LA-UR- 97-871

CONF-971005--7

STATIC BENCHMARKING OF THE NESTLE ADVANCED NODAL CODE

Russell D. Mosteller

Los Alamos National Laboratory
MS K551
Los Alamos, NM 87545

(505) 665-4879 (Voice)
(505) 665-3167 (FAX)
mosteller@lanl.gov

Submitted for Presentation at the
Joint International Conference on
Mathematical Methods and Supercomputing for Nuclear Applications
October 6-10, 1997 Saratoga Springs, NY

Preferred Session: Validation of Reactor Analysis Methods
Alternative Session: Benchmarks (poster session; paper will be re-written to conform to the guidlines for poster sessions)

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ABSTRACT

Results from the NESTLE advanced nodal code are presented for multidimensional numerical benchmarks representing four different types of reactors, and predictions from NESTLE are compared with measured data from pressurized water reactors (PWRs). The numerical benchmarks include cases representative of PWRs, boiling water reactors (BWRs), CANDU heavy water reactors (HWRs), and high-temperature gas-cooled reactors (HTGRs). The measured PWR data include critical soluble boron concentrations and isothermal temperature coefficients of reactivity. The results demonstrate that NESTLE correctly solves the multigroup diffusion equations for both Cartesian and hexagonal geometries, that it reliably calculates k_{eff} and reactivity coefficients for PWRs, and that — subsequent to the incorporation of additional thermal-hydraulic models — it will be able to perform accurate calculations for the corresponding parameters in BWRs, HWRs, and HTGRs as well.

I. INTRODUCTION

The NESTLE advanced nodal code[1,2] was developed at North Carolina State University with support from Los Alamos National Laboratory and Idaho National Engineering Laboratory. It employs the nodal expansion method (NEM)[3] in conjunction with a nonlinear iterative method[4,5] to solve the multigroup neutron diffusion equations in up to four energy groups. Furthermore, it can perform either two-dimensional (2D) or three-dimensional (3D) calculations in either Cartesian or hexagonal geometries. Finally, NESTLE can perform either steady-state or transient calculations. The results presented herein are limited to steady-state cases, however.

Because of its flexible geometric capabilities, NESTLE can be used to analyze several different types of reactors. The first part of this study compares results from NESTLE with those from other codes for four static numerical benchmarks. Those benchmarks include a pressurized water reactor (PWR), a boiling water reactor (BWR), a CANDU heavy-water reactor (HWR), and a high-temperature gas-cooled reactor (HTGR). The second part compares predictions from NESTLE with measured data from actual PWRs, including critical soluble boron concentrations and isothermal temperature coefficients of reactivity.

Currently, thermal-hydraulic feedback in NESTLE is based on a homogeneous equilibrium mixture (HEM) model that can account for two-phase flow but implicitly assumes that the pressure is constant. The HEM model is adequate for PWR steady-state conditions and for some PWR transients. However, additional thermal-hydraulic models need to be incorporated before it can perform at-power calculations for other types of reactors.

II. NUMERICAL BENCHMARKS

At the user's option, the nonlinear iterations can be omitted from NESTLE's solution strategy. In such cases, the solution degenerates to the standard finite-difference method (FDM). Although this feature is of no practical importance (it is well known that, in contrast to NEM, FDM requires a very fine mesh to produce an accurate solution), it allows the validation of NESTLE to proceed along two complementary paths: (1) comparison of its FDM solution with other FDM solutions, and (2) comparison of its NEM solution with reference solutions.

Four benchmarks from the *Argonne Benchmark Book* were studied: (1) the IAEA PWR case,[6] (2) the LRA BWR case,[7] (3) the CANDU HWR case,[8] and (4) the HTGR case.[9] None of these benchmarks account for variations in thermal-hydraulic conditions. Consequently, they are ideal tests of NESTLE's ability to solve the steady-state multigroup diffusion equations correctly.

Although the benchmark specifications include both 2D and 3D versions of each of these cases, only results for the 3D cases will be presented herein. The only exception is the 2D HTGR case, because the specifications for the 3D version of that benchmark are incomplete. Results for the 2D PWR, BWR, and HWR cases have been presented previously.[10] Unless otherwise noted, the results presented herein for all of the codes except NESTLE are taken directly from the solutions presented in the *Argonne Benchmark Book*.

A. IAEA 3D PWR Benchmark

The IAEA PWR benchmark contains 177 assemblies and has octant symmetry. The assembly pitch is 20 cm, which is typical of certain PWR designs. The height of the active core is 340 cm, which is slightly shorter than a typical PWR, and the core contains only two different types of assemblies. Control rods are fully inserted in nine assemblies and partially inserted in four. The benchmark specifications include two-group cross sections for both assembly types and the reflector.

Eigenvalues from NESTLE are compared with those from the VENTURE[11] and ARROTTA[12] codes in Table I, and the corresponding power distributions are presented in Figs. 1 and 2. VENTURE is based on FDM, while ARROTTA employs the analytic nodal method (ANM).[13] The ARROTTA results are taken from a presentation[14] at a meeting of the American Nuclear Society.

Table I
Results for 3D IAEA PWR Benchmark

Code	Method	Mesh Spacing (cm)		k_{eff}	Peak Relative Power
		Planar	Axial		
VENTURE	FDM	5	10	1.02864	2.504
		Extrap.	Extrap.	1.02903	2.354
ARROTTA	ANM	20	20	1.02899	NR[a]
NESTLE	FDM	5	10	1.02864	2.504
	NEM	20	20	1.02899	2.304
		5	10	1.02907	2.340

[a] Not reported

NESTLE's FDM calculation replicates k_{eff}, the assembly power distribution, and the nodal peak power from the VENTURE calculation with the same mesh. In addition, the NEM calculation produces the same value of k_{eff} as ARROTTA when the same mesh structure is employed. Furthermore, the power distributions from the NESTLE's NEM calculation are in excellent agreement with those from ARROTTA and from the extrapolated solution from VENTURE (the extrapolated solution was obtained by extrapolating the results from a series of finer and finer meshes to an infinitely fine mesh). There is only one exception to the generally excellent agreement, and that appears to be a typographical error for one of the assembly powers in the VENTURE solution for the 5-cm x 5-cm x 10-cm mesh

VENTURE, extrapolated mesh
VENTURE, 5-cm x 5-cm x 10-cm mesh
NESTLE (FDM), 5-cm x 5-cm x 10-cm mesh

					0.597 0.521 0.521		
				0.476 0.454 0.454	0.700 0.678 0.678	0.611 0.528 0.528	
			1.178 1.218 1.218	0 972 1 025 0 993	0.923 0.906 0.906	0.866 0.790 0.790	
		1.368 1.437 1.437	1.311 1.366 1.366	1 181 1.207 1.207	1.089 1.081 1.081	1.000 0.960 0.960	0.711 0.614 0.614
	1.397 1.490 1.490	1.432 1.516 1.516	1.291 1.356 1.356	1.072 1 112 1.111	1.055 1.058 1.058	0.976 0.944 0.944	0.757 0.685 0.685
0.729 0.750 0.750	1.281 1.378 1.378	1.422 1.508 1.508	1.193 1.266 1.266	0.610 0.606 0.606	0.953 0.963 0.963	0.959 0.932 0.932	0.777 0.706 0.705

Fig. 1. Relative Power by Assembly for 3D IAEA PWR Benchmark from FDM Calculations.

VENTURE, extrapolated mesh
ARROTTA, 20-cm x 20-cm x 20-cm mesh
NESTLE (NEM), 20-cm x 20-cm x 20-cm mesh
NESTLE (NEM), 5-cm x 5-cm x 10-cm mesh

					0.597 0.601 0.590 0.600		
				0.476 0.476 0.473 0.476	0.700 0.706 0.699 0.701	0.611 0.617 0.609 0.613	
			1.178 1.178 1.182 1.175	0 972 0.972 0.975 0 971	0.923 0.924 0.920 0.923	0.866 0.872 0.860 0.868	
		1.368 1.367 1.371 1.363	1.311 1.311 1.316 1.307	1.181 1 179 1 184 1 179	1.089 1.087 1.088 1.089	1.000 0.998 0.993 1.001	0.711 0.710 0.703 0.714
	1.397 1.394 1.401 1.391	1.432 1.429 1.436 1.426	1.291 1.288 1.294 1.286	1.072 1 070 1 075 1 069	1.055 1.051 1.054 1.055	0.976 0.971 0 970 0.977	0.757 0.752 0.749 0.760
0.729 0.732 0.733 0.727	1.281 1.277 1.285 1.278	1.422 1.419 1.426 1.417	1.193 1.190 1.198 1.190	0.610 0 610 0 610 0 610	0.953 0.950 0.955 0.953	0.959 0.954 0 955 0.961	0.777 0.770 0.770 0.780

Fig. 2 Relative Power by Assembly for 3D IAEA PWR Benchmark

Two additional points are worth noting. First, as a comparison of Figs. 1 and 2 demonstrates, NEM produces much more accurate power distributions than FDM for the same (coarse) mesh Second, the results from the NEM calculation change only slightly when a substantially finer mesh is employed.

B. LRA 3D BWR Benchmark

The linear-regression-analysis (LRA) BWR benchmark is an axially uniform BWR with 712 bundles and octant symmetry. The bundle pitch is 15 cm, which is only very slightly less than actual BWR designs. However, the height of the active core is 300 cm, which is approximately 20% shorter than a typical BWR, and it contains only two different types of fuel bundles. This benchmark specifies a control-rod ejection transient, but only the initial condition will be discussed herein. At that point, control rods are fully inserted in 40 of the bundles The benchmark specifications include two-group cross sections for both types of fuel bundles and the reflector.

Eigenvalues from NESTLE are compared with those from the QUANDRY code[13] in Table II, and the corresponding power distributions are presented in Fig. 3 QUANDRY, like ARROTTA, is based on ANM, and the QUANDRY results are taken from the thesis that describes the development of ANM.

Table II
Results for Static 3D LRA BWR Benchmark

Code	Method	Mesh Spacing (cm)		k_{eff}	Peak Relative Power
		Planar	Axial		
QUANDRY	ANM	7.5	25[a]	0.99639	NR[b]
		15	25[a]	0.99644	NR[b]
NESTLE	NEM	7.5	7.5	0.99638	3 462
		15	15	0.99627	3.210

[a] 15 cm in axial reflector
[b] Not reported

NESTLE produces excellent agreement with QUANDRY for both k_{eff} and the bundle power distribution. Although the results from both codes are relatively insensitive to the spatial mesh, the differences between the two sets of NESTLE results are more pronounced than those from QUANDRY. Such behavior is to be expected, however, because of basic differences between the ANM and NEM formulations.

C. 3D CANDU HWR Benchmark

The CANDU HWR Benchmark specifications also are for a transient. However, the discussion herein will be limited to the initial static condition. This benchmark contains three distinct regions: an inner core, an outer core, and a reflector. All three regions are axially uniform, and there is no top or bottom reflector The inner core is a parallelepiped that is 600 cm high and 360 cm across. The outer core encloses the sides of the inner core and is 120 cm thick, except at its jagged corners. The reflector, in turn, encloses the sides of the outer core and is 90 cm thick, except at its jagged corners In the jagged corners, the thickness of the outer core ranges from 60 to 120 cm , while the thickness of the reflector ranges from 60 to 90 cm The benchmark specifications include two-group cross sections for all three regions.

Table III compares results from NESTLE with two FDM codes, CERKIN[15] and CERBERUS [16] The *Argonne Benchmark Book* does not include detailed power distributions from CERKIN or CERBERUS but instead only reports power fractions for the inner core and for two separate portions of the outer core. Somewhat surprisingly, the results from the NESTLE NEM calculations are in better agreement with those from CERBERUS than are those from the NESTLE FDM calculation. This

QUANDRY, 7.5-cm x 7.5-cm x 25-cm mesh QUANDRY, 15-cm x 15-cm x 25-cm mesh NESTLE (NEM), 7.5-cm x 7.5-cm x 7.5-cm mesh NESTLE (NEM), 15-cm x 15-cm x 15-cm mesh							1.332 1.329 1.327 1.303	
						2.161 2.164 2.156 2.152	1.622 1.623 1.617 1.601	0.848 0.846 0.845 0.828
					1.852 1.853 1.849 1.852	2.051 2.053 2.047 2.048	1.680 1.680 1.676 1.665	0.972 0.973 0.969 0.957
				0.864 0.865 0.864 0.866	1.152 1.151 1.151 1.148	1.339 1.338 1.337 1.332	1.422 1.423 1.419 1.415	0.932 0.933 0.930 0.922
			0.552 0.553 0.553 0.556	0.678 0.678 0.678 0.681	0.843 0.843 0.843 0.843	1.022 1.022 1.022 1.020	1.221 1.220 1.220 1.217	0.853 0.853 0.852 0.845
		0.424 0.424 0.425 0.429	0.492 0.492 0.493 0.497	0.618 0.618 0.619 0.623	0.783 0.782 0.783 0.786	0.967 0.966 0.967 0.968	1.173 1.171 1.173 1.172	0.827 0.826 0.826 0.822
	0.400 0.400 0.401 0.407	0.407 0.406 0.408 0.413	0.490 0.490 0.492 0.496	0.670 0.671 0.672 0.678	0.940 0.939 0.941 0.946	1.151 1.149 1.152 1.155	1.281 1.281 1.282 1.286	0.870 0.867 0.867 0.864
0.612 0.612 0.615 0.628	0.440 0.440 0.442 0.448	0.413 0.413 0.415 0.419	0.512 0.511 0.513 0.518	0.790 0.789 0.792 0.797	1.384 1.385 1.388 1.404	1.660 1.662 1.663 1.681	1.481 1.481 1.483 1.487	0.924 0.923 0.924 0.922

Fig. 3. Relative Power by Bundle for 3D LRA BWR Benchmark.

behavior suggests that the finer mesh that CERBERUS uses near the fuel/reflector interface significantly improves the accuracy of the results. Overall, however, there is excellent agreement between the NEM calculations and the reference solutions from CERKIN and CERBERUS.

D. 2D HTGR Benchmark

The 2D HTGR problem is a sextant-symmetric HTGR with 247 fuel channels and 180 reflector channels The specifications for this benchmark differ from the others discussed above in two important respects: the geometry is hexagonal rather than Cartesian, and the cross sections are specified for four energy groups rather than two

Table IV compares the eigenvalues obtained from NESTLE with those from the VENTURE and GRIMHX[17] codes, and Fig. 4 compares the channel power distributions from NESTLE and GRIMHX. The *Argonne Benchmark Book* does not include detailed power distributions from VENTURE.

GRIMHX can solve the multigroup diffusion equations with either standard FDM or a higher-order coarse-mesh FDM (CMFDM).[18] The FDM option in NESTLE produces a value for k_{eff} that in is excellent agreement with those from the other

Table III
Results for 3D CANDU HWR Benchmark

Code	Method	Mesh Spacing (cm)		k_{eff}	Power Fraction		
		Planar	Axial		Outer Core, Front and Back	Outer Core, Sides	Inner Core
CERKIN	FDM	NR[a]	NR	1.00355	0.2752	0.3106	0.4142
CERBERUS	FDM	30 / 60[b]	60	1.00356	0.2752	0.3106	0.4142
NESTLE	FDM	30	60	1 00315	0.2739	0 3099	0.4162
	NEM	30	60	1 00357	0.2743	0 3114	0.4143
		15	60	1 00351	0.2742	0.3111	0.4147

[a] Not reported
[b] 30 cm near fuel/reflector interface, 60 cm elsewhere

Table IV
Eigenvalues for 2D HTGR Benchmark

Code	Method	Mesh Spacing (cm)	k_{eff}
VENTURE	FDM	36 2	1.12725
		Extrap	1.11835
GRIMHX	FDM	36 2	1.12725
	CMFDM	12.1	1.11863
NESTLE	FDM	36 2	1.12722
	NEM	36 2	1.11852

FDM calculations, and the NEM option in NESTLE produces a value for k_{eff} that in is excellent agreement with those extrapolated from VENTURE and calculated using CMFDM. Furthermore, the FDM and NEM power distributions from NESTLE are in excellent agreement with those from the FDM and CMFDM options, respectively, in GRIMHX. Although the values of k_{eff} extrapolated from VENTURE calculation and obtained from the CMFDM GRIMHX calculation differ slightly, the difference is sufficiently small that the power distribution that would be extrapolated from VENTURE should be quite similar to the one calculated with CMFDM.

III. MEASURED PWR DATA

Although it does not identify them by name, Ref 19 provides reasonably detailed descriptions of the core design and loading pattern for the first cycle of four PWRs. A succinct summary of the first cycle of each of those plants is provided in Table V All four have "out-in" loading patterns wherein the central portion of the core is a checkerboard of the two lower

0.369 0.315 0.315 0.372	GRIMHX (CMFDM), 12.1-cm mesh GRIMHX (FDM), 36.2-cm mesh NESTLE (FDM), 36.2-cm mesh NESTLE (NEM), 36.2-cm mesh							
1.093 1.281 1.282 1.090	1.093 1.281 1 282 1.090							
1.323 1.464 1.465 1.327	1.301 1.452 1 453 1 305	1.323 1.464 1.465 1.327						
1.269 1.392 1.392 1.274	1.307 1.434 1.435 1.312	0.856 1.023 1 024 0.858	1.269 1.392 1.392 1.274					
1.017 1.135 1.136 1.014	1.038 1.173 1.174 1.035	1.250 1.355 1.356 1.254	1.269 1.363 1.363 1.273	1.017 1.135 1.136 1.014				
0.947 1.017 1.017 0.943	0.326 0.259 0.259 0.328	0.995 1.097 1.098 0.993	1.229 1.286 1.287 1.232	1.189 1.242 1.242 1.193	0.947 1.017 1.017 0.943			
1.047 1.030 1.030 1.044	0.880 0.925 0.925 0.875	0.916 0.973 0.973 0.912	1.149 1.172 1.172 1.153	0.781 0.858 0.858 0.783	1.155 1.152 1.152 1.158	1.047 1.030 1.030 1..044		
0.642 0.648 0.648 0.638	0.994 0.947 0.946 0.991	1.000 0.977 0.977 1.000	1.080 1.054 1.054 1.080	1.160 1.127 1.127 1.164	1.159 1.116 1.116 1.162	1.083 1.029 1.029 1.084	0.648 0.648 0.648 0.638	
0.830 0.664 0.662 0.828	0.895 0.753 0.752 0.894	0.933 0.824 0.824 0.929	0.642 0.648 0.648 0.638	1.057 0.978 0.978 1.052	1.077 0.996 0.995 1.076	1.043 0.952 0.951 1.040	0.941 0.817 0.816 0.938	0.830 0.664 0.662 0.828

Fig. 4. Relative Power by Channel for 2D HTGR Benchmark

enrichments and the high enrichment is loaded on the periphery. In addition, all of them have discrete lumped burnable poison rods (LBPRs) in some of their assemblies. All of the measurements discussed herein were made at hot-zero-power (HZP) conditions at beginning of life (BoL), prior to ascension to power.

The assemblies in plants C and D are an "optimized" design, wherein the radius of the fuel rods has been decreased to provide a higher moderator-to-fuel ratio. This design, in combination with the wide range of enrichments and (in plant C) part-length LBPRs, produces a flux distribution that is very sensitive to changes in core operating conditions at BoL.

Table V
Plant Characteristics

Plant	Rated Power (MWt)	Number of Assemblies	Type of Assemblies	Type of LBPR	Fuel Enrichment (w/o)
B	3250	193	15 x 15	Full Length	2.248, 2.789, 3.292
C	2775	157	17 x 17	Part Length	1.6, 2 4, 3.1
D	3411	193	17 x 17	Full Length	1.6, 2.4, 3.1
F	2560	217	14 x 14	Part Length	1.9, 2 3, 2 8

The cross sections for NESTLE were not generated as part of this study. Instead, cross sections that had been generated previously[29] for ARROTTA simply were translated into NESTLE input format. Because of the similarity in the cross-section representation employed by the two codes, no approximations were required for the translation.

The NESTLE predictions for critical soluble boron concentrations are compared with the measured values in Table VI. The agreement is excellent: the largest difference between the measured concentration and that predicted by NESTLE is 38 PPM, and the average difference is only 9 PPM, which is comparable to the uncertainty in most measurements of PPM at HZP. The consistent agreement in the critical soluble boron concentration with different control-rod banks inserted also demonstrates that NESTLE accurately predicts control-rod worth. Furthermore, the values predicted by NESTLE are very similar to those predicted by ARROTTA, which suggests that much of the difference in critical PPM may be due to the cross sections rather than the code itself.

Table VI
Measured and Predicted Critical Soluble Boron Concentrations

Plant	Temperature (°F)	Control-Rod Banks Inserted	Critical Soluble Boron Concentration		
			Measured	ARROTTA[a]	NESTLE
B	547	None	1350	1314	1312
		D[b]	1348	—	1310
		C[c], D	1203	—	1170
		B[b], C, D	1085	—	1063
		A[c], B, C, D	940	—	914
C	557	None	1189	1190	1187
D	557	None	975	997	996
		D	902	933	932
		C, D	816	841	837
F	532	None	952	951	953
		5, 6, 7	844	813	823
		2, 3, 4, 5, 6, 7	606	580	605

[a] Values taken from Ref. 19
[b] 200 Steps Withdrawn (Fully Withdrawn at 220 Steps)
[c] 180 Steps Withdrawn (Fully Withdrawn at 220 Steps)

The NESTLE predictions for isothermal temperature coefficients of reactivity (ITCs) are compared with the measured values in Table VII. Once again, the values predicted by NESTLE are in excellent agreement with the measured ITCs and with those predicted by ARROTTA. The largest difference between the measurements and the corresponding NESTLE predictions is only 2.3 pcm/°F, and the average difference is only 0.4 pcm/°F. The agreement with the measurements from Plants C and D is particularly good, given the sensitivity of the flux distribution at these conditions. (Because of the large differences in enrichment and the out-in loading pattern, relatively small changes in the flux distribution can cause significant changes in the ITC.)

Table VII
Measured and Predicted ITCs at BoL

Plant	Temperature (°F)	Control-Rod Banks Inserted	Critical Boron Concentration (PPM)	ITC (pcm/°F)		
				Measured	ARROTTA[a]	NESTLE
B	547	D[b]	1348	-1.3 ± 0.3	-1.6	-1.6
		C[c], D	1203	-5.2 ± 0.3	-5.6	-5.3
		B[b], C, D	1085	-9.0 ± 0.9	-9.1	-8.5
		A[c], B, C, D	940	-10.3 ± 1.7	-10.7	-9.9
C	557	None	1189	3.5	1.3	2.1
D	557	None	975	-1.7	-3.2	-2.7
		D	902	-2.8	-4.3	-3.9
		C, D	816	-8.0	-8.9	-8.0
F	532	None	952	0.8	-0.8	-0.9
		5, 6, 7	844	-4.1	-5.4	-4.9
		2, 3, 4, 5, 6, 7	606	-10.4	-10.3	-8.1

[a] Values taken from Ref. 19
[b] 200 Steps Withdrawn (Fully Withdrawn at 220 Steps)
[c] 180 Steps Withdrawn (Fully Withdrawn at 220 Steps)

IV. CONCLUSIONS

The results from the numerical benchmarks demonstrate that NESTLE FDM calculations replicate the FDM calculations from other FDM codes almost identically and that the NESTLE NEM calculations produce excellent agreement with reference solutions. As expected, the NESTLE NEM calculations produce accurate results with spatial meshes that are much larger than those required for accurate FDM calculations. Furthermore, NESTLE produces accurate results not only for the PWR, BWR, and CANDU benchmarks in Cartesian geometry but also for the HTGR benchmark in hexagonal geometry.

NESTLE also has been shown to predict critical soluble boron concentrations, ITCs, and control-rod worth that are in excellent agreement with measured data from a variety of PWRs. The agreement with the corresponding values from ARROTTA demonstrates that, given the same input, the two codes produce very similar results for actual PWRs This behavior is, of course, consistent with their results for the IAEA PWR benchmark.

In summary, NESTLE has been shown to predict the behavior of a variety of PWRs very accurately at static conditions In addition, its ability to produce accurate results for numerical benchmarks representing BWRs, CANDUs, and HTGRs provides assurance that, after steady-state thermal-hydraulics modules for those types of reactors are installed in NESTLE, it can be used for calculations for them as well.

ACKNOWLEDGMENT

The cross sections for these comparisons with measured PWR data were obtained from the Electric Power Research Institute (EPRI). Special thanks are due to J. Chao and L. Agee of EPRI, who approved their use, and to L D Eisenhart of S Levy, Inc., who supplied the actual data.

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