure anisotropy, which comes from the reduction of the gyroviscous stress force.[7, 8, 9] The *linear* closure relations for the parallel stress force ($\mathbf{b} \cdot \nabla \cdot \tilde{\mathbf{\Pi}}_{\|}$) and heat flux ($\tilde{q}_{\|}$) are calculated in part I of this work by using a Chapman-Enskog-like approach and a moment approach. The simplified formulas are[1]

$$\mathbf{b} \cdot \nabla \cdot \tilde{\mathbf{\Pi}}_{\|e} = -n_0 m_e \mu_{\|e} \nabla_{\|}^2 \tilde{u}_{\|e} - l_e n_0 (\nabla_{\|} T_e)_1, \tag{6}$$

$$\mathbf{b} \cdot \nabla \cdot \tilde{\mathbf{\Pi}}_{\|i} = -n_0 m_i \mu_{\|i} \nabla_{\|}^2 \tilde{u}_{\|i} - l_i n_0 (\nabla_{\|} T_i)_1, \tag{7}$$

$$\tilde{q}_{\|e} = -h_e p_e \tilde{u}_{\|e} - n_0 \chi_{\|e} (\nabla_{\|} T_e)_1 + r_{\|} p_e (\tilde{u}_{\|e} - \tilde{u}_{\|i}), \tag{8}$$

$$\tilde{q}_{\|i} = -h_i p_i \tilde{u}_{\|i} - n_0 \chi_{\|i} (\nabla_{\|} T_i)_1, \tag{9}$$

where

$$(\nabla_{\|} T)_1 \equiv i k_{\|} \tilde{T} + \tilde{\mathbf{b}} \cdot \nabla T_0. \tag{10}$$

The electron parallel frictional force is given by

$$\tilde{R}_{\|e} = (\eta_s / 0.51) |e| n_0 \tilde{j}_{\|} + \frac{4}{5\sqrt{\pi}} n_0 m_e \nu_{ei} \frac{\tilde{q}_{\|e}}{p_e}, \tag{11}$$

where η_s is the Spitzer resistivity

$$\eta_s = 0.51 \frac{4}{3\sqrt{\pi}} \frac{m_e \nu_{ei}}{e^2 n_0},$$

and ν_{ei} (and later ν_{ii}) is the electron-ion (ion-ion) collision frequency.[2] In Eqs. (6)-(9) the pseudo-transport coefficients are given by ($j = e, i$):

$$\mu_{\|j} = -i \frac{2 v_{tj}}{5 k_{\|}} Z(\xi_1^j), \tag{12}$$

$$\chi_{\|j} = -i \frac{9 v_{tj}}{5 \pi k_{\|}} Z(\xi_3^j), \tag{13}$$

$$l_j \simeq h_j = -\frac{1}{5} Z'(\xi_2^j), \tag{14}$$

$$r_{\|} = -i \frac{48(3\sqrt{\pi} - 1)}{25 \pi \sqrt{\pi}} \frac{\nu_{ei}}{k_{\|} v_{te}} Z(\xi_4), \tag{15}$$

where $\mu_{\|}$ is the parallel flow viscosity coefficient; the l's are the coefficients of the stress forces induced by the perturbed parallel temperature gradients; $\chi_{\|}$ is the parallel thermal diffusivity coefficient; the h's are the heat pinch coefficients; and $r_\parallel$ is the coefficient of the electron frictional heat flux. The arguments (ξ's) in the Z and Z' functions are functions of mode frequency ω, collision frequency ν and $k_\parallel v_t$:

$$\begin{aligned}
\xi_1^j &= (3/5)(\omega + id_j\nu_{ji})/(k_\parallel v_{tj}),\\
\xi_2^j &= \sqrt{3/10}(\omega + ig_j\nu_{ji})/(k_\parallel v_{tj}),\\
\xi_3^j &= 36(\omega + ig_j\nu_{ji})/(25\pi k_\parallel v_{tj}),\\
\xi_4 &= 24(3\sqrt{\pi}-1)(\omega + ig_e\nu_{ei})/(25\pi k_\parallel v_{te}),
\end{aligned} \tag{16}$$

where d_j and g_j are constants:

$$d_e \simeq 0.90 + 0.64/z, \qquad d_i \simeq 0.32 \tag{17}$$

$$g_e \simeq 0.98 + 0.42/z, \qquad g_i \simeq 0.42 \tag{18}$$

with z being the ion charge state.

Asymptotic expressions of the pseudo-transport coefficients in both the fluid and adiabatic limits are very useful in the following analysis. They can be easily obtained by using the asymptotic expansion of the Z functions. The results are listed in Table 1.

The quasineutrality condition (which leads to a vorticity equation) that we will use in this paper is

$$\nabla \cdot \tilde{\mathbf{j}} = 0,$$

which, using the results obtained above, becomes

$$\begin{aligned}
&\nabla \cdot (\tilde{\mathbf{j}}_\parallel + \tilde{\mathbf{j}}_* + \tilde{\mathbf{j}}_p) = 0, \qquad (19)\\
&\tilde{\mathbf{j}}_\parallel = en_0(\tilde{u}_{\parallel i} - \tilde{u}_{\parallel e})\mathbf{b},\\
&\tilde{\mathbf{j}}_* = \frac{c}{B}\mathbf{b}\times\nabla(\tilde{p}_i + \tilde{p}_e) + \frac{c}{B}\tilde{\mathbf{b}}\times\nabla(p_i + p_e),\\
&\tilde{\mathbf{j}}_p = en_0\tilde{\mathbf{V}}_p.
\end{aligned}$$

As discussed in Ref. 1, this set of equations (1)-(11) are uniformly valid for arbitrary ratios of ω/ν and $\omega/k_\parallel v_t$. This unified feature allows us to study plasma drift instabilities of arbitrary collisionality and Landau damping strength uniformly within a fluid framework.

III. GENERALIZED PERTURBED OHM'S LAW

Generalizing the classical fluid Ohm's law $[\eta_s \mathbf{J} = \mathbf{E} + (1/c)\mathbf{V} \times \mathbf{B}]$ to the drift wave frequency regime (*i.e.*, $\nu \sim \omega \sim k_\| v_t$) is a key issue in plasma fluid theories. A widely used generalized Ohm's law is the Hazeltine, Dobrott and Wang's form[3] [HDW form, see Eq. (26)], which is valid for arbitrary ω/ν_{ei}. Unfortunately, due to the fluid ordering $\omega, \nu > k_\| v_t$ used in their derivation,[3] the HDW form is only valid in the fluid limit, or in the vicinity of a mode rational surface. The important electron Landau resonance effects are therefore missing in the HDW formula. This shortcoming can be overcome by using our unified fluid/kinetic equations. Using the electron parallel momentum equation (2) (where the finite Larmor radius effect induced scalar pressure anisotropy term can be neglected for electrons) together with the closure relations Eqs. (7), (8) and (11) we obtain a very general nonlinear perturbed parallel Ohm's law:

$$\begin{aligned} \hat{\eta}\tilde{j}_\| + \frac{m_e c}{e^2 n_0 B}[\tilde{\phi}, \tilde{j}_\|] &= \tilde{E}_\| + \frac{1}{en_0}(\nabla_\| p_e)_1 + \frac{1}{e}\kappa(\nabla_\| T_e)_1 \\ &+ \frac{m_e \nu_{ei}}{e}\left(\frac{4}{5\sqrt{\pi}}h_e + \frac{k_\|^2}{\nu_{ei}}\mu_{\|e} - i\frac{\omega}{\nu_{ei}}\right)\tilde{u}_{\|i} \\ &+ \frac{m_e c}{eB}[\tilde{\phi}, \tilde{u}_{\|i}]. \end{aligned} \tag{20}$$

Here, the generalized resistivity $\hat{\eta}$ is given by

$$\hat{\eta} = \eta_0 - i\frac{m_e \omega}{e^2 n_0} + \frac{m_e k_\|^2}{e^2 n_0}\mu_{\|e} - \frac{4}{5\sqrt{\pi}}\frac{m_e \nu_{ei}}{e^2 n_0}(r_\| - h_e), \tag{21}$$

with $\eta_0 = \eta_s/0.51$. This generalized resistivity extends the classical resistivity to include the contributions from electron inertia (the second term on the right side), viscous force ($\mu_{\|e}$), frictional heat flux effect ($r_\|$) and heat pinch (h_e) effects. In Eq. (20) the $\tilde{\mathbf{E}} \times \mathbf{B}$ nonlinear terms are denoted by the Poisson bracket

$$[\alpha, \beta] \equiv \mathbf{b} \cdot (\nabla\alpha \times \nabla\beta). \tag{22}$$

The first two terms on the right side of Eq. (20) are the usual parallel electric field and pressure gradient correction terms. The third $\nabla_\| T_e$ ("thermal force") term is from the electron thermal heat flux ($\chi_{\parallel e}$) and temperature gradient induced stress force (l_e) contributions expressed by the coefficient κ,

$$\kappa = \left(\frac{8}{5\sqrt{\pi}} \frac{\nu_{ei}}{v_{te}^2} \chi_{\parallel e} - l_e \right). \tag{23}$$

Note that since our closure relations are linearized, we cannot include the non-linear term from $(\nabla_\parallel T_e)_1$ as we did in Eq. (5). The last two terms in Eq. (20) are the ion contributions which can be neglected when $\tilde{u}_{\parallel i} \ll \tilde{u}_{\parallel e}$.

In the *fluid limit i.e.*, $\nu_{ei}, \omega \gg k_\parallel v_{te}$, the heat flux effects (terms with $\chi_\parallel$ and $r_\parallel$) become important while the viscous effects (terms with $\mu_\parallel$ and l) are small. By taking $k_\parallel \to 0$ we obtain

$$\hat{\eta} \to \frac{\eta_s}{0.51} \left(1 - i \frac{3\sqrt{\pi}}{4} \frac{\omega}{\nu_{ei}} - i \frac{6}{5\sqrt{\pi}} \frac{\nu_{ei}}{\omega + i g_e \nu_{ei}} \right), \tag{24}$$

$$\kappa \to \frac{2}{\sqrt{\pi}} \frac{i\nu_{ei}}{\omega + i g_e \nu_{ei}}. \tag{25}$$

Then, the generalized Ohm's law Eq. (20) can be reduced to the following form

$$\eta_s \tilde{j}_\parallel = \alpha_1 \tilde{E}_\parallel + \alpha_1 \frac{\tilde{\mathbf{B}}_\perp}{B} \cdot \left(\frac{\nabla p_e}{e n_0} + \alpha_2 \frac{\nabla T_e}{e} \right) \tag{26}$$

where

$$\begin{aligned} \alpha_1 &\equiv 0.51 \left[1 - \frac{i6\nu_{ei}/\omega}{5\sqrt{\pi}(1 + i g_e \nu_{ei}/\omega)} - \frac{3\sqrt{\pi}}{4} \frac{i\omega}{\nu_{ei}} \right]^{-1} \\ &\simeq 0.99 \frac{1 - 0.54 i\omega\tau_e}{1 - 2.98 i\omega\tau_e - 1.04\omega^2\tau_e^2} \qquad \text{(for } z = 1\text{)}, \end{aligned} \tag{27}$$

$$\begin{aligned} \alpha_2 &\equiv \frac{2i\nu_{ei}/\omega}{\sqrt{\pi}(1 + i g_e \nu_{ei}/\omega)} \\ &\simeq 0.80(1 - 0.54 i\omega\tau_e)^{-1} \qquad \text{(for } z = 1\text{)}. \end{aligned} \tag{28}$$

Equations (26), (27) and (28) are identical to the HDW formula [Eqs. (50)-(52) of Ref. 3], which was derived using a variational approach and solving the drift kinetic equation iteratively for large $\omega/k_\parallel v_{te}$. When $\nu > \omega$, further expansion of Eq. (26) leads to Hassam's formula[11]:

$$\begin{aligned} \eta_s[1 - i0.68(\omega/\nu_{ei})]\tilde{j}_\parallel &= \tilde{E}_\parallel + (e n_0)^{-1} (\nabla_\parallel p_e)_1 \\ &\quad + (0.80/e)(1 + i0.71\omega/\nu_{ei})(\nabla_\parallel T_e)_1. \end{aligned} \tag{29}$$

In contrast to the above formulas, since there is no assumption of large $\omega/k_{\parallel}v_t$ made in our Ohm's law Eq. (20), our formula remains valid for arbitrary $\omega/k_{\parallel}v_{te}$. Therefore, we can study the important electron Landau damping effects in the small $\omega/k_{\parallel}v_{te}$ adiabatic regime.

In the *adiabatic limit*, $\nu_{ei}, \omega \ll k_{\parallel}v_{te}$, we find that the viscous effects become dominant. Using Table 1 we have

$$\hat{\eta} \rightarrow \frac{m_e k_{\parallel}^2}{e^2 n_0}\mu_{\parallel e} \simeq \frac{2\sqrt{\pi}}{5}\frac{m_e k_{\parallel} v_{te}}{e^2 n_0}, \tag{30}$$

$$\kappa \rightarrow -l_e \simeq -2/5. \tag{31}$$

Therefore, Eq. (20) becomes (also neglecting the ion flow)

$$\left(\frac{2\sqrt{\pi}}{5}\frac{m_e k_{\parallel} v_{te}}{e^2 n_0}\right)\tilde{j}_{\parallel} = \tilde{E}_{\parallel} + \frac{1}{en_0}(\nabla_{\parallel}p_e)_1 - \frac{2}{5e}\kappa(\nabla_{\parallel}T_e)_1. \tag{32}$$

This result shows that away from the mode rational surface (where $k_{\parallel} = k_y x/L_s$ becomes large) the electron Landau damping effect replaces the Coulomb collisional dissipation. In the sheared slab geometry, this "Landau resistivity" increases linearly with the distance from the mode rational surface (x). The spatial behavior of $\hat{\eta}$ as a function of x for different ν_{ei}/ω values is shown in Fig. 1. First of all, the whole spatial region can be clearly divided into two regions: the fluid region where $x < x_e$ (x_e is defined by $\omega/k_{\parallel}(x_e)v_{te} = 1$) and the Landau damping region where $x > x_e$. In the fluid region the dynamic effect ($i\omega$) can enhance the real resistivity when ν_{ei}/ω becomes small. The change in the imaginary part of $\hat{\eta}$ also increases (becoming more negative) with decreasing ν_{ei}/ω. Second, in the adiabatic Landau region the Landau damping mechanism dominates the pseudo-resistivity even when ν_{ei}/ω is quite large (~ 10). We also find from Fig. 1 that the adiabatic region increases as the ratio ν_{ei}/ω decreases. For example, when $\nu_{ei}/\omega = 10$ the Landau damping starts to take over for $x \gtrsim x_e$, whereas when $\nu_{ei}/\omega = 0.1$ the Landau damping effects becomes significant for $x \lesssim 0.5x_e$.

The linearized form of Eq. (20) can be also expressed in terms of the electric and vector potentials $\tilde{\phi}$ and $\tilde{A}_{\parallel}$ by using the following electron density and temperature equations (from Eqs. (1), (3) and (8)):

$$\frac{\tilde{n}_e}{n_0} = -\frac{k_\parallel}{\omega}\frac{\tilde{j}_\parallel}{en_0} + \frac{\omega_{*e}}{\omega}\frac{e\tilde{\phi}}{T_e} + \frac{k_\parallel}{\omega}\tilde{u}_{\parallel i} \tag{33}$$

$$\begin{aligned} \frac{\tilde{T}_e}{T_e}\left(1 + i\frac{2}{3}\frac{k_\parallel^2}{\omega}\chi_{\parallel e}\right) &= \frac{\omega_{*e}\eta_e}{\omega}\left(\frac{e\tilde{\phi}}{T_e} + i\frac{2}{3}k_\parallel\chi_{\parallel e}\frac{e\tilde{A}_\parallel}{cT_e}\right) \\ &\quad -\frac{2}{3}(1 - h_e + r_\parallel)\frac{k_\parallel}{\omega en_0}\tilde{j}_\parallel \\ &\quad +\frac{2}{3}(1 - h_e)\frac{k_\parallel}{\omega}\tilde{u}_{\parallel i}. \end{aligned} \tag{34}$$

Here, $\tilde{\mathbf{b}} \cdot \nabla T_e = -i\omega_{*e}\eta_e e\tilde{A}_\parallel / c$ has been used. Substituting (33) and (34) into (20) and neglecting the ion parallel motion, we obtain an alternate form of the linear generalized Ohm's law

$$\begin{aligned} \left(\hat{\eta} + i\frac{m_e k_\parallel^2 v_{te}^2}{2e^2 n_0 \omega}H_2\right)\tilde{j}_\parallel &= -\left(1 - \frac{\omega_{*e}}{\omega} - \frac{3\omega_{*e}\eta_e}{2\omega}H_1\right) ik_\parallel\tilde{\phi} \\ &\quad + \left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}H_3\right) i\omega\frac{\tilde{A}_\parallel}{c}, \end{aligned} \tag{35}$$

where

$$H_1 \equiv \left(1 + \frac{8}{5\sqrt{\pi}}\frac{\nu_{ei}}{v_{te}^2}\chi_{\parallel e} - l_e\right)\left(1 + i\frac{2k_\parallel^2}{3\omega}\chi_{\parallel e}\right)^{-1}, \tag{36}$$

$$H_2 \equiv 1 + \frac{2}{3}(1 - h_e + r_\parallel)H_1, \tag{37}$$

$$H_3 \equiv 1 + \frac{8}{5\sqrt{\pi}}\frac{\nu_{ei}}{v_{te}^2}\chi_{\parallel e} - l_e - i\frac{2k_\parallel^2}{3\omega}\chi_{\parallel e}H_1. \tag{38}$$

This form will be used in the micro-tearing mode studies in Section VI.

IV. ELECTRON LINEAR DRIFT WAVES AND INSTABILITIES

To simplify the analysis we use the conventional cold ion, constant electron temperature and electrostatic mode assumptions. In this case, the quasineutrality condition Eq. (19) becomes

$$\frac{e^2 n_0}{T_e}\rho_s^2\frac{\partial}{\partial t}\nabla_\perp^2\tilde{\phi} - \nabla_\parallel\tilde{j}_\parallel = 0, \tag{39}$$

where $\rho_s = c_s/\Omega_i$, $c_s^2 = T_e/m_i$. From Eq. (33) the generalized Ohm's law becomes

$$\left(\hat{\eta} + i\frac{m_e k_\parallel^2 v_{te}^2}{2e^2 n_0 \omega}\right)\tilde{j}_\parallel = -(1-\omega_{*e}/\omega)ik_\parallel\tilde{\phi} + \left(\frac{k_\parallel^2 v_{te}^2}{2\omega^2} - 1 + \frac{k_\parallel^2}{i\omega}\mu_{\parallel e}\right)i\omega\frac{m_e}{e}\tilde{u}_{\parallel i}. \tag{40}$$

For cold ions the $\tilde{u}_{\parallel i}$ is mainly the ion acoustic motion:

$$\tilde{u}_{\parallel i} \simeq \frac{c_s^2 k_\parallel}{\omega}\frac{e\tilde{\phi}}{T_e}. \tag{41}$$

Using Eqs. (39)-(41) we obtain the following local ($\partial/\partial x \to 0$) dispersion equation for electron drift waves:

$$\left\{\omega b_s\left[1 - i\omega\hat{\eta}e^2 n_0/(k_\parallel^2 T_e)\right] + \omega - \omega_{*e} - k_\parallel^2 c_s^2/\omega\right\}\tilde{\phi} = 0, \tag{42}$$

with $b_s \equiv k_\perp^2 \rho_s^2$. Solving Eq. (42) for small growth rate modes and dropping the last ion sound wave term, we obtain

$$\omega_r = \frac{\omega_{*e}}{1+b_s}, \tag{43}$$

$$\gamma = \frac{2\omega_{*e}^2 b_s}{k_\parallel^2 v_{te}^2(1+b_s)}\frac{e^2 n_0}{m_e}Re(\hat{\eta}). \tag{44}$$

This result looks the same as the usual fluid result[6] except now the resistivity is a generalized one containing important dynamic and kinetic information. Therefore, dispersion equation (42) is valid for arbitrary collisionality and Landau damping strength. In the collisional limit, $\hat{\eta}$ reduces to the Spitzer resistivity (η_s). Then, we immediately obtain the well-known dissipative electron drift wave.[6, 12] In the adiabatic limit the resistivity arises from the electron Landau damping induced viscosity. Using Eq. (30) we then find

$$\frac{\gamma}{\omega_{*e}} \simeq \frac{4}{5}\sqrt{\pi}\frac{\omega_{*e}}{k_\parallel v_{te}}b_s. \tag{45}$$

This is the familiar collisionless drift instability (universal instability) result.[12] (The extra factor 4/5 in Eq. (45) is due to the assumption of constant temperature in the present simplified analysis.)

A generalized nonlocal electron drift wave eigenmode equation can be easily obtained from Eq. (42) by taking $\tilde{\phi}(\mathbf{x},t) = \tilde{\phi}(x)exp[i(k_y y - \omega t)]$. The result is

$$\rho_s^2 \left(\frac{\partial^2}{\partial x^2} - k_y^2\right)\tilde{\phi} - \frac{x^2}{x^2 - ix_R^2}\left(1 - \frac{\omega_{*e}}{\omega} - \frac{x^2}{x_s^2}\right)\tilde{\phi} = 0, \tag{46}$$

where

$$x_R^2 \equiv \frac{e^2 n_0 \omega}{T_e k_\parallel'^2}\hat{\eta}, \qquad x_s^2 = \frac{\omega^2}{k_\parallel'^2 c_s^2}. \tag{47}$$

As in the local case, Eq. (46) is identical to the dissipative electron drift eigenmode equation derived by Lui Chen, *et al.*,[13] except that the resistivity in x_R is a generalized one defined in Eq. (21). It is this replacement ($\eta_s \rightarrow \hat{\eta}$) that makes Eq. (46) valid for electron drift wave analyses in arbitrary collisionality and adiabaticity regimes. Detailed study of Eq. (46) requires a more sophisticated WKB treatment and numerical analysis,[13] and is beyond the scope of this paper.

V. ION LINEAR DRIFT TYPE MODES

Similar to the electron case, the generic equations for ion drift wave problems can be derived from Eqs. (1) - (9). When the electron-ion friction is not very large ($m_e \nu_{ei}/m_i < \omega_*$), the ion fluid/kinetic equation set decouples from that for the electrons. Substituting the closure equations (6) and (9) into (1) - (3), neglecting the vorticity induced pressure anisotropy in Eq. (2), we obtain the following three linearized equations for ion electrostatic instability studies (the species subscript i is suppressed for simplicity):

$$\omega\frac{\tilde{n}}{n_0} = (1+b)k_\parallel v_t + \omega_*\frac{e\tilde{\phi}}{T} - \omega b\left(\frac{e\tilde{\phi}}{T} + \frac{\tilde{p}}{p} - \frac{\tilde{\pi}_\parallel}{3p}\right), \tag{48}$$

$$(\omega + i\mu_\parallel k_\parallel^2)\frac{\tilde{u}_\parallel}{v_t} = \frac{1}{2}k_\parallel v_t\left[\frac{e\tilde{\phi}}{T} + \frac{\tilde{n}}{n} + \frac{\tilde{T}}{T}(1-l)\right], \tag{49}$$

$$\begin{aligned}(\omega + \frac{2}{3}ik_\parallel^2\chi_\parallel)\frac{\tilde{T}}{T} \quad = \quad & -\omega_*\eta\frac{e\tilde{\phi}}{T} + \frac{2}{3}(1-h+b)k_\parallel\tilde{u}_\parallel \\ & -\frac{2}{3}\omega b\left(\frac{e\tilde{\phi}}{T} + \frac{\tilde{p}}{p} - \frac{\tilde{\pi}_\parallel}{3p}\right),\end{aligned} \tag{50}$$

where $b \equiv (1/2)k_\perp^2\rho_i^2$ and $\tilde{\pi}_\parallel = (2/3)\mathbf{b}\cdot\nabla\cdot\tilde{\mathbf{\Pi}}_{\parallel i}$.

To illustrate how this set of unified equations works, we will discuss the ion temperature gradient modes (η_i modes). As usual, to simplify the analysis we assume the electrons are purely adiabatic. Thus, the quasineutrality condition leads to

$$\frac{\tilde{n}_i}{n_0} = \frac{\tilde{n}_e}{n_0} = \frac{e\tilde{\phi}}{\tau T_i}, \qquad \tau \equiv T_e/T_i. \tag{51}$$

Solving the closed equations (48)-(51) for the small finite Larmor radius effects ($k_\perp\rho_i \ll 1$) case, we obtain the following generalized dispersion equation

$$\omega^2\left(1+\frac{\omega}{\tau\omega_{*i}}\right)G_1 - \omega\frac{k_\parallel^2 v_{ti}^2}{2\omega_{*i}}G_2 + \frac{k_\parallel^2 v_{ti}^2}{2}\left[\eta_i - \frac{2}{3}(1-h_i)\right]G_3 = 0, \tag{52}$$

where

$$G_1 = 1 + i\frac{k_\parallel^2}{\omega}\mu_{\parallel i}, \tag{53}$$

$$G_2 = 1 + \frac{1}{\tau}\left[1 + \frac{2}{3}G_3(1-h_i)\right], \tag{54}$$

$$G_3 = (1-l_i)\left(1 + \frac{2}{3}i\frac{k_\parallel^2}{\omega}\chi_{\parallel i}\right)^{-1}. \tag{55}$$

The η_i mode can be easily addressed by taking $\omega \ll \omega_*$,[14] in which case Eq. (52) becomes

$$\omega^2 = -\frac{k_\parallel^2 v_{ti}^2}{2}\frac{G_3}{G_1}\left[\eta_i - \frac{2}{3}(1-h_i)\right]. \tag{56}$$

This is a generalized η_i mode dispersion relation valid for arbitrary collisionality and adiabaticity. In the collisional limit, $\nu_{ii} \gg \omega, k_\parallel v_{ti}$, we have $G_1, G_3 \to 1$ and the heat pinch term $h_i \to 0$. Then. Eq. (56) immediately reproduces the well-known fluid purely growing η_i mode result

$$\gamma = \frac{k_\parallel v_{ti}}{\sqrt{2}}(\eta_i - 2/3)^{1/2}. \tag{57}$$

Thus, the fluid threshold is $\eta_c = 2/3$. On the other hand, in the adiabatic limit, $k_\parallel v_{ti} \gg \omega, \nu_{ii}$, the ion Landau resonance effect becomes dominant. Using Table 1, we find

$$G_1 \rightarrow 1 + i\frac{2}{5}\sqrt{\pi}\frac{k_\parallel v_{ti}}{\omega},$$

$$G_3 \rightarrow -i\frac{\sqrt{\pi}}{2}\frac{\omega}{k_\parallel v_{ti}},$$

$$h_i \rightarrow \frac{2}{5}.$$

The dispersion equation (56) then becomes

$$\omega = i\frac{\sqrt{\pi}}{4}k_\parallel v_{ti}(\eta_i - 2), \tag{58}$$

which reproduces the kinetic η_i mode threshold[15, 16]

$$\eta_c = 2.$$

Therefore, our equation (56) does unify the fluid and kinetic features of the η_i modes. Using Muller's method for numerical root searching we obtain from Eq. (56) the threshold η_c as a function of ν_{ii} shown in Fig. 2. In the intermediate region where $\nu_{ii} \sim k_\parallel v_{ti}$, we find that the η_c is about 1.3.

VI. MICRO-TEARING MODE THEORY

Micro-tearing modes[3, 4, 17, 18, 19, 20, 21] are a set of electromagnetic modes driven by either magnetic free energy (Δ'–micro-tearing mode) or electron temperature gradient (η_e–drift tearing mode), with finite collisionality. As a possible candidate for plasma anomalous transport in tokamaks, these modes have often been invoked and intensively studied. In this section we first give the basic equations for these modes derived from our fluid/kinetic equation set. These equations are valid for arbitrary collisional regimes. In contrast to most of the previous works,[3, 4, 18, 20] the electron Landau damping effects are self-consistently included in our equations. We will prove that in the so-called semi-collisional regime the previous results (the Δ' driven and η_e driven modes) can be readily reproduced when the electron Landau damping effects are neglected.

Taking the perturbations as $(\tilde{A}_\|, \tilde{\phi}) \sim [A_\|(x), \phi(x)]\ exp[-i\omega t + ik_y y + ik_\| z]$, assuming cold ions and neglecting the ion acoustic effects, we have following three basic equations for electron electromagnetic mode studies.

Ampere's law:

$$\left(\frac{\partial^2}{\partial x^2} - k_y^2\right)\tilde{A}_\| = \frac{4\pi}{c}\tilde{j}_\|. \tag{59}$$

Quasineutrality condition [Eq. (39)]:

$$\omega\frac{e^2 n_0}{T_e}\rho_s^2\left(\frac{\partial^2}{\partial x^2} - k_y^2\right)\tilde{\phi} = -k_\|\tilde{j}_\|. \tag{60}$$

Generalized Ohm's law [Eq. (35)]:

$$\frac{1}{\hat{\sigma}}\tilde{j}_\| = -\left(1 - \frac{\omega_{*e}}{\omega} - \frac{3\omega_{*e}\eta_e}{2\omega}H_1\right)ik_\|\tilde{\phi} + \left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}H_3\right)i\omega\frac{\tilde{A}_\|}{c}, \tag{61}$$

where $\hat{\sigma}$ is a generalized conductivity

$$\hat{\sigma}(\omega, \nu_{ei}, k_\|(x)) \equiv \left(\hat{\eta} + i\frac{m_e k_\|^2(x) v_{te}^2}{2e^2 n_0 \omega}H_2\right)^{-1}. \tag{62}$$

The kinetic effects are included in the H's defined in Eqs. (36)-(38) and in $\hat{\eta}$. These coupled equations for $\tilde{\phi}$ and $\tilde{A}_\|$ are valid uniformly for all values of the ratios ω/ν_{ei} and $\omega/k_\| v_{te}$.

To simplify the analysis and show how to reproduce the previous result by Drake and Lee,[4] we restrict ourselves to the magnetic perturbations and neglect the electrostatic potential. This corresponds to the semi-collisional microtearing regime.[4] In this case, the eigenmode equation becomes

$$\frac{\partial^2 \tilde{A}_\|}{\partial x^2} - k_y^2\tilde{A}_\| = \frac{4\pi i\omega}{c^2}\hat{\sigma}\left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}H_3\right)\tilde{A}_\|. \tag{63}$$

Integrating Eq. (63) over the resistive layer (denoted by λ) and using the constant $\tilde{\psi}$ approximation we find

$$\begin{aligned}\Delta' &= \frac{1}{\tilde{A}_\|(0)}\frac{d\tilde{A}_\|}{dx}\bigg|_{-\lambda}^{\lambda} \\ &\simeq \frac{4\pi i\omega}{c^2}\int_{-\lambda}^{\lambda} dx\hat{\sigma}(x)\left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}H_3\right),\end{aligned} \tag{64}$$

where the discontinuity Δ' is the usual tearing mode stability parameter, and the assumption $\partial^2/\partial x^2 \gg k_y^2 \sim a^{-2}$ has been made with a being the plasma radius.

If the mode is well localized near the mode-rational surface, the fluid limit expressions can be used. They are

$$H_1 \simeq H_3 \simeq 1 + i\frac{2}{\sqrt{\pi}}\frac{\nu_{ei}}{\omega + ig_e\nu_{ei}}, \tag{65}$$

$$H_2 \simeq 1 + \frac{2}{3}H_3^2, \tag{66}$$

and Eq. (24) for $\hat{\eta}$. Extending λ to infinity to carry out the integration in Eq. (64), we find after some rearrangement

$$\gamma_k^2 \frac{k_0^2c^2}{2\pi^2\omega^3} i\hat{\eta}H_2 = -\left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}H_3\right)^2, \tag{67}$$

where following the notation of Drake and Lee[4]

$$\gamma_k \equiv \frac{k_y v_{te}\Delta' a}{2\sqrt{\pi}k_0^2 a L_s}$$

is the collisionless growth rate, $k_0^{-1} = c/\omega_{pe}$ is the collisionless skin depth, c is the speed of light and ω_{pe} is the electron plasma frequency. Dispersion relation (67) describes the local fluid micro-tearing modes for arbitrary ω/ν values and thus generalizes the Drake and Lee result.[4]

A. Collisional MHD limit case $\nu_{ei} \gg \omega \gg \omega_*$

Under this ordering, $\hat{\eta} \to \eta_s$, $H_2 \simeq 3.17$, and thus Eq. (67) produces a purely growing mode with growth rate

$$\begin{aligned} \gamma &= \left(\frac{H_2}{2\pi^2}\right)^{1/3} \gamma_k^{2/3}\eta_s^{1/3}(k_0c)^{2/3} \\ &\simeq 1.01\gamma_k^{2/3}\tau_e^{-1/3}, \end{aligned} \tag{68}$$

where τ_e is the electron-ion collision time of Braginskii.[2] This is identical to the semi-collisional micro-tearing mode of Drake and Lee:

$$\begin{aligned} \gamma|_{DL} &= \left(\frac{3\pi^{1/4}}{4\Gamma(11/4)}\right)^{2/3} \gamma_k^{2/3}\tau_e^{-1/3} \\ &\simeq 0.95\gamma_k^{2/3}\tau_e^{-1/3}. \end{aligned}$$

B. Collisional drift case $\nu_{ei} \gg \omega \sim \omega_* \gg \gamma_k^{2/3} \tau_e^{-1/3}$

In this case Eq. (67) reduces to

$$\omega - \omega_{*e} - \omega_{*e}\eta_e H_3 + \gamma_k \frac{k_0 c}{\sqrt{2}\pi\omega^{1/2}} \sqrt{i\eta_s H_2} = 0, \tag{69}$$

where $H_3 \simeq 1.81$. Choosing the unstable branch for $\sqrt{i}$ $(= -\sqrt{2}/2 - i\sqrt{2}/2)$ and supposing that the growth rate is smaller than the mode frequency, we reproduce the η_e driven drift-tearing mode[3, 4]

$$\omega_r \simeq \omega_{*2} \equiv \omega_{*e}(1 + 1.81\eta_e),$$

$$\gamma = 0.58\omega_{*e}\eta_e\omega_{*2}/\nu_{ei} + 0.28\gamma_k k_0 c \eta_s^{1/2} \omega_{*2}^{-1/2}.$$

When the collision frequency is small compared with the mode frequency (but still in the fluid region), Eq. (67) will also reproduce the inertially driven tearing mode discussed by Hazeltine, *et al.*[3]

We note that in above analysis the electron Landau damping effects have been totally neglected. As we discussed in Section III, the generalized resistivity will be dominated by the electron Landau damping effects in the $x > x_e$ region. Including the Landau damping effects properly requires a more careful treatment of the integral in Eq. (64). Numerical analysis and a more complicated study of the coupled equations (59) and (60) will then be needed. However, these are beyond the scope of this paper.

VII. NONLINEAR MODE COUPLING EQUATIONS — GENERALIZED HASEGAWA-WAKATANI EQUATIONS

The nonlinear fluid moment equations (1), (2) and (3) plus the linear fluid/kinetic closure relations (7)-(9) can be used to study plasma drift type microturbulence if the nonlinearities from the closure relations are not important. In this section we derive a set of three-field ($\tilde{\phi}$, $\tilde{n}$, $\tilde{T}$) mode coupling equations for electron drift wave turbulence studies. These equations generalize the Hasegawa-Wakatani two-field ($\tilde{\phi}$, $\tilde{n}$) turbulent equations[5] to arbitrary collisionality and adiabaticity.

For simplicity, we adopt the usual cold ion assumption. Using following normalization

$$t \rightarrow \frac{tc_s}{L_n}, \quad (\nabla_\perp, \nabla_\parallel) \rightarrow (\rho_s \nabla_\perp, L_n \nabla_\parallel),$$

$$(\phi, n, T, v_d, j, q) \rightarrow \frac{L_n}{\rho_s}\left(\frac{e\tilde{\phi}}{T_e}, \frac{\tilde{n}}{n_0}, \frac{\tilde{T}_e}{T_e}, \frac{v_d}{c_s}, \frac{\tilde{j}_\parallel}{en_0c_s}, \frac{\tilde{q}_{\parallel e}}{p_e c_s}\right),$$

$$\nu \rightarrow \frac{m_e}{m_i}\frac{L_n}{c_s}\frac{e^2 n_0}{m_e}\hat{\eta},$$

where $v_d \equiv -(cT_e/|e|B)L_n^{-1}$ is the electron diamagnetic flow velocity, we obtain the three-field (ϕ, n and T) mode coupling equations

$$\frac{\partial}{\partial t}\nabla^2\phi - \nabla_\parallel j = [\nabla_\perp^2\phi, \phi], \tag{70}$$

$$\frac{\partial n}{\partial t} + v_d\frac{\partial h}{\partial y} - \nabla_\parallel j = [n, \phi], \tag{71}$$

$$\frac{\partial T}{\partial t} + v_d\eta_e\frac{\partial \phi}{\partial y} - \frac{2}{3}\nabla_\parallel(j + q) = [T, \phi], \tag{72}$$

$$\nu j = -\nabla_\parallel[\phi - n - (1+\kappa)T]. \tag{73}$$

Equations (71)-(73) represent the vorticity, electron density, electron temperature and Ohm's law equations from Eqs. (19), (1), (3) and (20), respectively. The nonlinear Poisson brackets are defined in (22). An m_e/m_i order nonlinear term in the Ohm's law has been dropped. In Eq. (73), the generalized resistivity $\nu(\hat{\eta})$ and the temperature gradient induced current denoted by κ are given by Eqs. (21) and (23). The normalized parallel heat flux q in Eq. (72) is given by Eq. (8).

In the collisional fluid limit and constant temperature case, $\nu \rightarrow \nu_s$ (the normalized Spitzer resistivity), the Ohm's law becomes

$$\nu_s j = -\nabla_\parallel[\phi - n].$$

Then, our three-field equations reduce to the *Hasegawa-Wakatani* equations[5]:

$$\frac{\partial}{\partial t}\nabla^2\phi - \frac{1}{\nu_s}\nabla_\parallel^2(n - \phi) = [\nabla^2\phi, \phi]$$

$$\frac{\partial n}{\partial t} + v_d\frac{\partial \phi}{\partial y} - \frac{1}{\nu_s}\nabla_\parallel^2(n - \phi) = [n, \phi].$$

Although Eqs. (70)-(73) are valid for arbitrary collisionality and adiabaticity, some special care needs to be taken before these equations can be used for numerical simulations since the quantities ν, κ and q contain the mode frequency ω. To advance the equations explicitly in time, we need to do the inverse Laplace transformations. This is usually a subtle problem. Here, using a procedure suggested in Section IV. C of part I,[1] we suggest the following approximations.

Instead of using the generalized Ohm's law (73), we use the electron parallel momentum equation (2) for the parallel perturbed current ($\tilde{j}_\parallel \simeq -en_0\tilde{u}_{\parallel e}$). After writing this in terms of normalized variables we obtain

$$\begin{aligned}\frac{m_e}{m_i}\left(\frac{\partial j}{\partial t}+[\phi,j]\right) &= -\nabla_\parallel(\phi-n-T) \\ &\quad +\Pi-\bar{\nu}(j+3q/5),\end{aligned} \tag{74}$$

where

$$\bar{\nu}\rightarrow\frac{4}{3\sqrt{\pi}}\frac{m_e}{m_i}\frac{L_n}{c_s}\nu_{ei},\qquad \Pi\rightarrow\frac{L_n^2}{\rho_s p_e}\mathbf{b}\cdot\nabla\cdot\tilde{\mathbf{\Pi}}_{\parallel e}.$$

The closure terms Π and q are still functions of ω. They can be rewritten in a decomposed form

$$\mathbf{b}\cdot\nabla\cdot\tilde{\mathbf{\Pi}}_{\parallel e}=\Pi_V+\Pi_T,$$
$$\tilde{q}_{\parallel e}=Q_V+Q_T+Q_f.$$

Using the simplest one-pole approximations to the Z and Z' functions,[1] we find that the components satisfy the following evolution equations (in unnormalized form, see Ref. 1):

$$\left(\frac{\partial}{\partial t}+d_e\nu_{ei}+\frac{5}{3\sqrt{\pi}}k_\parallel v_{te}\right)\Pi_V=-\frac{2m_e}{3e}(k_\parallel v_{te})^2\tilde{j}_\parallel, \tag{75}$$

$$\left(\frac{\partial}{\partial t}+g_e\nu_{ei}+\frac{25\sqrt{\pi}}{36}k_\parallel v_{te}\right)Q_T=-\frac{5}{4}v_{te}^2 n_0 ik_\parallel\tilde{T}_e, \tag{76}$$

$$\left(\frac{\partial^2}{\partial t^2}+\lambda_1\frac{\partial}{\partial t}+\lambda_2\right)\begin{pmatrix}\Pi_T\\ Q_V\end{pmatrix}=-\frac{2}{3}(k_\parallel v_{te})^2\begin{pmatrix}n_0 ik_\parallel\tilde{T}_e\\ -p_i\tilde{j}_\parallel/en_0\end{pmatrix}, \tag{77}$$

$$\left(\frac{\partial}{\partial t} + g_e\nu_{ei} + \frac{25\sqrt{\pi}}{24(3\sqrt{\pi}-1)}k_\parallel v_{te}\right) Q_f = -\frac{2}{\sqrt{\pi}}\frac{\nu_{ei}T_e}{e}\tilde{j}_\parallel, \tag{78}$$

where

$$\lambda_1 \equiv 2g_e\nu_{ei} + \sqrt{5\pi/6}k_\parallel v_{te},$$

$$\lambda_2 = (g_e\nu_{ei}^t)^2 + \sqrt{5\pi/6}g_e\nu_{ei}k_\parallel v_{te} + (5/3)(k_\parallel v_{te})^2.$$

Equations (70), (71), (74), (75)-(78) provide closed fluid/kinetic equations for numerical simulation of electron drift turbulence. Unfortunately, as we can see, in order to include the Landau damping effects in a real time variable formulation, we need to solve additional evolution equations (75)-(78) for the closure terms. However, since the closure term evolution equations (75)-(78) are linear, the numerical treatment should be much easier than the other nonlinear equations. While this procedure provides one numerically viable approach, precisely how best to embed the Landau damping effects in an efficient numerical simulation scheme remains an unsolved problem.

VIII. PRELIMINARY CONSIDERATION OF TRAPPED PARTICLE EFFECTS

So far our discussion has been restricted to a sheared slab magnetic geometry. Other kinds of kinetic effects arise in toroidal plasmas, for example the magnetic trapped particle effects. Due to the new free energy sources they make accessible, the magnetic trapped particle effects play an important role in determining the plasma stability and turbulence of toroidal magnetized plasmas. Since trapped particle effects are basically single particle motion kinetic effects, the question of how to include them self-consistently in the fluid equations is still a challenging problem. The usual way of treating trapped particle problems is to separate a plasma species (electrons or ions) into trapped and untrapped species and have two sets of fluid equations for each components (*e.g.* the usual Kadomtsev and Pogutse approach,[22] or a fluid equation for the untrapped component and a kinetic equation for the trapped component). Therefore, the number of the equations needed for describing trapped particle modes is nearly doubled from that needed for ordinary drift wave instabilities. This makes the analytic or numerical treatment very difficult. In this section we will show that the separation of the fluid equations into trapped and untrapped components can be omitted when the trapped particle effects are included in the closure relations. To see how this procedure works, for simplicity we use the drift kinetic approach for calculating the closure relations instead of the full Chapman-Enskog-like approach.

The closure terms in Eqs. (2) and (3) can be written for an inhomogeneous magnetic field case as

$$\mathbf{b}\cdot\nabla\cdot\mathbf{\Pi}_{\parallel} = \frac{2}{3}\mathbf{b}\cdot\nabla\tilde{\pi}_{\parallel} - \tilde{\pi}_{\parallel}\mathbf{b}\cdot\nabla\ln B, \tag{79}$$

$$\nabla_{\parallel}\cdot\tilde{\mathbf{q}} = \mathbf{b}\cdot\nabla\tilde{q}_{\parallel} - \tilde{q}_{\parallel}\mathbf{b}\cdot\nabla\ln B, \tag{80}$$

where the kinetic definitions of $\tilde{\pi}_{\parallel}$ and $\tilde{q}_{\parallel}$ are

$$\tilde{\pi}_{\parallel} = m\int d^3v\left(v_{\parallel}^2 - \frac{v_{\perp}^2}{2}\right)\tilde{f},$$

$$\tilde{q}_{\parallel} = \frac{m}{2}\int d^3v v_{\parallel} v^2 \tilde{f}.$$

Using the usual drift kinetic equation, we can solve for the perturbed distribution function $\tilde{f}$ for the trapped and untrapped particles. The detailed derivation of $\tilde{\pi}_{\parallel}$ and $\tilde{q}_{\parallel}$ is given in Appendix A where we assume electrostatic modes and adopt a Krook collision model. Substituting the solutions (A.1) and (A.2) into the above definitions, we find that the parallel stress force is composed of three parts: trapped particle contribution $\tilde{\pi}_{\parallel t}$ Eq. (A.3), untrapped particle Landau resonance effect $\tilde{\pi}_{\parallel u}^{L}$ Eq. (A.7) and magnetic resonance effect $\tilde{\pi}_{\parallel u}^{M}$ Eq. (A.10). The total $\tilde{\pi}_{\parallel}$ (see Eq. (A.12) of Appendix A) is given by

$$\begin{aligned}\tilde{\pi}_{\parallel} &= \tilde{\pi}_{\parallel t} + \tilde{\pi}_{\parallel u}^{L} + \tilde{\pi}_{\parallel u}^{M} \\ &\simeq nmv_t^2\frac{e\tilde{\phi}}{T}\left[\left(1 - \frac{\omega_*}{\omega}\right)S_1 - \frac{\omega_*\eta}{\omega}S_2\right],\end{aligned} \tag{81}$$

where

$$S_l \equiv \sqrt{\epsilon}I_l + (1-\sqrt{\epsilon})(L_l + M_l), \qquad l = 1, 2.$$

Here, a small inverse aspect ratio $\epsilon \simeq a/R \ll 1$ has been assumed. The trapped particle contribution is denoted by I_l given in Eqs. (A.4), the Landau resonance effect L_l in Eqs. (A.8) and (A.9), and the magnetic drift resonance effect M_l in Eqs. (A.11). (Note that the magnetic drift resonance effect due to the trapped particles can be neglected due to the $\sqrt{\epsilon} \ll 1$ ordering.)

Similar expression for $\tilde{q}_{\|}$ is given in Eq. (A.13) except that due to the odd velocity moment there is no trapped particle generated parallel heat flux, just like the plasma current has no direct contributions from the trapped particles.

A. Dissipative trapped electron instability

To demonstrate the validity of our single species treatment for trapped particle modes, we discuss the simplest trapped-particle mode — the dissipative trapped electron mode.[23] For cold ions we have

$$\frac{\tilde{n}_i}{n_0} = \left(\frac{k_{\|}^2 c_s^2}{\omega^2} + \frac{\omega_{*e}}{\omega} - b_s \right) \frac{e\tilde{\phi}}{T_e}. \tag{82}$$

For electrons (as an entire species) the density and parallel momentum equations can be combined together to yield,

$$\frac{\tilde{n}_e}{n_0} \left(1 - \frac{k_{\|}^2 v_{te}^2}{2\omega^2} \right) = \frac{k_{\|}^2 v_{te}^2}{2\omega^2} \left(-\frac{e\tilde{\phi}}{T_e} + \frac{2}{3} \frac{\tilde{\pi}_{\|}}{p_e} \right) + \frac{\omega_{*e}}{\omega} \frac{e\tilde{\phi}}{T_e}, \tag{83}$$

where temperature fluctuations have been neglected. The kinetic effects (trapped electrons, electron Landau and magnetic drift resonance effects) have been all included in the closure parallel stress force term $\tilde{\pi}_{\|}$. For electron drift type modes, $\omega/k_{\|} \ll v_{te}$, using Eq. (81) and the quasineutrality condition $\tilde{n}_i = \tilde{n}_e$, we obtain the following dispersion relation:

$$1 + b_s - \frac{\omega_{*e}}{\omega} - \frac{4}{3} \left[\left(1 - \frac{\omega_*}{\omega} \right) S_1 - \frac{\omega_* \eta}{\omega} S_2 \right] = 0. \tag{84}$$

Here, terms of order $\omega^2/k_{\|}^2 v_{te}^2 \ll 1$ and $c_s^2/v_{te}^2 = m_e/m_i \ll 1$ have been dropped. When the untrapped particle contributions are neglected ($L = M = 0$), Eq. (A.6) yields the dispersion relation

$$1 + b_s - \frac{\omega_{*e}}{\omega} + i\epsilon^{3/2} \frac{\omega}{\nu_{ei}} \frac{4}{\sqrt{\pi}} \left[\left(1 - \frac{\omega_{*e}}{\omega} \right) - \frac{5\omega_{*e}\eta_e}{2\omega} \right] = 0. \tag{85}$$

To the lowest order we have

$$\omega_r = \omega_{*e}/(1 + b_s),$$

$$\frac{\gamma}{\omega_{*e}} \simeq \epsilon^{3/2}\frac{\omega_{*e}}{\nu_{ei}}\frac{4}{\sqrt{\pi}}\left(5\eta_e/2 + b_s\right).$$

This is the same as the typical kinetic result,[23] apart from some inconsequential differences in the numerical coefficients.

This simple example shows that analogous to the inclusion of Landau damping effects discussed in part I of this work, the trapped (and magnetic drift resonance) effects can be also included in the fluid equations through closure terms $\tilde{\pi}_\parallel$ and $\tilde{q}_\parallel$. Consequently, it is not necessary to separate the trapped and untrapped components and have separate fluid moment equations for them as is done in most treatments[22] of trapped-particle instabilities. Rather, in our model one need only treat the electrons and ions as whole species with regular fluid moment equations, with the trapped-particle effects entering through the closure relations.

IX. SUMMARY AND DISCUSSION

We have investigated various plasma microinstabilities using the set of fluid/kinetic moment equaitons developed in Part I of this work. A new generalized perturbed Ohm's law has been derived which is valid for arbitrary collisionality ω/ν_{ei} and adiabaticity $\omega/k_\parallel v_{te}$. Therefore, both the dynamic (finite $i\omega$) and kinetic (Landau damping) effects are uniformly included. In the fluid limits the new Ohm's law reproduces the previous Hazeltine, Dobrott and Wang result [Eq. (50) of Ref. 3].

The advantages of using the unified equations [Eqs. (1)-(11)] have been exhibited through applications to the major plasma drift type microinstabilities — the electron drift modes, η_i modes and micro-tearing modes. Due to its fluid characteristics, the analytic procedure is analogous to the usual analysis of the classical Braginskii equations. The result, however, is valid for any ratio of ω/ν_{ei}. The dynamic and kinetic effects are automatically included through the new pseudo-transport coefficients in the closure relations. As we have shown in this paper, generalized theories can be easily developed using our unified equations for the major plasma microinstabilities by following the usual fluid analysis. These generalizations and unifications not only build a bridge connecting the fluid and kinetic descriptions, but also opens many new areas for both linear and nonlinear plasma microinstability and turbulence studies. The application of these equations to various major microinstability problems described in this paper has just begun. Further effort needs to be devoted to solving the generalized equations that have been derived in this paper in order to explore the physics in the intermediate parameter regime.

The application of the unified equations to nonlinear electron drift instabilities and turbulence has been addressed. The resultant generalized Hasegawa-Wakatani equations [Eqs. (70)-(73)] involve electron temperature fluctuations and include electron Landau damping effects. An inverse Laplace transformation of the closure relations to the real time space is suggested for numerical simulation purposes. Application to nonlinear ion drift waves (η_i mode turbulence for example) can proceed in the same manner.

The additional kinetic effects induced by non-slab magnetic geometry effects (*e.g.* magnetic trapped particle, and magnetic drift resonance effects) have been studied using the same kinetic closure idea. Preliminary results show that in contrast to the usual separation treatment for trapped and untrapped components of a plasma species, our model need only treat the electrons or ions as whole species with regular fluid moment equations where the trapped particle effects entering through the closure relations. A great simplification is therefore achieved.

Through this work (parts I and II) we have shown that a unification of the fluid and kinetic descriptions of plasmas can be achieved by using the lowest order fluid moment equations and careful kinetic calculation of the needed moment closure relations. The unified equations possess the attractive simplicity and consistency features we desired. Since this work has been concerned primarily with a sheared slab geometry, the resultant equations can be considered as a generalization of the classical Braginskii equations.[2] Much work needs to be done to extend this work to include neoclassical effects (viscous damping, bootstrap current, etc.) and full trapped-particle effects within the Chapman-Enskog-like formalism. Also, how to self-consistently include the nonlinearity effects and other intrinsical kinetic effects (nonthermal high energetic particle effects, for example) in the Chapman-Enskog closure procedure is certainly another important and challenging topic. Nonetheless, the present work (parts I and II) has laid out the basic approach and demonstrated its power for a number of important plasma problems.

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APPENDIX A: CALCULATION OF $\tilde{\pi}_\parallel$ AND $\tilde{q}_\parallel$ USING THE DRIFT KINETIC APPROACH

The calculations for both $\tilde{\pi}_\parallel$ and $\tilde{q}_\parallel$ are similar. We only present the detailed algebra for the $\tilde{\pi}_\parallel$ evaluation, but give results for both closure equations.

Using the following conventional gyrokinetic equation:

$$\tilde{f} = -\frac{e\tilde{\phi}}{T} f_M + g,$$

$$\left(\omega - \omega_D - k_\parallel v_\parallel - iC + i\frac{v_\parallel}{qR}\frac{\partial}{\partial\theta}\right) g = \frac{ef_M}{T}(\omega - \omega_*)\tilde{\phi},$$

where $\mathbf{b}\cdot\nabla = ik_\parallel + (1/qR)(\partial/\partial\theta)$, θ is the poloidal angle, q the safety factor, and R the plasma major radius. Here, we have assumed electrostatic perturbations and small FLR effects. In low collisionality regimes we have

$$\omega_b \equiv \frac{v_t}{qR} \gg \omega, k_\parallel v_t, \nu.$$

Thus, the lowest order g does not depend on θ. Bounce-averaging the gyrokinetic equation for trapped particles leads to

$$g_t \simeq \frac{\omega - \omega_*}{\omega - \bar{\omega}_D + i\bar{\nu}_{eff}} f_M \frac{e\tilde{\phi}}{T}, \tag{A.1}$$

where a Krook collision model has been used with $C(g_t) = -\nu_{eff} g_t$, in which for trapped electrons $\nu_{eff} = \nu_{ei}(v)/\epsilon$, ($\epsilon \equiv r/R \ll 1$). The θ dependence in $\tilde{\phi}$ has been assumed to be small. The bounce average is defined by

$$\bar{A} \equiv \left(\int_{-\theta_0}^{\theta_0} d\theta \frac{qR}{|v_\parallel|} A\right)\left(\int_{-\theta_0}^{\theta_0} d\theta \frac{qR}{|v_\parallel|}\right)^{-1}.$$

Similarly, for the untrapped particles we have

$$g_u \simeq \frac{\omega - \omega_*}{\omega \mp k_\parallel |\bar{v}_\parallel| - \bar{\omega}_D + i\bar{\nu}} f_M \frac{e\tilde{\phi}}{T}, \tag{A.2}$$

where the transit average is identical to the bounce average except for the replacement of θ_0 by π.

The trapped particle contribution to $\tilde{\pi}_{\parallel}$ can thus be expressed as

$$\begin{aligned} \tilde{\pi}_{\parallel t} &= m \int d^3v \left(v_{\parallel}^2 - \frac{v_{\perp}^2}{2} \right) g_t \\ &= nmv_t^2 \frac{e\tilde{\phi}}{T} \left[\left(1 - \frac{\omega_*}{\omega} \right) I_1 - \frac{\omega_* \eta}{\omega} I_2 \right]. \end{aligned} \tag{A.3}$$

Here, we have defined

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \frac{1}{nv_t^2} \int_t d^3v \frac{(v_{\parallel}^2 - v_{\perp}^2/2)}{1 - \bar{\omega}_D/\omega + i\nu_{eff}/\omega} f_M \begin{pmatrix} 1 \\ v^2/v_t^2 - 3/2 \end{pmatrix}. \tag{A.4}$$

To evaluate the integrals, we transform the velocity space variables $(v_{\parallel}, v_{\perp}) \rightarrow (\lambda, E)$, where $E = mv^2/2$, $\lambda = \mu/E$ and $v_{\parallel} = \sigma v \sqrt{1 - \lambda B}$ with $\sigma \equiv sign(v_{\parallel})$. Then, we have

$$v_{\parallel}^2 - \frac{v_{\perp}^2}{2} = \frac{2E}{m} \left(1 - \frac{3}{2} \lambda B \right),$$

$$\int d^3v = \sum_{\sigma} \frac{\sqrt{2}\pi}{m^{3/2}} \int_0^{\infty} \sqrt{E} dE \int_0^{\lambda_c} \frac{B d\lambda}{\sqrt{1 - \lambda B}},$$

where $\lambda_c = 1/B_{max}$ corresponds to the turning points of banana orbits. The integration over λ in I_1 and I_2 can be readily worked out (neglecting the slight λ dependence in $\bar{\omega}_D$) to yield

$$\int_0^{\lambda_c} \frac{B d\lambda}{\sqrt{1 - \lambda B}} \left(1 - \frac{3}{2} \lambda B \right) = \frac{1}{2} \left(1 + \frac{2B}{B_{max}} \sqrt{1 - \frac{B}{B_{max}}} \right) \simeq \frac{1}{2}. \tag{A.5}$$

Thus, the integrals reduce to (for electrons)

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \simeq \frac{\sqrt{\epsilon}}{2\sqrt{\pi}} \int_0^{\infty} \frac{x^{3/2} e^{-x} dx}{1 - \bar{\omega}_D/\omega + i x^{-3/2} (\nu_{ei}/\epsilon\omega)} f_M \begin{pmatrix} 1 \\ x - 3/2 \end{pmatrix}.$$

The $\sqrt{\epsilon}$ factor has been introduced here based on the fact that the fraction of trapped particles is $\sim \sqrt{\epsilon}$.

For $\nu_{ei}/\epsilon \gg \omega \gg \bar{\omega}_D$, the I integrals can be performed to yield

$$\tilde{\pi}_{\parallel t}^e \simeq -\epsilon^{3/2} n m_e v_{te}^2 i \frac{\omega}{\nu_{ei}} \frac{3}{\sqrt{\pi}} \left[\left(1 - \frac{\omega_{*e}}{\omega} \right) - \frac{5\omega_{*e}\eta_e}{2\omega} \right] \frac{|e|\tilde{\phi}}{T_e}. \tag{A.6}$$

(Note however that the integrals I_1 and I_2 also contain the trapped particle magnetic drift resonances for $\omega = \bar{\omega}_D(x)$ and these resonance effects would become important for the opposite limit of $\nu_{ei}/\epsilon \ll \omega$.)

For untrapped particles, both Landau damping and magnetic drift resonance effects need to be included. Since the $k_\| v_\|$ and ν corrections to the ω_D term are small near the $\omega \simeq \omega_D$ resonance, the two resonances can be separated:[24]

$$\frac{1}{\omega - k_\| v_\| - \omega_D + i\nu} \simeq \frac{1}{\omega - k_\| v_\| + i\nu} + \frac{\omega_D}{\omega(\omega - \omega_D)}.$$

Therefore, we can write

$$\tilde{\pi}_{\|u} \simeq \tilde{\pi}^L_{\|u} + \tilde{\pi}^M_{\|u},$$

where the L superscript represents the Landau resonance part and the M superscript represents the magnetic drift resonance. Assuming that the energy dependence of the effective collision frequency can be omitted for the calculation of Landau resonance effects, we can use our slab geometry calculation result (see Ref. 1). Multiplied by the fraction of untrapped particles $(1 - \sqrt{\epsilon})$, the Landau resonance part becomes

$$\tilde{\pi}^L_{\|u} = (1 - \sqrt{\epsilon}) n m v_t^2 \frac{e\tilde{\phi}}{T} \left[\left(1 - \frac{\omega_*}{\omega}\right) L_1 - \frac{\omega_* \eta}{\omega} L_2 \right], \tag{A.7}$$

where

$$L_1 = \frac{1}{2} \frac{\omega}{k_\| v_t} (Z + \zeta Z'), \tag{A.8}$$

$$L_2 = \frac{1}{2} \frac{\omega}{k_\| v_t} \left(\zeta^3 Z' - \zeta Z' + \frac{Z}{2} - \zeta \right), \tag{A.9}$$

with $\zeta = (\omega + i\nu)/k_\| v_t$ being the argument of the Z and Z' functions.

The magnetic drift resonance part can be also similarly written as

$$\tilde{\pi}^M_{\|u} = (1 - \sqrt{\epsilon}) n m v_t^2 \frac{e\tilde{\phi}}{T} \left[\left(1 - \frac{\omega_*}{\omega}\right) M_1 - \frac{\omega_* \eta}{\omega} M_2 \right], \tag{A.10}$$

with

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} \equiv \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{\bar{\omega}_D x^{3/2} e^{-x} dx}{\omega - \bar{\omega}_D} f_M \begin{pmatrix} 1 \\ x - 3/2 \end{pmatrix}. \tag{A.11}$$

The total $\tilde{\pi}_\parallel$ is thus the sum of its three parts:

$$\begin{aligned}\tilde{\pi}_\parallel &= \tilde{\pi}_{\parallel t} + \tilde{\pi}^L_{\parallel u} + \tilde{\pi}^M_{\parallel u} \\ &\simeq nmv_t^2 \frac{e\tilde{\phi}}{T}\left[\left(1 - \frac{\omega_*}{\omega}\right) S_1 - \frac{\omega_* \eta}{\omega} S_2\right], \end{aligned} \quad (A.12)$$

where

$$S_l \equiv \sqrt{\epsilon} I_l + (1 - \sqrt{\epsilon})(L_l + M_l), \qquad l = 1, 2.$$

The $\tilde{q}_\parallel$ equation is much simpler. Since $\tilde{q}_\parallel$ is an odd $v_\parallel$ moment, trapped particles have no net effect on it, just like the plasma current has no direct contributions from the trapped particles. The untrapped particle magnetic resonance effect will also vanish (when the boundary-layer effect[24] is ignored ω_D does not depend on $v_\parallel$), since the particles moving in the opposite directions carry the same amount of heat. Therefore, the main contribution to $\tilde{q}_\parallel$ will come from the untrapped particle Landau damping effect. That is, we have

$$\begin{aligned}\tilde{q}_\parallel &\simeq \tilde{q}^L_{\parallel u} \\ &= (1 - \sqrt{\epsilon}) p v_t \frac{e\tilde{\phi}}{T}\left[\left(1 - \frac{\omega_*}{\omega}\right) K_1 - \frac{\omega_* \eta}{\omega} K_2\right], \end{aligned} \quad (A.13)$$

where again the result from a slab geometry calculation[1] has been used to yield

$$K_1 = \frac{1}{2}\frac{\omega}{k_\parallel v_t}(\zeta^2 Z' - 3Z'/2 - 1),$$

$$K_2 = \frac{1}{2}\frac{\omega}{k_\parallel v_t}(\zeta^4 Z' - 2\zeta^2 Z' - 3\zeta Z/2 - \zeta^2 - 1).$$

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Tables

Figures

	Fluid limit $(\omega, \nu \gg k_\parallel v_t)$	*Adiabatic limit* $(\omega, \nu \ll k_\parallel v_t)$
$\mu_{\parallel j}$	$i\frac{2}{3}\frac{v_{tj}^2}{\omega + i d_j \nu_{ji}}$	$\frac{2\sqrt{\pi}}{5}\frac{v_{tj}^2}{k_\parallel v_{tj}}$
$\chi_{\parallel j}$	$i\frac{5}{4}\frac{v_{tj}^2}{\omega + i g_j \nu_{ji}}$	$\frac{9}{5\sqrt{\pi}}\frac{v_{tj}^2}{k_\parallel v_{tj}}$
l_j, h_j	$-\frac{2}{3}\frac{k_\parallel^2 v_{tj}^2}{(\omega + i g_j \nu_{ji})^2}$	$\frac{2}{5}$
$r_\parallel$	$\frac{2}{\sqrt{\pi}}\frac{i\nu_{ei}}{\omega + i g_e \nu_{ei}}$	$\frac{48}{25\pi}(3\sqrt{\pi} - 1)\frac{\nu_{ei}}{k_\parallel v_{te}}$

Table 1: Asymptotic expressions for the pseudo-transport coefficients. The subscript $j = e$ for electrons and $j = i$ for ions. The parameters d and g are given by Eqs. (17) and (18).

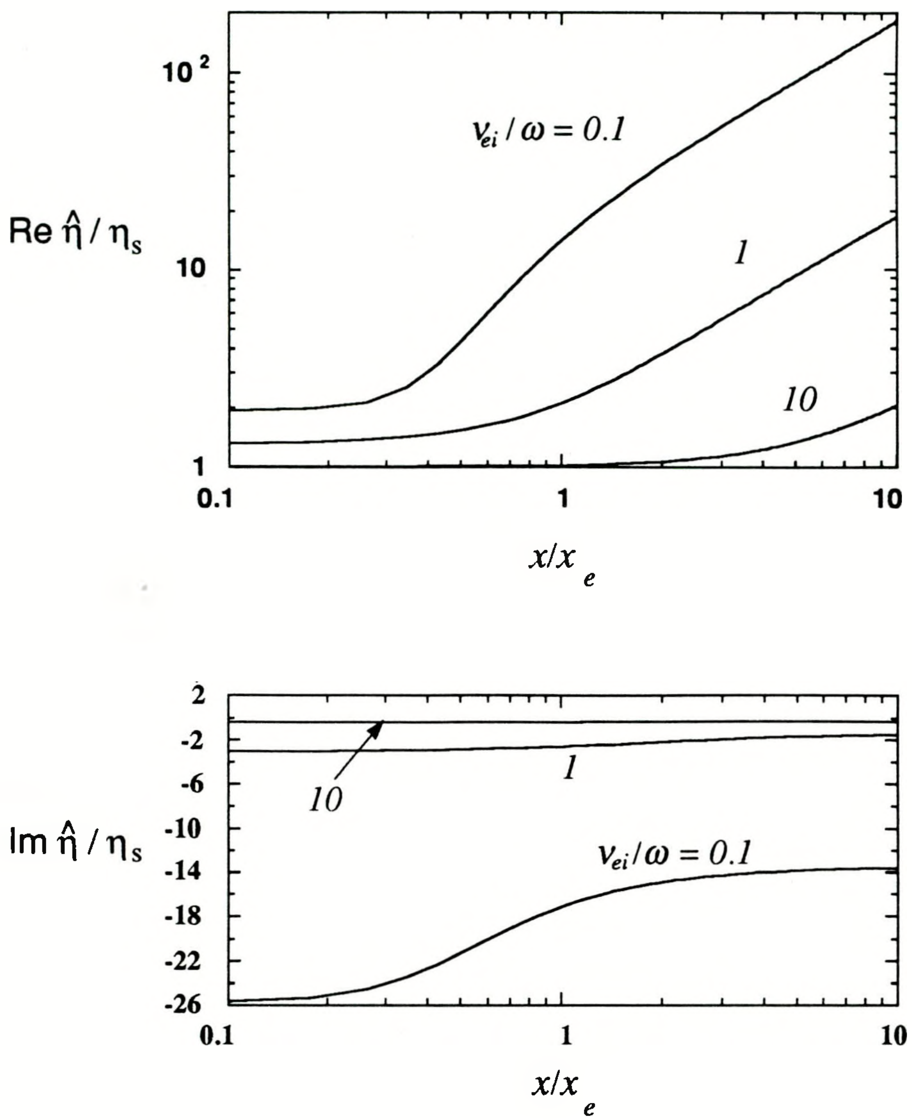

Fig. 1: The generalized resistivity $\hat{\eta}$ as a function of x (distance from the mode rational surface). Here, η_s is the Spitzer resistivity, $x_e \equiv \omega L_s / k'_{\|} v_{te}$, and the results for three different ratios of ν_{ei}/ω are shown.

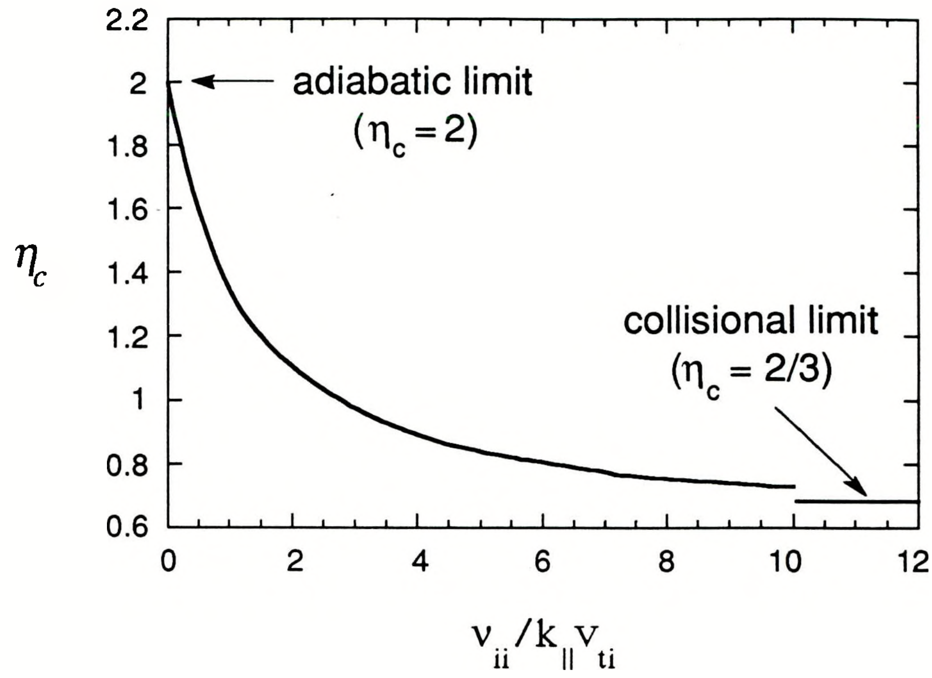

Fig. 2: The η_i mode threshold value η_c as a function of the ion collision frequency ν_{ii} normalized by $k_{\parallel}v_{ti}$.