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AMPLIFIED-RESPONSE-SPECTRUM ANALYSIS OF SODIUM-WATER REACTION PRESSURE WAVES

by

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1.0 INTRODUCTION

At the semi annual Large Leak Sodium-Water Reaction Test and Analysis Review Meeting held at GE/ARSD in August 1981 (Reference 1), GE was asked by DOE to prepare a plan for upgrading and validating the Argonne National Laboratory (ANL) developed SWAAM I sodium water reaction computer code (Reference 1). To emphasize the need for the work outlined in that plan (Reference 2), GE has evaluated the SWAAM I pressure history predictions of the LLTR Series II A-2 test using an Amplified Response Spectrum analysis (ARS) technique.

GE has determined, in its CRBRP-IHTS piping dynamic stress analysis work, that the piping loads created by the pressure wave produced during double rupture disc assembly operation are design limiting. Therefore, a SWR code which fails to adequately characterize* the pressure wave produced by double rupture disc assembly operation should not be used to analytically design the piping for LMFBR/Intermediate Heat Transport Systems which use double rupture discs.

This report deals with a frequency spectrum evaluation of the SWAAM I predicted double rupture disc assembly operation pressure wave generated in the LLTR Series II A-2 test. It also evaluates the same wave predicted by the TRANSWRAP II code and the pressure wave actually measured upstream of the rupture disc assembly by the test instrumentation in Test A-2.

The SWAAM I and TRANSWRAP II codes currently use the same analytical model** to characterize the rupture discs until the disc strikes the knife edges. Thereafter, the SWAAM I code relies on analytical techniques to characterize the phenomena, whereas the TRANSWRAP II code uses empirical parameters based on A-2 test data to represent the disc behavior (See Figure 1).

Any differences in the predicted dynamic pipe loads caused by double rupture disc assembly operation, using the forcing functions predicted by the codes can, therefore, be traced to this difference.

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* To produce conservative yet realistic piping load predictions for the design base sodium water reaction transient.

** The elastic-plastic model developed at ANL for the SWAAM I Code.

Instead of directly comparing the pressure waves predicted by the computer codes with the test data, the comparisons in this report are based upon the effects of the wave on the IHTS piping. Direct comparisons of forcing functions usually involve comparing the peak pressures only, but the frequency content of the wave is as or more important than amplitude in assessing the effects of a forcing function on dynamic piping loads. Amplified Response Spectrum Analysis described in Section 4.3 was used in this study as the means of evaluating forcing functions from the viewpoint of the affected piping.

This report deals only with the effects that double rupture disc assemblies have on LMFBR-IHTS axial piping loads. Since those phases of rupture disc operation which display the most non-linear behavior are associated with the use of double rupture disc assemblies, single rupture disc systems can probably be modeled adequately using analytical techniques only.

2.0 CONCLUSIONS

1. The SWAAM I SWR computer code, as it is presently constituted, will underpredict peak IHTS piping axial dynamic loads resulting from double rupture disc assembly operation. The under-prediction varies from 20 to 500 percent in the LMFBR-IHTS pipe component natural frequency range 10 to 30 Hz for the A-2 test.

2. The TRANSWRAP II SWR Code will overpredict these loads for piping components with natural frequency below 15 Hz and above 20 Hz. The over-prediction are from 0 to 280%. In the intermediate frequency range it will under-predict these loads by up to 25%.

3. The difference, in the predictions of the double rupture disc assembly operation forcing function using SWAAM I and TRANSWRAP II, is the use of empirical parameters by TRANSWRAP to characterize rupture disc behavior after the disc strikes the knife edges.

4. Given the current state of the technology and computer limitations, it is not possible to accurately predict the pressure waves, created by double rupture disc assembly operation using analytical means only.

5. The ARS technique can be used to evaluate the effects of LMFBR-IHTS piping design changes and conditions on dynamic axial piping loads. It can also be used as an aid in analytically designing these systems.

3.0 RECOMMENDATIONS

1. The SWAAM I Computer Code should not be used for the analytical design of piping for LMFBR-Intermediate Heat Transport Systems, which use double rupture disc assemblies, until its simulation of the rupture discs has been improved.

2. A dynamic stress model of the LLTR should be created to determine the system fundamental natural frequencies.

3. The empirical modeling of double rupture disc assemblies should be upgraded, using the LLTR Series II test data, to more closely predict the rupture disc operation pressure wave.

4. The ARS technique should be used in LMFBR-IHTS piping analytical design to assess the effects of alternative designs and conditions on dynamic axial pipe loads.

4.0 DISCUSSION

Amplified Response Spectrum Analysis is used in this study as a means of evaluating a given forcing function from the point of view of the piping affected by it. ARS evaluates the dynamic load response of a single degree of freedom system to a given forcing function. The forcing functions in this analysis are the pressure waves generated in the piping by rupture disc operation. The technique has been used extensively in other dynamic analyses (e.g., seismic analysis) where the forcing function is externally applied. Appendix A contains a brief description of the Amplified Response Spectrum Analysis technique as it has been applied in this study to the dynamic analysis of sodium water reaction transients and Section 4.3 contains a description of the ARS technique.

4.1 Background

GE has determined during its CRBRP-IHTS work that the design base sodium-water reaction transients are limiting for the IHTS piping in that plant. It has also determined that the pressure waves created by rupture disc operation produce the greatest IHTS piping loads. Further, the greatest of these dynamic loads are produced by the pressure differences which occur between consecutive 90° pipe elbows as a pressure wave passes through the pipe section connecting the elbows. See Figures 2 and 3. This study does not attempt to evaluate all piping loads induced by SWR, but rather was limited to the axial piping loads caused by these pressure differences.

The passage of a pressure wave through a pipe section creates a pressure difference at the ends. This pressure difference creates a force in the direction of the lower pressure. The pipe attempts to move in the direction of the force but is restrained from doing so by the pipe support system. As a result, pipe response loads of equal magnitude but opposing the pressure force are created. See Figure 2.

Dynamic loads increase greatly when the forcing function contains considerable energy content at or near the natural frequency of the piping*.

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* The natural frequency range of interest for the IHTS piping is 10 to 30 Hz (especially in the lower end of that range).

It is, therefore, necessary to reasonably characterize the frequency content of the forcing function, as well as its amplitudes, to accurately predict the dynamic loads produced. In order to evaluate the validity of a predicted forcing function, it is necessary to look at the forcing function from the point of view of the affected piping. ARS provides a means of looking at the forcing function as the piping sees it. If prototypical test data of the subject forcing function is analyzed in the same way using the same assumptions, a valid comparison can be made between the predicted forcing function and the measured one with respect to the piping loads the forcing function might be expected to cause.

This report presents a comparison of the SWAAM I predicted forcing function with the one measured in the LLTR Series II test A-2.* It also presents a similar comparison between TRANSWRAP II and the same test data. The GE proprietary Acceleration Response Spectrum Analysis Code SECA04 (Reference 3), is used as the vehicle of comparison. While it is true that the shape of a pressure wave is altered as it passes through the piping**, the basic character of the wave remains fundamentally the same until its shape has been altered considerably. Therefore, it can be expected that the trends shown in this study will persist through piping sections somewhat removed from the immediate region of the rupture disc assembly.

This study considers a range of wave transit times of from 2 to 32 milliseconds. Given a sonic velocity of 5000 fps the pipe lengths considered range from 10 to 160 feet. For other sonic velocities the pipe section lengths evaluated can be determined from the equation

$$L = S\theta$$

where L = the pipe section length
S = Sonic Velocity in the fluid
θ = the wave transit time

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* Rupture disc operation measured at pressure transducer P-525.

** Due to frictional losses, and other pipe fitting losses, as well as by reflected and rarefaction waves from elsewhere in the piping system, etc.

All of the ARS cases use a damping ratio of 2%. This is a value normally used for dynamic analysis of this type of system. The boundary conditions applied to each of the three cases are the same with the exception of the forcing function used. The forcing functions used in this analysis were the pressure histories predicted and measured upstream of the double rupture disc assembly in LLTR. See Figure 4.

4.2 Forcing Functions

The sources and contents of the various forcing functions used in this analysis are discussed below.

MEASURED FORCING FUNCTION

The A-2 measured forcing function used is the pressure history measured by pressure transducer P-525. The data are taken directly from the GE Honeywell conversion of the A-2 test data tape supplied by ETEC. The curve of the data is shown in Figure 5.

SWAAM I FORCING FUNCTION

The SWAAM I predicted forcing function is the SWAAM I pressure history prediction at the location of pressure transducer P-525, presented in Reference 4 Figure 51, and shown in Figure 6. The wave was divided into 90 segments as shown in Figure 6. The point pairs used to characterize these data are given in Table I. Linear interpolations were made every 0.1 millisecond to produce 1000 data points to characterize the forcing function over the 100 msec range considered.

TRANSWRAP II FORCING FUNCTION

The TRANSWRAP II predicted forcing function is the TRANSWRAP II pressure history predicted at the location of pressure transducer P-525, using the revised standard methodology, and presented in Reference 5. The curve is shown in Figure 7. This wave was divided into 29 segments as shown in that figure. The point pairs used are given in Table II. Linear interpolations were made on the curve in the same manner as described above for the SWAAM I forcing function.

4.3 ARS Analysis Description

The effects of the forcing functions described above are evaluated for frequency content using the ARS technique. ARS is a proven dynamic analysis technique which generates acceleration response spectrum, for an arbitrary range of frequencies from a given forcing function, to compute maximum response loads for single degree of freedom systems. The response spectrum of each of the predicted forcing functions is ratioed to that produced by the LLTR test A-2 measured forcing function. This provides a direct means of evaluating the validity of the predicted forcing functions over the dominant piping natural frequency range.

The ARS technique separates evaluation of the forcing function from the structure affected by it. The forcing function is evaluated using the ARS code to obtain the response spectrum. The structure is evaluated, using the dynamic analysis code, to obtain the natural frequencies of each degree of freedom* of interest. The dynamic analysis code need not be rerun for a change in the forcing function but only for changes in the structure. The ARS code need not be rerun for changes in the structure but only for changes in the forcing function. From the output of the dynamic analysis code, the natural frequency of the degree of freedom of interest is used to select the pertinent structural response load from amoung those generated by the ARS code for the given forcing function. A graphic description of the application of ARS to dynamic analysis is given in Figure 8.

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* Each element in a dynamic model has 6 degrees of freedom, 3 translational and 3 rotational.

The ARS technique can also be used to determine the effects of changes in the forcing function, for a range of structure natural frequencies, by comparing the respective response spectra generated by the forcing functions. This is the way the method was used in this study, since the natural frequencies of the LLTR structure are not currently known.

Individual forcing functions can be defined to characterize the response spectrum for each type of dynamic loading on a single degree of freedom system. The results of any number of such applicable forcing functions can be combined to determine the net response spectrum for the given single degree of freedom system. Or, conversely the forcing functions can be combined to produce a composite forcing function.

For a simple mass, spring, damper system driven by a general dynamic loading, the acceleration response gives the maximum amplitude of the force transmitted to the support. This amplitude depends on the natural frequency and on the relative damping of the system.

If the system is driven first by one forcing function, then by another, the two general dynamic forcing functions may be compared in terms of the maximum load delivered to the support. A response spectrum is developed by repeating the process as the natural frequency of the system is systematically varied.

The dynamic response of a multiple degree of freedom system may be expressed in terms of equivalent single degree of freedom responses of the various harmonic modes. Thus, the ARS gives the maximum loading at the supports for any vibration mode of a complex system. These features of an ARS make it a very useful tool for evaluating dynamic loads. The GE proprietary ARS analysis computer code SPECA04 (Reference 3) is used in this study.

The application of the ARS method in this study is described in Appendix A.

4.4 ARS Application

It cannot be expected that the ARS technique, as it has been used in the study, will provide quantitative information on the dynamic piping loads created by sodium-water reaction, since the natural frequencies of the LLTR structural components have not yet been determined. Also this study focused on only one type of pipe loading. However, the technique can be used now to compare the effects of various forcing functions on pipe response. It can therefore, be used to forecast the validity of predicted forcing functions and the effects of changes in forcing function wave shape on piping response. The cost and time involved in using this technique is considerably less than would be required for a quantitative thermal-hydraulic/dynamic stress analysis of the phenomena. However, determination of the LLTR structure natural frequencies is needed to compare analytically determined peak response loads with those measured in the test.

5.0 RESULTS

Three cases were analyzed using Amplified Response Spectrum Analysis. The output is given in maximum response force per unit pipe flow area versus frequency. In addition, the ARS analyses using predicted forcing functions were ratioed to the ARS analysis of the test data (Case I). The ARS cases run and the comparisons made are defined in Table III below.

TABLE III

CASE NO.	ARS ANALYSIS AND COMPARISONS DESCRIPTION	FORCING FUNCTION SHOWN IN FIGURE NO.	ARS GIVEN IN FIGURE NO.
I	ARS Analysis of the Measured Forcing Function for Test A-2	5	9
II	ARS Analysis of the SWAAM I predicted Forcing Function for Test A-2	6	10
III	ARS Analysis of the TRANSWRAP II predicted Forcing Function for Test A-2	7	11
Comparison		ARS ANALYSIS RESULTS SHOWN IN FIGURE NO.	COMPARISON GIVEN IN FIGURE NO.
A	of SWAAM I Predicted for A-2 to Test Measured ARS Analysis	9 & 10	12
B	of TRANSWRAP II Predicted for A-2 to Test Measured ARS Analysis	9 & 11	12

The fundamental natural frequencies of the IHTS piping sections and supports to excitation along the pipe axis is in the range of 10 to 30 Hz, and therefore, the ARS analysis is only of interest in that range of frequencies. In order to apply the ARS analysis results to a given section of pipe, support or component, the natural frequencies of each of its degrees of freedom of interest would have to be known.

5.1 ARS Analysis

Case I ARS Analysis of the Measured Forcing Function in Test A-2:

As can be seen in Figure 9, the piping response increases rapidly with increased frequency for all pipe segment lengths considered to around a frequency of 20 Hz and then decreases to nearly the end of the frequency range of interest (30 Hz). For most of the frequency range, as the pipe length increases, the piping loads increase.

Case II ARS Analysis of the SWAAM I Predicted Forcing Function in Test A-2:

As can be seen in Figure 10, the ARS analysis predicts piping response to the SWAAM I forcing function (Fig. 6) is considerably lower in amplitude, than that for the measured forcing function, throughout the frequency range of interest. This indicates that the SWAAM I predicted forcing function is deficient in energy content for that frequency range. The piping response loads increase gradually throughout the range 10-30 hz with no intervening peaks. Use of the SWAAM I predicted forcing function could be expected to greatly under-predict SWR induced axial piping loads.

Case III ARS Analysis of the TRANSWRAP II Predicted Forcing Function in Test A-2

As can be seen in Figure 11, the ARS analysis prediction of piping response to the TRANSWRAP II forcing function (Fig. 7) shows a rise to a sharp peak at 23 Hz* with much lower peaks occuring down to 10 Hz. The ARS analysis of the measured forcing function shows a larger rise in amplitude in the frequency range from 15 to 20 Hz than predicted for the TRANSWRAP II forcing function. This indicates that the TRANSWRAP II model of the rupture discs produces forcing functions deficient in energy content in this frequency range. This would produce under-predictions of pipe response loads for piping components whose fundamental natural frequencies are in the range 15 to 20 Hz. For all other frequencies in the range of interest the energy content is greater than predicted for the measured pressure pulse.

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* Compared with 20 Hz for the measured pulse.

The TRANSWRAP II computer code could be expected to produce rupture disc operation forcing functions which would result in the over-prediction of piping loads except for components with fundamental natural frequencies in the range 15 to 20 Hz.

5.2 Comparison of Predictions with Test Data

In order to aid the reader in understanding the significance of the results of this analysis, direct comparisons are made between the ARS analysis of each of the predicted forcing functions and the forcing function measured in the test A-2. The results of the comparisons follow:

Comparison A: Case II (SWAAM I Predicted A-2) ÷ Case I (Measured A-2) -

As can be seen in Figure 12, the forcing function predicted by SWAAM I for rupture disc operation in test A-2 is deficient in energy content throughout the frequency range of interest. This will result in under-prediction of axial piping loads caused by rupture disc operation of from 20 to 500% for piping components whose natural frequencies are below 30 Hz.

Comparison B: Case III (TRANSWRAP II Predicted A-2) ÷ Case I (Measured A-2) -

As can be seen in Figure 12, the forcing function predicted by TRANSWRAP II for rupture disc operation in test A-2 contains excess energy content in the frequency range of interest except between 15 and 20 Hz where it is deficient. This will result in over-prediction of axial piping loads caused by rupture disc operation of from 0 to 260% for pipe components with fundamental natural frequencies below 15 Hz and from 0 to 280% for those whose fundamental natural frequencies are above 20 Hz. For pipe components with fundamental natural frequencies from 15 to 20 Hz it would cause under-prediction of the axial piping loads by from 0 to 25%.

TABLE I

SWAAM I PREDICTED FORCING FUNCTION
CHARACTERIZING RUPTURE DISC OPERATION

(SWAAM I Predicted Pressure History @ Pressure Transducer P-525 for LLTR Series II Test A-2 Characterized by 90 Linear Segments)

POINT NO.	TIME ~msec	PRESS ~psig	POINT NO.	TIME ~msec	PRESS ~psig	POINT NO.	TIME ~msec	PRESS ~psig
0	0.	125	32	25.1	107	64	59.8	105
1	1.7	125	33	25.9	30	65	61.	11
2	2.5	182	34	26.4	39	66	62.1	61
3	2.8	125	35	26.9	0	67	62.4	23
4	3.2	200	36	27.9	75	68	64.	102
5	4.4	289	37	35.3	120	69	65.2	23
6	5.4	118	38	36.7	239	70	66.1	27
7	5.9	180	39	37.5	18	71	67.4	84
8	6.5	59	40	38.	68	72	68.4	25
9	7.4	211	41	39.	2	73	69.9	25
10	8.3	118	42	40.	86	74	70.4	2
11	8.6	173	43	40.7	27	75	71.7	91
12	9.9	202	44	41.4	91	76	73.8	68
13	10.9	45	45	42.6	-15	77	74.9	91
14	11.4	130	46	45.7	134	78	76.2	84
15	12.	45	47	46.1	-15	79	77.8	23
16	12.8	152	48	46.2	57	80	81.5	64
17	13.4	68	49	47.4	57	81	82.5	23
18	14.	123	50	47.7	0	82	83.5	23
19	14.8	23	51	48.6	64	83	85.7	91
20	16.9	164	52	49.5	15	84	87.4	30
21	17.6	45	53	50.7	159	85	88.4	52
22	18.3	100	54	51.	95	86	92.6	30
23	18.9	45	55	51.4	91	87	94.6	82
24	19.6	114	56	51.7	27	88	97.	30
25	20.1	34	57	52.4	68	89	98.	68
26	21.	100	58	53.1	5	90	100	64
27	21.5	68	59	54.3	93			
28	22.2	68	60	55.3	75			
29	23.	41	61	56.8	-145			
30	23.9	68	62	58.	66			
31	24.4	45	63	59.1	34			

TABLE II

TRANSWRAP II PREDICTED FORCING FUNCTION
CHARACTERIZING RUPTURE DISC OPERATION

(TRANSWRAP II Predicted Pressure History @ Pressure Transducer P-525 for LLTR Series II Test A-2 Characterized by 29 Linear Segments)

POINT NO.	TIME ~msec	PRESS ~psia
0	0	150
1	2	150
2	2.4	205
3	2.6	100
4	3.7	371
5	4.	269
6	5.	232
7	6.3	136
8	9.8	55
9	16.8	94
10	18.	133
11	21.6	154
12	37.2	153
13	38.6	110
14	41.5	55
15	42.5	71
16	50.2	77
17	60.	133
18	62.9	183
19	65.5	275
20	67.2	390
21	68.	271
22	69.	169
23	73.7	24
24	80.3	44
25	85.1	41
26	87.2	50
27	92.6	50
28	96.8	64
29	100	49

6.0 REFERENCES

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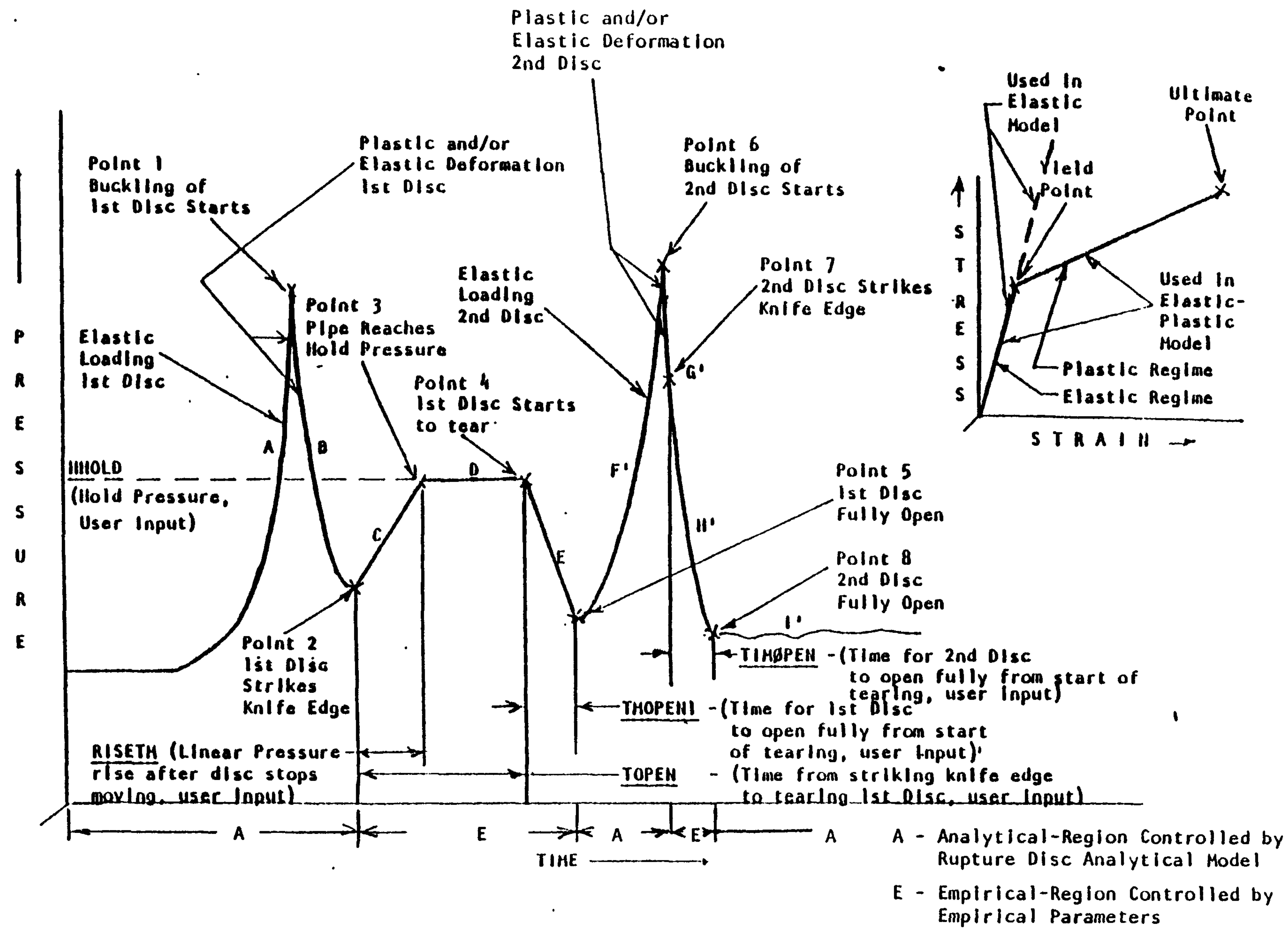
5. D. E. Knittle et al., "Evaluation of LLTR Series II Test A-7 Results," Sept. 1981, transmitted by GE letter XL-898-10478 dated Sept. 30, 1981.

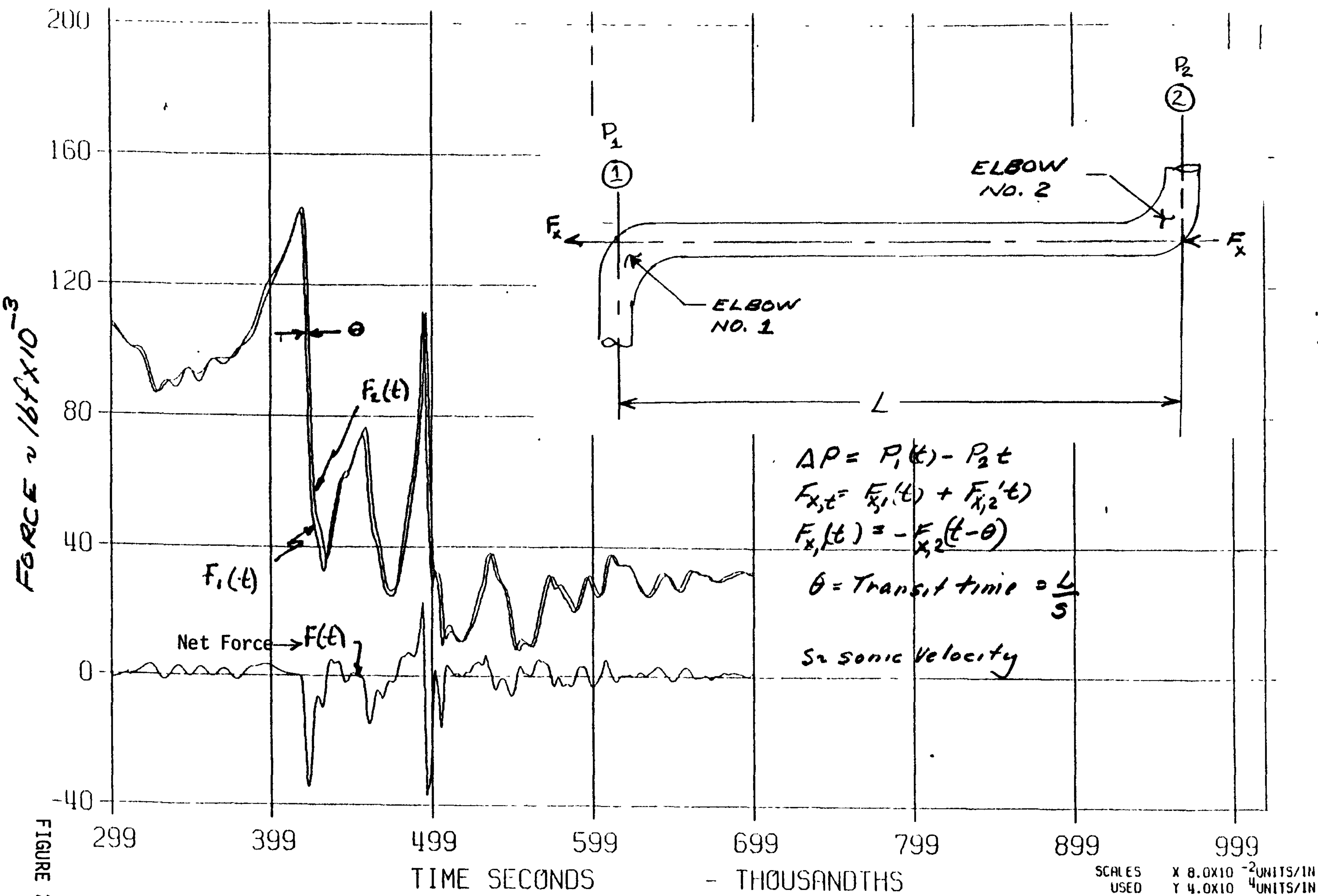
6. Clough R. W., & Penzien, "Dynamics of Structures," 1975, McGraw-Hill Inc., N.Y., Chapters 6 & 26.



* The behavior of the second disc differs somewhat from that of the first

FIGURE 1

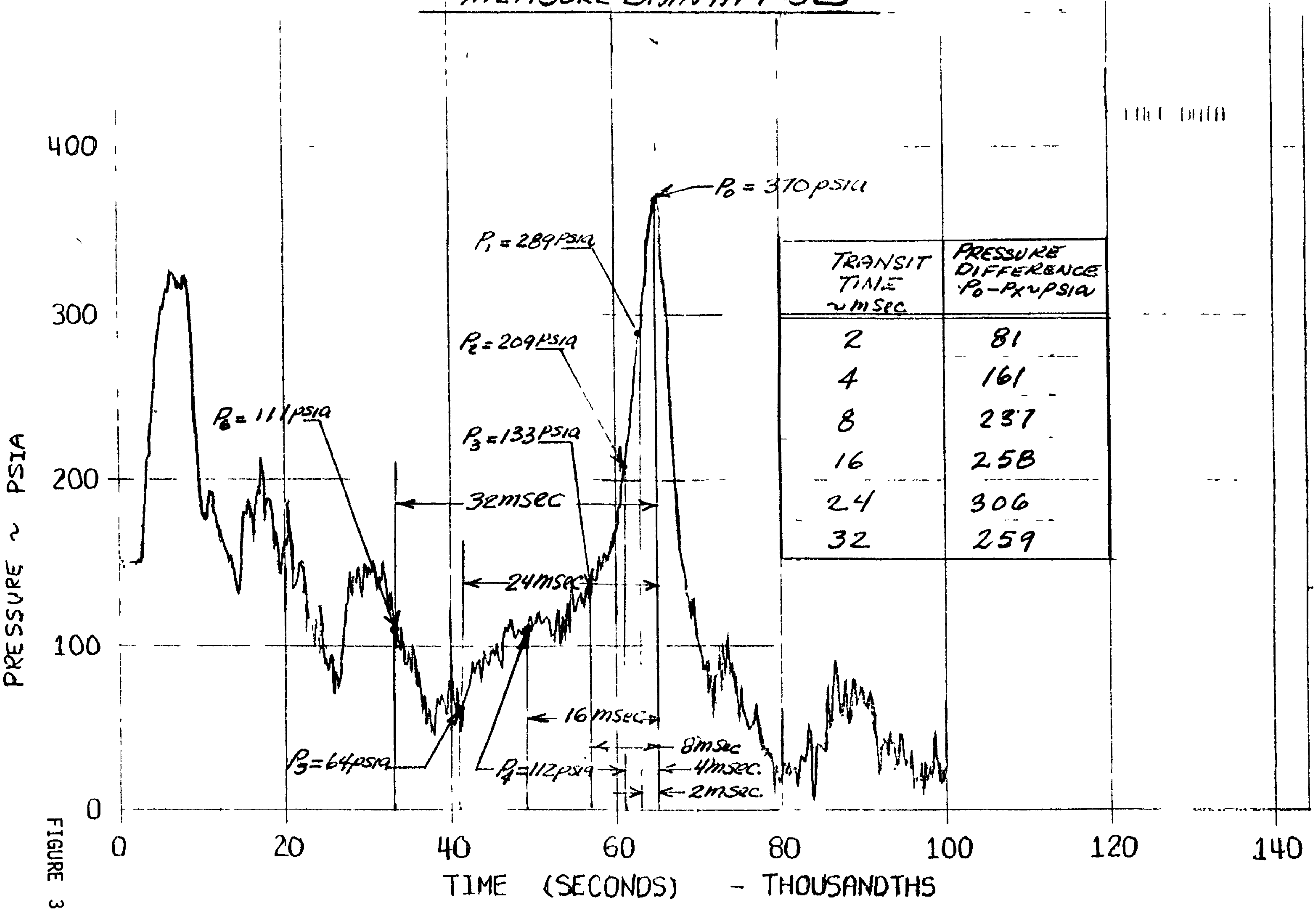
RUPTURE DISC DYNAMIC MODEL USED BY TRANSWRAP II



200
160
120
80
40
0
-40
FORCE ~ lbf X 10^-3
299
399
499
599
699
799
899
999
TIME SECONDS - THOUSANDTHS
SCALES USED
X 8.0X10 -2 UNITS/IN
Y 4.0X10 4 UNITS/IN
θ
F2(t)
F1(t)
Net Force → F(t)
P1
1
P2
2
ELBOW NO. 2
ELBOW NO. 1
Fx
Fx
L
ΔP = P1(t) - P2 t
Fx,t = Fx,1(t) + Fx,2(t)
Fx,1(t) = - Fx,2(t-θ)
θ = Transit time = L/S
S = sonic velocity

FIGURE 2

PRESSURE HISTORY UPSTREAM OF RUPTURE DISC
LLTR SERIES II TEST A-2
MEASURE DATA AT P-525



TRANSIT TIME ~ mSec	PRESSURE DIFFERENCE $P_0 - P_x$ ~ PSIA
2	81
4	161
8	237
16	258
24	306
32	259

FIGURE 3

LLTR SERIES II
TRANSWRAP MODEL

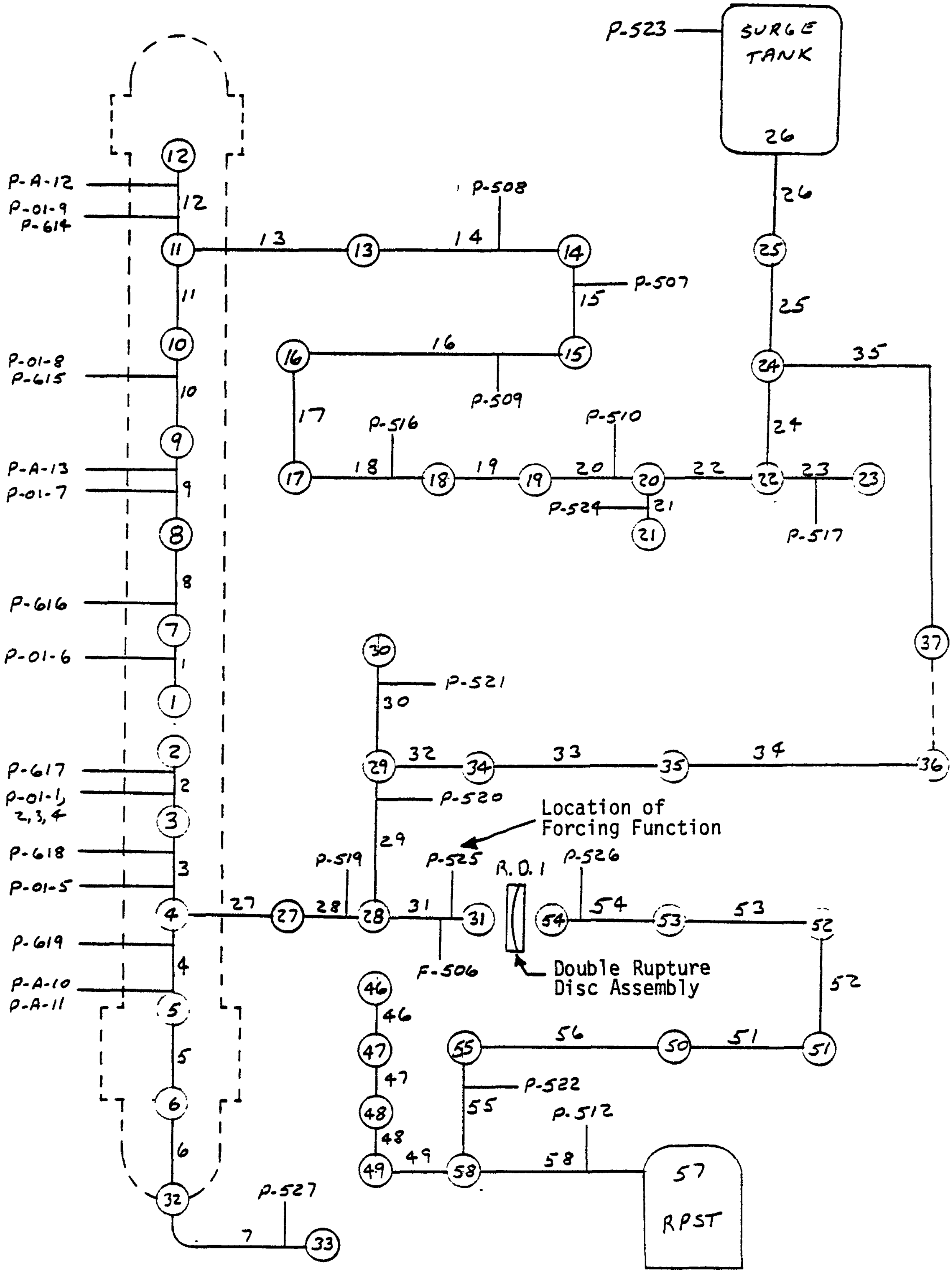


FIGURE 4

CASE I

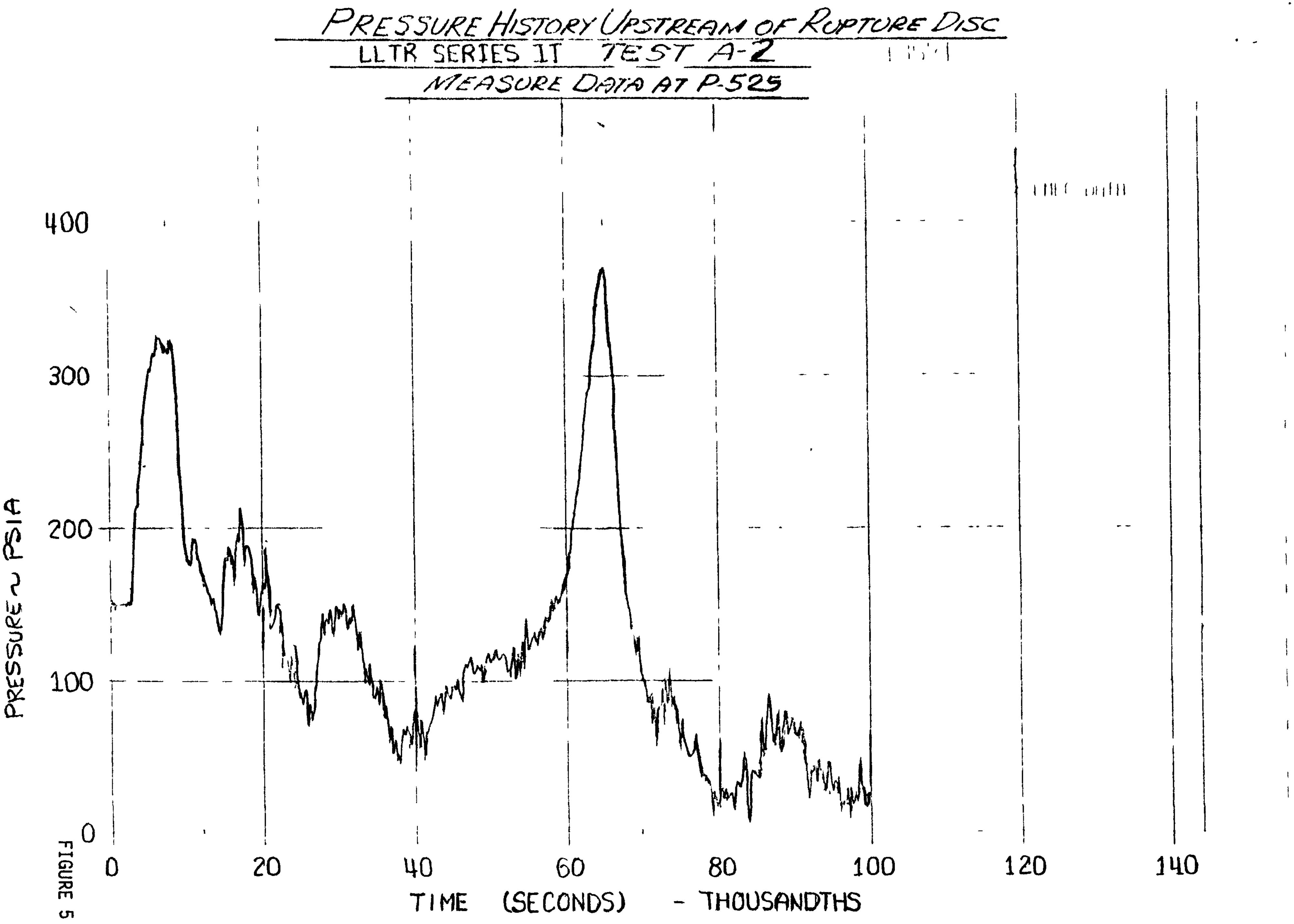


FIGURE 5

CASE II

PRESSURE HISTORY UPSTREAM OF RUPTURE DISC

PRESSURE HISTORY OF LLTR-II A2 AT P525A

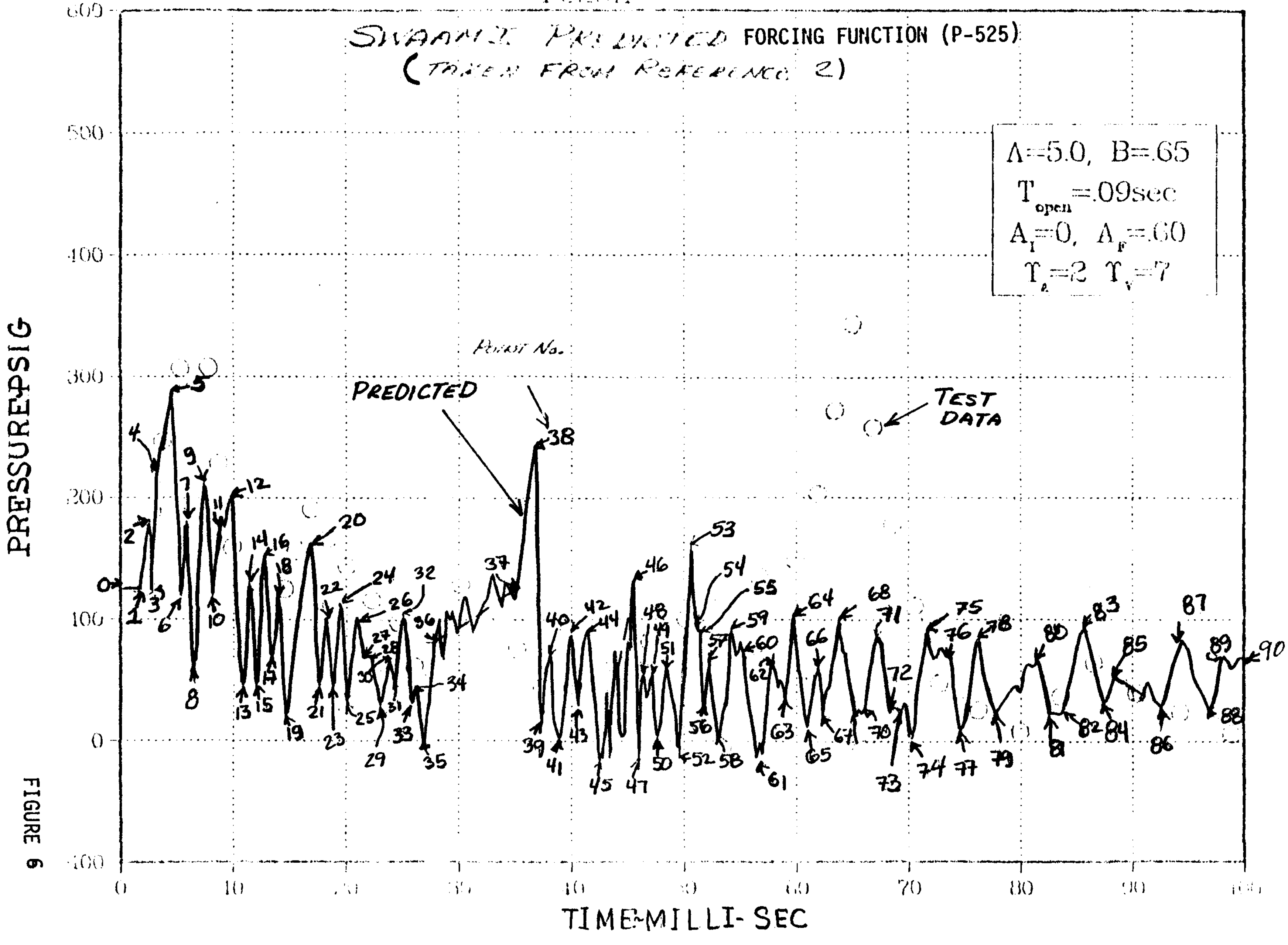


FIGURE 6

CASE III

PRESSURE HISTORY UPSTREAM OF RUPTURE DISC
LLTR SERIES II – TEST A2
TRANSWRAP II PREDICTED DATA AT P-525*

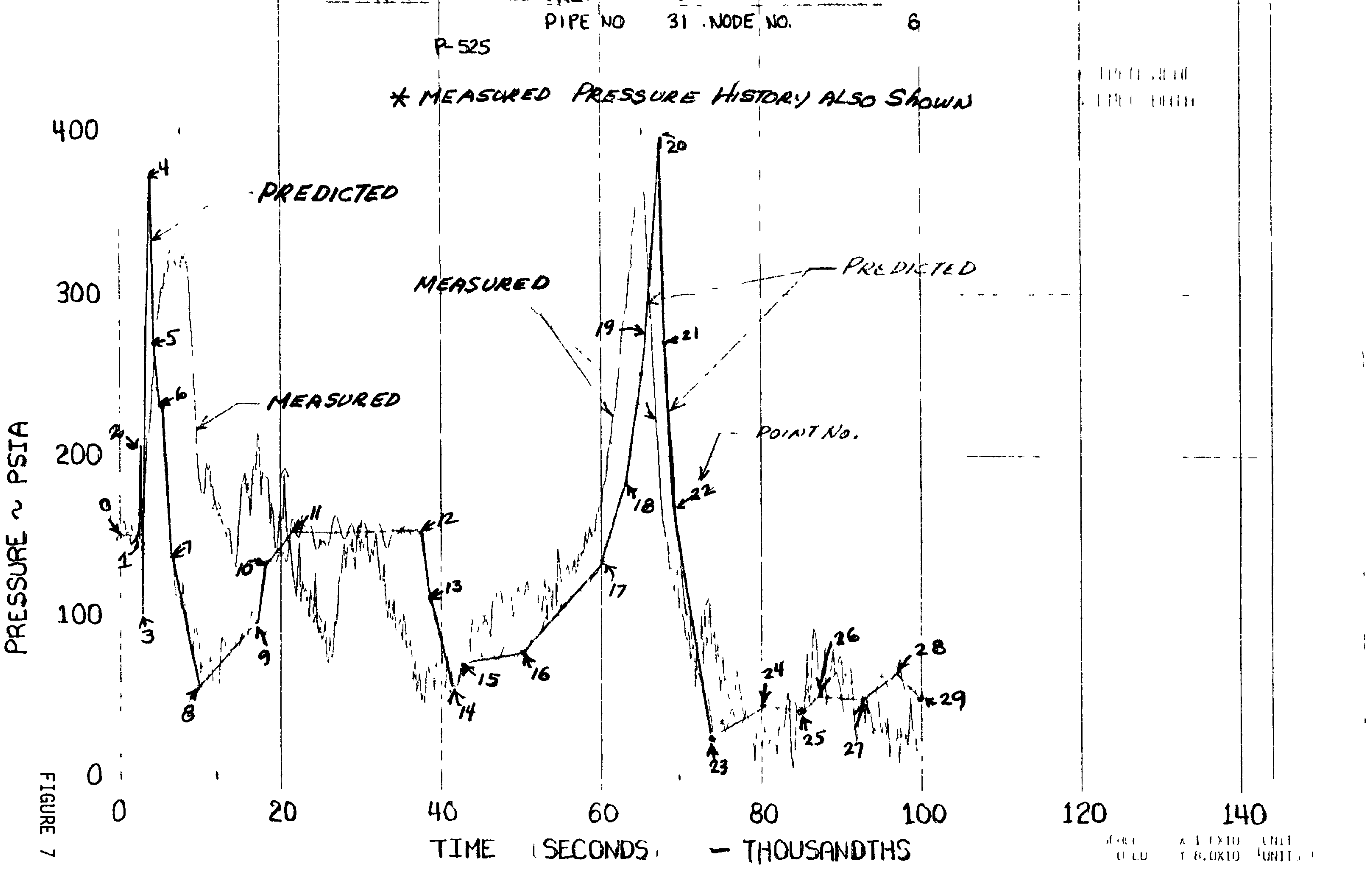
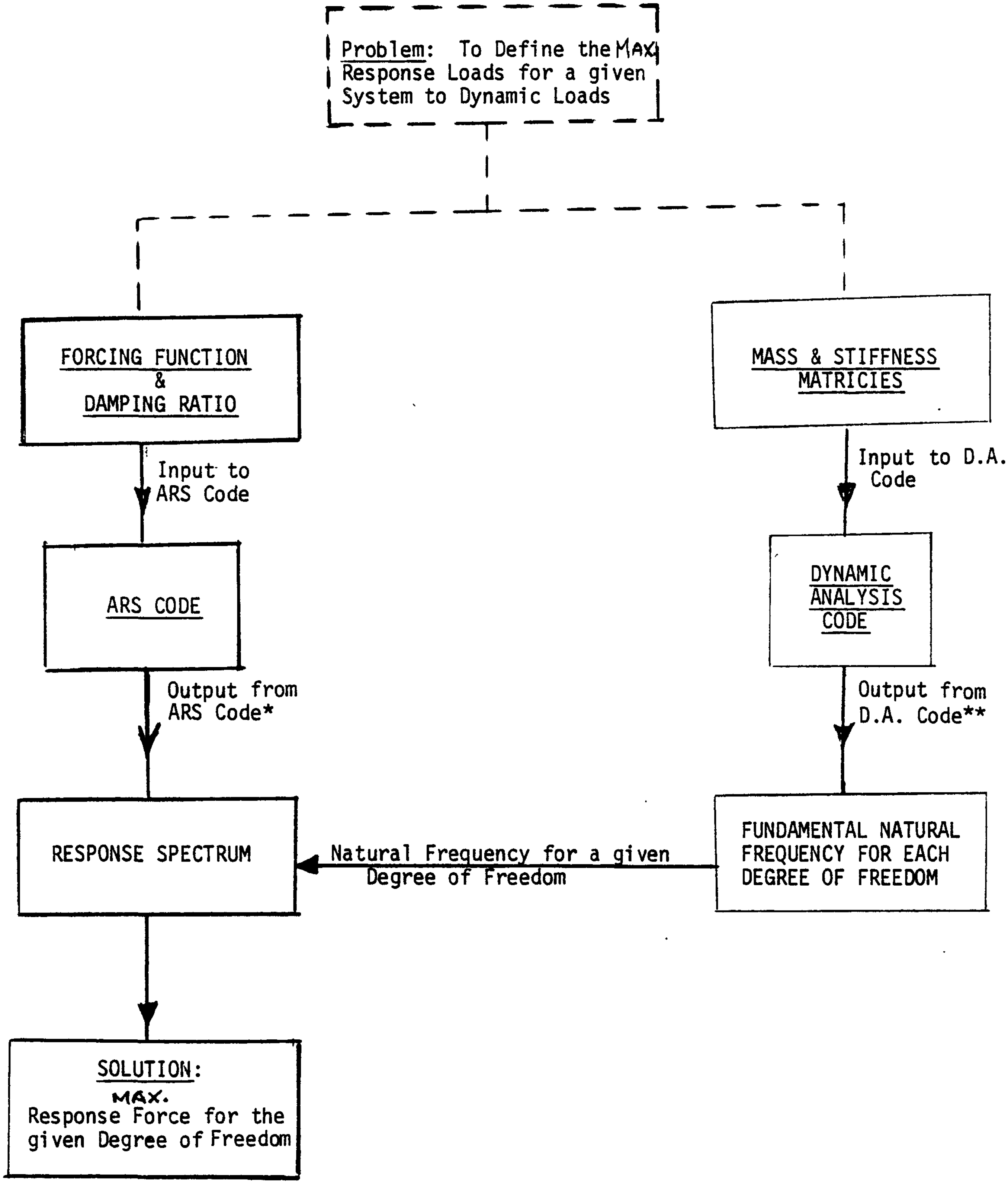


FIGURE 7

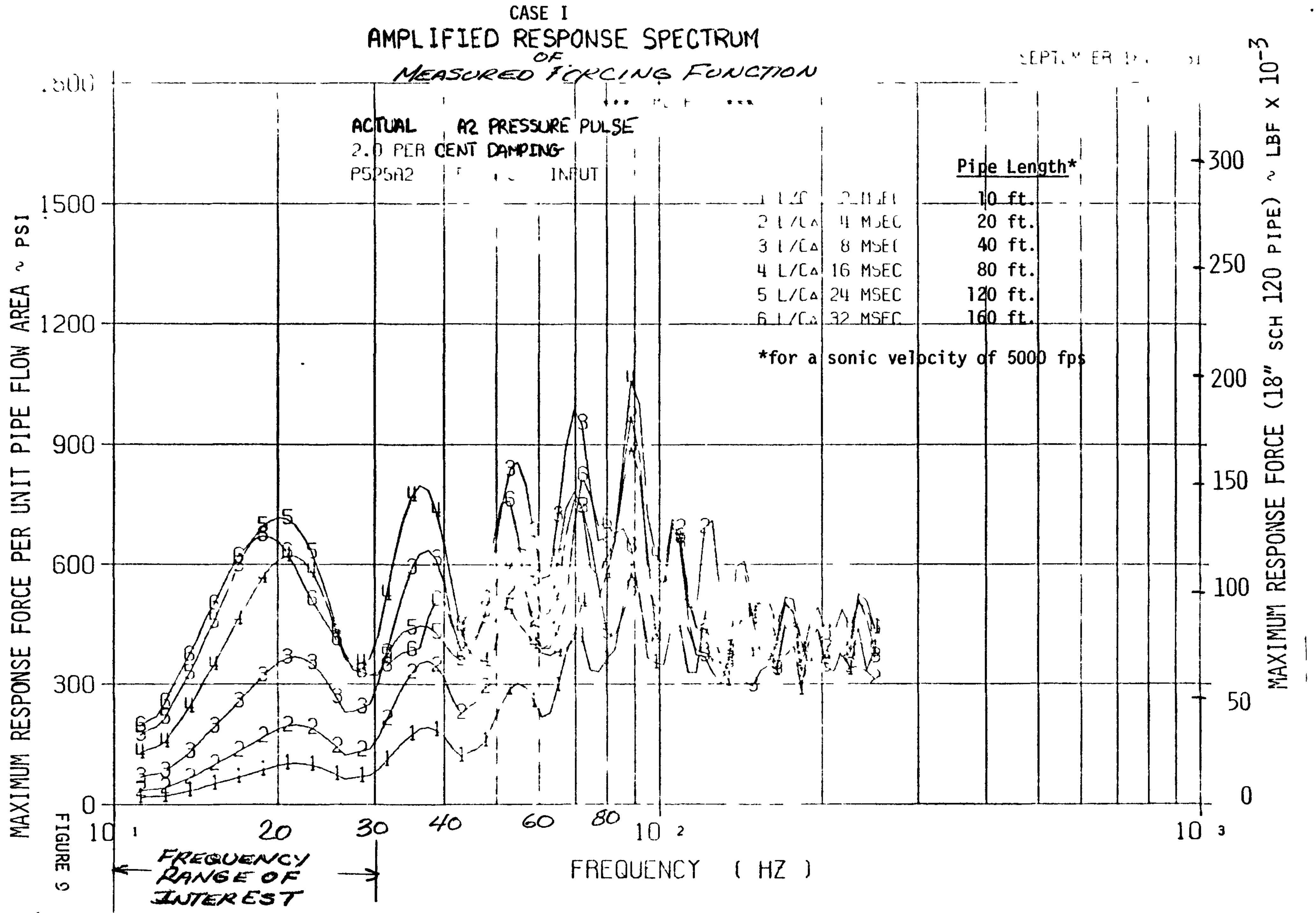
GRAPHIC REPRESENTATION OF THE APPLICATION OF ARS TO DYNAMIC ANALYSES



* Results do not change unless the forcing function changes

** Results do not change unless changes are made in the structure

FIGURE 8



CASE I
AMPLIFIED RESPONSE SPECTRUM
OF
MEASURED FORCING FUNCTION
ACTUAL A2 PRESSURE PULSE
2.0 PER CENT DAMPING
P5P5A2 INPUT
Pipe Length*
2 L/CΔ 4 MSEC 20 ft.
3 L/CΔ 8 MSEC 40 ft.
4 L/CΔ 16 MSEC 80 ft.
5 L/CΔ 24 MSEC 120 ft.
6 L/CΔ 32 MSEC 160 ft.
10 ft.
*for a sonic velocity of 5000 fps
MAXIMUM RESPONSE FORCE PER UNIT PIPE FLOW AREA ~ PSI
MAXIMUM RESPONSE FORCE (18" SCH 120 PIPE) ~ LBF x 10^-3
1500
1200
900
600
300
0
300
250
200
150
100
50
0
20
30
40
60
80
FREQUENCY (HZ)
FREQUENCY RANGE OF INTEREST

FIGURE 6

PL 1

CASE II

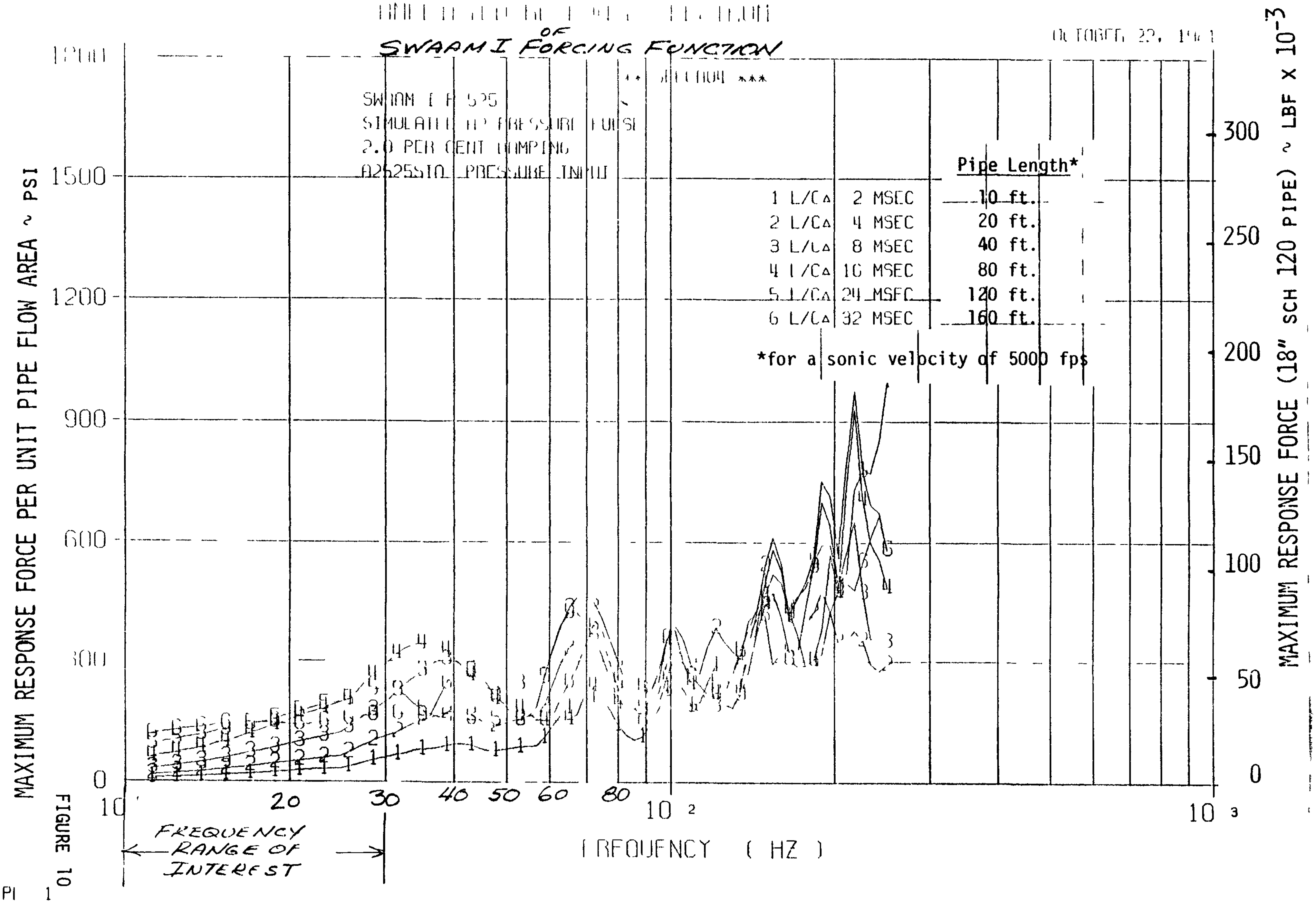


FIGURE 10

CASE III

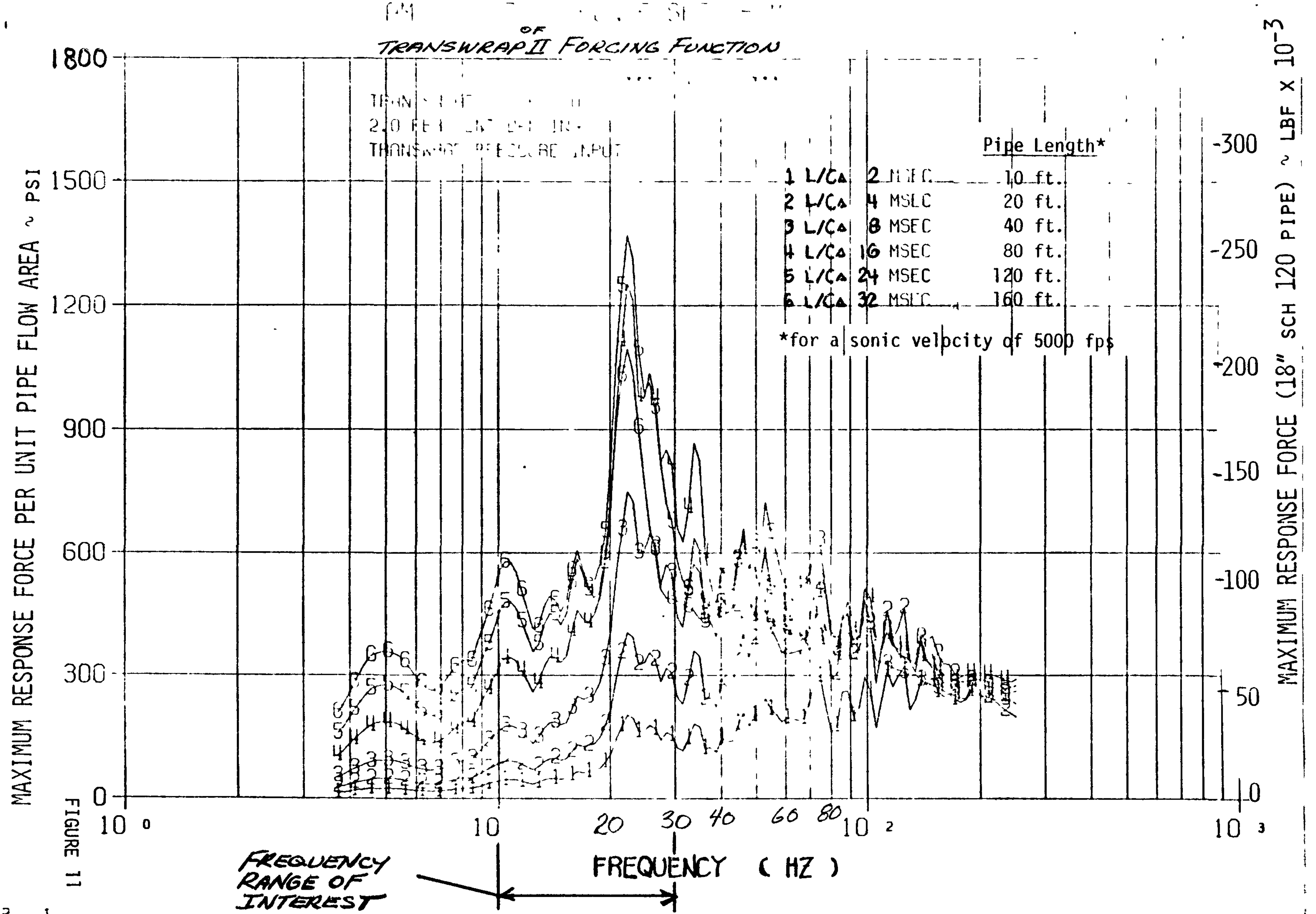
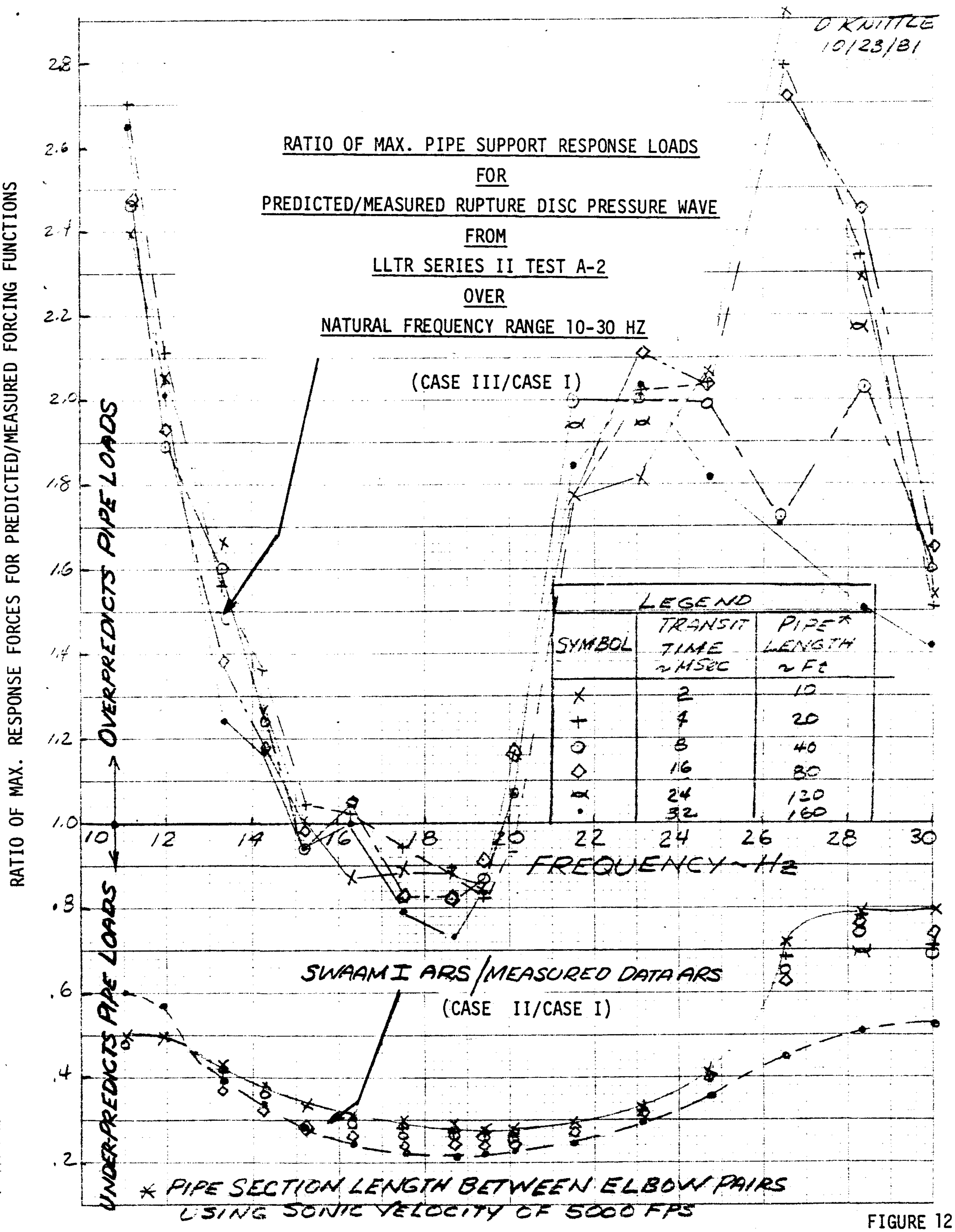


FIGURE 11



D KNITTLE
10/23/81
RATIO OF MAX. PIPE SUPPORT RESPONSE LOADS
FOR
PREDICTED/MEASURED RUPTURE DISC PRESSURE WAVE
FROM
LLTR SERIES II TEST A-2
OVER
NATURAL FREQUENCY RANGE 10-30 HZ
(CASE III/CASE I)
RATIO OF MAX. RESPONSE FORCES FOR PREDICTED/MEASURED FORCING FUNCTIONS
OVERPREDICTS PIPE LOADS
UNDER-PREDICTS PIPE LOADS
FREQUENCY ~ Hz
LEGEND
SYMBOL
TRANSIT TIME ~ MSEC
PIPE* LENGTH ~ Ft
2
4
8
16
24
32
10
20
40
80
120
160
SWAAM I ARS / MEASURED DATA ARS
(CASE II/CASE I)
* PIPE SECTION LENGTH BETWEEN ELBOW PAIRS USING SONIC VELOCITY OF 5000 FPS

FIGURE 12

APPENDIX A: DESCRIPTION OF THE APPLICATION OF AMPLIFIED RESPONSE SPECTRUM ANALYSIS TO SWR DYNAMIC ANALYSIS

Amplified Response Spectrum Analysis

Acceleration response spectra can be calculated for the force applied to the mass of a single degree of freedom system. See Reference 6. ARS analysis gives the maximum value of the restoring force amplitude during application of the applied load (i.e., forcing function).

Derivation of the ARS method follows.

A.1 FORCE APPLIED TO A MASS

The displacement of a single degree of freedom system due to a force applied at the mass is the solution to the equation:

Single Degree of Freedom System

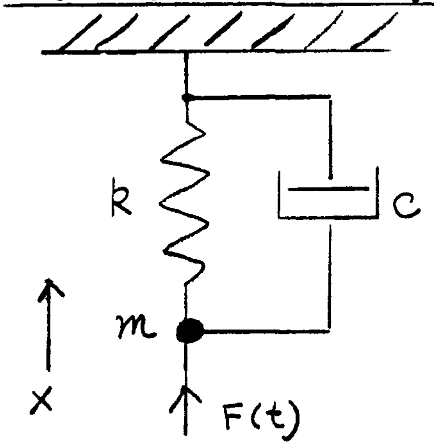


Fig. A.1

(1) $m\ddot{x} + c\dot{x} + kx = F(t)$
(See figure A.1 for nomenclature.)
or its equivalent

(2) $\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = \frac{F(t)}{m}$

from (1), the restoring force at any instant of time is:

(3) $c\dot{x} + kx = F(t) - m\ddot{x}$

The maximum amplitude of the restoring force is the spectral displacement for the system when driven by F(t). This value depends only on the natural frequency and the damping fraction of the system.

To eliminate system mass as a parameter, the displacement using the Duhamel equation is written as follows:

(4) $x(t) = \frac{1}{\omega m} \int_0^t F(\tau)\, e^{-\xi\omega(t-\tau)} \sin\omega(t-\tau)\, d\tau$

The term $m\ddot{x}$ on the right hand side of (3) may be computed independently of m using this equation and differentiating twice. The spectral displacement is then:

$$(5)\quad S_d(\omega) \triangleq {}_t\mathrm{Max}|F(t) - m\ddot{x}|$$

where $0 \leq t \leq t_{max}$.

The computational procedure is to compute the acceleration using the Duhamel integral and subtract it from the applied force at each time point. By definition the result is the maximum restoring force during application of the load.

A.2 DISPLACEMENT OF A PIPE SECTION SUBJECT TO PRESSURE WAVE LOADINGS

To determine the piping response to sodium water reaction pipe loadings, Figure 2, a single degree of freedom differential equation can be written which separates the loading function, pipe response characteristics, and pipe filter characteristics into separate functions.

$$(6)\quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overbrace{H(\bar{\omega})\; C(\bar{\omega})\; e^{i\bar{\omega}\theta}}^{A}\; [1 + 2\, e^{-i\bar{\omega}\theta}]\, d\bar{\omega}$$

Pressure Wave Characteristics

Piping Response

Pipe Filter Characteristics (Figure A.2)

A - Is the Response of the pipe section due to a wave at one end of pipe.

Nomenclature $x \sim$ displacement

α = relative force factor $\sim$-1 for lateral loads due to opposing elbows.

θ = time for wave to travel from one end of the pipe section to the other (i.e., transit time)

Equation 6 can be solved to determine the pipe section response to a given imposed loading once the values for the individual functions have been determined. The derivation of equation 6 follows.

A.3 DISPLACEMENT OF PIPE DUE TO END FORCES GENERATED BY A PRESSURE WAVE

For a wave traveling from end 1 to end 2, Figure 2, the response may be obtained in terms of the loading on one end only. The Duhamel equation for displacement due to a general dynamic load is:

(7) $$x(t) = \frac{1}{m\omega} \int_0^t p(\tau)\, h(t-\tau)\, d\tau$$

where:

$x(t)$ = displacement of a single degree of freedom system

$p(t)$ = applied force (net end force)

$h(t-\tau)$ = response at time t due to unit impulse at time τ

The net force is the sum of two end forces:

(8) $$p(t) = p_1(t) + p_2(t)$$

The force at end 2 is a multiple of the force at end 1 with the time delay, θ, for the wave to move from end 1 to end 2.

(9) $$p(t) = p_1(t) + \alpha\, p_1(t-\theta).$$

Assume $p(t) = 0,\ t \leq 0$

Then

(10) $$x(t) = \frac{1}{m\omega} \{\int_0^t p_1(t)\, h(t-\tau)\, d\tau + 2\int_\theta^t p_1(t-\theta)\, h(t-\tau-\theta) d\tau\}$$

(11) $$x(t) = x_1(t) + \alpha\, x_1(t-\theta)$$

where $x_1(t)$ = displacement due to the force on end 1.

A.4 FREQUENCY ANALYSIS

(12) $$\chi_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\bar{\omega})\ C(\bar{\omega})e^{1\bar{\omega}t}\ d\bar{\omega}$$

where

$H(\bar{\omega})$ = Response to unit frequency input

$C(\bar{\omega})$ = Fourier density coefficient for p (t)

$$= \int_{-\infty}^{\infty} p_1(t)e^{-1\bar{\omega}t}\ dt$$

and

$$C^1(\bar{\omega}) = \int_{-\infty}^{\infty} p_1(t - \theta)e^{-1\bar{\omega}t}\ dt$$

$$= \int_{-\infty}^{\infty} p_1(\tau)e^{-1\bar{\omega}(\tau - \theta)}dt$$

$$= e^{-1\bar{\omega}\theta}\int_{-\infty}^{\infty} p_1(\tau)e^{\ 1\bar{\omega}\tau}d\tau$$

$$\therefore \quad C^1(\bar{\omega}) = C(\bar{\omega})e^{-1\bar{\omega}\theta}$$

Substitute in $\chi(t) = \chi_1(1) + \alpha\ \chi_1(t - \theta)$:

$$\chi(t) = \frac{1}{2\pi}\{\int_{-\infty}^{\infty} H(\bar{\omega})\ C(\bar{\omega})\ [e^{1\bar{\omega}t} + \alpha\ e^{1\bar{\omega}(t - \theta)}]d\bar{\omega}$$

(13) $$= \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\bar{\omega})\ \ C(\bar{\omega})e^{1\bar{\omega}t}\ [1 + \alpha\ e^{-1\bar{\omega}\theta}]\ d\bar{\omega}$$

The term, $e^{-1\bar{\omega}\theta}$, has a real and imaginary part. The real part ranges in magnitude between +1 and -1. The factor, α, may be positive or negative, depending on whether the end forces act in the same or opposing directions when the pressure is increased i.e. the sign of $\frac{dp}{dt}$ is the sign of α.

The magnitude of α depends on the shape of the end fittings.

For symmetric end fittings, the value of α will be -1. The real part of the displacement will be alternately magnified by a factor of 2 or suppressed to 0 as the frequency ranges from 0 to ∞.

PIPE FILTER CHARACTERISTICS

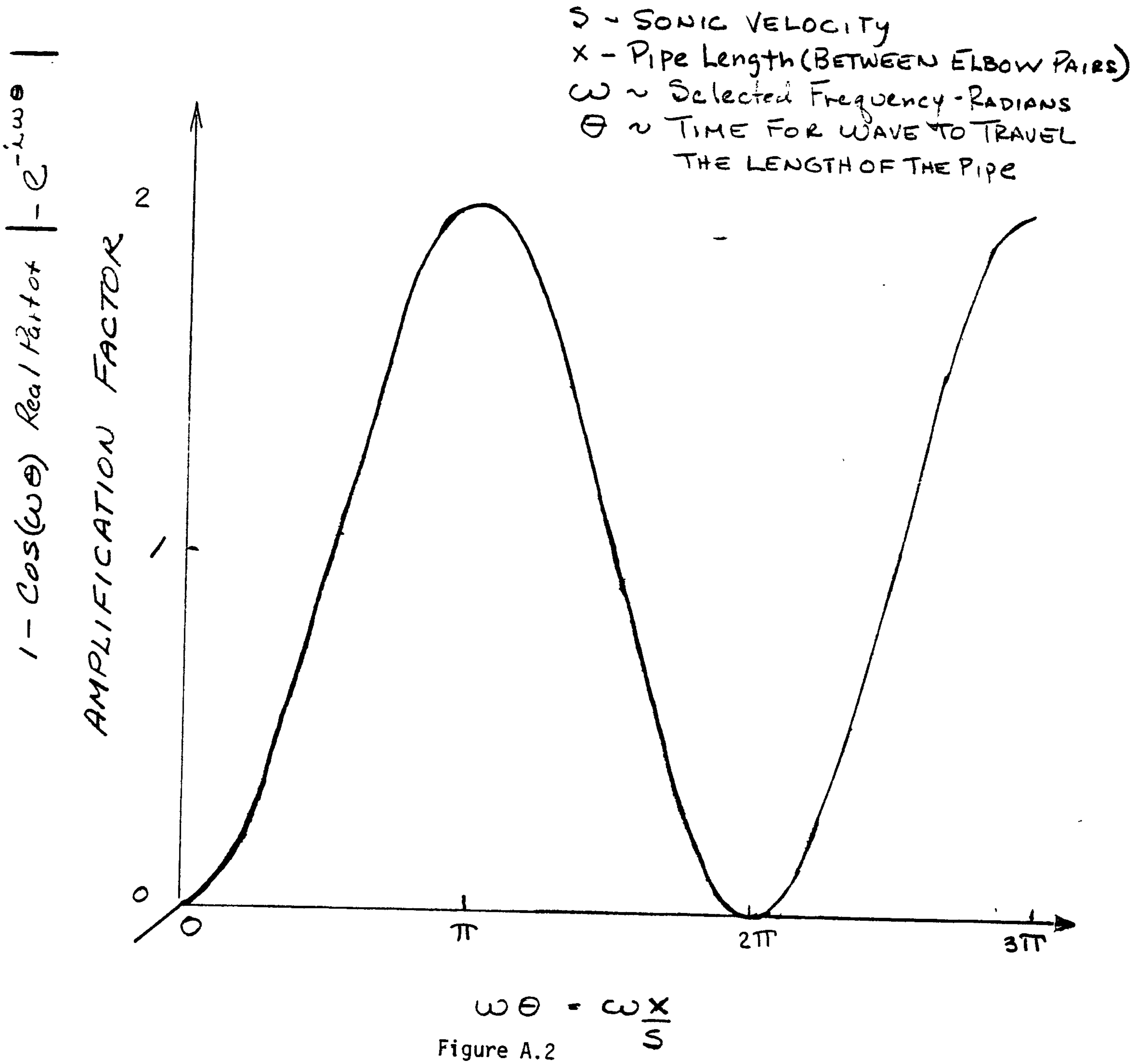


Figure A.2