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TITLE: MULTI-DIMENSIONAL HYBRID-SIMULATION TECHNIQUES IN PLASMA PHYSICS

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MULTI-DIMENSIONAL HYBRID-SIMULATION TECHNIQUES IN PLASMA PHYSICS

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Summary

Multi-dimensional hybrid simulation models have been developed for use in studying plasma phenomena on extended time and distance scales. The models make fundamental use of the small Debye length or quasi-neutrality assumption. The ions are modeled by particle-in-cell (PIC) techniques while the electrons are considered a collision-dominated fluid. The fields are calculated in the nonradiative Darwin limit. Some electron inertial effects are retained in the Finite Electron Mass model (FEM). In this model, the quasi-neutral counterpart of Poisson's equation is obtained by first summing the electron and ion momentum equations and then taking the quasi-neutral limit. The resulting elliptic equation gives ambipolar electrostatic potentials but neglects the short-range ω_p fields. In the Zero Electron Mass (ZEM) model explicit use is made of the axisymmetric properties of the model to decouple the components of the model equations. Equations to self-consistently advance the electron temperature have recently been added to the scheme. The model equations which result from these considerations are two coupled, nonlinear, second order partial differential equations. These two equations are integrated in time by a noniterative ADI procedure along with the explicit PIC ion time integration procedure. The resulting nearly implicit electron-field algorithm treats wide variations in the local signal velocity without instability; this consideration is most important since arbitrary intermixing of plasma and pure vacuum regions are allowed.

Introduction

A plasma simulation method taht can describe macroscopic phenomena while including particle ion effects in high-β plasmas has considerable utility in magnetic fusion research. Of particular interest are plasma phenomena with scale lengths comparable to ion gyro-radii on the order of centimeters and time scales on the order of a few tens of μs or longer. Examples of plasmas with such parameters abound in high-β controlled fusion research with typical plasma parameters ranging between 10^{12} to 10^{15} particles/cm^3, between 50eV and 2 keV for temperatures, and between 1 and 15 kG for magnetic fields. In this hybrid regime, the density, temperatures, and magnetic field are such that the ions are essentially collisionless and have orbits allowing them to experience large variations in electromagnetic fields--requiring a Vlasov treatment of the ions. Electrons experience many more collisions than do ions and/or have relatively small Larmor orbits. Electron behavior is adequately modeled as a collision dominated, thermal fluid. Multidimensional simulation of such plasmas in the past was restricted to models describing only MHD behavior and did not include all the physical effects desired. Conversely, full electron and ion particle-in-cell (PIC) techniques provide more details about plasma behavior than are needed for macroscopic studies. What is needed are hybrid models which describe plasmas with at least two levels of detail between these two extremes.

The first level of detail is still microscopic in that elimination of plasma and/or electron cyclotron oscillation is more important than considerations of macroscopic geometery. The important questions in this fast hybrid (FH) regime concern the behavior of microinstabilities on the lower-hybrid or slower time scale. Some remnants of electron inertia must be retained in order to correctly model composite frequencies faster than simple ion oscillations.

Several potentially useful multidimensional models have been developed for simulation in this FH regime. These models fall into three classes: implicit PIC techniques, hybrid models based upon the small electron Larmor radius r_{ce}, and hybrid models based upon the small Debye length λ_D. Implicit particle methods are the most recent and ambitious methods designed for the FH regime. Such methods will no doubt become important techniques for microinstability simulation in the future. The assumption of small r_{ce} is used in electron guiding center models which work well for low-β plasma phenomena dominated by strong magnetic fields. Limitations arise for small magnetic fields in a high-β plasma where the more appropriate expansion parameter is the Debye length λ_D--equivalent to assuming quasi-neutrality since plasmas deviate from charge neutrality only on scale lengths small compared to λ_D. Our quasi-neutral finite-electron-mass (FEM) hybrid model[1] has been designed to operate in this FH regime. Through separation of the electron current and electric field into irrotational and solenoidal components, electron plasma oscillations and the associated time step restrictions are eliminated from the model. Other electron inertial effects necessary to describe most microinstabilities requiring finite ω_{ce} are retained.

In the slow, macroscopic, hybrid regime (SH), the ω_{ce} time scale is still too restrictive since there is no question that all electron inertial effects can be neglected. This regime requires a model with massless fluid electrons and ions represented by either a standard PIC technique or as a finite-mass, thermal fluid---the first important non-MHD effects arise from simply representing the plasma as a two component fluid. The model by Byers et al.[2] provides an avenue into this parameter regime--particularly as extended recently by Harned[3]. A difficulty is that these hybrid techniques all require division by density in the electron and field equations in each plasma computational cell. This feature produces sensitivity to fluctuations in regions of low but finite density and will not describe regions with zero density. Recently a new method of solution for the relevant combination of electron and radiationless field equations has been developed[4]. This formulation greatly reduces the low-density-fluctuation problem and allows arbitrary plasma-vacuum intermixing without having to follow plasma-vacuum interfaces.

Ion Time Advance

The ion plasma component is represented by a discrete set of particles. Associated with each particle are a mass, a charge, two position coordinates, and three velocity coordinates. By appropriately initializing these particles in velocity and position, arbitrary low-order velocity moments of the ion distribution can be represented. The ions are advanced in time by stepping forward each particle with the local self-consistent Lorentz force using PIC techniques. The particle-stepping algorithm[5] uses the following equations which are second order accurate in the time step Δt. Explicitly,

D. HEWETT

$$v^0 = v^{-1/2} + \frac{h}{2} E^0$$

$$v^{1/2} = f v^{-1/2} + h\,(E^0 + g B^0 + v^0 \times B^0) \qquad (1)$$

$$x^1 = x^0 + \Delta t\, v^{1/2}$$

where $h = q\,\Delta t/m$, $f = 1.-(hB^0)^2/2$ and $g = h\, v^{-1/2} \cdot B^0/2$. $v^{1/2}$ and x^1 are correct to order Δt^2. At the end of each time step, the velocity moments, ρ (density), J_i (current), and K_i (divergence of the ion kinetic tensor), are calculated by averaging over the new positions and velocities of the particles.

The Finite Electron Mass Hybrid Model

The FEM model makes use of quasi-neutrality and retains some aspects of finite electron mass. The solenoidal part of the electron current J_e retains finite electron inertia and is a fundamental part of the time integration scheme--allowing this model to display many phenomena that require finite electron cyclotron frequency ω_{ce}. In order to eliminate electron plasma oscillations, the strong coupling between the irrotational part of the electric field and the electron current must be removed. The decoupling is accomplished by obtaining the irrotational J_e from the quasi-neutral continuity equation $\nabla \cdot (J_i + J_e) = 0$. This equation leads to $\nabla^2 V = \nabla \cdot J_i$ with $J_{e\ell} = -\nabla V$. The result is a procedure which for suitable boundary conditions determines $J_{e\ell}$ from J_i. The subscript ℓ denotes irrotational; the subscript t, used later, denotes solenoidal. This technique obviates the need for the irrotational part of the electron momentum equation--thus excluding the strong short-range coupling between $J_{e\ell}$ and E_ℓ.

J_{et} is advanced explicity in time by direct evaluation of the electron momentum equation

$$\dot{J}_e = K_e + \frac{q^2}{m_e} n_e E + \frac{q}{m_e c} J_e \times B + \nu_{ei}(J_i + J_e) \qquad (2)$$

where ν_{ei} is the electron-ion collision frequency and K_e is the divergence of the electron kinetic energy tensor. K_e is obtained by assuming the electron fluid has the entropy of an ideal gas. After calculation of the vector field $\dot{J}_e$, $\dot{J}_{et}$ can be obtained by subtracting the irrotational part. J_{et} is advanced in time by a second order in Δt scheme that is analagous to the ion scheme eqs. (1).

A new technique is required to determine E_ℓ (or equivalently the electrostatic potential ϕ) consistent with the quasi-neutral assumption. Exact charge neutrality requires the electrostatic field to be identically zero. Quasi-neutrality implies only that the difference between the ion and electron charge densities be everywhere small in a relative sense. For these situations, a quasi-neutral "Poisson" equation can be obtained using only the quasi-neutral continuity and the sum of the electron and ion momentum equations. The resulting elliptic equation has the form

$$\frac{c^2}{4\pi} \nabla \cdot (\mu \nabla\phi) = \nabla \cdot (K_e + K_i + \mu E_t + \xi \times B) \qquad (3)$$

where $\mu = e^2\rho\,(1/m_i + 1/m_e)$, $c\xi = e\,(J_i/m_i - J_e/m_e)$, and of course $E_\ell = -\nabla\phi$.

The other fields, E_t and B, are generated in the Darwin limit. (In the FEM model, this limit is achieved by neglecting E_t in Ampere's law.) Working in the Coulomb gauge, the vector potential A is given by

$$\nabla^2 A = -\frac{4\pi}{c} J_t \qquad (4)$$

where $B = \nabla \times A$. Using the relation $E_t = -\dot{A}/c$ with eq. (4) gives

$$\nabla^2 E_t = \frac{4\pi}{c^2} \dot{J}_t \qquad (5)$$

where $c^2\dot{J} = K_e + K_i + \mu(E_\ell + E_t) + \xi \times B$. The E_t calculation is somewhat more complicated than the preceding calculation of the magnetic field. It has been demonstrated that the calculation of E_t needs to be fully implicit in time[5] since this model exhibits instantaneous propagation. To avoid finite differencing in time, notice that $\dot{J}_{et}$ was obtained for use in eq. (5) by summing the ion and electron momentum equations.

One additional complication is that the quasi-neutral Poisson equation, eq. (3), requires E_t as a source term and the equation for E_t, eq. (5), requires E_ℓ as a source term. A fully implicit solution therefore requires iteration over both E_ℓ and E_t calculations; in practice, only two or three iterations are required.

Results of the application of this model to the lower-hybrid-drift instability[4,6] will be presented. Shown below are B_z contours of β dynamic lower-hybrid-drift instability at $t=200\ \omega_{pe}^{-1}$ that were simulated with a full ion and electron PIC model, Fig. 1, compared to the FEM model result, Fig. 2.

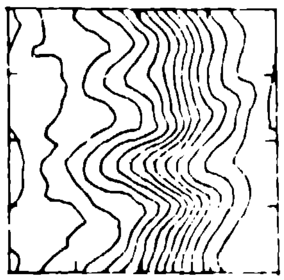


Fig. 1

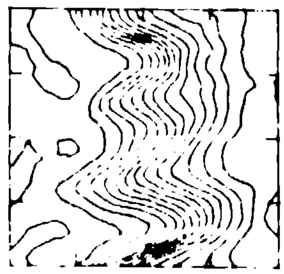


Fig. 2

The Zero Electron Mass Hybrid Model

The basic equations for advancing the electromagnetic fields in any simulation are of course Maxwell's equations. However, the ZEM assumption provides additional constraints which must be utilized to self-consistently advance the electron quantities, such as current J_e and temperature T_e, simultaneously with the field components. What results is a mixture of the electron momentum equation and Maxwell's equations which must be advanced in time along with the ion particle advance. These electron-field equations are configured for this work to allow nearly implicit time advance of all field and electron quantities in axisymmetric cylindrical geometry[4].

In the limit of small electron inertia, the electron momentum equation is

$$E = -\frac{\nabla \rho T_e}{e\rho} - \frac{u_e \times B}{c} + \eta J \qquad (6)$$

where T_e and u_e are the electron temperature and drift velocity, respectively. J is the total current and η is the resistivity. Quasi-neutrality is also assumed in this model so that the electron density is nearly equal to the ion density and both will be denoted by the symbol ρ.

As with the FEM model, an immediate consequence of quasi-neutrality is that the total current J must be nearly solenoidal, $J \cong J_t$. This result follows from the charge continuity equation which, in the quasi-neutral limit, is $\nabla \cdot J_e \cong -\nabla \cdot J_i$. This equation suggests the choice of equal and opposite irrotational currents $J_{e\ell} = -J_{i\ell}$ which is fully general as long as the boundary conditions on the currents are consistent with quasi-neutrality.

To complete the set of equations governing the time advance of electron-field quantities, the radiation-free or Darwin limit of Ampere's law $\nabla \times B = 4\pi/c\ J_t$ and Faraday's law $\nabla \times E_t = -1/c\ \partial B/\partial t$

must be combined with the electron momentum equation (6). Introducing the magnetic vector potential $\underset{\sim}{A}$ in the Coulomb gauge, equation (6) takes the form

$$\dot{A}_\theta - \frac{c^2}{4\pi} \eta \nabla^2 \underset{\sim}{A}|_\theta + u_{er} \frac{1}{r} \frac{\partial (rA_\theta)}{\partial r} + u_{ez} \frac{\partial A_\theta}{\partial z} = 0 \qquad (7)$$

where axisymmetry has been exploited for $E_{\theta L}=0$. A_θ and, consequently B_r and B_z, are advanced in time using this equation with u_{er} and u_{ez} coming from the curl of $B_\theta \hat{e}_\theta$, ρ, u_{ir}, and u_{iz}. The B_3 equation can be obtained by first taking the curl of equation (6) and then replacing $\nabla \times \underset{\sim}{E}$ with $\dot{\underset{\sim}{B}}$ from Faraday's law. The θ-component of the resulting equation is

$$\dot{B}_\theta - \frac{c^2}{4\pi} \left[\frac{\partial}{\partial r} \frac{\eta}{r} \frac{\partial (rB_\theta)}{\partial r} + \frac{\partial}{\partial z} \eta \frac{\partial B_\theta}{\partial z} \right]$$

$$+ \frac{\partial}{\partial r} \left[u_{er} B_\theta\right] + \frac{\partial}{\partial z} \left[u_{ez} B_\theta\right]$$

$$= B_r \left[\frac{\partial u_{e\theta}}{\partial r} - \frac{u_{e\theta}}{r} \right] + B_z \frac{\partial u_{e\theta}}{\partial z}$$

$$- \frac{c}{e\rho} \left[\frac{\partial \rho}{\partial z} \frac{\partial T_e}{\partial r} - \frac{\partial \rho}{\partial r} \frac{\partial T_e}{\partial z} \right]. \qquad (8)$$

To complete this model, a mechanism for advancing T_e which can proceed along with the other time integration procedures is required. A procedure has recently been developed that provides nearly correct cross field conduction and still a nonphysically low but somewhat realistic parallel conductivity that is between 10^2 and 10^3 larger than the cross-field conduction. This procedure and eqs. (7) and (8) thus comprise a set of coupled nonlinear partial-differential equations which can be used to advance the magnetic field in time along with the associated electron current.

Regions of Small or Zero Density in the ZEM

The most common difficulty with codes of this type are low density fluctuations. Density fluctuations always occur due to the stochastic nature of PIC simulation. Low density fluctuations produce spikes in density-dependent signal velocities which can exceed the local stability limit causing the simulation to terminate. In our method, a density cutoff is introduced to put limits on these signal velocities in the plasma. Those cells whose density is below this threshold are considered pure vacuum cells--not cells filled with cold background density--so that the proper nearly "instanteous" vacuum signal velocities are achieved. This creates a disparity in signal velocities in adjacent cells that introduces an overwhelming time step constraint in explicit codes. Consequently, the field advance must be implicit to some degree. Due to the explicit time integration of the ion component, a time step restriction is already imposed on the simulation model. Thus, no advantage is gained by requiring the field calculations to be fully implicit. Our field equations are advanced in time semi-implicitly with noniterative ADI so that unwanted high frequencies decay exponentially with time. A cutoff is imposed that is consistent with the Δt constraint resulting from the explicit ion time advance. Practically, the cutoff value is such that all but two or three percent of the particles are included in "plasma" cells. The first use of the cutoff concept is found in the equations for $\underset{\sim}{u}_e$,

$$\underset{\sim}{u}_e = \underset{\sim}{u}_i + \frac{c}{4\pi e\rho} \underset{\sim}{\nabla} \times \underset{\sim}{B} \qquad (9)$$

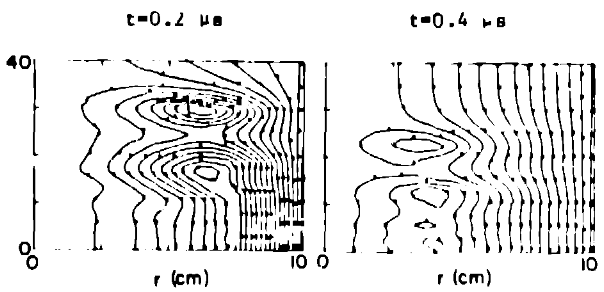
where the second term is ignored if ρ is less than the cutoff value.

The density cutoff must also be considered in eqs. (7) and (8). Monitoring the plasma-vacuum interfaces in these equations would, in highly turbulent situations, become too time consuming. Fortunately, with the exception of eq. (9), the implicit time advance of the electron-field equations removes the need for separate treatments of plasma-vacuum regions. Equations (7) and (8) automatically respond with the appropriate physics in regions of low or zero density by setting η to a large ($\eta_{plasma} \times 10^{10}$) value in any cell in which the density drops below the cutoff value--eliminating any restrictions on the position or number of plasma-vacuum interfaces.

The total $\underset{\sim}{E}$ field in finite density cells is now calculated from equation (6) using the newly updated $\underset{\sim}{B}$ field and $\underset{\sim}{u}_e$. The field calculations are completed by solving $\nabla^2 \underset{\sim}{E} = 0$ in the vacuum region.

Examples of ZEM Simulations

The ZEM model has been used to investigate the properties of magnetic reconnection in field reversed configurations such as the FRX series in Los Alamos.[7] Shown below are magnetic flux contours at two different times during a simulation of FRX formation.



Results and further details will be presented.

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