LA-UR -91-3063

Conf-9105251--2

LA-UR--91-3063

DE92 000187

TITLE: PHYSICAL INTERPRETATION OF SUPERCOHERENT STATES AND THEIR ASSOCIATED GRASSMANN NUMBERS

AUTHOR(S): MICHAEL MARTIN NIETO

SUBMITTED TO: PROCEEDINGS OF WORKSHOP ON FOUNDATIONS OF QUANTUM MECHANICS HELD IN SANTA FE, NM FROM MAY 27-31, 1991.

Physical Interpretation of Supercoherent States and their Associated Grassmann Numbers

Michael Martin Nieto

Theoretical Division, Los Alamos National Laboratory,
Los Alamos, New Mexico 87545, U.S.A.

ABSTRACT

A physical interpretation of supercoherent states is suggested. It is based upon the observation that an ordinary coherent state is an eigenstate of a specific mode of the radiation field. A supercoherent state is viewed as a photino coherently combined with photons of the same mode. An interpretation of the associated Grassmann-valued numbers of the state is also discussed.

The introduction of graded Lie algebras was an important milestone in the study of combined internal and space-time symmetries. This led to the development of supersymmetric theories which predict the existence of boson and fermion partner states: e.g., for the photon there is a partner photino, etc.

Up to now none of the supersymmetric partners have been found, so that supersymmetry, if extant, is broken. All evidence for supersymmetry has been found in the low energy regeme: e.g., in nuclei,[1] in atomic systems,[2] and in WKB calculations.[3] But these results, although exciting and indicative, do not unequivocally prove the need for a fundamental supersymmetry. Rather, they are tantalizing hints. The proof would be in the discovery of supersymmetric particles, perhaps at the SSC.

However, if supersymmetric partners were found, then there would be a need to give a physical interpretation to unusual quantities in the full supersymmetry mathematics: Grassmann numbers. We will return to Grassmann numbers below, but first we point out that such a situation would be similar to the problem of the physical interpretation of imaginary numbers when quantum mechanics was discovered.

When Schrödinger described his "coherent states",[4] he thought that the physics was contained in the real part of his wave solutions. It was only later that the interpretation of the wave function as a probability amplitude was formulated. The realization that the complex phase had physical information followed. Similarly, it was understood that the decay time is the modulo of a pure imaginary number. No experiment ever measures an imaginary number. Rather, an imaginary number in the theory is interpreted in terms of a real physical process.

Returning to the coherent states (of the harmonic oscillator),[4–6] they can be defined in many equivalent ways.[7] Among them are a) the position-momentum minimum-uncertainty states of the harmonic oscillator, b) the eigenstates of the destruction operator, and c) those states obtained by applying the displacement operator on the ground state:

$$D(\alpha)|0>= exp[\alpha a^+ - \alpha^* a]|0>= |\alpha> . \tag{1}$$

In the number-state basis, $|\alpha>$ has the decomposition

$$|\alpha>= exp[-|\alpha|^2/2]\Sigma_n[\alpha^n/(n!)^{1/2}]|n> \tag{2}$$

$$= \Sigma_n a_n |n> . \tag{3}$$

The a_n represent the relative occupation in each energy level; i.e., the probability that the particle is in the eigenstate with energy $(\hbar\omega)$ is

$$P(n) = |a_n|^2. \tag{4}$$

There are also many ways to give a physical interpretation of these coherent states. One is that they describe the most classically- possible motion of a quantum-mechanical particle in a harmonic oscillator potential.

The displacement operator method has been generalized to arbitrary semisimple Lie Groups by a number of authors.[8] Recently, FKNT[9] further generalized this method to supergroups, yielding supercoherent states.[9] As with Lie groups, one applies a unitary irreducible representation of the supergroup to the ground state to obtain its coherent states. In particular, for the super Heisenberg-Weyl group defined by

$$[a, a^+] = I, \quad \{b, b^+\} = I, \tag{5}$$

one has

$$T(g) = exp[-\overline{A}a + Aa^+ + BI + \theta b^+ + \overline{\theta} b], \tag{6}$$

$$T(g)|0,0>= |Z> . \tag{7}$$

Technically, A and $\overline{A}$ are complex, even, Grassmann numbers. Because a^+ and a are pure even elements, we associate A and $\overline{A}$ with the α and α^* of the ordinary coherent states $(A = \alpha I)$. B is an imaginary, even, Grassmann number. θ and $\overline{\theta}$ are odd Grassmann numbers. An odd Grassmann number can be written as a complex number, c, multiplying an odd Grassmann basis vector ζ; that is, $\theta = c\zeta$. However, these odd Grassmann numbers are nilpotent, since they satisfy anticommutation relations among themselves: $\{\zeta_1, \zeta_2\} = 0$.[10-12] If there is a fundamental supersymmetry in nature, obtaining a physical interpretation of these vectors would be analogous to having obtained an interpretation of imaginary numbers in early quantum mechanics.

The two labels of $|0,0>$ in Eq. (7) represent the even (bosonic) and odd (fermionic) sectors. Explicit calculation yields

$$|Z>= [1-(1/2)\overline{\theta}\theta]|A,0> +\theta|A,1>, \tag{8}$$

with the bosonic sector being a coherent state $|A>$ and the fermionic sector having zero or one fermions. The supersymmetric Hamiltonian and its expectation value are

$$H = a^+a + b^+b = \{Q, Q^+\}, \quad Q = ab^+, \tag{9}$$

$$< Z|H|Z >= \overline{A}A + \overline{\theta}\theta. \tag{10}$$

The question now arises as to what is the physical interpretation of these states. To understand our suggestion, it is useful to review another, more field-theoretic, interpretation of the ordinary coherent states. It is that the coherent states describe a coherent radiation field. Then a coherent state describes only one mode of the possible many modes of the radiation field, the commutation relations more precisely being

$$[a_{kl}, a_{jr}^{+}] = \delta_{kj}\delta_{lr}. \tag{11}$$

In general there is a product of coherent states for all possible momentum k-vectors, corresponding to $Ek = \omega k$, and all possible polarizations λ. (We could also be describing other bosons, such as scalar mesons, for example.)

Ignoring the k-vector and the polarization of the photon, $P(n)$ signifies the probability that there are n photons with energy E in the coherent field. There is a definite phase relation to the probability amplitude that there are m photons in the coherent field. It is given by the phase of a_n relative to any other a_m.

Now return to the supercoherent states of Eq. (8). We suggest that the odd part describes the existence of a coherent, massless fermion; i.e., a "photino" with energy E. (If one will, one could consider the cavity to be the early universe at 10^{-16} sec when even the mass of a 1 TeV photino would be insignificant.)

As to the Grassmann numbers, we now make a suggestion for their interpretation. To begin, we propose that the fermion sector "phase" relative to the boson sector is defined by the c in $\theta = c\zeta$, ζ labeling the fermionic part of the state. The probability of finding a supercoherent state that has a bosonic sector coherent with one photino is c^*c, with the $\overline{\zeta}\zeta$ in $\overline{\theta}\theta$ labeling the probability as being for a fermion. Thus, the probability of finding a bosonic sector without a coherent photino is $(1 - c^*c)$, from $(1 - \overline{\theta}\theta)$. Note that one must have $|c| \leq 1$. Then the probabilities for the coherent state having or not having a fermion are both ≤ 1. (It is to be observed that our restriction on the value of c is analogous to physical restrictions placed on ordinary quantum mechanics. For example, one demands that all $|a_n| \leq 1$ and one disallows unnormalizable solutions of the Schrödinger equation.)

The above restriction on the value of c is consistent with the expectation value of the Hamiltonian, given in Eq. (10). The amount of fermion energy is $|c|^2 \leq 1$. The maximum energy that there can be in the fermion sector is 1 since there can be at most one fermion in a given mode. The fact that when one squares a Grassmann number one obtains zero is simply a reflection that one can't have two identical fermions.

Observe that one also give an ansatz for the number of Grassmann basis vectors that a particular system should have. In general, this number is determined by the number of independent fermion operators. Here we have two (b and b^+, so the basis vectors are $I, \zeta, \overline{\zeta}$, and $\overline{\zeta}\zeta$.

This suggestion would be a field-theoretic generalization of the physical interpretation that FKNT[9] gave to the supercoherent states for a quantum-mechanical electron in a magnetic field. There one has two towers of states separated by the magnetic field energy eB/M. In the expectation value of the Hamiltonian, this energy is multipled by $\bar{\theta}\theta$, and so $\bar{\theta}\theta$ can be taken as a measure of the separation energy of the two towers; i.e., $|c| = 1$. Here we could also say we have two towers (each of which is comprised of the number states which define the ordinary coherent states). However, instead we are indentifying $\bar{\theta}\theta$ with the probability, $|c|^2 \leq 1$, that there is a fermionic quanta of the system, a massless photino, which is supercoherent with the even, bosonic-sector, coherent state.

At this point it is apropriate to comment on the simplicity of the supercoherent states in Eq. (8). This simplicity allowed our interpretation to be clearly described. These coherent states can be written as a direct product of boson and fermion coherent states. Therefore, the reader might wonder if such a direct product is always the case. The answer is, "No." As demonstrated in FKNT,[9] there are other supercoherent states, such as for $osp(1|2)$, which do mix fermions and bosons.

Therefore, the simplicity of these present states comes about because the algebra of Eq. (5) does not have any nontrivial (anti-) commutation relations which explicitly involve both even and odd operators. In particular, note that when the odd Grassmann numbers associated with the fermion sector are zero, there is no "soul" in the Grassmann number associated with the bosonic probability amplitude. We suspect that if our suggested interpretation has some validity, then this restriction will be true in general, even for more complicated coherent states This is one of many aspects of our proposal which will have to be investigated further.

To this last, remember that there are many questions which remain to be understood. In particular, we have advocated a finite basis set for systems with a finiute number of fermion modes (states). To date, mathematical studies have not rigorously yielded either a mathematical norm nor an integral-differential procedure for such systems. Even so, we hope that our suggestions are steps in the right direction, which will aid in the search for a physical interpretation of multimode Grassmann systems.

I thank P. Ginsparg, A. S. Goldhaber, T. Goldman, and J. R. Klauder for illuminating questions and discussions on this topic. Special thanks go to V. A. Kostelecký and D. R. Truax. In our collaborative work on supergroups,[9,13] we have continuously kept in mind and discussed the possibility of there being a proper physical interpretation of Grassmann-valued quantities

References

[1] F. Iachello, Phys. Rev. Lett. **44** (1981) 772; Nucl. Phys. **A370** (1981) 28[illegible].

[2] V. A. Kostelecký and M. M. Nieto, Phys. Rev. Lett. **53** (1984) 2285; V. A. Kostelecký, M. M. Nieto, and D. R. Truax, Phys. Rev. A **38** (1988) 4413 and references therein.

[3] A. Comptet, A. D. Bandrauk, and D. K. Campbell, Phys. Lett. B **150** (1985) 159.

[4] E. Schrödinger, Naturwiss. **14** (1926) 664.

[5] R. J. Glauber, Phys. Rev. Lett. **10** (1963) 84; Phys. Rev. **130** (1963) 2529; **131** (1963) 2766.

[6] J. R. Klauder, J. Math. Phys. **4** (1963) 1058; E. C. G. Sudarshan, Phys. Rev. Lett. **10** (1963) 277; J. R. Klauder and E. C. G. Sudarshan, Fundamentals of Quantum Optics (W. A. Benjamin, New York, 1968).

[7] M. M. Nieto and L. M. Simmons, Jr. Phys. Rev. D. **20** (1979) 1321; M. M. Nieto, in: Coheent States, Applications in Physics and Mathematical Physics, Eds. J. R. Klauder and B.-S. Skagerstam (Worlds Scientific, Singapore, 1985) p. 29, and references therein.

[8] J. R. Klauder, J. Math. Phys. **4** (1963) 1058; A. O. Barut and L. Girardello, Commun. Math. Phys. **21** (1971) 41; A. M. Perelomov, Commun. Math. Phys. **26** (1972) 222; J. R. Gilmore, Lie Groups, Lie Algebras, and Some of Their Applications (John Wiley & sons, New York, 1974).

[9] B. W. Fatiga, V. A. Kostelecký, M. M. Nieto, and D. R. Truax, Phys, Rev. D **43** (1991) 1403.

[10] A. Rogers, J. Math. Phys **21** (1980) 352; **22** (1981) 443; **22** (1981) 939; **26** (1985) 385.

[11] B. DeWitt, Supermanifolds (Cambridge University Press, Cambridge, 1984) Ch. 1.

[12] V. A. Kostelecký and J. M Rabin, J. Math. Phys. 25 (1984) 2744.

[13] D. R. Truax, V. A. Kostelecký, and M. M. Nieto, J. Math. Phys. **27** (1986) 354; V. A. Kostelecký, M. M. Nieto, and D. R. Truax, J. Math. Phys. **27** (1986) 1419.

END

DATE FILMED

10 / 28 / 91