LOW FREQUENCY NOISE IN FREELY SUSPENDED TIN FILMS AT THE SUPERCONDUCTING TRANSITION

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Abstract

~~We have measured~~ the spectral density, $S_V(f)$, of the voltage noise across 1-D current-biased tin films at the superconducting transition. Each film was freely suspended between two thermal clamps a distance L apart in a vacuum can. A thin layer of lead was evaporated on the outer portion of the films to leave an uncoated middle region of length ℓ. $S_V(f)$ was flat at frequencies below $f_L \approx D/L^2$, where D is the thermal diffusivity. At frequencies between f_L and $f_\ell \approx (L/\ell)^2 f_L$ the slope was typically -0.8, while at frequencies above f_ℓ the slope was somewhat less steep than -1.5. The shape and magnitude of $S_V(f)$ were in good agreement with an equilibrium temperature fluctuation model in which the temperature fluctuations are spatially uncorrelated. Measurements of the autocorrelation function also strongly supported this model. These results are in contrast with those obtained for normal films and films at the superconducting transition supported by substrates, for which a model was required with spatially correlated fluctuations. We conclude that the 1/f noise for films on substrates is mediated by an interaction between the substrate and the film.

INTRODUCTION

Voss and Clarke[1] used a model involving equilibrium temperature fluctuations to account for the 1/f voltage noise observed at room temperature in metal films deposited on glass substrates. The temperature fluctuations generate resistance fluctuations that give rise to voltage fluctuations with a mean square value $\langle(\Delta V)^2\rangle = I^2(dR/dT)^2 k_B T^2/C_V$, where I is the constant bias current, $(1/R)dR/dT$ is the temperature coefficient of resistance, and C_V is the heat capacity of the film. This model was supported by the observation that the spectral density of the voltage noise, $S_V(f)$, was proportional to $I^2(dR/dT)^2/\Omega$, where Ω is the volume of the film. Furthermore, at a given frequency the 1/f noise was found to be spatially correlated over a distance $\lambda(f) \approx (D/f)^{\frac{1}{2}}$, where D is the thermal diffusivity of the film. The usual Langevin treatment[2] of temperature fluctuations leads to the equation

$$\frac{\partial T(\vec{r},t)}{\partial t} - D\nabla^2 T(\vec{r},t) = \frac{1}{c_V}\vec{\nabla}\cdot\vec{F}(\vec{r},t), \qquad (1)$$

where c_V is the specific heat, and $F(\vec{r},t)$ represents a fluctuating energy flux of the form $\langle\vec{F}(\vec{r}+\vec{s},\, t+\tau)\cdot\vec{F}(\vec{r},t)\rangle = (2\pi)^3 F_0^2\delta(\vec{s})\delta(\tau)$. The resulting equal-time temperature fluctuations are spatially uncorrelated, $\langle\Delta T(\vec{r}+\vec{s},\, t)\Delta T(\vec{r},t)\rangle \propto \delta(s)$. Unfortunately, this formulation does not lead to a 1/f spectral density.

Voss and Clarke[1] therefore proposed an alternative treatment leading to the equation

$$\frac{\partial T(\vec{r},t)}{\partial t} - D\nabla^2(\vec{r},t) = \frac{1}{c_V} P(\vec{r},t), \qquad (2)$$

where $P(\vec{r},t)$ represents fluctuating sources and sinks of energy, and is of the form

$\langle P(\vec{r} + \vec{s}, t + \tau)P(\vec{r},t)\rangle = (2\pi)^3 P_0^2 \delta(\vec{s})\delta(\tau)$. The resulting equal-time temperature fluctuations are correlated, $\langle \Delta T(\vec{r} + \vec{s}, t)\Delta T(\vec{r},t)\rangle \propto |s|^{-1}$. For a rectangular region of a large body Eq. (2) leads to a range of frequencies $D/L^2 \ll f \ll D/w^2$ where the spectral density is proportional to $1/f$ (L and w are the length and width of the region). Furthermore, the magnitude of the observed noise was in good agreement with the predictions of the P-model. However, difficulties remain with the model: the physical origin of P is unclear; if the fluctuations are assumed to be intrinsic to the metal, one finds that the magnitude of P_0 depends on the volume of the sample; and, experimentally, there is no change in slope of the spectral density at a frequency of order D/L^2, as expected. Nevertheless, a final set of experiments further supported the P-model. An alternative way to determine $S_T(f)$ is to perturb the system from equilibrium and to measure the subsequent change of the spatially averaged temperature, $\bar{T}(t)$. For the appropriate perturbation the change represents the autocorrelation function, $c_T(\tau)$, which is related to $S_T(f)$ by

$$S_T(f) = 4\int_0^\infty c_T(\tau)\cos(2\pi f\tau)d\tau. \tag{3}$$

The autocorrelation function is normalized by setting $c_T(0) = k_B T^2/C_v$. It can be shown[1] that if a delta-function of power is applied to the sample the cosine transform of the change in $\bar{T}(t)$ is equal to the spectral density of the fluctuations calculated from Eq. (1), whereas if a step-function of power is applied the transform is equal to the spectral density calculated from Eq. (2). It was found that the cosine transform of the response to a step-function was in excellent agreement with the measured spectral density, implying that the P-model describes the temperature fluctuations.

An unresolved problem in this work was the role of the substrate. The model calculations were performed for a thermally homogeneous medium, whereas in the experiments the substrate and film had very different thermal properties. To investigate the properties of a system that had well-defined thermal properties, we performed experiments on freely-suspended tin films at the superconducting transition. In contrast to the films on substrates, the spectral density of the noise was in excellent agreement with the $\vec{\nabla}\cdot\vec{F}$-model.

THEORY

A film of length L is suspended between two thermal clamps at temperature T_0 (Fig. 1). The transverse dimensions of the film are much smaller than $(D/f)^{\frac{1}{2}}$ at frequencies of interest. We wish to find the spectral density of the fluctuations in the average temperature of the length ℓ, $\bar{T}(t) = \ell^{-1}\int_{-\ell/2}^{\ell/2} T(x,t)dx$, subject to the constraint $T(-L/2, t) = T(L/2, t) = T_0$. The spectral density can be calculated from a 1-D version of Eq. (1) by noting that the boundary condition allows only discrete k-values: $k_n = (n\pi/L)$, n = 1, 2, ... The allowed modes associated with n even are of the form $\sin(n\pi x/L)$, and do not contribute to $\bar{T}(t)$; the allowed modes associated with n odd are of the form $\cos(n\pi x/L)$. The spectral density is given by

$$S_T(f) = \frac{32\, k_B T^2 D}{\ell^2 L c} \sum_{n_{odd}} \frac{\sin^2(n\pi\ell/2L)}{(n^2\pi^2 D/L^2)^2 + (2\pi f)^2}. \tag{4}$$

$S_T(f)$ (Fig. 2) is white at frequencies below $f_L \approx D/L^2$, and varies as $f^{-3/2}$ at frequencies greater than $f_\ell \approx D/\ell^2$. In the range $f_L \ll f \ll f_\ell$, $S_V(f)$ varies as $f^{-1/2}$ for $L/\ell \gtrsim 100$, while for $L/\ell \lesssim 10$ the slope is considerably steeper.

Similarly, one can obtain $S_V(f)$ for the P-model from Eq. (2). $S_V(f)$ is white at frequencies below f_L, varies as f^{-2} at frequencies greater than f_ℓ, and, for large L/ℓ, varies as $f^{-3/2}$ for $f_L \ll f \ll f_\ell$.

EXPERIMENT

We performed experiments on a total of 15 samples, using both pure tin and tin with 3% indium. The In-doped samples were 30 mm long, 50 μm wide, and 1.5 μm thick. The lead coating (Fig. 1) was 0.1 μm thick. The tabs at the ends of each sample were fastened down with superconducting clamps to two copper blocks that were attached to but electrically isolated from a copper plate. The copper plate, which had a long thermal time constant, was suspended in a vacuum can immersed in liquid helium at about 1.7 K. The temperature of the plate was raised by electrical heating to be within the width of the superconducting transition of the tin film. A current I_1 was applied to the film, and the voltage fluctuations measured with a dc SQUID voltmeter.[3] The output of the voltmeter was digitized, and stored in a PDP-11 computer that subsequently computed $S_V(f)$.

We measured the noise spectral densities of 15 samples. In all cases $S_V(f)$ was proportional to the square of the current through the film, provided that self-heating was not significant, and to $(dR/dT)^2$. Figure 3 shows three representative power spectra with $\ell = 3.2$ mm (A) and $\ell = 6.4$ mm (B and C). In each case the spectral density is white below $f_L \approx 2$ Hz, and a second knee is evident at $f_\ell \approx (L/\ell)^2 f_L$. The heavy lines are calculated from Eq. (3) with the value of D deduced from f_L, typically 30 $cm^2 s^{-1}$, and with the handbook value of the specific heat. The slopes below f_L and between f_L and f_ℓ are in excellent agreement with the theory. The slopes above f_ℓ are less steep than predicted, and consequently the knees at f_ℓ are less pronounced than expected. The slopes are never steeper than the prediction of the $\vec{\nabla}\cdot\vec{F}$-model. The slopes are therefore in considerably greater disagreement with the predictions of the P-model, for which the slopes above f_L are steeper than those of the $\vec{\nabla}\cdot\vec{F}$-model. In the frequency range investigated the measured spectral densities are within a factor 2 or 3 of the values predicted by Eq. (3).

The experimental results provide strong evidence that the observed voltage noise is generated by thermal fluctuations. In addition, the measured power spectra agree much more closely with the $\vec{\nabla}\cdot\vec{F}$-model than with the P-model. To further distinguish between the two models we applied a step-function in power to a sample with $\ell = 2.2$ mm, and measured the subsequent change in the average temperature with time. We computed the spectral densities from the cosine transforms of this transient and of its time-derivative. The step-function in power was applied by suddenly decreasing the bias current in the film. The feedback loop of the SQUID was unable to track the sudden change in voltage, and instantaneously become unlocked. The SQUID feedback loop then recovered lock, and the subsequent voltage change across the film was measured in the usual way. Because the dissipation in the film had been lowered the temperature of the film decayed to a lower value on the superconducting transition. The resulting voltage decay was measured. The film resistance was very nearly a linear function of temperature along the decay path. When the decay was complete, we measured $S_V(f)$. The spectral density $S_T(f) = S_V(f)/I_1^2(dR/dT)^2$ is plotted in Fig. 4. The measured decay curve was normalized by setting $c_T(0) = k_B T^2/C_V$. The cosine transforms of this curve and of its normalized time-derivative are shown in Fig. 4. The shape and magnitude of the cosine transform of the delta-function response are in excellent agreement with the measured spectral density, $S_T(f)$, whereas the shape and magnitude of the cosine transform of the step-function response differ substantially from $S_T(f)$. We conclude that the delta-function response has the shape of the true autocorrelation function, and that $\vec{\nabla}\cdot\vec{F}$ is the appropriate driving term for the diffusion equation.

DISCUSSION

The measured power spectra of the freely suspended films correspond more nearly to the predictions of the $\nabla \cdot F$-model than to the predictions of the P-model. In addition, the spectral density given by the cosine transform of the response to a delta-function of power is in excellent agreement with the measured spectral density. This result is in sharp contrast with the work of Voss and Clarke[1] on films on substrates, where the cosine transform of the response to a step-function of power was in excellent agreement with the measured spectral density. We conclude that the 1/f noise in metal films on substrates must be strongly influenced by an interaction between the film and the substrate. This claim is further supported by the observation that a very thin layer of aluminum between the film and the substrate can dramatically change the spectral density of noise in tin films at the superconducting transition.[3] The mechanism of this interaction is unknown.

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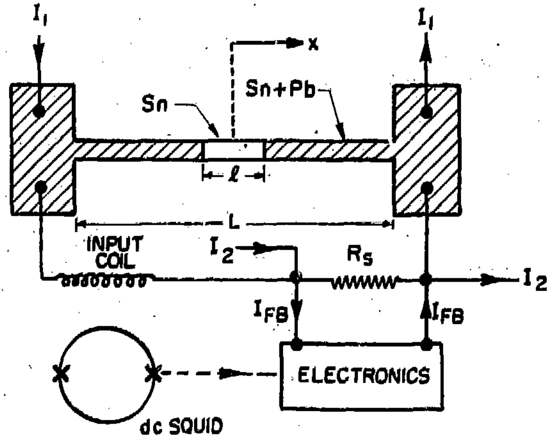

Fig. 1. Configuration of tin film and four-terminal measurement circuit.

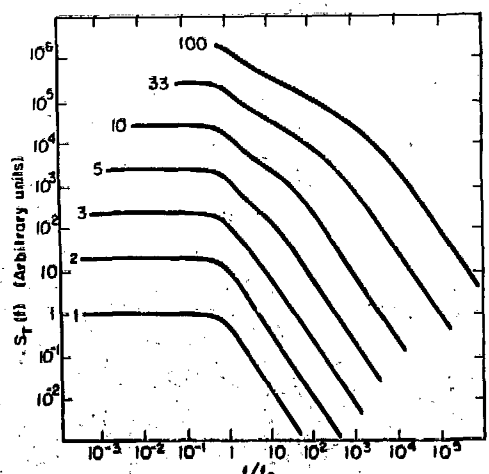

Fig. 2. Computed power spectra for the configuration of Fig. 1 for several values of L/ℓ, with frequency normalized to $f_0 = \pi D/2L^2$. For clarity, as L/ℓ increases each spectrum has been displaced upwards by one decade relative to the spectrum below.

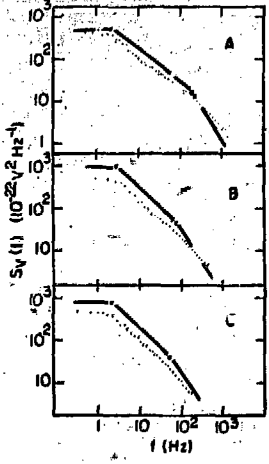

Fig. 3. Spectral densities for 3 samples: For A, ℓ = 3.2 mm, and for B and C, ℓ = 6.4 mm. The solid lines are the predictions of Eq. (4).

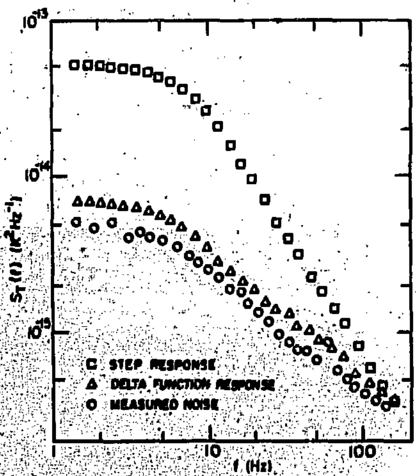

Fig. 4. $S_T(f)$ obtained from direct measurement (o), and cosine transforms of response to step-function (□) and delta-function (Δ).