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A LINEAR-PROGRAMMING APPROACH TO ELECTRICITY  
DEMAND-CURTAILMENT PLANNING

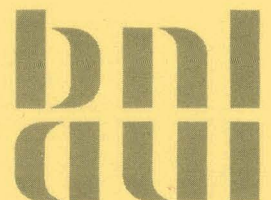
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## I. INTRODUCTION

The threat of electricity supply shortages has been frequently alluded to by spokesmen for the electric public utilities as a consequence of our alleged failure to permit growth in generating capacity to keep pace with estimated demand. Recent events such as the Arab oil embargo of 1973-74, the coal strike of 1977-78, the drought in the Northwest of 1977-78 and the shut down of selected nuclear power plants before and after the incident at Three Mile Island have focused attention on the possibility of electricity supply shortages from long term (one to 12 months) energy (as opposed to capacity) deficiencies.

There are a number of supply side responses which could be adopted to mitigate the disruptive impacts of fuel-type shortages on the generation of electric energy. For example, utility systems may reduce system voltages, switch from economic dispatch to a fuel conserving mode, switch to alternate fuels where a dual fired capability exists, defer normal maintenance or retirements, relax emission standards for air pollutants and exploit transfer capabilities of interties. When all possible supply side initiatives have been taken, any potential shortage of electricity which remains must somehow be allocated among the utilities' customers. An orderly approach to such allocation is via the curtailment plan.

Curtailment planning at a generally rudimentary level has been undertaken by the governments of some twenty states. Many utilities have demand curtailment plans, however, these are often incorporated in plans for meeting capacity shortages. In at least five states, there are apparently no demand curtailment plans either at the state government level

or at the utility level. Moreover, none of the existing electricity demand curtailment plans which were examined included an explicit statement of the planners' objective in arriving at a specified sharing of the burdens of curtailment among consumer classes. Yet clearly the actual allocations of such burdens will affect the cost of the shortage.

In practice it appears that decision-makers are motivated by a wide variety of considerations in assigning the burdens of curtailment to consumer classes. One of these may be the minimization of economic loss, however defined, from demand curtailment. Another may be the minimization of the number of voters who are seriously inconvenienced by curtailment. Yet a third may reflect some subjective or popularly conceived criterion of equity. Additional considerations are not difficult to imagine. It is likely that electricity demand curtailment planners starting with a clearly defined objective will early in the planning process be faced with problems of technical feasibility, of legality and of enforceability. Satisfactory solutions of problems of one kind are frequently incompatible with solutions of the other.

Since a study of state planning failed to yield a clear-cut indication of which of many possible curtailment allocation schemes would best serve as a point of departure for the design of an "optimal" curtailment strategy to deal with prolonged supply deficiencies, it was then decided to use a linear programming approach. Such an approach has advantages which we examine in the next section. In Section III some important conceptual and practical problems in the design of a specific linear programming model are addressed. Section IV contains a mathematical statement of the model. In Section V we briefly review the principal

methodological short-comings of the linear programming approach. Finally, in Section VI we discuss how the analysis might be expanded to account for inter-regional and other secondary effects of curtailment.

## II. LINEAR PROGRAMMING: THE DEMAND CURTAILMENT CONTEXT

Linear programs are concerned with minimizing objective functions in the face of certain constraints. In the context of electricity demand curtailment a linear program might be written to minimize the loss in a socio-economic account, for example, income, or employment, subject to certain constraints. The value of the "objective function" measures the socioeconomic loss, in terms of such an account, from electric demand curtailment. The changes in the value of this function as constraints are varied indicate the cost of the constraints or the cost of deviating from the minimum cost allocation. Such deviations are likely since in the real world of institutions, politics and technological limitations, a minimum cost allocation is impossible. On the other hand, the change in the value of the objective function that arises from a deviation from the loss minimizing allocation of demand curtailment is an indication of the economic price which is being paid for achieving a different allocation. For example, the implicit dollar value in terms of production (and corresponding decrease in the unemployment rate) from a reallocation of electricity supply to industry by allowing an increase in curtailment in the residential sector (or commercial sector) can be shown in the solution.



Finally, it is likely that much of the data upon which we might base our design of a curtailment plan are less than perfect, or even unavailable. The approach that we adopt permits us to test the sensitivity of results across a range of possible values for these data.

In summary, the reasons for our decision to base our initial design of an "optimal" allocation strategy on the linear program approach are, to a large extent, pragmatic. There is no single existing approach to demand curtailment allocation for which the claim may be made that it minimizes socio-economic impacts, however defined. A linear program based on the minimization of the loss in a specified economic account provides an estimate of the loss, the "shadow prices" of constraints and the cost of deviating from the loss minimizing allocations. The availability of these accounting values enables the planner to answer such questions as "What does it cost if legally you might not require Industry A to curtail more than Industry B?" or "What is the cost of constraining the curtailment of electricity to Industry C to be less than k%?" Linear programming thus provides a means for estimating the cost of departure from an allocation designed to minimize losses from demand curtailment. In addition, the data assembled in a linear programming format may easily be used in an accounting mode to measure the cost of specific curtailment plans such as equal percentage curtailment to all sectors.

### III. DESIGN OF A SPECIFIC MODEL

In general terms, our linear programming model minimizes economic loss by allocating curtailments to utility customer classes according to

some value measure of "economic loss" per kWh subject to constraints on curtailment to sensitive or strategic sectors; within those constraints the customer class with the lowest economic loss per kWh is curtailed first, up to some specified maximum curtailment level, then the customer class with the next lowest economic loss per kWh is curtailed up to its' maximum, and so forth until the total required regional curtailment is achieved.

The procedure for allocating the burden of curtailment, as formulated here, takes carefully into account in principle and sometimes in practice, the differential nature of socio-economic impacts across a disaggregated set of utility customer classes. In the application of a linear programming approach to curtailment, an appropriate level of disaggregation means that total required regional curtailment may be achieved without overshooting the need (thereby making the economic effects more damaging than is necessary), yet due consideration can be given to intangible costs from curtailment in sensitive or strategic sectors such as hospitals or communications. Disaggregation in the commercial sector is not conceptually difficult, although in some cases there are data problems. Appropriate disaggregation of the industrial sector is conceptually more difficult; the classification scheme which seems "best" in principle, has intractable data problems.

In practice, we divide the commercial sector into eight subsectors: communications, retail trade, wholesale trade, services, other utilities, schools, hospitals, public administration (We assume that public transportation and street lighting are not curtailed.) The industrial sector is divided into twenty subsectors corresponding to the two digit SIC

code, plus mining. These classification systems were chosen mainly because the data -- shipments or value added, electricity consumed, employment and so forth -- are most readily available in that form, particularly at the state level and, in many cases, similar information can be obtained or estimated for counties. Although this classification system makes data collection possible, there is a real question as to whether or not the allocation strategy as it is so far conceived would actually minimize total economic loss in real situation.

The linear programming model curtails sectors with the smallest economic loss per kWh, therefore, the sum of economic losses over all sectors is, in the mathematics of the model, the smallest sum, hence, the model by implication is choosing an optimal allocation plan. There is no apparent reason to doubt that an "optimal strategy" chosen in the linear programming approach will do a significantly better job, in terms of minimizing total economic loss, than the strategy of equal curtailment across sectors -- the prototype state curtailment plan.

Nevertheless, the linear programming solution is not, in principle, the best strategy. The true optimal strategy would take into account the concept of complementarity.

Complementarity means that when the output of one establishment declines there is a corresponding decrease in output in other businesses. This happens because the other companies are either suppliers to the first, require the curtailed output as input for their own production, or, make products which are used in conjunction with the curtailed output. When the curtailment plan does not take complementarity into account the results are likely to be more disruptive than is necessary.

An optimum plan would combine establishments into complementary groups and use this classification scheme as part of the overall strategy. In practice, however, we have no way at the present time of classifying utility customers according to complementarity. One possible approach to this problem would be to combine a linear programming model with a regional input-output model. We expand further on this idea in the final section of this paper.

Another problem with the classification of commercial and industrial sectors by the aforementioned SIC categories is that there is a wide disparity in energy intensiveness, that is, output per kWh, within many 2 digit sectors. In fact, energy intensiveness often varies significantly even among companies or establishments within narrower product code definitions. This problem, however, is not as intractable as the previous one; it could be solved in principle by an extensive data collection effort.

Finally, the design of a specific linear programming model must be concerned with the choice of appropriate measures of economic loss in each sector.

As residential levels of electricity utilization begin to drop, there is some loss in comfort and convenience and we quantify that loss by tracking the decline in utility revenues. It may be argued that the amount paid by the residential consumers (the revenue received by the utility) is less than the consumer would be willing to pay to avoid a supply deficiency. It has been suggested that the appropriate measure is actually the latter price. The fact remains however that the vast majority of consumers who have experienced outages have not purchased standby

power generators. For this reason and a number of others, especially the difficulty in obtaining data on the "cost" of curtailment to the residential sector,\* we use lost revenues to the utility from curtailment of demand in the residential sector as the proxy for economic loss from such curtailment. To measure the sensitivity of the results to this assumption, weighting factors may be applied in the objective function. In fact, we can employ the same device to enable sensitivity tests of assumptions in the commercial and industrial sectors.

In the commercial and industrial sectors, we also assume that the price paid for electricity reflects the value of its loss, but only when curtailment in these sectors does not cause curtailment of operations. When we project curtailment levels which imply production losses, we switch to value added as the measure of economic loss. Using value added instead of output helps to avoid double counting when total economic loss from a given curtailment scenario is calculated. In addition, value added reflects industrial ability to maintain wages, another important perspective in minimizing the economic impacts of electricity shortages.

Schools, cultural institutions, such as museums, and hospitals have outputs that cannot be quantified in monetary terms. For such establishments, economic loss from demand curtailment is assumed to be lost payroll. For these establishments careful consideration must be given to an appropriate choice of constraints - the upper bounds on allowable curtailment - for each level of required regional curtailment. Due to the paucity of data, these constraint choices are necessarily subjective.

In the residential sector, an upper bound to curtailment at the level of the individual dwelling may also be imposed as a constraint to reflect intangible costs or political considerations.

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\*More extensive discussion of this point appears in another paper currently in draft form, "Modeling the Intangible Costs of Prolonged Electricity Demand Curtailment."

#### IV. MODEL STATEMENT

The linear programming curtailment model establishes a quantitative value for the minimum economic loss in terms of the economic accounts described above and shows how it may be attained. This is accomplished within a set of bounds on curtailment policy reflecting: 1) the requirement that total regional electricity reduction is met, 2) sectoral target reductions are met in residential, commercial and industrial sectors, 3) maximum curtailment standards are set for specific categories of electricity users in the commercial and industrial sectors, and, possibly, 4) unemployment may not exceed certain limits. The model also provides, with respect to 1) for example, the additional dollar economic loss associated with another percentage point curtailment of regional electricity supply.

Model description begins with a statement of the regional total electricity curtailment to be put into effect and constraints on curtailments in the residential, commercial and industrial sectors. The optimization procedure selects appropriate sectoral curtailments that achieve a minimum economic loss. These are expressed as the fractional (or percentage) reduction in electricity use on a monthly basis imposed upon specific customer categories. While one might wish to determine likely end-use cutbacks in space heating, lighting, etc. in the residential and commercial sectors electricity utilization is discretionary; we do not specify what end-uses in these sectors are actually cut back. Consequently, the curtailment variable is defined on a customer category, for example retail stores in the commercial sector.

In the mathematical statement of the model, the variable of interest is the percentage reduction in electricity use of customer group "i" in month "j"

$$x_{ij} = 1 - X_{ij}/X_{ij}^0$$

where  $X_{ij}$  is "actual" (curtailed) kWh electricity use over the month and  $X_{ij}^0$  is used in the absence of any curtailment.

Accordingly, the objective function is defined as the "cost" of the change in the level of electricity use by all customers in the residential, commercial and industrial sectors.

$$\text{Min } W = \sum_{i,j} \alpha_{ij} c_{ij} X_{ij}^0 x_{ij} \quad (1)$$

where  $c_{ij}$  is the "value" of one kWh of electricity use and  $\alpha_{ij}$  is a weighting parameter which may be varied to reflect importance of industrial vs commercial vs residential losses.

The curtailment of electricity use must at least meet the prescribed regional generation limit:

$$\sum_i \left( \frac{X_{ij}^0}{E_j^0} \right) x_{ij} \geq \Delta E_j \quad (2)$$

where  $E_j$  is the required regional electricity curtailment (%) in a given month and  $(X_{ij}^0/E_j^0)$  is the contribution of each customer group to the demand for electricity in the region in that month.

Additional constraints, corresponding to maximum service levels, are imposed upon the residential, commercial, and industrial sectors:

$$\sum_{i = \text{indus}} \left( \frac{X_{ij}^0}{E_j^0} \right) x_{ij} > \Delta E_j^I \quad (3a)$$

$$\sum_i = \text{com} \quad \left( x_{ij}^o / E_j^o \right) x_{ij} \geq \Delta E_j^C \quad (3b)$$

$$\sum_i = \text{resid} \quad \left( x_{ij}^o / E_j^o \right) x_{ij} \geq \Delta E_j^R \quad (3c)$$

where the  $\Delta E_j$  are minimum electric energy reductions to be imposed.

In any attempt to meet the regional (Eq. 2) and sectoral (Eq. 3) electricity reduction targets, individual users within a sector may suffer disproportionately when electricity loss becomes severe. In the interest of minimizing those types of losses, which usually are difficult to quantify, reductions in supply to certain categories of customers may be limited. For example, sufficient energy delivery to residential consumers to maintain minimum interior space temperatures during heating seasons might be maintained. Similarly, we might want to limit reduction of service to hospitals, schools, or essential industry groups. Accordingly,

$$x_{ij} \leq D_{ij} \quad (4)$$

where  $D_{ij}$  is the maximum cutback assigned to specific customers "i" in month "j". Appropriate averages of these constraints give us minimum service levels for each sector.

As indicated earlier in this report it is possible to include in the model a variety of constraints which reflect unquantifiable social goals. For example, although unemployment may be quantified, no measure of unemployment reflects either the psychological hardship imposed on individuals or the potentially destructive effect on the social fabric of the country. Accordingly, it may be desirable to set limits on overall unemployment effects.



## V. A BRIEF REVIEW OF SOME METHODOLOGICAL SHORTCOMING

The linear programming approach to electricity demand curtailment planning for long-term electricity supply shortages shares a variety of shortcomings with other methods. The most significant of these are listed below.

- a) Planning must be made in a specified macro-economic context. Production curtailed as a result of an electricity supply deficiency can more easily be made up when there is a general slack in the economy. A related problem is the matter of capacity utilization by specific industry and region. Depending on the availability of excess capacity, production might be shifted to regions not experiencing electricity shortages or rescheduled to periods when supplies are adequate.
- b) The methodology adopted here assumes that value added in industry varies linearly with electric energy input to the production process. We do not know that this is true. In the 1-12 month period with which are concerned it would appear that this assumption does not lead us seriously astray.
- c) Our approach as outlined above ignores secondary effects. Reduced output of Product A may have serious consequences for production of Product B of which it may be a component part. Thus curtailment of production due to electricity supply shortages may force curtailed production of products which are not directly affected by such shortages.

- d) We do not apply specific value measures to particular intangible costs such as lost education, decreased amenities, or feelings of insecurity or fear which may arise as a result of curtailed electric supply. It may however be claimed that the approach which we have taken which relates such costs to lost utility revenues from curtailment does in fact provide a measure of societal estimate of the worth of the services of electric energy which we forego and includes intangible cost. Our dissatisfaction with this approach is moderated by our ability to test the sensitivity of our results to the values which we assume.

## VI. ANALYTICAL FRONTIERS

The linear programming approach to demand curtailment, as described above, has proven extremely useful for estimating the first round of costs from a prolonged electricity shortage, assuming we wish to minimize some objective function whose arguments are measurable. With this approach we can adjust minimum service level constraints to individual sectors as well as classes of users, thereby replicating allocation strategies across a wide range of institutional considerations. The model can then be used to estimate the magnitude of the change in cost, however defined, which results as we move further away from the "optimal" plan. Actual applications of this methodology on hypothetical curtailments in New York State, in the Mid-West, and in other regions within the United States, have indicated that total first round economic losses

could more than double when "nonoptimal" allocation plans are projected. Moreover, differences of this magnitude occur even when intangible costs such as loss of health and security are minimized by limiting curtailment in sensitive and strategic sectors. When a shortage is prolonged however, economic impacts beyond first order effects become significant and the linear programming approach does not take these secondary effects into account.

Typical second order effects from long-term electricity curtailment may be classified as either structural change, demand-level changes, or supply-type effects.

1. Structural Change

To the extent to which curtailment does not result in lost production or sales, a decline in utility revenue means an increase in cash flow for curtailed customers. The effect of this structural change may be an increase in expenditures for products which directly replace electricity, e.g., candles and kerosene lamps, an increase in expenditures for unrelated products, or a temporary increase in savings. These shifts in demand would be very difficult to model.

2. Demand Level Changes

Industries which must curtail production (or which lose sales) as a result of electricity demand curtailment will reduce orders for intermediate goods and services, i.e., those goods and services used in their production processes. In addition, lost wages imply a reduction in consumer spending. Either of these demand level changes can cause decrease in production (income) either within the

affected region or outside of the region. To the extent to which this type of secondary effect falls on businesses within the region which have themselves been forced to curtail output as a result of the energy shortage, there will be no increase in total economic loss. However, if the changes in demand affect industries hitherto uncurtailed, and this category would include all industries operating outside of the region experiencing the electricity shortage, then total economic loss increases.

### 3. Supply-type Effects

When the lost output from electricity demand curtailment is an intermediate product required in the production process of a non-curtailed industry within the region, or required by an industry located outside of the region, then, if substitute capacity is not available for production of these intermediate products, there will be a ripple effect which will increase total economic loss.

As curtailment continues income and employment effects will continue to spread. In order to model secondary and inter-regional effects of the demand level changes and supply-type effects discussed above, the study would have to include multiplier analysis, appropriate regional and multi-regional analysis including information on inter-regional trade relationships, and appropriate input-output data. At this time we are in the process of examining possible methodologies for combining the linear programming approach with a regional input-output model in order to minimize the sum of first order and second order effects from curtailment.