ELECTROMAGNETIC TORQUES AND FORCES DUE TO MISALIGNMENT EFFECTS AND EDDY CURRENTS IN THE HOMOPOLAR GENERATOR, POWER SUPPLY FOR THE TEXAS EXPERIMENTAL TOKAMAK (TEXT)

M. D. Driga, W. L. Bird, K. M. Tolk, W. F. Weldon
H. G. Rylander, H. H. Woodson
Center for Electromechanics
The University of Texas at Austin
Austin, Texas 78712

Abstract

Asymmetries in the applied magnetic field due to manufacturing tolerances and rotor-stator misalignments can cause significant forces and moments in a homopolar generator. Parasitic eddy-currents in the rotor, brushes and bearings are also important effects of such asymmetries. The finite element method is used to calculate the magnetic flux distributions in the TEXT homopolar generators. The axial magnetic thrust force and the magnetic tilt moment acting on the rotor are calculated. Eddy-current torques opposing rotor motion are determined using the theory for eddy-current brakes.

The results have been used in the design of the TEXT homopolar generator which have been proposed to provide the energy store and conversion for the toroidal field and ohmic heating coils of the new Texas Experimental Tokamak.

Introduction

The general design of a homopolar machine assumes a geometric and magnetic symmetry. However, the effects of normal manufacturing tolerances can effect such symmetry and have important consequences on the performance of such a machine.

It has been proposed that the new TEXT (Texas Experimental Tokamak) will use homopolar generators as power source for both toroidal and ohmic heating coil systems. The technical features and the general design of the generators are presented in a companion paper [1]. Here only forces and torques of electromagnetic origin due to misalignments, eccentricity and eddy-currents will be treated. As it will be seen, they remain within reasonable bounds, provided that the manufacturing tolerances are maintained within normal limits. One factor which keeps the problem from becoming a major one is a relatively low value of flux density (1.6 T compared with 4.0-6.0 T in a fast discharging homopolar machine [2]). On the other hand the presence of massive pieces of relatively unsaturated iron (rotor, yoke) affects it adversely.

In this paper the techniques of generalized forces (Lagrangian) are used for evaluating the forces and torques due to the misalignment. The general theory of eddy-currents brakes is employed to calculate the parasitic torques produced by eddy-currents due to eccentricities in the machine. For this purpose, the formula for the starting torque of an induction motor is an alternative, considering the rotating magnetic field due to the eccentricity of the rotor.

Misalignment Axial Forces Due To Magnetic Field Coupling

The axial mechanical force can be expressed [3] in terms of the complementary energy of the system

$$(f_e)_k = \frac{\partial W'_m(i_1,\dots i_n,\dots x_1,\dots x_m)}{\partial x_k} - \sum_{i=1}^{n} \lambda_i \frac{\partial i_i(\lambda_1,\dots \lambda_n, x_1,\dots x_m)}{\partial x_k}$$

where $\frac{\partial W'_m}{\partial x_k}$ is the variation of coenergy with the chosen coordinate. In the case of the TEXT homopolar machine, the generalized coordinate is a translation, x, having a value equal to zero when the rotor is in the center of the machine. Then

$$f = \frac{\partial W'_m(N_f i_f, x)}{\partial x}$$

Starting from the median position of the rotor, several values for the displacement of the rotor are given. These values, in per-unit (referred to the value of the air gap) are 0, 1/8, 1/4. Through a finite element

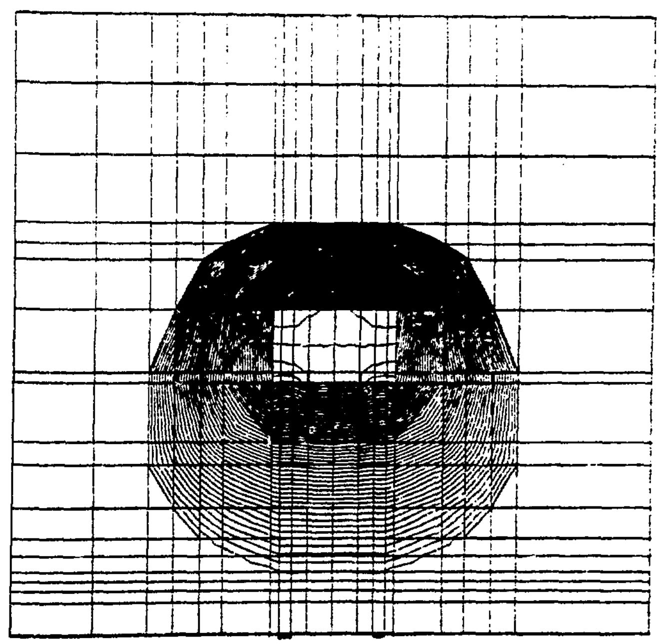

Figure 1a: Magnetic Field Plot, Misalignment M = 0.

method, briefly outlined in Appendix 1, the magnetic field is calculated and plotted (Figures 1a, 1b, 1c) for each of the positions of the rotor, for a constant magnetomotive force given by the field winding $N_f i_f = 5 \times 10^4$ At.

The force is obtained as the ratio between the increment in coenergy versus the corresponding increment in the generalized coordinate

$$f = \frac{W'_{mII} - W'_{mI}}{x_{II} - x_I} = \frac{\Delta W'_m}{\Delta x}$$

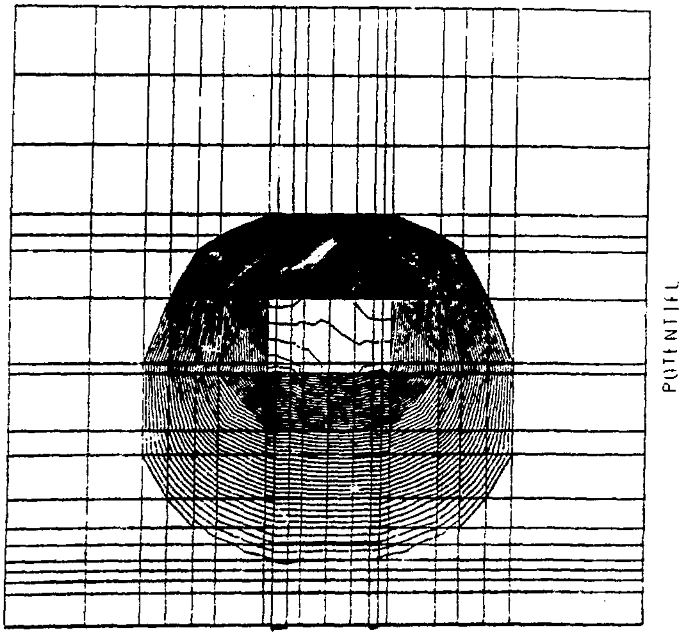

Figure 1b: Magnetic Field Plot, Misalignment $M = \frac{\delta}{8}$.

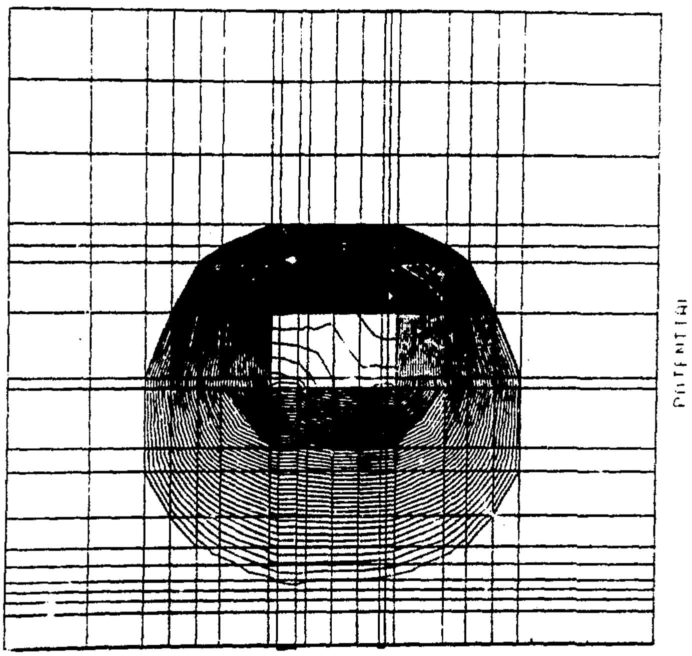

Figure 1c: Magnetic Field Plot, Misalignment $M = \frac{\delta}{4}$.

It is assumed that this value of the axial force occurs at the middle of distance x_{II} and x_I. Table 1 shows the values obtained. These values are plotted as a function of distance in Figure 2.

Table 1

$\Delta\Psi$ (Wb)	8.775×10^{-3}	11.558×10^{-3}
$N_f i_f$ (At)	5×10^4	5×10^4
Δx (m)	4.763×10^{-3}	4.763×10^{-3}
Force N	92,120	121,340
Force lb	20,659	27,212
x_I, x_{II}	0 i $\delta/8$	$\delta/8$ i $\delta/4$

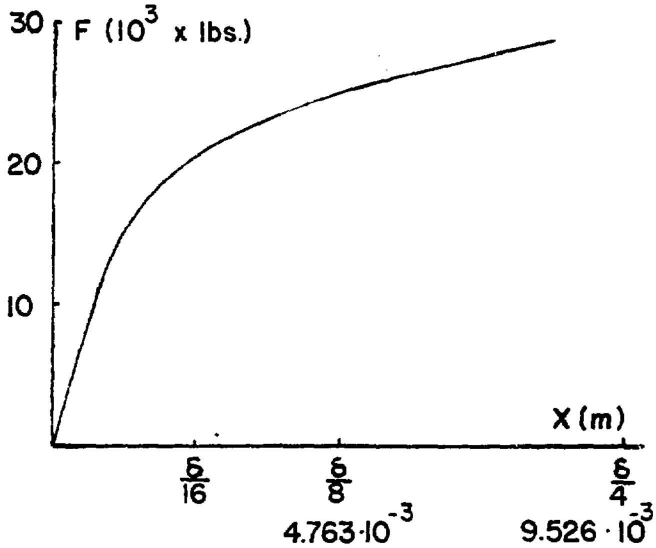

Figure 2: Axial Force Versus Misalignment.

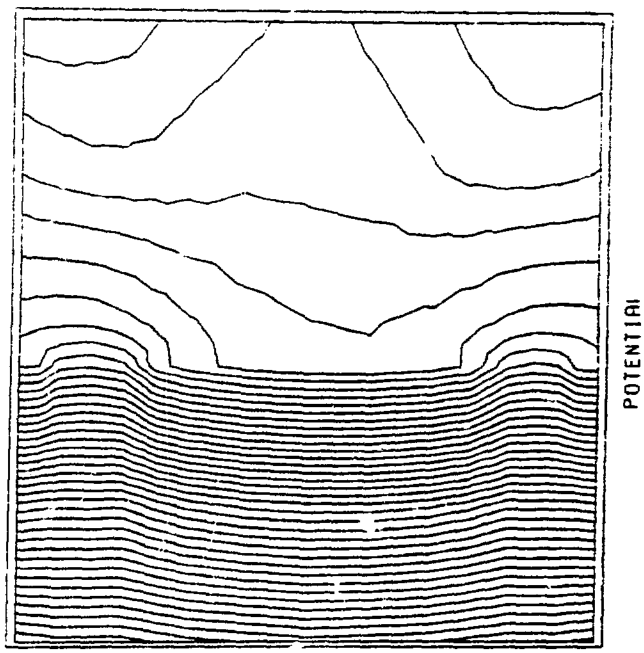

Figure 3a: Rezoned Flux Plot for Leakage Region (a).

In Figures 3a and 3b, the magnetic field is calculated more accurately in the leakage region using the "rezone" method of the finite element approach. It has resulted that the majority of modifications of the energy stored which are responsible for the axial forces, occur in this region.

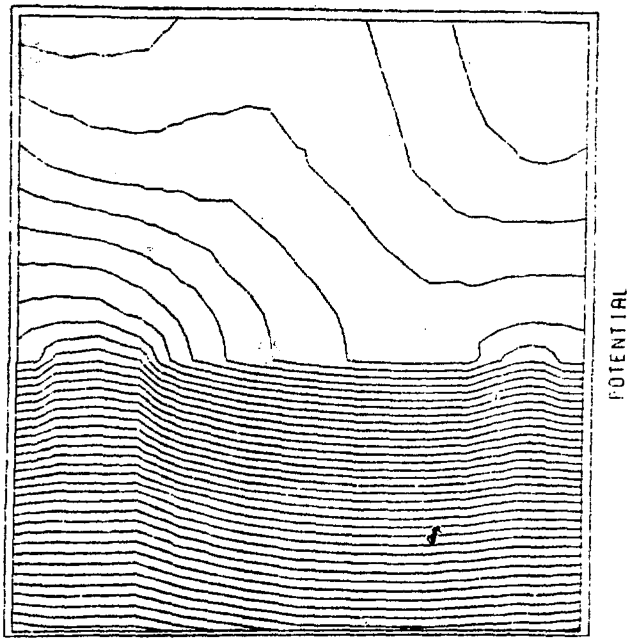

Figure 3b: Rezoned Flux Plot for Leakage Region (b).

Torque Caused by Tilt of the Rotor Due to Magnetic Field Coupling

If the angle of rotation α *(Figure 4) is chosen* as generalized coordinate, the torque, trying to tilt the rotor is obtained as

$$M = \frac{\partial W'_m\,(N_f i_f,\ \alpha)}{\partial \alpha}$$

where W'_m is the complementary magnetic energy. An alternative to this calculation is given in Appendix 2, which solves the problem analytically, assuming an infinite magnetic permeability of the iron, and neglecting the influence of the energy stored in the leakage field. The magnetic lines are compared to the symmetric situation and the distortion is assumed to vary cosinusoidally along the periphery of the rotor. If an eccentricity $E = \frac{1}{8}$ is assumed, for a flux density B = 1.65 T and for a radius of the rotor *R = 0.812 M, a torque M = 227, 750 Newton-meters is* found. If the generalized force formula is used and an increment of 5.865 x 10^{-3} radians is given to α coordinate, then the increment of coenergy stored in the leakage field (neglected in the previous approach) is 187 J.

This yields a value of 31,880 N·m, which represents the contribution of the leakage to the torque. It amounts to 14% of the value given by the analytical method outlined in Appendix 2. This increment in the value of the torque was calculated using an approximative method discussed by Bewley in [4].

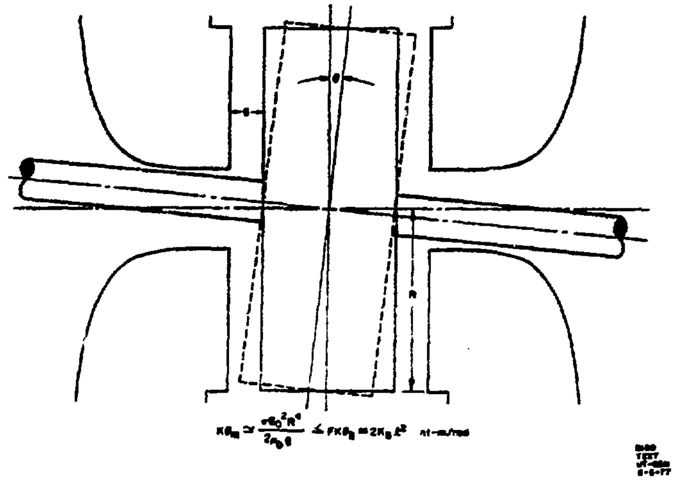

Figure 4: Magnetic Tilt Instability.

Torques Due to the Eddy-Current Brake Effects

Different methods for finding the magnitude of such type of torques are outlined by Gibbs and Davies in [5], [6], [7]. Any misalignment in bearings combined with excessive tolerance within the bearing, or in general, anything which will permit a "wobbling" motion of the rotor equivalent with a rotating magnetic field. Also, assymmetry of the air gap due to an imperfect planarity of the surfaces of the rotor and yoke will create an equivalent revolving magnetic field.

Only the fundamental wave (2_p = 2) will be considered and the machine will be divided in several circular bands (stator-rotor) in the assumption that within each band the magnetic field H and the relative permeability μ_r can be assumed constant. For each band Gibbs' formula can be applied [5,6]:

$$(\mu_r \mu_0)^{1/4}\ H_m = \sqrt{\frac{8W^2}{\rho\omega}}$$

where W = specific losses (watts/m^2) and ρ = resistivity (Ωm).

If instead of using Gibbs' curves for $(\mu_r \mu_0)^{1/4}\ H_m$ plotted against H [6], the empirical formula is used:

$$(\mu_r \mu_0)^{1/4}\ H_m = 0.97\ H^{0.77}$$

under the assumption of a strong skin effect

$$\sqrt{2}\ \alpha >> \frac{2\pi}{\lambda}$$

where $\alpha = \sqrt{\mu_r \mu_0\ \omega/2\rho}$ is high for ferromagnetic materials and λ is also high (2p = 2). Then the torque is given by the expression

$$T^{0.35} = k\,\frac{p^{0.675}\ n^{0.325}}{\rho^{0.325}\ L^{0.65}}\,\{D_1^{0.35}\phi_1 + D_2^{0.35}\phi_2 + D_3^{0.35}\phi_3\}$$

where p = number of pair of poles (p = 1 for the fundamental wave)
- n = r/s of the rotor
- L = length of the respective band (m) (equal spacing is assumed)
- $D_1 \ldots D_3$ = mean diameter of the band (m)
- $\phi_1 \ldots \phi_3$ = flux per pole for a given band (wb)

considering a total tolerance of 10^{-4}m and neglecting the armature reaction of the induced currents, the amplitudes for the flux density B in the three bands are 1.71×10^{-3}, 1.18×10^{-3} and 0.643×10^{-3} T. The total torque resulting is 3.22 Nm which corresponds to a power loss of 0.918 Kw for a conductivity of the steel of 12% or a power loss of 1.2 Kw for a conductivity of 16%.

If the amplitude of the "wobbling" of the rotor is doubled, the losses will rise to 6.64 Kw and 8.72 Kw respectively.

The same eddy-current brake effects are produced by the holes for the bolts used for mounting the compensation plates. The same formula applies, this time the number of poles per band being determined by the number of holes. For a conductivity of 12% for the rotor material, the losses are 1.45 Kw, rising to 1.9 Kw for a 16% conductivity of the steel.

Conclusion

The calculations have shown that the forces and torques due to misalignment effects and eddy-currents remain within reasonable bounds for the manufacturing tolerances prescribed for the homopolar generators used for TEXT.

Acknowledgement

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References

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Appendix 1

The finite element method was used to find the magnetic fields and subsequently for solving the problem of parasitic forces and torques with considerable accuracy. The partial differential equation is

$$\underset{\sim}{\nabla} \times \frac{1}{\mu(B)} \underset{\sim}{\nabla} \times \bar{A} = \bar{J}$$

subject to the constraint $\nabla \cdot \bar{A} = 0$.
Across the boundary, the vector potential $\bar{A}$ satisfies the condition

$$\bar{n} \times \left| \frac{1}{\mu(B)} \nabla \times \bar{A} \right| = 0$$

and $A \to 0$ when $r \to \infty$

The problem has cylindrical symmetry. The partial differential equation is satisfied if the fields are such as to minimize the functional

$$F(A) = \int_{\Omega} (W - JA)\, d\Omega$$

where W is the density of the energy stored in the magnetic fields.

$$W = \int_0^B H\,dB$$

The current and the vector potential are in the direction of Θ coordinate. The unknown potential A_Θ is approximated by polynomials in terms of

$$A_\Theta = \sum_{i=1}^{n} N_i(r,z)\, A_e^{\,i}(t)$$

The element equations are formed and assembled into a set of nonlinear equations.

$$\underline{\underline{K}}(B)\, \underline{A} = \underline{J}$$

The methodology is to linearize the problem locally, the nonlinearity of ferromagnetic materials being included by iterative corrections to the linearized equations through a Newton-Raphson iterative process [8]. The equations are solved using one of the classical methods and flux densities, obtained as curl $\bar{A}$ are calculated. The magnetic flux lines are plotted (see Figures 1,3).

Appendix 2

$$h = g\left(1 + \frac{\gamma r}{g}\cos\Theta\right)$$

$$\frac{Bh}{\mu_0} = N_f I_f \quad \text{(The iron is infinitely permeable)}$$

$$B = \frac{\mu_0 N_f I_f}{h}$$

$$B = \frac{\mu_0 NI}{g + \gamma r\cos\Theta} \quad \text{but } \gamma = \frac{e}{R}$$

$$M = -2 \int_0^R \int_0^{2\pi} \frac{B^2\, r\cos\Theta}{2\mu_0}\, r\,d\Theta\,dr$$

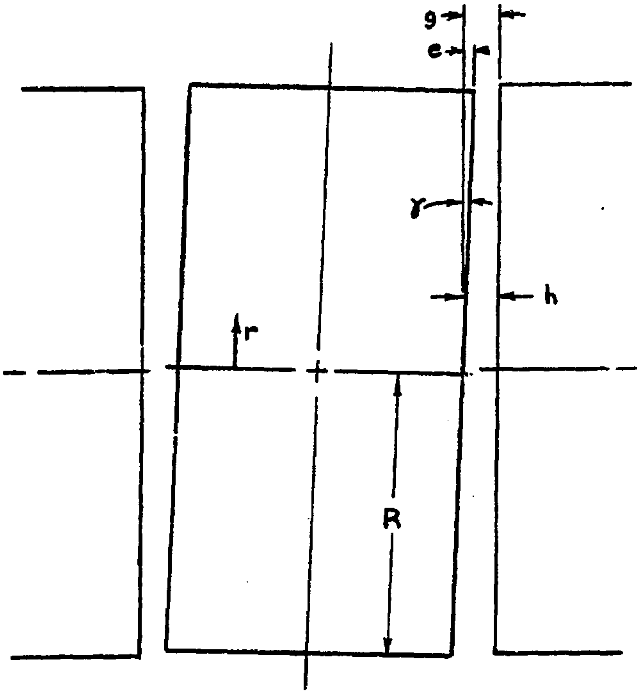
$$= -\int_0^R\int_0^{2\pi} \frac{\mu_0 N^2 I^2 r^2 \cos\Theta}{(g + \frac{e}{R} r\cos\Theta)^2}\, d\Theta dr$$

Let $B_0 = \frac{\mu_0 NI}{g}$

$$M = -\int_0^R\int_0^{2\pi} \frac{B_0^2\ r\ \cos\Theta}{\mu_0(1 + \frac{\varepsilon}{g}\frac{r}{R}\cos\Theta)}\, d\Theta dr$$

Let $\varepsilon = \frac{e}{g}$ $\quad \xi = \frac{r}{R}$ $\quad d = \frac{dr}{R}$

$$M = -\int_0^1\int_0^{2\pi} \frac{B_0^2 R^3}{\mu_0}\ \frac{\xi^2\ \cos\Theta}{(1 + \varepsilon\xi\cos\Theta)^2}\, d\Theta d\xi$$



After integration and several transformations

$$M = \frac{\pi B_0^2 R^3}{2\mu_0}\left[\frac{\varepsilon + \frac{1}{2}\varepsilon^3 + \ldots}{1 - \frac{1}{2}\varepsilon^2 - \frac{1}{8}\varepsilon^4 - \ldots}\right]$$

If $\varepsilon \ll 1$

$$M = \frac{\pi B_0^2 R^3}{2\mu_0}\,\varepsilon \left(\frac{1 + \frac{1}{2}\varepsilon^2}{1 - \frac{1}{2}\varepsilon^2}\right)$$

$$M = \frac{\pi B_0^2 R^3}{2\mu_0}\,\varepsilon\ (1 + \varepsilon^2)$$

$$= \frac{\pi B_0^2 R^3}{2\mu_0}\,\frac{e}{g}\left(1 + \frac{e^2}{g^2}\right)$$