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NEUTRAL PION PHOTOPRODUCTION ON THE NUCLEON NEAR THRESHOLD*

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Neutral pion photoproduction on the nucleon near threshold is investigated using a dynamical model. It is shown that the commonly used procedure, based on an analytical continuation of the K-matrix to the unphysical region, is not compatible to a dynamical approach. We show that the final state interaction(FSI) amplitude derived from a dynamical model could involve large cancellation between the different pion photoproduction mechanisms. This leads that the FSI due to the intermediate $\pi^0 p$ state can be as important as that due to the $\pi^+ n$ intermediate state. At threshold, we obtain $E_{0+} = -1.92 \times 10^{-3}/m_{\pi^+}$. This number is close to the measured value of $E_{0+} = -1.5 \times 10^{-3}/m_{\pi^+}$. No violation of the low energy theorem is required to obtain good agreement between the calculated total cross sections and experimental data from threshold to about 400 MeV incident photon energy.

I. INTRODUCTION

It has been reported for some time that the E_{0+} amplitude extracted from analyses[1,2] of threshold neutral pion photoproduction data seems incompatible with the well-established low energy theorem.[3] In this talk we explore this problem within a dynamical model[4-6] in which the pion-nucleon interaction is described by $\pi N \leftrightarrow N$ and $\pi N \leftrightarrow \Delta$ vertices, and a two-body potential. The complete results will be given elsewhere[7].

Let us first recall what has been done in arriving at that "surprising" result in the experimental analyses[1,2]. A non-trivial problem is how to treat the rescattering correction due to the charge exchange process: $\gamma p \to \pi^+ n \to \pi^0 p$. To evaluate this contribution, one needs to define the *off-shell* πN charge-exchange scattering amplitude. Evidently this cannot be obtained from the experimental data of πN scattering without making theoretical assumptions. In the analyses of Refs. 1 and 2, it is assumed that the s-wave multipole amplitude can be written as

$$E_{0+} = \hat{E}_{0+}(\gamma p \to \pi^0 p) + (FSI). \tag{1.1a}$$

Here $\hat{E}_{0+}(\gamma p \to \pi^0 p)$ is extracted from fitting the data by assuming that the final state interaction (FSI) is given by

$$(FSI) = i\, k_t\, a_{\pi^+ n \to \pi^0 n}\, E_{0+}(\gamma p \to \pi^+ n). \tag{1.1b}$$

In Eq. (1.1b), the $\pi^+ n$ relative momentum k_t is evaluated from the on-energy-shell condition $W = E_n(k_t) + E_{\pi^+}(k_t)$, where $E_n(k) = \sqrt{m_n^2 + k^2}$ and $E_{\pi^+}(k) = \sqrt{m_{\pi^+}^2 + k^2}$. At the π^0 production threshold we have $k_t = i\,0.19 fm^{-1}$. By using $a_{\pi^+ n \to \pi^0 n} = 0.129/m_{\pi^+}$ and the value of $E_{0+}(\gamma p \to \pi^+ n) = 28.3$ (all multipoles are in unit $10^{-3}/m_{\pi^+}$), we find that $(FSI) = -1.0$. Inserting the reported value[1,2] $\hat{E}_{0+}(\gamma p \to \pi^0 p) = -0.5$ into Eq. (1.1), one then obtains

$$E_{0+} = -0.5 - 1.0 = -1.5 \tag{1.2}$$

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It is clear from Eq. (1.2) that the significance of the often-mentioned "unexpected value" -0.5, depends strongly on the validity of using the large FSI correction, -1.0.

We here emphasize that Eq. (1.1) results from a particular choice[8] for the analytical continuation of the K-matrix to the unphysical region, which is not unique unless the underlying dynamics is specified. In a dynamical approach[4-6], the FSI can be explicitly calculated from the underlying Hamiltonian. The objective of this work is to reveal this dynamical feature, using the model we have recently proposed.[6] This will allow us to examine whether Eq. (1.1) is compatible with a dynamical model. We also mention here that a similar study has been reported in the contributed paper of this conference by Yang[9].

II. ANALYTICAL PROPERTIES OF THE PHOTOPRODUCTION AMPLITUDE

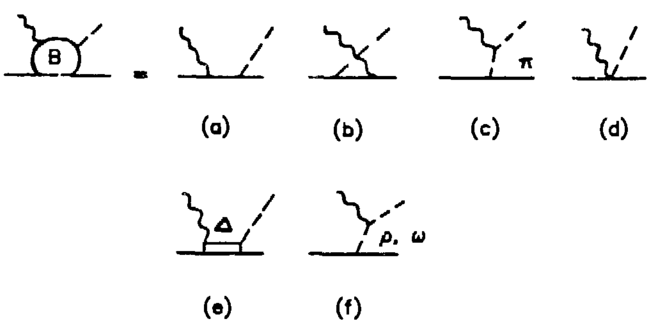
For the considered $\gamma p \rightarrow \pi^0 p$ reaction, each pion photoproduction multipole amplitude is determined by the following equation (all partial wave quantum numbers are suppressed)

$$M_{\pi^0 p \leftarrow \gamma p}(k_0, q) = B_{\pi^0 p \leftarrow \gamma p}(k_0, q) + (FSI)_{\pi^0 p} + (FSI)_{\pi^+ n} \tag{2.1}$$

where k_0 and q are the momenta of the pion and the photon in the center of mass frame, respectively. In Eq.(2.1) B is the Born term calculated from the Feynman amplitudes of Fig.1. The on-shell momentum k_0 is calculated from $W = E_p(k_0) + E_{\pi^0}(k_0)$. The FSI term involves an integration over the half-off-shell πN t-matrix (as illustrated in Fig. 2)

$$(FSI)_{\pi N} = \int_0^\infty dk\, k^2 \frac{t_{\pi^0 p \leftarrow \pi N}(k_0, k, W) B_{\pi N \leftarrow \gamma p}(k, q)}{W^+ - E_N(k) - E_\pi(k) + i\epsilon}, \tag{2.2}$$

where πN denotes $\pi^0 p$ or $\pi^+ n$.



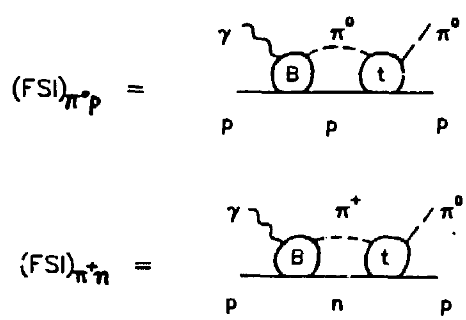


Fig. 1 The pion photoproduction mechanisms. Fig. 2 Graphical representation of the FSI.

To extract an overall πN phase from Eq. (2.1), we recall the well-known relation between the t- and the K-matrix (called the R-matrix in Refs. 6)

$$t(E) = K(E) - i\pi t(E)\delta(E - H_0)K(E), \tag{2.3}$$

where H_0 is the sum of the free energy operators of the pion and the nucleon. After a slightly lengthy algebraic manipulation(, see Ref. 7 for detail,) Eq.(2.3) is written as

$$t_{\pi^0 p \leftarrow \pi N}(k_0, k, W) = e^{i\delta_{\pi^0 p}} cos\delta_{\pi^0 p} \hat{K}_{\pi^0 p \leftarrow \pi N}(k_0, k, W) \tag{2.4}$$

where

$$\hat{K}_{\pi^0 p-\pi N}(k_0,k,W) = K_{\pi^0 p-\pi N}(k_0,k,W)$$
$$-i\theta(W-m_n-m_{\pi^+})F_{\pi^0 p-\pi^+ n}(k_0,k_t,W)K_{\pi^+ n-\pi N}(k_t,k,W) \quad (2.5a)$$

with

$$F_{\pi^0 p-\pi^+ n}(k_0,k_t,W) = \frac{\rho_{\pi^+ n}(k_t)K_{\pi^0 p-\pi^+ n}(k_0,k_t,W)}{1+i\theta(W-m_n-m_{\pi^+})\rho_{\pi^+ n}(k_t)K_{\pi^+ n-\pi^+ n}(k_t,k_t,W)} \quad (2.5b)$$

Here $\rho_{\pi N}(k) = \pi k E_N(k)E_\pi(k)/[E_N(k)+E_\pi(k)]$ and we have defined $\theta(x) = 1$ for $x \geq 0$ and $= 0$ otherwise. Note that the well known relationship between the phase shift and the on-shell t-matrix

$$\rho_{\pi N}(k_0)t_{\pi N-\pi N}(k_0,k_0,W) = -e^{i\delta_{\pi N}}sin\delta_{\pi N} \quad (2.6)$$

is used in the derivation of Eq.(2.4). Substituting Eq. (2.4) into Eq. (2.2) and splitting the propagator of Eq.(2.2) into a principal-value part and the δ-function part, Eq. (2.1) is written as

$$M_{\pi^0 p-\gamma p}(k_0,q) = e^{i\delta_{\pi^0 p}}cos\delta_{\pi^0 p}[B_{\pi^0 p-\gamma p}(k_0,q)$$
$$-\,i\,\theta(W-m_n-m_{\pi^+})F_{\pi^0 p-\pi^+ n}(k_0,k_t,W)B_{\pi^+ n-\gamma p}(k_t,q)$$
$$+\sum_{\pi N=\pi^0 p,\pi^+ n} P\int_0^\infty dk\,k^2\frac{\hat{K}_{\pi^0 p-\pi N}(k_0,k,W)B_{\pi N-\gamma p}(k,q)}{W-E_N(k)-E_\pi(k)}] \quad (2.7)$$

Eq. (2.7) is our main result. Below $\pi^+ n$ threshold, the multipole amplitude of Eq. (2.7) becomes a purely real number. As the energy moves from below to above the $\pi^+ n$ threshold, we will see a sudden change in the imaginary part of the amplitude. This is the dynamical formulation of the cusp effect.

Before we leave this section We note here that if we drop the principal value integral term and ignore the restriction due to the presence of the θ function in Eq. (2.7) and (2.5a), we then have a formula identical to that of Davidson and Mukhopadhyay.[10] If we further neglect the difference between $\hat{K}$ and K matrices, defined in Eq. (2.5a), and extend the usual relationship between the K-matrix and the scattering length to assume that $\rho_{\pi^+ n}(k_t)K_{\pi^0 p-\pi^+ n}(k_0,k_t,W) \rightarrow -a_{\pi^0 p-\pi^+ n}k_t$, we then get the commonly employed Eq. (1.1).

III. RESULTS AND DISCUSSIONS

With the given choice of πN Hamiltonian, there are three free parameters in Ref. 6: (1) the cutoff Λ of a monopole form factor $\Lambda^2/(\Lambda^2+k^2)$ which regularizes the Born term, (2) the coupling strengths G_M and G_E for the $\Delta \leftrightarrow \gamma N$ transition. In Ref. 6, by fitting the $M_{1+}(3/2)$ and $E_{1+}(3/2)$ multipole amplitudes one obtained Λ = 650 MeV/c, G_M = 2.28 and G_E = 0.07. The resulting model is able to give an overall good description of the cross section data for the $\gamma p \rightarrow \pi^0 p$, $\gamma p \rightarrow \pi^+ n$ and $\gamma n \rightarrow \pi^- p$ reactions up to about 400 MeV incident photon energy.

In Fig. 3 we compare the full calculation(solid curve) of the present model(Eq.(2.7)) with the data[1,2]. In the same figure the result from the Born term alone(dashed curve) and the contribution from E_{0+} multipole alone(dash-dotted curve) are also displayed. By comparing the solid and the dashed curves, it is clear that the FSI indeed improves the agreement with the data. The cusp effect, due to the opening of the $\pi^+ n$ channel, is visible in the E_{0+} contribution.

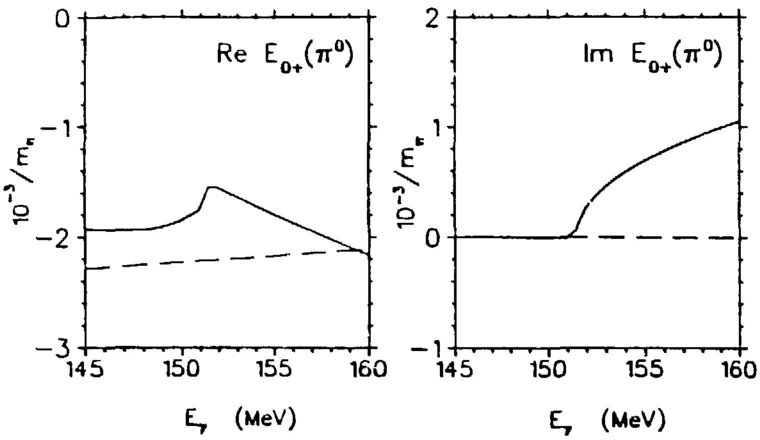


Fig. 3 Calculated total cross section of $\gamma p \rightarrow \pi^0 p$ reaction near threshold.

In Fig. 4 we show the E_{0+} amplitude from the full calculation(solid curve) and Born calculation(dashed curve). Here the cusp effect is emerging from the full calculation. It is important to emphasize here that the "cusp" effect exhibited here is generated dynamically from a πN model Hamiltonian, and is radically different from that based on Eq. (1.1).

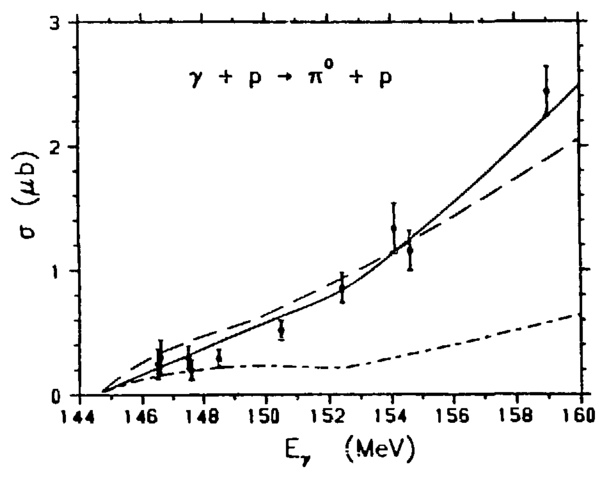


Fig. 4 E_{0+} multipole amplitude of $\gamma p \rightarrow \pi^0 p$ reaction.

To see the dynamical content of our calculation, we now focus on the results at the threshold. In Table 1, we show the FSI effects in determining the multipole amplitudes for three reactions. The FSI effect in the $\pi^\pm$ production is small and hence our full calculations do not deviate significantly from the predictions of the low energy theorem. The FSI effect on the E_{0+} for π^0 production (first row) is +0.37. It is of the opposite sign to the second term of Eq. (1.1) employed in the analyses of Ref. 1 and 2. Our value $E_{0+} = -1.92$ is close to the *measured* value -1.5 of Eq. (1.2). We therefore have demonstrated that with a dynamical treatment of the FSI, it is possible to describe the π^0 photoproduction data without introducing any modification of the low energy theorem.

Table 1: Real parts of the multipole amplitudes at the threshold energies E_{th}. B is the contribution from the Born term, and $FSI = (FSI)_{\pi^0 p} + (FSI)_{\pi^+ n}$ is the final state interaction contribution.

reaction (threshold energy)		Re $\{E_{0+}\}$ $10^{-3}/m_{\pi^+}$	Re $\{E_{1+}\}$ $10^{-3}qk/m_{\pi^+}^3$	Re $\{M_{1+}\}$ $10^{-3}qk/m_{\pi^+}^3$	Re $\{M_{1-}\}$ $10^{-3}qk/m_{\pi^+}^3$
$\gamma p \to \pi^0 p$	B	−2.29	−0.15	5.94	−5.47
($E_{th} = 144.7MeV$)	$B + FSI$	−1.92	−0.18	6.45	−5.14
$\gamma p \to \pi^+ n$	B	27.3	5.21	−9.74	5.71
($E_{th} = 151.4MeV$)	$B + FSI$	26.9	5.30	−10.1	5.48
$\gamma n \to \pi^- p$	B	−31.2	−5.36	11.4	−7.66
($E_{th} = 148.5MeV$)	$B + FSI$	−29.7	−5.43	11.7	−7.44

To further understand our result, it is necessary to examine the role of each mechanism, shown in Fig. 2. This is presented in Table 2. We note that there are large cancellations between diagrams (c) and (d) in the $(FSI)_{\pi^+ n}$ term. This explains why we have a very surprising result that the $(FSI)_{\pi^0 p}$ is *larger* than $(FSI)_{\pi^+ n}$. This is seen in comparing the numbers in the last column of Table 2. If the prescription Eq. (1.1) is used, the contribution from each mechanism will be scaled by the same factor "$k_t a$" and we would expect the otherwise, as commonly assumed in the analyses. The results shown in Table 2 clearly indicates the fundamental difference between a dynamical model and the approach based on Eq. (1.1). In Table 3 we also show that the cancellation between the diagrams (c) and (d) also occurs in the charged pion productions.

Table 2: Contributions of mechanisms shown in Fig. 1 to the E_{0+} amplitude(real part) of the $\gamma p \to \pi^0 p$ reaction at the threshold energy. All amplitudes are in unit of $10^{-3}/m_{\pi^+}$ B denotes the contribution from the Born alone. (FSI) is final state interaction term defined in Eq. (2.1).

$\gamma p \to \pi^0 p$ ($E_{th} = 144.7MeV$)	diagrams in Figs. 1 and 2 (a)	(b)	(c)	(d)	(e)	(f)	sum
B	−1.26	−1.25	0.0	0.0	0.0	+0.22	−2.29
$(FSI)_{\pi^0 p}$	−0.10	+0.57	0.0	0.0	0.0	+0.05	+0.52
$(FSI)_{\pi^+ n}$	−0.53	−0.24	−3.07	+3.57	0.0	+0.10	−0.15
$B + FSI$	−1.89	−0.92	−3.07	+3.57	0.0	+0.37	−1.92

Table 3: Same as Table 2, except for the charged pion production reactions.

reaction (threshold energy)		diagrams in Figs. 1 and 2 (a)	(b)	(c)	(d)	(e)	(f)	sum
$\gamma p \to \pi^+ n$	B	−1.75	−0.30	−0.04	+29.2	0.0	+0.16	+27.3
	FSI	−1.00	+0.36	−3.43	+3.40	0.0	+0.27	−0.40
($E_\gamma = 151.4MeV$)	$B + FSI$	−2.75	+0.06	−3.47	+32.6	0.0	+0.43	+26.9
$\gamma n \to \pi^- p$	B	−0.29	−1.77	+0.06	−29.3	0.0	+0.15	−31.2
	FSI	−0.15	+1.29	+3.48	−3.30	0.0	+0.13	+1.45
($E_\gamma = 148.5MeV$)	$B + FSI$	−0.44	−0.48	+3.54	−32.6	0.0	+0.28	−29.7

We should emphasize here that this somewhat unexpected result is very much related to our "three-dimensional reduction" introduced in Ref. 6 to deduce from the Feynman amplitudes unitary and gauge invariant current matrix elements which are consistent with the considered πN scattering theory. It is well known that the three-dimensional reduction of a field theory is not unique. Hence further investigations are needed to determine whether this cancellation is specific to our approach or is a general property of the basic dynamics.

At threshold, the FSI is only due to the πN interaction in the S_{11} and S_{31} channels. We simply mention here that the FSI effects from both the S_{31} and S_{11} channels are important. Again there is a large cancellation between these two FSI effects.

To close, we note that the FSI calculation is sensitive to the off-shell behavior of the employed πN model. This has been pointed out by Yang.[9] The phenomenological πN model employed in our study is certainly not very satisfactory theoretically, although it can accurately describe the πN phase shifts up to 500 MeV. In the future, it is necessary to investigate the problem using a πN model constructed also from an effective lagrangian which accommodates the low energy theorems. Only using such a fully consistent description of both the hadronic and electromagnetic matrix elements, the low energy theorem can be truly tested.

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