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CAPILLARY SURFACES IN EXOTIC CONTAINERS

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ABSTRACT

A survey is presented of results to date for capillary surfaces in "exotic" containers. These containers have the property that each one admits a continuum of distinct equilibrium free surfaces, all bounding with the container walls the same volume of fluid, making the same contact angle at the triple interface curve, and having identical mechanical energies. The containers can be so designed that they are themselves axially symmetric but that the fluid configurations of minimizing energy cannot be axially symmetric.

Key words: *capillary surfaces, free surfaces, microgravity, symmetry breaking, space experiments*

The free surface of a liquid that partly fills a container under the action of surface and gravitational forces may assume, in general, one of several possible equilibrium configurations. An example for which only one configuration is possible is a vertical homogeneous cylindrical container of general cross-section, with gravity either absent or directed downward into the liquid; if the boundary of the free surface lies entirely on the cylindrical walls, then the surface is determined uniquely by its contact angle and the liquid volume.[1,2] Examples of other containers can be given for which there exist two or more distinct equilibrium configurations. Our interest here is in certain container shapes having the striking property that there is an entire continuum of such configurations.

Axially symmetric containers having this property can be constructed for any prescribed contact angle and Bond number, in such a way that all configurations in the family have identical volumes and yield identical mechanical energies, and so that no two of the free surfaces in the continuum are congruent to each other.[3,4,5] The procedure requires integration of a system of differential equations, in which the coefficients are determined as solutions of an auxiliary (nonlinear) differential equation system. Nevertheless, the global existence can be proved rigorously. It is shown in Refs. 4 and 6 that the families are all unstable, in the sense that the horizontal flat surface, which all families contain, can be deformed locally into a surface of smaller mechanical energy. It is possible to construct such "exotic" containers in such a way that no symmetric configuration can be energy minimizing. The configurations in Figure 1 show seven surface meridians of the continuum (dashed lines) in the particular case of contact angle 60°, at varying Bond numbers B; the solid curves are container meridians. The curves in the figures have been normalized so that the containers have maximum radius unity.

Further particular cases are calculated and displayed in Ref. 5. It would be difficult to observe the phenomenon in the earth's gravitational field, as the dimensions of the apparatus would have to be so small as to preclude ac-

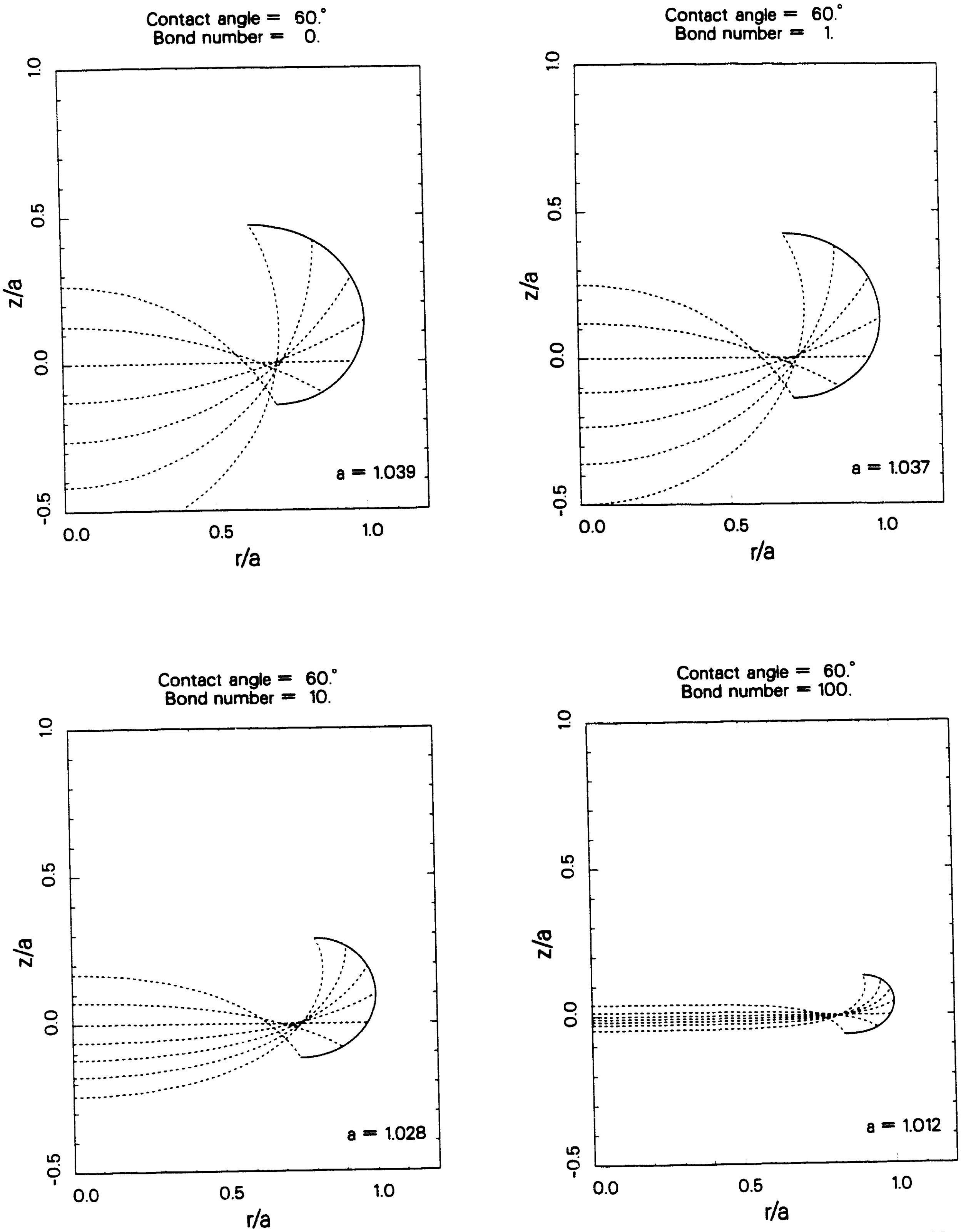

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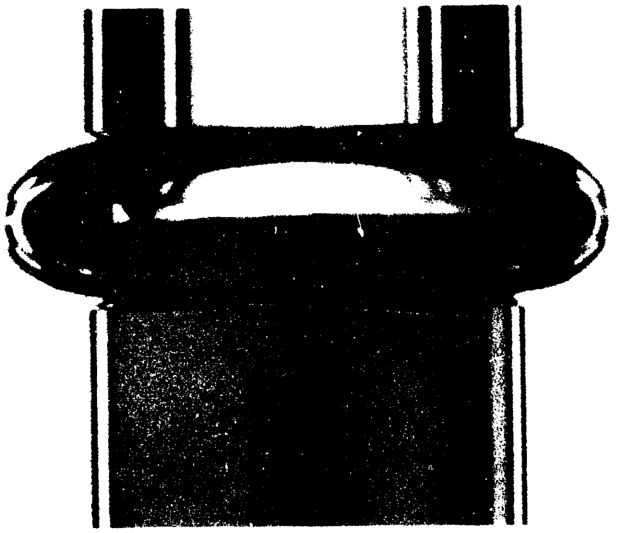
Figure 1. Meridian of container (solid curve) for contact angle 60° and several Bond numbers showing meridians of some of the symmetric equilibrium solution surfaces (dashed curves), all having the same contact angle and energy, and enclosing the same volume of liquid.



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Figures 2a-b. Drop-tower experiment: 2a. (upper) Flat interface initial configuration; 2b. (lower) Non-symmetric terminal configuration.

curate measurements. However, if gravity is absent the equations describing the containers are scale invariant, and experiments can be designed of any size, subject only to the usual laboratory constraints. Even in a microgravity environment that is not strictly zero-gravity, adequately large length scales for accurate observation and measurement are possible. An extended space experiment has been planned and is scheduled to be carried out on the NASA United States Microgravity Laboratory flight (USML-1) in 1992.

Preliminary drop tower experiments have been conducted by M. Weislogel on the five-second drop tower at NASA Lewis Research Center in Cleveland, Ohio.[7] An exotic container was fabricated corresponding to a contact angle of 80° (the contact angle for the materials used – acrylic plastic and water) and $B = 0$. The container, initially vertical, was filled with a volume of liquid corresponding to a horizontal planar equilibrium free surface meeting the container wall with contact angle 80°, as shown in Figure 2a. This is an equilibrium solution for any gravity level, but for zero g the interface is unstable in this container according to the mathematical theory, as are all members of the symmetric equilibrium family corresponding to the ones depicted in Figure 1. A resulting configuration after a five-second period of free fall subsequent to release of the container in the drop tower is depicted in Figure 2b. The liquid appeared essentially to have settled down to an equilibrium configuration by that time. The free surface exhibits a shape that is obviously non-symmetric, in accordance with the mathematical theory.

Independently, numerical studies were carried out in collaboration with M. Callahan. These studies are based on the Surface Evolver program developed by K. Brakke,[8] which deforms a given surface in such a way as to decrease (if that is possible) a discrete approximation to the total energy. Three local minima are suggested by the numerical investigation, of which the one with least energy looks remarkably like the surface in Figure 2b observed by Weislogel. It meets the container wall at the join between the bulge and the upper cylindrical wall for a good portion of the circumference, drops rapidly to the join with the lower cylindrical wall, and meets the lower join for the remainder of the circumference. The other locally energy-minimizing solutions are similar in nature, but have two or three excursions from top to bottom of the bulge (and back), instead of just one.[9]

Detailed derivations and further discussion can be found in the references.

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REFERENCES

1. R. Finn, *Equilibrium Capillary Surfaces* (Springer-Verlag, New York 1986), Chapter 5. Russian edition: (Mir Publishers, Moscow 1988).
2. T. I. Vogel, Uniqueness for certain surfaces of prescribed mean curvature, *Pacific J. Math.* **134** (1988) 197–207.
3. R. Gulliver and S. Hildebrandt, Boundary configurations spanning continua of minimal surfaces, *Manuscr. Math.* **54** (1986) 323–347.
4. R. Finn, Nonuniqueness and uniqueness of capillary surfaces, *Manuscr. Math.* **61** (1988) 347–372.
5. P. Concus and R. Finn, Exotic containers for capillary surfaces, *J. Fluid Mech.* **224** (1991) 383–394; Corrigenda, *J. Fluid Mech.* (1991), to appear.
6. P. Concus and R. Finn, Instability of certain capillary surfaces, *Manuscr. Math.* **63** (1989) 209–213.
7. P. Concus, R. Finn, and M. Weislogel, Drop-tower experiments for capillary surfaces in an exotic container, *AIAA J.* (1991), to appear.
8. K. A. Brakke, *Surface Evolver Program*, Geometry Supercomputer Project, University of Minnesota, Minneapolis, MN, 1990.
9. M. Callahan, P. Concus, and R. Finn, Energy minimizing capillary surfaces for exotic containers, in *Computing Optimal Geometries* (with accompanying videotape), J. E. Taylor, ed., AMS Selected Lectures in Mathematics (Amer. Math. Soc., Providence, RI, 1991).

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