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SECTIONING IN IMAGE PROCESSING*

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ABSTRACT

For reasons of mathematical tractability, simplifying assumptions are usually made about the statistical nature of images that are to be processed. Among these assumptions are stationarity of the signal and noise processes and the space invariance of the blur. These assumptions rarely hold over a large image but can be considered accurate over small subsections of an image. With this idea in mind, it is natural to divide an image into small, possibly overlapping sections and process each section with consideration to its peculiar characteristics. This technique has been used with success on problems involving iterative numerical image restorations, signal-dependent noise and space variant point spread functions.

An image is made up of many parts. In pictorial photography we usually distinguish the foreground, background, and subject. The fact that such distinctions can be made indicates that the different parts have different characteristics. In image processing these characteristics are usually ignored. The picture is processed as a unit. The purpose of this paper is to show the advantages of using these characteristics by sectioning in image processing.

Processing an entire image by processing small sections of it and concatenating the processed sections is not new. Laboratories with computing facilities much smaller than those of Los Alamos and Livermore find that sectional processing is the only way to solve the problems on their machines. This type of processing achieves the same result as global processing since the sections are operated upon in an identical manner. This use of sectioning is merely for computational convenience and does not make use of the different properties of the different sections.

In most image processing work the difference between different parts of the image is minimized. All global signal restoration algorithms which

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are statistically based (e.g., Wiener filtering) assume that the image represents a stationary process. If the image contains almost any visual information, this assumption is false. The assumption that the image is statistically uniform permits the use of algorithms which are soundly based in theory. The results of applying these algorithms usually have been quite good. However, the fact that images do not have the properties which are required for these algorithms to be optimal indicates that better results are possible.

Computationally, sectional processing represents more overhead than global processing. To avoid a blocking effect the sections must be overlapped. The minimum bound required for smooth transition between sections is determined by the size of the point spread function in the case of signal restoration and by the eye and experience in some of the enhancement algorithms. The ratio of the overlap area to section size determines the sectioning overhead (or efficiency) of the method. The ideal section size is the smallest size that contains enough data to define the desired characteristics and is larger than the minimum size required for the elimination of boundary effects. Since smaller section sizes require a larger proportion of overlap, they are more inefficient. Thus, computational limitations have a very real effect on the practical choice of section size.

There are two basic ways to take advantage of sectional processing to produce better images. The first use is for a better convergence criterion for numerical solution methods. The second use is changing processing parameters according to the characteristics of the section to be processed.

Several signal restoration methods use iterative numerical algorithms to achieve a solution. The most prominent methods of this type are maximum a posteriori (MAP) restoration and maximum entropy (ME) restoration. The MAP method uses a modified Picard's iteration; the ME method usually uses a modified Newton-Raphson algorithm. These numerical methods require a well-defined convergence criterion. This criterion is usually a Euclidean norm of some function of the image. Such a norm is an average value. Thus, large values may be balanced by small values.

For example, the MAP restoration method converges when

$$\| \underset{\sim}{g} - s(\underset{\sim}{H}\,\underset{\sim}{f}_k) \|^2 = \| \underset{\sim}{n} \|^2 \tag{1}$$

where the imaging system is modeled by

$$\underset{\sim}{g} = s(\underset{\sim}{H}\,\underset{\sim}{f}) + \underset{\sim}{n} \tag{2}$$

where

- $\underset{\sim}{g}$ is the recorded image;
- $\underset{\sim}{f}$ is the ideal image;
- $\underset{\sim}{H}$ is the blur matrix;
- s is a nonlinear memoryless transformation;
- $\underset{\sim}{n}$ is random noise.

The criterion (1) states that the residual image, $\underset{\sim}{g} - s(\underset{\sim}{H}\,\underset{\sim}{f})$, should have the same variance as the noise, $\underset{\sim}{n}$. It is desirable for the residual to also have the same correlation as the noise, but this is an impractical test. It suffices to output the residual image and let the user's eye test for correlation or visibly detectable patterns. Since the criterion (1) is an average, it should be beneficial to take the average over smaller sections. This conjecture has been shown to be true.[1] In the smaller sections large values are less likely to be cancelled by small values. The sectioned residual appears much more uniform as we would expect noise to be. As a result the restoration is improved.

In the case of sectioned MAP we have processed each section identically. The convergence criterion has remained constant. This assumes that the noise, $\underset{\sim}{n}$, is stationary. It is now an easy step to modify the MAP method to handle the case of signal-dependent noise. Film grain noise is a signal-dependent random process which is often modeled by

$$d_r = d + \sigma\, d^{\beta}\eta$$

where

- d_r is the recorded optical film density;
- d is the ideal density;
- σ is a scaling parameter dependent on scanning aperture and film type;
- β is an experimentally determined value dependent on film type;
- η is a standard normally distributed random variable (i.e., $\eta \epsilon N[0,1]$).

If the sections are small enough then d^{β} is fairly uniform over the section, and this value can be replaced by an average value, $\bar{d}^{\beta}$. In practice this modification of the sectioned MAP method allows the algorithm to approach the true solution where there is little noise and prevents noise from dominating the solution in areas of high noise.[2]

The next extension of sectional processing in image restoration is to space variant point-spread functions. Unlike the case of signal-dependent noise for film grain, there is no a priori model to describe the changing point-spread function. Each restoration problem must be handled individually. The blur detection algorithm of Cannon[3] would not yield good results because of its need to average over many sections. Still, a function is needed to describe the positional dependence of the point-spread function. In computer simulations this is easy. In a real world application we must rely on a priori knowledge, visual inspection, and trial and error. Once a descriptive function is found the sectional processing proceeds just as before. The actual results have been good.

The implementation of the sectional restoration of space variant degradations can be relatively simple or complex. If all sections and overlap are constant, the algorithm is a simple modification of the signal-dependent noise case. The section size and overlap are chosen according to the largest possible point-spread function. Computational savings can probably be made by varying the size of the sections and overlap according to the variation of the point-spread function. This would increase the bookkeeping in the computer program. It is doubtful that any visual improvement would be obtained by such an approach.

Sectional processing can be used in enhancement as well as restoration. In the case of image enhancement the gains achieved by this technique will have to be judged on a qualitative rather than quantitative basis. The methods which follow are presented as examples of sectional enhancement methods. These do not portend to exhaust the field of possibilities but merely give a flavor of what can be accomplished.

A method of enhancement was devised by using a method from numerical analysis of solving the diffusion equation backwards in time.[4] The main parameter used by this algorithm is the signal-to-noise ratio (SNR) of the degraded image. To apply sectional processing with this algorithm it is necessary to determine the SNR for eacn section. If the noise model is known, then the variance of the noise can be computed for each section. The variance of the signal plus noise can be estimated from the appropriate section of the recorded image. From this the SNR on the section to be processed can be estimated. The result of sectional processing with this method is an aggressive enhancement in areas of high SNR and smoothing in areas of low SNR. Heuristically, this is the desired result.

The method described above to estimate the signal-to-noise ratio can be used in an algorithm for noise smoothing. It is known that the human observer will tolerate more noise in an area of high signal strength. Conversely, the eye will not accept much noise in areas of low signal strength. A heuristic algorithm has been devised which is based on this property of the eye.[5]

The original noisy image is blurred by a point-spread function of the user's choice. The enhanced image is produced by taking a section-by-section linear combination of the original image and the smoothed image. Again, it is assumed that the noise model is known. If the variance of the difference between the smoothed section and the original section is less than the variance of the noise for that section, then the enhanced section is just the smoothed section. If this variance is greater than the noise variance, then the enhanced section is the linear combination given by

$$N_i = \Theta_i B_i + (1 - \Theta_i) O_i$$

and

$$\Theta_i = \text{minimum} \left\{ 1.0, \frac{\sigma^2_{n_i}}{\sigma^2_{D_i}} \right\}$$

where

N_i is the enhanced i^{th} section;

B_i is the blurred i^{th} section;

O_i is original noisy i^{th} section;

$\sigma^2_{n_i}$ is the variance of the noise on the i^{th} section;

$\sigma^2_{D_i}$ is the variance of the difference between the blurred and original on the i^{th} section.

The algorithm passes the smoothed signal in areas of low SNR and passes the original signal in areas of high SNR. With proper section size and overlap results have been subjectively good.

Several techniques have been described which use sectional processing. The advantages of such techniques are heuristically obvious. Whether the effort to process the image by sections is worth the effort is a matter of judgment. If the image is being processed in sections at the present because of computational restrictions, then sectional processing is perhaps an easy way to a better image. If the very best possible restoration is desired, then perhaps sectional processing can be used to achieve it. It is clear that sectional processing costs more, but in some cases it may well be worth it.

PROFESSIONAL BIOGRAPHY

H. J. Trussell received the B.S. in applied mathematics from Georgia Institute of Technology in 1967, the M.S. in mathematics from Florida State University in 1968, and the Ph.D. in Electrical Engineering and Computer Science from the University of New Mexico in 1976. Since 1969 he has been a staff member at the Los Alamos Scientific Laboratory. In 1977 he became an adjunct professor at the University of New Mexico. He is a member of Tau Beta Pi, Phi Kappa Phi, Sigma Xi. He is also a member of the IEEE.

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