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DEVELOPMENT AND VERIFICATION OF MULTICYCLE DEPLETION PERTURBATION THEORY*

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Recently, Williams[1] has developed a coupled neutron/nuclide depletion perturbation theory (DPT) applicable to multidimensional and multigroup reactor analysis problems. This theoretical framework has been verified using the newly developed DEPTH[2] module within the context of the VENTURE modular code system.[3] The accuracy and usefulness of this alternate calculational method for burnup analyses has been demonstrated for a variety of final-time response functionals. However, these examples were restricted to single-cycle depletion analyses due to the theoretical assumption that the nuclide density field was continuous in time. Clearly, in multicycle problems, the nuclide concentrations must vary discontinuously with time to model refueling and shuffling operations or discrete control rod movements. Thus, the purpose of this work is to generalize the original DPT framework to include nuclide discontinuities and to verify that this generalization can be employed in realistic multicycle applications.

The discontinuity in the nuclide field due to refueling and shuffling operations can be represented as

$$\underline{N}(\vec{r},t_i) = \underline{\underline{P}}_{sd}(\vec{r},t_i)\underline{N}(\vec{r},t_i^-) + \underline{M}(\vec{r},t_i) \qquad (1)$$

where $\underline{N}(\vec{r},t_i)$ is the nuclide density vector following the refueling operation,

$\underline{N}(\vec{r},t_i^-)$ is the nuclide density vector immediately preceding the refueling operation,

$\underline{\underline{P}}_{sd}(\vec{r},t_i)$ is the shuffling/discharge operator at time t_i, and $\underline{M}(\vec{r},t_i)$ is a vector describing the addition of feed material to the reactor at t_i.

With only slight interpretational differences, the above equation also describes nuclide discontinuities other than the refueling process.

Inclusion of the condition expressed by Eq. (1) in the forward burnup equations introduces a discontinuity in the adjoint equations required for DPT. Williams[1] has shown that the discrete power normalization and flux shape discontinuities of the quasi-static burnup approximation introduce the "P* effect" and "Γ* effect" jump conditions on the adjoint importance function, $\underline{N}^*(\vec{r},t)$. In much the same way, the forward refueling discontinuity manifests itself as an "adjoint refueling" jump condition. These time-step boundary conditions on the adjoint importance function are given by

$$\underline{N}^*(\vec{r},t_i) = \underline{N}^*(\vec{r},t_i^+) + \underline{P}^*(\vec{r},t_i^+)_{effect} + \underline{\Gamma}^*(\vec{r},t_i^+)_{effect} \qquad (2)$$

and

$$\underline{N}^*(\vec{r},t_i^-) = \underline{\underline{P}}^*_{sd}(\vec{r},t_i)\underline{N}^*(\vec{r},t_i) \qquad (3)$$

where $\underline{\underline{P}}^*_{sd}$ is the adjoint refueling operator.

Two points regarding Eqs. (2) and (3) deserve discussion. First, it should be noted that the time boundaries are reversed relative to Eq. (1), which is consistent with the time direction for the solution of the adjoint burnup equations. Second, the use of zone-averaged quantities (as in most depletion codes) results in a matrix representation of the $\underline{\underline{P}}_{sd}$ operator; describing the zone to zone shuffling and discharge operations. Hence the adjoint operator, $\underline{\underline{P}}^*_{sd}$, becomes simply the transpose of the $\underline{\underline{P}}_{sd}$. Thus, if zone ℓ is moved to zone m in the forward calculation, the importance associated with the nuclide in zone m would be shuffled to zone ℓ in the adjoint

case. Additionally, if zone m was discharged in the forward computation, the corresponding adjoint operation would be to set the zone m importance functions to zero.

The central theme in the above development is that a fuel management operation in a multicycle forward calculation implies that a corresponding importance management operation be performed in the multicycle adjoint case. Equation (1) describes the forward fuel management operation while Eq. (3) describes its adjoint.

Numerical calculations directed at confirming the validity and usefulness of multicycle DPT have been performed using the DEPTH module. A two-dimensional (R,Z), nine-group calculational model of a GCFR was utilized. One-third of each of the four core zones and associated axial blankets were refueled on an annual basis. In addition, one-sixth of the radial blanket zones were replaced each cycle. The ^{239}Pu inventory at the end of the fourth cycle (EOC4) was selected as a representative response function to examine the capability of the multicycle DPT generalization.

Table 1 contrasts the predicted EOC4 ^{239}Pu inventory based on multicycle DPT with direct calculation for several perturbations. The nuclide perturbations to the reference system included both isolated changes such as the 10 percent ^{239}Pu increase to core zone 1 and multiple nuclide variations over the four cycles as in the fission product perturbations (addition of 10^{21} atoms/cm^3 to initial and reload batches in CZ1). As indicated by Table 1, multicycle DPT agrees extremely well with direct forward calculation.

One noteworthy capability of DPT is its ability to account for the "memory" of the reactor. Assemblies initially loaded in the core regions

are not physically present during the fourth cycle due to the refueling scheme specified. Yet, the effects of perturbations to the initial core are predicted with remarkable accuracy. For example, the effect of the isolated initial core ^{239}Pu perturbation on the EOC4 ^{239}Pu inventory was predicted to within 1.7 percent!

The above development and verification of multicycle DPT greatly extends the potential useful application of time-dependent perturbation theory. The effect of various design options on important "equilibrium cycle" responses can now be investigated without several costly multicycle forward burnup calculations. In addition, the ability of DPT to quantify the time-dependent importance of a nuclide through several reactor cycles may prove invaluable in fuel management and core design optimization studies.

References

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Table 1. DPT Versus Direct Calculation For An EOC4 ^{239}Pu Inventory Response in a 2-D, Nine-Group GCFR Calculational Model

Design Variation	Time of Perturbation	Change in Response (kg) DPT Result	Direct Result	Diff. (%)
Δ FISSP	initial core	56.23		
	1st reload	30.61		
	2nd reload	21.46		
	3rd reload	10.97		
	SUM	119.27	115.98	2.8
Δ ^{239}Pu	initial core	-27.68	-27.21	1.7
Δ ^{239}Pu	1st reload	- 2.35	- 2.43	-3.3