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DISCRETE CAVITY MODEL OF A STANDING-WAVE FREE-ELECTRON LASER

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A standing-wave free-electron laser (SWFEL) has been proposed for use in a two-beam accelerator (TBA). Unlike a conventional microwave free-electron laser, the SWFEL has a wiggler that is divided by irises into a series of standing-wave cavities, and the beam is reaccelerated by induction cells between cavities. We introduce a one-dimensional discrete-cavity model of the SWFEL. In contrast to the continuum model that has been extensively used to study the device, the new model takes into account time-of-flight effects within the cavity and applies the reacceleration field only between cavities, where the ponderomotive force is absent. As in previous SWFEL models, only a single signal frequency is considered. Using this model, effects of finite cavity length are investigated. For moderately long cavities, it is shown that there are no adverse effects on the phase stability of the device.

I. INTRODUCTION

A standing-wave free-electron laser (SWFEL) has been proposed[1, 2] as a power source for a high gradient structure in a configuration known as the "two beam accelerator" (TBA)[3]. In this device, irises are placed along the FEL wiggler to form a series of microwave cavities, and induction cells are placed between cavities to reaccelerate the beam (see Figure 1). The standing-wave signal that builds up in the cavities as the beam passes through is coupled to a parallel high-gradient radio-frequency accelerator.

Previously, a continuum model had been used to study the device[1, 2, 4]. In this model, an infinitesimal cavity length was assumed and the particles were reacclerated continuously. In this paper, we study the effects of finite cavity length on phase fluctuations and output power. Moreover, we reaccelerate the particles only between the cavities. In Section II, we describe our model in some detail. In Section III, we summarize the results obtained using this model. Section IV contains our conclusions.

II. DISCRETE CAVITY MODEL

In this section, we develop a one-dimensional discrete cavity model of the SWFEL. First, we obtain particle and field equations within a single cavity. We assume that the one-dimensional beam couples only to the TE_{01} waveguide mode[1]. For a waveguide with height h and width w, the signal wavenumber k_s is given as follows

$$k_s = (w_s^2/c^2 - \pi^2/h^2)^{1/2}. \tag{1}$$

The wiggler field is generated using an idealized linear wiggler with a vector potential

$$\vec{A}_w = \frac{m_e c^2}{e} a_w \cos(k_w z)\hat{x}. \tag{2}$$

Vector potential for the signal field is given as follows

$$\vec{A}_s = \frac{m_e c^2}{e} a_s \sin(\pi y/h)\cos(k_s z - \omega_s t + \phi)\hat{x}. \tag{3}$$

We assume that the energy $\gamma m_e c^2$ of all beam electrons is sufficiently high that $a_w/\gamma \ll 1$. The signal amplitude a_s is taken to be small compared with a_w. Both a_s and field phase ϕ are assumed to be slowly varying compared with $k_s z$ and $\omega_s t$.

With these approximations, the wiggle-averaged particle equations are identical to those in a conventional single-mode microwave FEL (due to this wiggle-averaging, the cavity needs to be at least as long as a wiggler wavelength for the equations to apply rigorously within the cavity). Denoting the particle phase $(k_s + k_w)z - \omega_s t$ by θ_j and taking z to be the independent variable, the equations are given as follows

$$\frac{d\theta_j}{dz} = k_w + k_s - \frac{w_s}{c} - \frac{w_s}{2c\gamma_j^2}\left[1 + \frac{a_w^2}{2} - 2D_x a_w(\hat{a}_r \cos\theta_j - \hat{a}_i \sin\theta_j)\right], \tag{4}$$

$$\frac{d\gamma_j}{dz} = -D_x \frac{w_s}{c}\frac{a_w}{\gamma_j}(\hat{a}_r \sin\theta_j + \hat{a}_i \cos\theta_j) \tag{5}$$

where $\hat{a}_r$ and $\hat{a}_i$ are the real and imaginary parts of the complex field amplitude $\hat{a} \equiv \hat{a}_r + i\hat{a}_i = a_s \exp(i\phi)$. The coupling coefficient D_x for a TE_{01} mode is given by

$$D_x = [J_o(\xi) - J_1(\xi)]/2, \tag{6}$$

where $\xi = \omega_s a_w^2/(8ck_w\gamma_j^2)$. The complex field amplitude satisfies the following equation (again given by conventional FEL theory)

$$\frac{d\hat{a}}{dz} = i\eta\langle\frac{\exp(-i\theta_j)}{\gamma_j}\rangle, \tag{7}$$

where the coefficient η is given by

$$\eta = \frac{8\pi}{hw}\frac{eI_b}{m_e c^3}\frac{D_x a_w}{k_s}. \tag{8}$$

The novel features of a SWFEL come into play when we consider the interaction of an electron beam with a series of cavities. The electron beam is divided into beam slices. Each beam slice is a uniform distribution of particles with initial spreads $\Delta\theta_0$ and $\Delta\gamma_0$ in θ and γ respectively. As in previous SWFEL simulations[1, 2, 4], the average θ and γ for a given beam slice are prescribed as follows:

$$\langle \theta_0 \rangle_k = \alpha + \beta(k-1), \qquad \langle \gamma_0 \rangle_k = \gamma_r, \tag{9}$$

where k is the beam slice index, $\gamma_r^2 = w_s(1 + a_w^2/2)/2c(k_w + k_s - \omega_s/c)$ is the resonant energy and α, β are constants. Typically, we take α equal to zero and

$$\beta = -\pi/(K-1). \tag{10}$$

Here, K is the total number of beam slices and is related to the total beam length L_b by the relation

$$L_b = \lambda_s(K + 1/2), \tag{11}$$

where λ_s is the signal wavelength. The spacing between the centers of beam slices ΔL_b is no longer λ_s in this case but is slightly larger:

$$\Delta L_b = \lambda_s \left(1 + \frac{\lambda_s}{2L_b}\right). \tag{12}$$

Now, consider the first beam slice as it propogates through the SWFEL structure. The particles within this slice are distributed according to Eq. (9) with $k = 1$. There is a small input field $\hat{a}_0$ within each cavity. As the beam slice propogates throught the first cavity, it evolves according to Eqs. (4) and (5) and generates radiation field through Eq. (7). When it exits the cavity, the average energy lost in this cavity is restored to all particles within the beam slice. However, nothing is done to the particle phases. The beam slice now enters the second cavity which is also given the same input field $\hat{a}_0$ as the first cavity. As the beam exits this cavity, we add the average energy lost in the first cavity to all particles in the beam slice rather than the average energy lost in the second cavity[1]. This process is continued for all subsequent cavities. Hence, the recceleration field is independent of the cavity number.

Next, consider the second beam slice. As it enters the first cavity, it still sees only the initial input field $\hat{a}_0$ and not the additional field generated by the passage of the first

beam slice. This can be seen as follows. Electrons in the beam slice can interact only with a forward traveling wave. But the field generated by the first slice takes a finite amount of time T_c to make a round trip within the first cavity since the cavity has a finite length L_c. This time is given as

$$T_c \approx 2L_c/c. \tag{13}$$

Within this time period, K' beam slices have already passed through the cavity where [cf. Eqs. (12) and (13)]

$$K' \approx 2L_c/\Delta L_b. \tag{14}$$

The above situation applies to all the cavities. Hence the first K' beam slices all see the same input signal field amplitude. However, there could be differences in their evolution due to differences in other initial conditions like θ_0 and beam current.

Next, we consider the $(K'+1)$th beam slice. By the time this enters a cavity, the signal field generated by the first beam slice has already made a round trip within the cavity, and the electrons therefore see an enhanced input signal field amplitude. For the sake of simplicity, in our present model we neglect losses due to reflections and due to leakage through the iris. To solve the FEL equations within the cavity, we also require the initial field phase, which is determined as follows. Since we want the field to set up a standing wave within each cavity after a few reflections, we take the cavity length L_c to be an integer multiple of the signal wavelength λ_s. We also assume that the field phase ϕ changes by π radians during each reflection. With these assumptions, ϕ at the beginning of the cavity after one round trip is the same as ϕ at the end of the cavity after the passage of the first beam slice. Now that we have determined all initial conditions, we can solve the FEL equations for the $(K'+1)$th beam slice within each cavity. As usual, we add the average energy lost by the beam slice in the first cavity to all the particles before they enter a new cavity.

We can repeat the above arguments for subsequent beam slices. In general, the input signal field for the kth beam slice at the beginning of the lth cavity is given by the output signal field for the $(k - K')$th beam slice at the end of the lth cavity. The full interaction of the electron beam with the SWFEL structure can therefore be represented symbolically by the following recursion relations:

$$\theta_{k,l} = \theta_{k,l-1} + F(\theta_{k,l-1}, \gamma_{k,l-1} + \Delta\gamma_k, \hat{a}_{k-K',l}) \tag{15}$$

$$\gamma_{k,l} = \gamma_{k,l-1} + G(\theta_{k,l-1}, \gamma_{k,l-1} + \Delta\gamma_k, \hat{a}_{k-K',l}) \tag{16}$$

$$\hat{a}_{k,l} = \hat{a}_{k-K',l} + H(\theta_{k,l-1}, \gamma_{k,l-1} + \Delta\gamma_k, \hat{a}_{k-K',l}) \tag{17}$$

Here, θ and γ are n-vectors where n is the number of particles within a beam slice. The quantity $\Delta\gamma_k$ is the average energy lost in the first cavity by the kth beam slice. For k equal to 1, we take $\hat{a}_{k-K',l} = \hat{a}_0$ for all l.

III. NUMERICAL RESULTS

We have numerically studied the discrete cavity model explained in the previous section. This section contains a brief summary of our results.

The parameters used for the simulation, listed in Table I, are the optimized values obtained previously[4] using the continuum model of the SWFEL. In our model, we have an additional parameter that we can vary – the cavity length. First, we set the cavity length equal to a signal wavelength. In this limit, our model should go over into the continuum model. The output microwave energy and field phase ϕ are shown in Figures 2 and 3. The results are from a multi-particle simulation. We have assumed an initial spread of 10% in θ and 1% in γ. We have also employed a linearly increasing current. The results are seen to agree with those given in Ref. [4] for the continuum model. Similar agreement was found for single-particle simulations and for different parameter sets. These calculations thus served as a benchmark for our numerical code.

Next, we set the cavity length to a realistic value of 14.7 cm. Other parameter values remain unchanged from the short cavity case. The results are again shown in Figures 2 and 3 so as to facilitate an easy comparision with the short cavity results. The ripple seen in these figures is at the synchrotron frequency and can be explained analytically[4]. We see that the magnitude of field phase fluctuations is still not significantly larger than in the continuum case. This shows that the SWFEL concept still holds promise as a stable microwave power source.

At present, we are studying in detail the variation of various physical quantities as a function of cavity length. Sensitivity of the device to errors in input energy and current is also being investigated.

IV. SUMMARY

We have developed a discrete cavity model of a standing-wave free-electron laser. A numerical code has been built to study this model. This code has been benchmarked against the code for the continuum model by taking the cavity length to be small. We found that a cavity length of 14.7 cm has no deleterious effect on the magnitude of fluctuations in the field phase.

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TABLES

TABLE I. Optimized simulation parameters for the standing-wave FEL

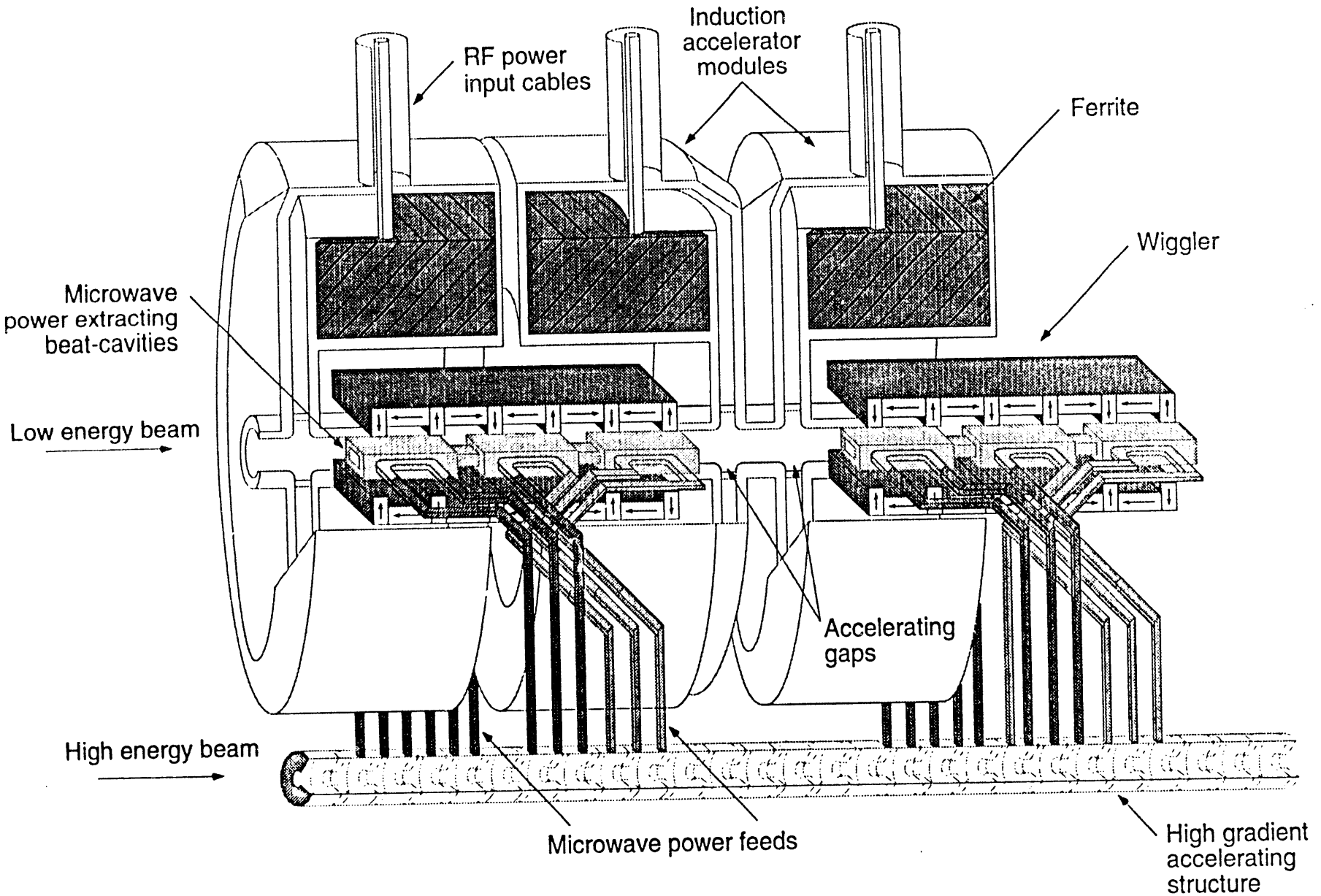
average beam current	I_b	3.5 kA
beam length	L_b	180.0 cm
resonant energy	γ_r	16.4
wiggler strength	a_w	1.4
wiggler wavelength	λ_w	37 cm
wiggler length	L_w	40 m
waveguide height	h	3 cm
waveguide width	w	10 cm
signal frequency	$\omega_s/2\pi$	17.1 GHz
cavity Q	Q	10^4
input power	P_{in}	80 kW/m
output energy	W_{out}	10 J/m

FIGURES

FIG. 1. Conceptual layout of one section of a standing-wave TBA

FIG. 2. Output microwave energy as a function of distance z along the wiggler. Results from both the short and long cavity cases are shown.

FIG. 3. Field phase ϕ as a function of distance z along the wiggler. Results from both the short and long cavity cases are shown.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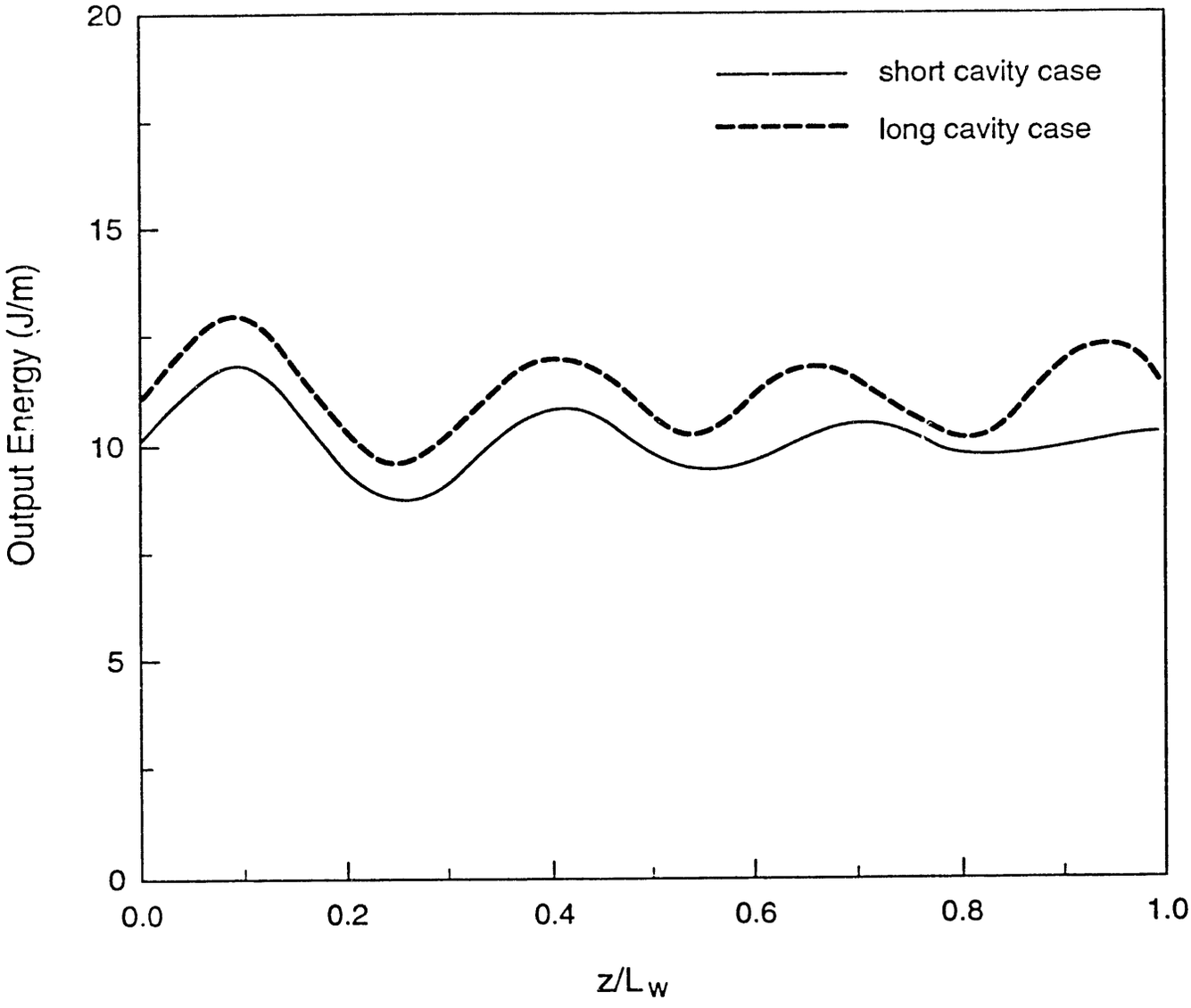
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Output Energy (J/m)
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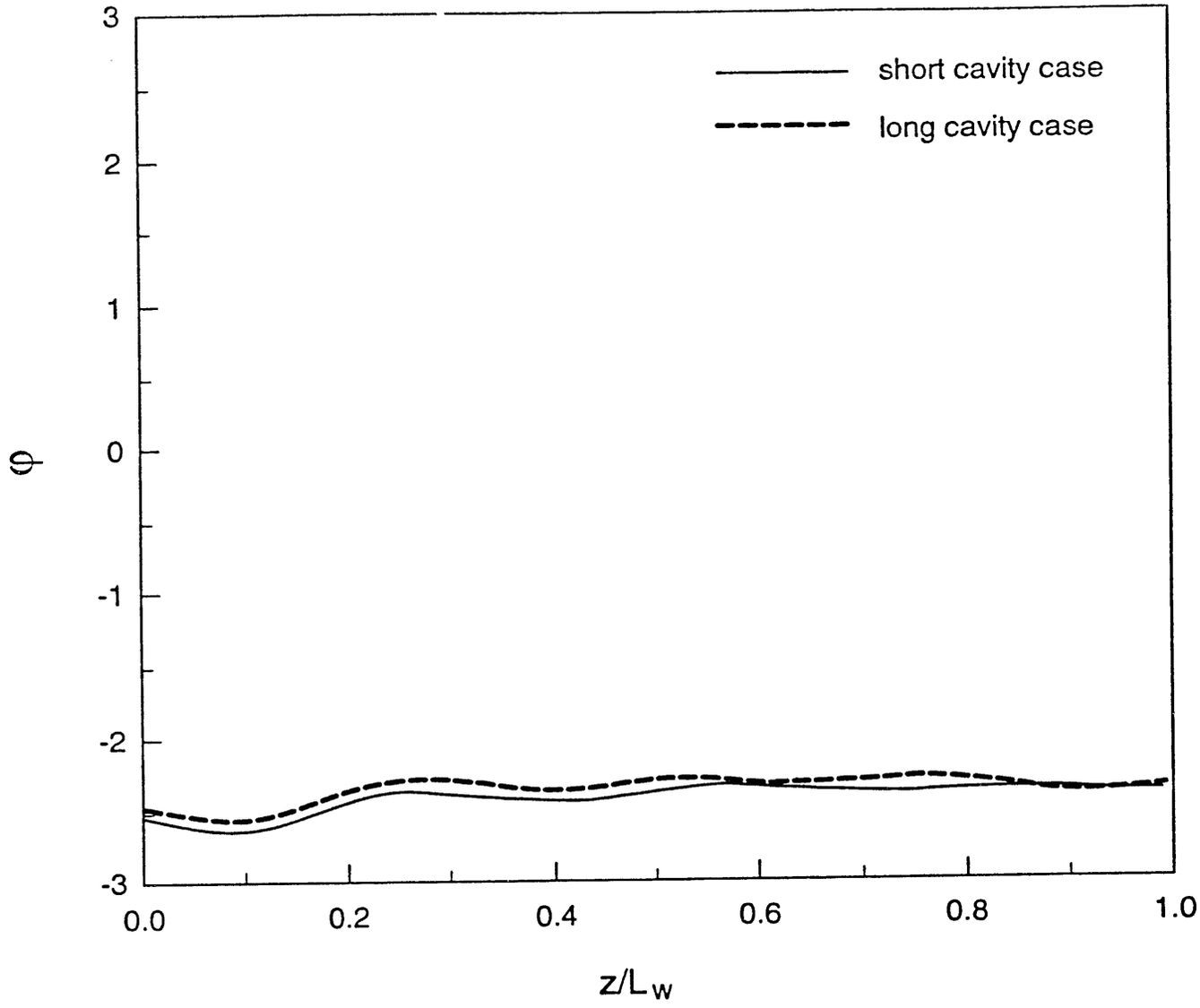
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