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Accuracy of CDC 6600/7600 Fortran Library Functions

Carl B. Bailey, Rondall E. Jones

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ACCURACY OF CDC 6600/7600 FORTRAN LIBRARY FUNCTIONS

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ABSTRACT

This report documents the results of tests of accuracy performed on Fortran library functions corresponding to: version 3.0 (PSR 326) of the Fortran Extended Library and version V337 of the CDC 6600 SCOPE 3.3 operating system; version 4.6 (PSR 439) of the Fortran Extended Library and the NOS operating system; and version 4.6 (level 420) of the Fortran Extended Library and level 221C of the 7000 SCOPE 2.1 operating system. It is emphasized that corresponding routines of the same name in different libraries may--and often do--yield different results, so that those reported apply only to the particular system on which they were obtained. They are not intended as endorsements or criticisms of the CDC routines tested, but as information for users of Sandia Laboratories 6600/7600 computer systems.

CONTENTS

	Page
1. Background	7
2. Overview of Test Procedure	8
2.1 Test Setup	8
2.2 Test Processing	8
2.3 Test Output	9
3. Method for Measuring Accuracy	10
3.1 Introduction	10
3.2 Validity Checks on Function Values	11
3.3 Gross Errors	11
3.4 Computation of the Error	12
4. Generation of Arguments	14
4.1 Introduction	14
4.2 Linear Distribution	15
4.3 Exponential Distribution	16
5. Explanation of Tables and Graphs	18
6. Summary	42
7. Bibliography	43

FIGURES

Figure		
1	Equally Spaced Distribution, Plot 19	21

CONTENTS (cont)

TABLES

Table		Page
I	Test Parameters and Statistics on the CDC 6600 Under Scope	22
II	Number of Comparisons with a Given Difference on the CDC 6600 Under Scope	24
III	Number of Comparisons Requiring a Given Number of Bits on the CDC 6600 Under Scope	26
IV	Test Parameters and Statistics on the CDC 6600 Under NOS	28
V	Number of Comparisons with a Given Difference on the CDC 6600 Under NOS	30
VI	Number of Comparisons Requiring a Given Number of Bits on the CDC 6600 under NOS	32
VII	Test Parameters and Statistics on the Control Data 7600	34
VIII	Number of Comparisons with a Given Difference on the Control Data 7600	36
IX	Number of Comparisons Requiring a Given Number of Bits on the Control Data 7600	38
X	Results Which Have Either Different Exponents or Different Signs	40
XI	Results Which Have the Largest Observed Differences	41

ACCURACY OF CDC 6600/7600 FORTRAN LIBRARY FUNCTIONS

1. Background

A prototype program for experimentally measuring the accuracy of a single precision function by comparing values of that function with values of a corresponding double precision function was developed by one of the authors in 1967 and 1968. That program provided useful results but because it contained assembly language subroutines and other highly machine-dependent characteristics, it would run only on the Univac 1108.

The authors then collaborated to write a more flexible and general program with machine-dependent characteristics parameterized and localized. The result is an all Fortran program which has been easily converted and run on several different computer systems despite the machine-dependent nature of some of the computations and a few deviations from ANSI Fortran.

The program used to obtain these test results is described herein only in enough detail to explain the methods used and thereby to enable the reader to understand the results. The program was designed primarily for use by the authors and is largely undocumented. However, the program has other applications and is not terribly difficult to use. For completeness, a microfiche listing of the program is enclosed inside the back cover of this report, but interested readers should consult the authors.

2. Overview of Test Procedure

2.1 Test Setup

The setup for testing functions on a computer system consists primarily of defining machine description parameters, a pair of corresponding function subprograms for each test, and a set of test description parameters for each test. The machine-dependent parameters are defined in the main program -CRACK. The function names and the test description parameters are defined in subroutine TESTER (or in subroutines called by TESTER).

The machine description parameters are:

BASE - the base of the machine's number system

NDIGIT - the number of significant digits in a real (single precision) value

NBITS - the equivalent number of significant bits in a real value.

The pair of corresponding functions are, generically:

RFUN - the single precision function being tested

DFUN - the double precision function used for comparison.

The test description parameters are described in Section 4.

Because of the different valid ranges of arguments on different computer systems and the different functions that are available, TESTER is a machine-dependent (but standard) Fortran subroutine. Since we are measuring the accuracy of 11 library functions here, our version of TESTER simply calls 11 lower level subroutines that define parameters and call TABLES and GRAPHS (see 2.2 Test Processing).

The CDC Fortran library has no double precision functions corresponding to TAN, ASIN, ACOS, and TANH. We have written DTAN, DASINT, DACOST, and DTANHQ to provide double precision function values for testing the library. For the other single precision functions we have used the corresponding double precision functions in the CDC library for comparison.

2.2 Test Processing

Two key subroutines evaluate the test functions at the test arguments, tabulate or plot the errors, and write out the results. Subroutine TABLES writes five sets of tabulated results on units 1 through 5. Subroutine GRAPHS writes printer plots on unit 6. TABLES and GRAPHS independently generate NUMFUN argument values (RA) and function values:

$$RV = RFUN\ (RA)$$

$$DA = RA$$

$$DV = DFUN\ (DA).$$

Each pair of corresponding function values (RV, DV) is compared to measure the error in RV. The accumulated errors are written out in the form of statistics, tables, and plots.

2.3 Test Output

The test description parameters and overall test statistics are tabulated and written on unit 1. The number of comparisons with a given difference in the coefficients (see 3.4) is tabulated and written on unit 2. The number of comparisons requiring a given number of bits to represent the difference in the coefficients is tabulated and written on unit 3. All detected gross errors (see 3.3) are written on unit 4. The 25 largest observed errors in each test are written on unit 5. A printer plot of accuracy for each test is written on unit 6.

3. Method for Measuring Accuracy

3.1 Introduction

We approach the question of accuracy of a function by measuring the error (as we will define it) at a relatively small number of argument values. By using some care in selecting the arguments so that we probe the function in neighborhoods that we believe are likely to be critical, we believe that we are able to obtain a generally reliable picture of the accuracy of that function. To measure the accuracy at any given argument value, we compare the values of two corresponding functions evaluated at that argument.

A single precision function and a corresponding double precision function are evaluated at the same value of the argument. The double precision function value is converted to single precision to give a "true" single precision value for the function. The "accuracy" of the single precision function is measured by comparing its value with the "true" value.

A nonzero difference between the two single precision values guarantees that at least one of the values is in error. However, the "true" value is not necessarily correct to full single precision and even could be in error by more than the error in the single precision function being tested. Both values could be in error by an amount much greater than the difference between them.

We will make the plausible supposition that the error in the "true" value is less than the error in the single precision function value (and usually smaller by at least a factor of two) so that the difference between the single precision value and the "true" value indicates the order of magnitude of the actual error in that single precision function value. The errors we report here are obtained using this assumption.

We have sampled the accuracy of 11 single precision library functions using several distributions, each with 1001 arguments, for each of the functions. This gives a general picture of the accuracy of those functions. It is possible that there exist much larger errors than those which we have observed. Samples made using 5001 arguments yielded a few somewhat larger errors and, of course, required five times as long to run without providing much more information. Applying additional brute force to obtain a more precise determination of accuracy is not very productive.

Additional knowledge of the accuracy of these functions might be obtained by thoroughly analyzing each computational procedure but that is beyond the scope of our work. In cases where precise knowledge of accuracy is extremely important, additional, independent tests would be in order.

3.2 Validity Checks on Function Values

The single precision function value and the "true" value are checked for validity. On the CDC 6000/7000 systems we use the LEGVAR function to detect infinite and indefinite forms which are invalid at least as far as performing additional computations on them.

A less machine-dependent concept is that of normalization. A nonzero value is said to be normalized if the leftmost significant digit is nonzero. We attempt to detect unnormalized values by making assumptions about the internal machine representations of real and integer forms in Fortran. We assume:

(1) That a positive integer is represented as a right-justified string of digits with zeros at the left,

(2) that a positive real value containing NDIGIT digits of precision is represented by an exponent followed by those NDIGIT digits, and

(3) that through nonstandard use of EQUIVALENCE we can operate on a real value as an integer.

Thus, we assume that a positive real value V is represented as

$$V = X \ldots X\,YD \ldots D,$$

where X . . . X is the exponent, and YD . . . D are the significant digits. V is normalized if and only if Y is nonzero.

Now on a base B machine

$$\frac{V}{B^{NDIGIT-1}} = X \ldots XY$$

and

$$X \ldots XY = Y \text{ modulo } B.$$

This potentially machine-dependent computation, which is not central to the program, has detected unnormalized values on at least one computer system. None have been detected on the CDC 6000/7000 systems.

3.3 Gross Errors

Four kinds of gross errors are excluded from statistical computations and tabulations when ever they are detected:

- The single precision value (RV) is an invalid form (infinite, indefinite, or unnormalized)
- The "true" value (TV) is an invalid form.
- RV and TV are both nonzero and of unlike sign.
- RV and TV differ in magnitude by more than a factor of two.

These kinds of gross errors are listed in the fourth of the five kinds of tables produced. Subroutine RCHECK detects such gross errors.

3.4 Computation of the Error

After the occurrence of a gross error has been ruled out, the error is computed in a machine-independent, standard-conforming way.

We suppose quite generally that a nonzero real value is represented as NDIGIT significant digits in the base (BASE) of the machine together with an exponent (say, k). Thus, a value may be represented as:

$$\underbrace{.\,dd\,.\,.\,.\,.\,d}_{\text{NDIGIT digits}} * \text{BASE}^{k}$$

The string of significant digits is called the coefficient of the value. In effect, we are assuming only that all real values in a computer can be represented in a form of scientific notation. The exact internal machine representation is immaterial.

Given a real test function value (RV) and a corresponding "true" function value (TV), we define the error in the test function value to be

$$\frac{\text{RV - TV}}{\text{BASE}^{k}} * \text{BASE}^{\text{NDIGIT}},$$

where k is the exponent in the "true" value. That is, the error is the difference expressed in terms of units in the least significant digit position of the coefficient of the "true" value. Computationally, we rewrite the error as

$$\frac{\text{RV - TV}}{\text{BASE}^{k-1}} * \text{BASE}^{\text{NDIGIT-1}}.$$

The computation of BASE^{k-1} is done as follows:

Let ARV = |RV|.

Let ATV = |TV|.

In Fortran we compute the value of k-1 as

KEXP = ALOG(ATV)/ALOG(BASE),

i.e., logarithm to the base BASE truncated to an integer, and BASE^{k-1} as

BTV = BASE**KEXP,

i.e., BTV is of the form .1 * BASE^{k} and BTV and ATV have the same exponent when expressed in scientific notation. Then, in their internal machine representations, BTV and ATV should have like exponents and BTV should be normalized with leading digit one and trailing zeros.

Then BTV * BASE ** (1-NDIGIT) is the value of one unit in the least significant digit position in the coefficient of ATV. Therefore, the error as we have defined it is

$$\text{KDF} = \frac{(\text{ARV - ATV})}{\text{BTV}} \text{BASE}^{(\text{NDIGIT-1})}.$$

The equivalent number of bits of accuracy lost is the number of times that KDF must be divided by two using Fortran integer arithmetic before the resulting quotient is zero.

For example, consider an eight bit binary machine (BASE = 2, NDIGIT = 8). Suppose

$$ARV = .11010010 * 2^{20}$$

and

$$ATV = .11001111 * 2^{20}$$

We note that $19 < \log_2 ATV < 20$. Therefore,

$$\begin{aligned}
KEXP &= 19 \\
BTV &= 2^{19} = .10000000 * 2^{20} \\
ARV - ATV &= .00000011 * 2^{20} \\
\frac{ARV - ATV}{BTV} &= .00000110 \\
BASE^{(NDIGIT-1)} &= 2^{7} \\
KDF &= 11 \text{ (binary, i.e., KDF is 3).}
\end{aligned}$$

Thus, the error in ARV is equal to three units in the least significant position of ATV. Two bits of accuracy are lost.

This computation, which makes no assumptions about the internal representation of values and avoids masking and other machine-dependent, bit-picking techniques, has computed the error reliably on several different computer systems.

4. Generation of Arguments

4.1 Introduction

One test argument is generated each time subroutine GETARG is called by TABLES or by GRAPHS. As each argument is generated, the single precision test function and the corresponding double precision function are evaluated, the error is computed, statistics on the errors are accumulated, and gross errors are written out. Except for plotting the error, there is no need to store all the arguments in memory. Several different sequences of arguments are producible by GETARG. Use of more than one kind of argument sequence (especially when carefully selected) tends to avoid overlooking errors because of a too simplistic sampling procedure and thereby adds to the credibility of the results.

There are two general forms of argument distributions that we call Linear (LIN) and Exponential (EXP). Within each form are six specific kinds that we call:

- Equally spaced (EQU)
- Uniform randomly spaced (RAN)
- Normal random about left end point (NDL)
- Normal random about midpoint (NOR)
- Normal random about right end point (NDR)
- Incremental (or decremental) (INC).

The following test description parameters are required for each test:

NUMFUN - the number of argument values

MFORM - the general form of the argument distribution, either linear or exponential

MDIST - one of six specific kinds of argument distributions within the general form.

Depending on the choices of MFORM and MDIST, one of the following groups of test description parameters must be defined:

1. For linear distributions:
   a. For equally spaced and random distributions
      VBEG - initial (usually left) end of range
      VEND - final (usually right) end of range
   b. For incremental distributions
      VBEG - initial end of range
      INC - number of units in the least significant digit position by which to increment (or decrement)
2. For exponential distributions:
   a. For equally spaced and random distributions
      KSIGN - sign of the arguments
      IEXP - initial (usually lower) end of exponent range
      LEXP - final (usually upper) end of exponent range

b. For incremental distributions

KSIGN - sign of the arguments

IEXP - initial exponent

INC - number of units in the least significant digit position by which to increment (or decrement).

4.2 Linear Distribution

For equally spaced and random distributions we define an interval of the real numbers from VBEG through VEND and a number of arguments, NUMFUN. From these values we derive:

WIDTH = VEND - VBEG

STEP = WIDTH/FLOAT(NUMFUN-1)

For incremental distributions we define VBEG and NUMFUN as above and INC, the number of units in the least significant digit position by which to increment (or decrement). We derive:

AV = ABS(VBEG)

KEXP = ALOG(AV)/ALOG(BASE)

Then

XINC = BASE ** KEXP * BASE ** (1-NDIGIT),

corrected as necessary for round-off, is the value of one unit in the least significant digit position of VBEG.

On the N-th call to GETARG, we compute one of the following values for the test argument:

- Equally spaced
  VBEG + STEP * FLOAT(N-1)

- Uniform random
  VBEG + WIDTH * URAND(0)
  where URAND is a uniform random number generator

- Normal random about left end point
  VBEG + 2.0 * ABS(GRAND(0) - 0.5) * WIDTH
  where GRAND is a Gaussian random number generator with mean 1/2 and standard deviation 1/12

- Normal random about midpoint
  VBEG + GRAND(0) * WIDTH

- Normal random about right end point
  VEND - 2.0 * ABS(GRAND(0) - 0.5) * WIDTH

- Incremental
  VBEG + FLOAT(INC) * FLOAT(N-1) * XINC.

4.3 Exponential Distribution

If the range of arguments extends over several different values of the exponent in the internal machine representation of those arguments, then--out of the machine representable numbers in that range--large values are sampled much more frequently than small values. For example, of the values in the interval 1 to 1,000,000, roughly 1/20 of the values exactly representable in a binary machine lie between 1 and 2 versus 1/999,999 of the real numbers in that interval.

In order to sample machine-representable values more uniformly, we define an exponential form of distribution with the exponents in the machine representation more or less uniformly distributed. With the exception of the incremental distribution, we compute the coefficient (COEF) and the exponent (JEXP) independently and then combine them to form an argument,

COEF * BASE ** JEXP.

We take minus this value if the parameter KSIGN is -1. Here COEF is not a fraction. Instead $1 \leq \text{COEF} < \text{BASE}$.

For equally spaced and random distributions, we define a sign for the argument, KSIGN, an initial exponent in the base of the machine, IEXP, a last exponent, LEXP, and a number of arguments, NUMFUN. We derive

```
WIDTH = BASE - 1.0
 STEP = WIDTH/FLOAT(NUMFUN-1).
```

On the N-th call to GETARG we compute the next value in sequence for JEXP, running cyclically from IEXP to but not including LEXP, and one of the following values for COEF:

- Equally spaced
  1.0 + FLOAT(N-1) * STEP
- Uniform random
  1.0 + URAND(0) * WIDTH
- Normal random about 1.0
  1.0 + 2.0 * ABS(GRAND(0) - 0.5) * WIDTH
- Normal random about midpoint of COEF
  1.0 + GRAND(0) * WIDTH
- Normal random about BASE
  BASE - 2.0 * ABS(GRAND(0) - 0.5) * WIDTH

For incremental distributions we define KSIGN, IEXP, and NUMFUN as above and INC as in the case of linear distributions.

We derive

VALUE = BASE ** IEXP,

or minus this value if KSIGN is -1 and

XINC = BASE ** (IEXP - NDIGIT + 1).

On the N-th call to GETARG we compute the following value for the argument:

- Incremental

  VALUE + FLOAT(INC) * FLOAT(N - 1) * XINC.

5. Explanation of Tables and Graphs

Seventy five tests were performed during one run using a given computer system. Three separate corresponding runs were made for the three different computer systems--CDC 6600 SCOPE, CDC 6600 NOS, and CDC 7600. Each test consisted of tabulating and plotting errors for one pair of functions and one distribution of 1001 arguments. The five tables and the plots are related by the test number and plot number.

The tables titled "Test Parameters and Statistics on Differences in the Coefficients of the Results" summarize the test description parameters and test statistics. (See Tables I, IV, and VII for CDC 6600 SCOPE, CDC 6600 NOS, and CDC 7600 test results, respectively.) Each test is summarized on one line in these tables. The column headings and corresponding tabulated data are:

N	- the test number (used to relate to other tables and plots)
FUNCT1	- the name of the single precision function, RFUN
FUNCT2	- the name of the double precision function, DFUN
TYP	- the general form of argument distribution (LIN or EXP)
DIS	- the specific kind of distribution (EQU, RAN, NDL, NOR, NDR, or INC)
SGN	- the sign for exponential distributions
BEGIN	- the initial endpoint of the argument range
END	- the final endpoint of the argument range
NUM	- the number of argument values
ERS	- the number of gross errors
MIN DIF	- the algebraically minimum observed error, KDF, excluding gross errors (see 3.3 and 3.4)
MAX DIF	- the algebraically maximum observed error
MEAN DIF	- the average observed value of KDF
MEAN ABS	- the average observed value of |KDF|
STAN DEV	- the standard deviation of KDF.

For the exponential form of distribution, the values listed under BEGIN and END are the initial (IEXP) and last (LEXP) values of the exponent in the argument and are the power to which two (the number base of the machine) is raised.

The incremental distribution is especially helpful for carefully examining a function in the immediate neighborhood of a pole or zero. Indeed, many of the largest errors have been observed using an incremental distribution. The value of INC, which is not listed in the tables, was either 1 or -1 in all cases.

The tables titled "Number of Comparisons With a Given Difference in the Coefficients of the Results" show a frequency distribution for errors from -8 to 8 units in the least significant digit position. Larger errors are tallied in the columns headed MORE and LESS. (See Tables II, V, and VIII for the 6600 SCOPE, 6600 NOS, and 7600 test results, respectively.) These tables show well the frequency distributions of the errors for very accurate functions.

The tables titled "Number of Comparisons for Which the Difference in the Coefficients of the Results Requires a Given Number of Bits" show a frequency distribution for errors in intervals of the number of bits (from 0 to 17) required to represent the error. (See Tables III, VI, and IX for the 6600 SCOPE, 6600 NOS, and 7600 test results, respectively.) Errors requiring more than 17 bits to represent are tallied in the column headed MORE. The number of bits required to represent the largest error (excluding gross errors) is listed in the column headed MAX. These tables show the frequency distributions of the errors better for inaccurate functions or for functions with a wide range of errors.

A table titled "Results which Have Either Different Exponents or Different Signs on the Coefficients" is produced for each test in which gross errors are detected. Up to 50 gross errors are listed for a test. Note that the table heading is not strictly correct in that two corresponding values with different exponents in their internal representations are not necessarily treated as gross errors. The real criterion is whether they differ in magnitude by more than a factor of two. The tabulated data are the sequential number of the argument and the octal representations of the argument of the real function, the real function value, the argument of the double precision function value, the "true" function value obtained from the double precision function value, and the full double precision function value. At the right hand end of each line (without any column heading) is a code that indicates the kind of gross error. Table X shows an example of this type of table for Test 38 on the CDC 6600 SCOPE system. The ACOS (1.0) is computed to be $\sim$ .7E-14 instead of zero.

A table titled "Results which Have the Largest Observed Differences in the Coefficients" is produced for each test. Each table lists the 25 largest errors observed in that test. The data listed in the table are the sequential number of the argument, the octal representation of the argument, the octal representation of the real function value, the octal representation of the full double precision function value, and the error. Gross errors are not included in this table.

Table XI shows an example of this kind of table for Test 19 on the CDC 6600 SCOPE system. This example is not necessarily typical of library function accuracy but was selected because it shows both extremes of accuracy and is convenient for relating the tables and plot for a given test. Table IX showed that in 833 cases out of 1001, the error in TAN affects only the last two bits and in 928 cases affects only the last three bits. However, in eight cases, the error affects more than 17 bits of the function value. These eight cases are listed in Table XI as argument numbers 1, 126, 251, 376, 626, 751, 876 and 1001. These eight arguments have the values in the current distribution nearest to -2π, $-3/2\pi$, $-\pi$, $-1/2\pi$, $1/2\pi$, π, $3/2\pi$, and 2π.

A printer plot titled "Number of Bit Positions of Agreement in the Coefficients of RFUN and DFUN on the CDC 6600 Under SCOPE" is produced for each test where "RFUN" and "DFUN" are the pair of functions compared and "CDC 6600 under SCOPE" is the name of the computer system used for the test. For equally spaced and incremental distributions, the same data are used for the plot as for the tables. For random distributions, a similar set of arguments is generated for the plots as for the tables, but not the exact same values (due to different random numbers). We define the number of bits of agreement to be

NBITS - LOST,

i.e., the equivalent precision of the machine in bits minus the number of bits required to represent the error. Gross errors are indicated as zero bits of agreement.

Figure 1 shows an example of a printer plot for Test 19 on the CDC 6600 SCOPE system. The eight largest errors mentioned above for Test 19 appear clearly near the bottom of the plot. The agreement for these eight values varies from one bit to eight bits (the error varies from 40 to 47 bits). Elsewhere, the TAN function generally has 46 to 48 bits of agreement, as is indicated by the tables and was mentioned above.

The tables of gross errors and largest observed errors and the printer plots (as well as copies of Tables I through IX) are recorded on the enclosed 48X microfiche to reduce the volume of this report. Also included on the microfiche is a Fortran compilation listing of the program that produced those tables and plots. There is one microfiche for each of the three systems tested.

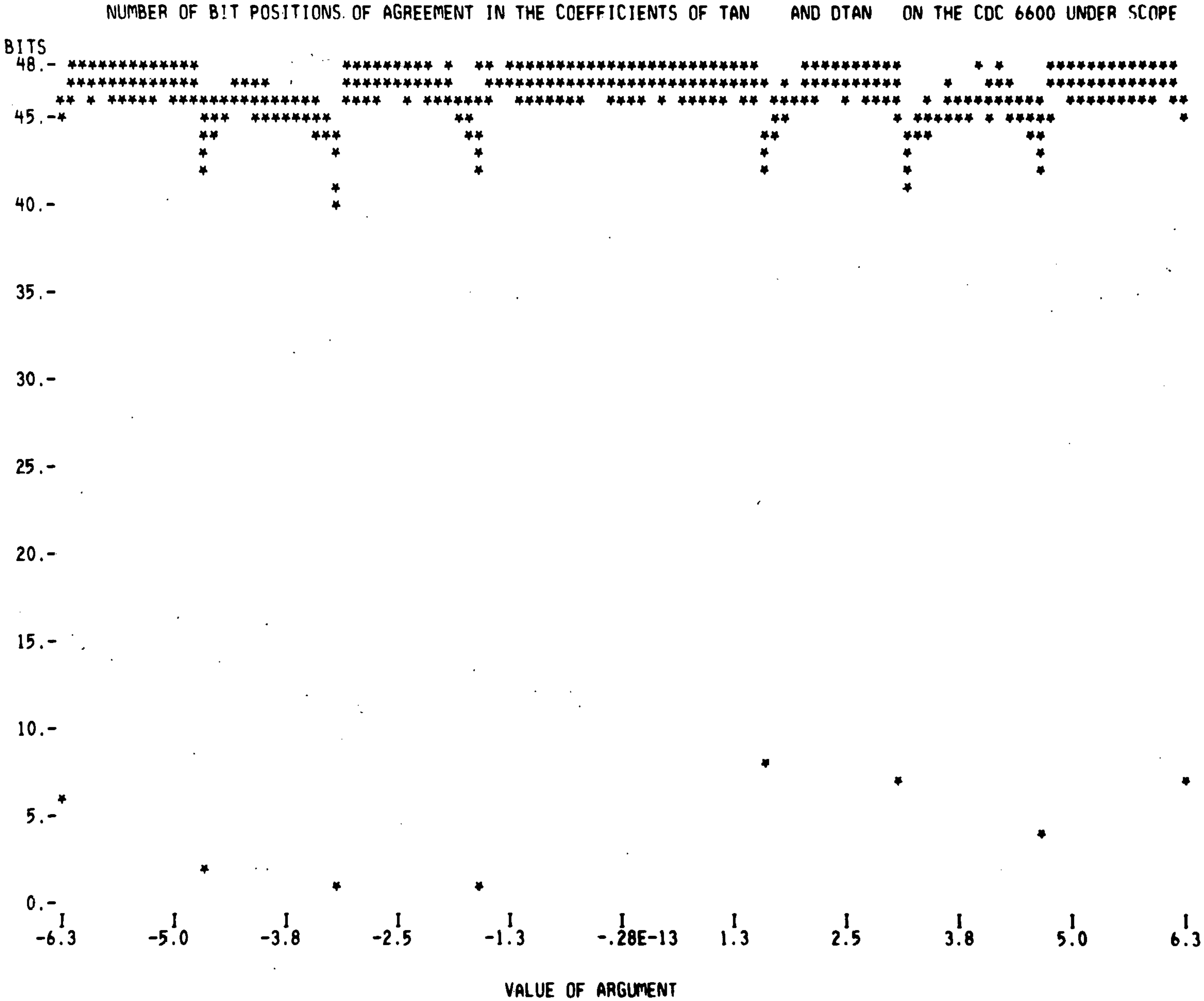

Figure 1. Equally Spaced Distribution, Plot 19

TABLE I

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CDC 6600 UNDER SCOPE

TEST PARAMETERS AND STATISTICS ON DIFFERENCES IN THE COEFFICIENTS OF THE RESULTS

N	FUNCT1	FUNCT2	TYP	DIS	SGN	BEGIN	END	NUM	ERS	MIN DIF	MAX DIF	MEAN DIF	MEAN ABS	STAN DEV
1	SIN	DSIN	LIN	EQU		-6.283185	6.283185	1001	0	-7	6	.18E-01	.10E+01	.17E+01
2	SIN	DSIN	LIN	RAN		0.	1.570796	1001	0	-8	2	-.10E+01	.12E+01	.15E+01
3	SIN	DSIN	LIN	RAN		3141.593	3147.876	1001	0	-8	8	-.17E-01	.11E+01	.17E+01
4	SIN	DSIN	LIN	RAN		3141593.	3141599.	1001	0	-7	8	.17E+00	.99E+00	.16E+01
5	SIN	DSIN	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	-8	8	.40E-02	.10E+01	.16E+01
6	SIN	DSIN	LIN	NOR		3.000000	3.300000	1001	0	-9	9	.11E+01	.56E+01	.57E+01
7	SIN	DSIN	LIN	INC		3.141593	3.141593	1001	0	-6	12	.62E+01	.62E+01	.13E+01
8	SIN	DSIN	LIN	INC		3.141593	3.141593	1001	0	-8	0	-.62E+01	.62E+01	.12E+01
9	SIN	DSIN	EXP	RAN	NEG	-5.000000	45.00000	1001	0	-7	8	.34E+00	.12E+01	.18E+01
10	COS	DCOS	LIN	EQU		-6.283185	6.283185	1001	0	-8	8	.16E-01	.11E+01	.17E+01
11	COS	DCOS	LIN	RAN		0.	1.570796	1001	0	-9	2	-.81E+00	.11E+01	.14E+01
12	COS	DCOS	LIN	RAN		3141.593	3147.876	1001	0	-8	9	.44E-01	.11E+01	.18E+01
13	COS	DCOS	LIN	RAN		3141593.	3141599.	1001	0	-8	8	.80E-02	.11E+01	.17E+01
14	COS	DCOS	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	-7	7	.51E-01	.11E+01	.18E+01
15	COS	DCOS	LIN	NOR		1.500000	1.650000	1001	0	-9	9	.13E+01	.60E+01	.60E+01
16	COS	DCOS	LIN	INC		1.570796	1.570796	1001	0	-6	12	.62E+01	.62E+01	.13E+01
17	COS	DCOS	LIN	INC		1.570796	1.570796	1001	0	-8	0	-.62E+01	.62E+01	.12E+01
18	COS	DCOS	EXP	RAN	NEG	-5.000000	45.00000	1001	0	-8	7	-.65E-01	.11E+01	.17E+01
19	TAN	DTAN	LIN	EQU		-6.283185	6.283185	1001	0	-131607567434331	31924754468871	.23E+12	.27E+12	.50E+13
20	TAN	DTAN	LIN	RAN		0.	1.570796	1001	0	-7	3	-.28E+00	.75E+00	.10E+01
21	TAN	DTAN	LIN	RAN		3141.593	3147.876	1001	0	0	5466595	.51E+05	.51E+05	.26E+06
22	TAN	DTAN	LIN	RAN		3141593.	3141599.	1001	0	0	2346032904	.16E+08	.16E+08	.85E+08
23	TAN	DTAN	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	0	1516616752882	.16E+11	.16E+11	.70E+11
24	TAN	DTAN	LIN	NOR		1.500000	1.650000	1001	0	-139151	3159	-.15E+03	.24E+03	.44E+04
25	TAN	DTAN	LIN	NOR		3.000000	3.300000	1001	0	-7679	8705	.14E+02	.11E+03	.49E+03
26	TAN	DTAN	LIN	INC		1.570796	1.570796	1001	1	*37535446845295	48128195293719	-.14E+12	.86E+12	.55E+13
27	TAN	DTAN	LIN	INC		1.570796	1.570796	1001	0	-70846690961574	0	-.89E+11	.89E+11	.22E+13
28	TAN	DTAN	LIN	INC		3.141593	3.141593	1001	1	*31924754468871	37387555560445	-.16E+12	.83E+12	.52E+13
29	TAN	DTAN	LIN	INC		3.141593	3.141593	1001	0	*31924754468871	0	-.15E+12	.15E+12	.42E+13
30	TAN	DTAN	EXP	RAN	NEG	-5.000000	45.00000	1001	23	*94375636849246	7	-.29E+13	.29E+13	.15E+14
31	ASIN	DASINT	LIN	EQU		0.	1.000000	1001	0	-1153	400	-.16E+02	.73E+02	.13E+03
32	ASIN	DASINT	LIN	EQU		-1.000000	0.	1001	0	-400	1207	.18E+02	.73E+02	.14E+03
33	ASIN	DASINT	LIN	INC		-1.000000	-1.0000000	1001	0	0	2	.13E+01	.13E+01	.44E+00
34	ASIN	DASINT	LIN	INC		1.0000000	1.000000	1001	0	-2	0	-.13E+01	.13E+01	.44E+00
35	ACOS	DACOST	LIN	EQU		0.	1.000000	1001	1	-399	411	.60E+01	.56E+02	.11E+03
36	ACOS	DACOST	LIN	EQU		-1.000000	0.	1001	0	-104	100	-.19E+01	.15E+02	.28E+02
37	ACOS	DACOST	LIN	NOR		-.1000000	.1000000	1001	0	-24	37	-.17E+00	.79E+00	.28E+01
38	ACOS	DACOST	LIN	INC		1.0000000	1.000000	1001	1	0	16777216	.12E+07	.12E+07	.11E+07
39	ACOS	DACOST	LIN	INC		-1.000000	-1.0000000	1001	0	-1	0	-.77E+00	.77E+00	.42E+00
40	ATAN	DTAN	LIN	RAN		-1.000000	1.000000	1001	0	-1	1	.20E-02	.30E+00	.55E+00
41	ATAN	DTAN	EXP	RAN	POS	0.	20.00000	1001	0	-1	1	.77E-01	.10E+00	.31E+00
42	ATAN	DTAN	EXP	RAN	NEG	0.	20.00000	1001	0	-1	1	-.89E-01	.12E+00	.33E+00
43	ATAN	DTAN	EXP	RAN	POS	0.	1069.000	1001	0	-1	1	.60E-02	.80E-02	.89E-01
44	ATAN	DTAN	EXP	RAN	NEG	0.	1069.000	1001	0	-1	0	-.60E-02	.60E-02	.77E-01

TABLE I
(cont)

N	FUNCT1	FUNCT2	TYP	DIS	SGN	BEGIN	END	NUM	ERS	MIN DIF	MAX DIF	MEAN DIF	MEAN ABS	STAN DEV
45	SQRT	DSQRT	LIN	EQU		1.000000	2.000000	1001	0	-1	0	-.46E+00	.46E+00	.50E+00
46	SQRT	DSQRT	EXP	RAN	POS	0.	1069.000	1001	0	-1	0	-.46E+00	.46E+00	.50E+00
47	SQRT	DSQRT	EXP	RAN	POS	-974.0000	0.	1001	0	-1	0	-.46E+00	.46E+00	.50E+00
48	EXP	DEXP	LIN	EQU		-1.000000	0.	1001	0	-3	2	-.27E+00	.70E+00	.89E+00
49	EXP	DEXP	LIN	RAN		0.	1.000000	1001	0	-3	2	-.34E+00	.69E+00	.90E+00
50	EXP	DEXP	LIN	RAN		0.	741.0000	1001	0	-3	2	-.36E+00	.72E+00	.93E+00
51	EXP	DEXP	LIN	RAN		-675.0000	0.	1001	0	-2	2	-.31E+00	.66E+00	.85E+00
52	ALOG	DLOG	LIN	EQU		1.000000	2.000000	1001	0	-2	1	-.22E+00	.36E+00	.57E+00
53	ALOG	DLOG	LIN	NOR		1.000000	1.500000	1001	0	-2	1	-.34E+00	.46E+00	.61E+00
54	ALOG	DLOG	LIN	NOR		.5000000	1.500000	1001	0	-2	1	-.92E-01	.41E+00	.65E+00
55	ALOG	DLOG	LIN	NOR		.5000000	1.000000	1001	0	-2	2	.21E+00	.40E+00	.61E+00
56	ALOG	DLOG	LIN	RAN		.5000000	1.000000	1001	0	-2	2	.15E+00	.34E+00	.57E+00
57	ALOG	DLOG	LIN	INC		1.000000	1.000000	1001	0	-2	0	-.50E+00	.50E+00	.52E+00
58	ALOG	DLOG	LIN	INC		1.0000000	1.000000	1001	0	0	1	.21E+00	.21E+00	.41E+00
59	ALOG	DLOG	EXP	RAN	POS	0.	1069.000	1001	0	0	1	.10E-02	.10E-02	.32E-01
60	ALOG	DLOG	EXP	EQU	POS	-974.0000	0.	1001	0	0	1	.10E-02	.10E-02	.32E-01
61	ALOG10	DLOG10	LIN	EQU		1.000000	2.000000	1001	0	-3	1	-.77E+00	.80E+00	.71E+00
62	ALOG10	DLOG10	LIN	NOR		1.000000	1.500000	1001	0	-3	1	-.98E+00	.99E+00	.71E+00
63	ALOG10	DLOG10	LIN	NOR		.5000000	1.500000	1001	0	-3	3	-.68E-01	.88E+00	.11E+01
64	ALOG10	DLOG10	LIN	NOR		.5000000	1.000000	1001	0	-1	3	.81E+00	.84E+00	.70E+00
65	ALOG10	DLOG10	LIN	RAN		.5000000	1.000000	1001	0	-1	3	.70E+00	.74E+00	.66E+00
66	ALOG10	DLOG10	LIN	INC		1.000000	1.000000	1001	0	-3	0	-.11E+01	.11E+01	.64E+00
67	ALOG10	DLOG10	LIN	INC		1.0000000	1.000000	1001	0	0	2	.74E+00	.74E+00	.52E+00
68	ALOG10	DLOG10	EXP	RAN	POS	0.	1069.000	1001	0	-2	1	-.56E+00	.58E+00	.54E+00
69	ALOG10	DLOG10	EXP	EQU	POS	-974.0000	0.	1001	0	-1	2	.56E+00	.58E+00	.54E+00
70	TANH	DTANHQ	LIN	RAN		-.1000000E-01	.1000000E-01	1001	0	-1	1	-.10E-02	.90E-02	.95E-01
71	TANH	DTANHQ	LIN	RAN		-1.000000	1.000000	1001	0	-11	10	-.26E+00	.14E+01	.21E+01
72	TANH	DTANHQ	EXP	RAN	POS	-5.000000	5.000000	1001	0	-18	11	-.57E+00	.94E+00	.19E+01
73	TANH	DTANHQ	EXP	RAN	NEG	-5.000000	5.000000	1001	0	-6	21	.22E+00	.98E+00	.18E+01
74	TANH	DTANHQ	EXP	RAN	POS	-48.00000	48.00000	1001	0	-9	4	-.56E-01	.88E-01	.54E+00
75	TANH	DTANHQ	EXP	RAN	NEG	-48.00000	48.00000	1001	0	-3	8	.34E-01	.10E+00	.57E+00

TABLE II

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CDC 6600 UNDER SCOPE

NUMBER OF COMPARISONS WITH A GIVEN DIFFERENCE IN THE COEFFICIENTS OF THE RESULTS

				DIFFERENCES IN THE COEFFICIENTS															
N	FUNCT1	FUNCT2	LESS	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	MORE
1	SIN	DSIN	0	0	6	7	6	18	1	18	284	311	288	23	3	15	8	8	4	1	0
2	SIN	DSIN	0	6	11	10	20	37	4	32	604	216	60	1	0	0	0	0	0	0	0
3	SIN	DSIN	0	1	7	10	10	16	0	17	278	291	314	18	3	20	12	2	1	1	0
4	SIN	DSIN	0	0	4	3	8	8	0	5	283	303	326	17	4	14	10	10	5	1	0
5	SIN	DSIN	0	1	5	4	6	10	9	23	288	320	285	11	3	16	8	3	8	1	0
6	SIN	DSIN	1	40	71	94	114	68	10	0	0	0	0	1	28	110	156	150	115	42	1
7	SIN	DSIN	0	0	0	1	0	0	0	0	0	0	0	0	0	65	257	256	256	163	3
8	SIN	DSIN	0	170	255	258	255	63	0	0	0	0	0	0	0	0	0	0	0	0	0
9	SIN	DSIN	0	0	2	6	5	14	4	9	249	296	318	18	5	30	17	19	8	1	0
10	COS	DCOS	0	2	2	10	6	15	3	20	305	259	321	20	1	19	8	3	4	3	0
11	COS	DCOS	1	1	8	11	13	47	3	19	496	281	119	2	0	0	0	0	0	0	0
12	COS	DCOS	0	2	7	6	8	13	6	12	309	268	296	27	0	23	10	7	3	3	1
13	COS	DCOS	0	4	4	5	6	15	3	16	303	303	277	20	1	26	9	4	1	4	0
14	COS	DCOS	0	0	8	5	13	17	5	11	291	264	320	19	1	18	15	8	6	0	0
15	COS	DCOS	3	43	81	104	108	54	4	0	0	0	0	0	3	68	154	149	139	83	8
16	COS	DCOS	0	0	0	1	0	0	0	0	0	0	0	0	0	65	257	256	256	163	3
17	COS	DCOS	0	170	255	258	255	63	0	0	0	0	0	0	0	0	0	0	0	0	0
18	COS	DCOS	0	1	3	6	9	14	4	16	365	257	265	21	3	17	1	6	10	0	0
19	TAN	DTAN	30	6	5	4	20	17	40	78	180	234	179	83	39	25	12	4	8	4	33
20	TAN	DTAN	0	0	1	0	2	2	11	75	309	409	154	31	7	0	0	0	0	0	0
21	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
22	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
23	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
24	TAN	DTAN	368	7	11	14	22	30	40	80	83	42	8	3	0	0	0	0	0	0	293
25	TAN	DTAN	341	12	25	5	3	25	47	63	69	61	17	0	0	0	0	0	0	0	333
26	TAN	DTAN	501	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	499
27	TAN	DTAN	1001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	TAN	DTAN	501	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	499
29	TAN	DTAN	1001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	TAN	DTAN	823	5	3	1	2	0	2	4	27	49	44	10	0	4	3	0	1	0	0
31	ASIN	DASINT	391	16	12	13	16	18	18	38	49	45	23	32	15	9	10	4	15	14	263
32	ASIN	DASINT	248	9	10	14	11	12	11	22	32	43	51	30	20	18	12	15	16	15	412
33	ASIN	DASINT	0	0	0	0	0	0	0	0	0	0	739	262	0	0	0	0	0	0	0
34	ASIN	DASINT	0	0	0	0	0	0	0	262	739	0	0	0	0	0	0	0	0	0	0
35	ACOS	DACOST	187	8	12	12	19	12	26	35	63	85	53	60	57	43	18	21	12	7	270
36	ACOS	DACOST	208	13	6	15	17	35	37	73	132	120	61	38	34	26	14	12	9	11	140
37	ACOS	DACOST	10	1	0	4	3	4	6	9	235	677	17	10	3	2	5	2	1	1	11
38	ACOS	DACOST	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1000
39	ACOS	DACOST	0	0	0	0	0	0	0	0	768	233	0	0	0	0	0	0	0	0	0
40	ATAN	DTAN	0	0	0	0	0	0	0	0	151	697	153	0	0	0	0	0	0	0	0
41	ATAN	DTAN	0	0	0	0	0	0	0	0	14	896	91	0	0	0	0	0	0	0	0
42	ATAN	DTAN	0	0	0	0	0	0	0	0	104	882	15	0	0	0	0	0	0	0	0
43	ATAN	DTAN	0	0	0	0	0	0	0	0	1	993	7	0	0	0	0	0	0	0	0
44	ATAN	DTAN	0	0	0	0	0	0	0	0	6	995	0	0	0	0	0	0	0	0	0

TABLE II
(cont)

N	FUNCT1	FUNCT2	LESS	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	MORE
									DIFFERENCES IN THE COEFFICIENTS												
45	SQRT	DSQRT	0	0	0	0	0	0	0	0	462	539	0	0	0	0	0	0	0	0	0
46	SQRT	DSQRT	0	0	0	0	0	0	0	0	457	544	0	0	0	0	0	0	0	0	0
47	SQRT	DSQRT	0	0	0	0	0	0	0	0	460	541	0	0	0	0	0	0	0	0	0
48	EXP	DEXP	0	0	0	0	0	0	2	67	348	380	194	10	0	0	0	0	0	0	0
49	EXP	DEXP	0	0	0	0	0	0	6	86	326	416	156	11	0	0	0	0	0	0	0
50	EXP	DEXP	0	0	0	0	0	0	8	97	323	402	160	11	0	0	0	0	0	0	0
51	EXP	DEXP	0	0	0	0	0	0	0	68	345	424	153	11	0	0	0	0	0	0	0
52	ALOG	DLOG	0	0	0	0	0	0	0	10	271	652	68	0	0	0	0	0	0	0	0
53	ALOG	DLOG	0	0	0	0	0	0	0	9	382	547	63	0	0	0	0	0	0	0	0
54	ALOG	DLOG	0	0	0	0	0	0	0	9	234	598	160	0	0	0	0	0	0	0	0
55	ALOG	DLOG	0	0	0	0	0	0	0	4	85	606	304	2	0	0	0	0	0	0	0
56	ALOG	DLOG	0	0	0	0	0	0	0	2	90	666	241	2	0	0	0	0	0	0	0
57	ALOG	DLOG	0	0	0	0	0	0	0	11	478	512	0	0	0	0	0	0	0	0	0
58	ALOG	DLOG	0	0	0	0	0	0	0	0	0	794	207	0	0	0	0	0	0	0	0
59	ALOG	DLOG	0	0	0	0	0	0	0	0	0	1000	1	0	0	0	0	0	0	0	0
60	ALOG	DLOG	0	0	0	0	0	0	0	0	0	1000	1	0	0	0	0	0	0	0	0
61	ALOG10	DLOG10	0	0	0	0	0	0	4	133	508	341	15	0	0	0	0	0	0	0	0
62	ALOG10	DLOG10	0	0	0	0	0	0	14	192	561	228	6	0	0	0	0	0	0	0	0
63	ALOG10	DLOG10	0	0	0	0	0	0	5	81	295	273	293	51	3	0	0	0	0	0	0
64	ALOG10	DLOG10	0	0	0	0	0	0	0	0	18	298	553	123	9	0	0	0	0	0	0
65	ALOG10	DLOG10	0	0	0	0	0	0	0	0	21	349	546	82	3	0	0	0	0	0	0
66	ALOG10	DLOG10	0	0	0	0	0	0	6	245	596	154	0	0	0	0	0	0	0	0	0
67	ALOG10	DLOG10	0	0	0	0	0	0	0	0	0	295	669	37	0	0	0	0	0	0	0
68	ALOG10	DLOG10	0	0	0	0	0	0	0	12	549	430	10	0	0	0	0	0	0	0	0
69	ALOG10	DLOG10	0	0	0	0	0	0	0	0	10	434	547	10	0	0	0	0	0	0	0
70	TANH	DTANHQ	0	0	0	0	0	0	0	0	5	992	4	0	0	0	0	0	0	0	0
71	TANH	DTANHQ	3	2	6	8	16	18	51	78	226	312	148	67	28	17	11	3	3	2	2
72	TANH	DTANHQ	8	5	7	9	13	21	34	42	182	556	102	9	6	2	3	0	0	0	2
73	TANH	DTANHQ	0	0	0	2	1	6	15	86	123	485	139	80	27	12	12	4	2	3	4
74	TANH	DTANHQ	1	0	0	2	1	2	2	6	20	954	12	0	0	1	0	0	0	0	0
75	TANH	DTANHQ	0	0	0	0	0	0	2	8	13	945	20	5	3	2	0	0	2	1	0

TABLE III

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CDC 6600 UNDER SCOPE

NUMBER OF COMPARISONS FOR WHICH THE DIFFERENCE IN THE COEFFICIENTS OF THE RESULTS REQUIRES A GIVEN NUMBER OF BITS

				NUMBER OF BITS																		
N	FUNCT1	FUNCT2	MAX	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	MORE
1	SIN	DSIN	4	311	572	45	72	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	SIN	DSIN	4	216	664	37	78	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	SIN	DSIN	4	291	592	38	78	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	SIN	DSIN	4	303	609	26	62	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	SIN	DSIN	4	320	573	46	60	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	SIN	DSIN	4	0	0	39	878	84	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	SIN	DSIN	4	0	0	0	835	166	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	SIN	DSIN	4	0	0	0	831	170	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	SIN	DSIN	4	296	567	36	101	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	COS	DCOS	4	259	626	44	67	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	COS	DCOS	4	281	615	24	79	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	COS	DCOS	4	268	605	45	77	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	COS	DCOS	4	303	530	40	70	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	COS	DCOS	3	264	611	36	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	COS	DCOS	4	0	0	7	857	137	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	COS	DCOS	4	0	0	0	835	166	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	COS	DCOS	4	0	0	0	831	170	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	COS	DCOS	4	257	630	44	69	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	TAN	DTAN	47	234	359	240	95	37	18	7	2	1	0	0	0	0	0	0	0	0	0	8
20	TAN	DTAN	3	409	463	124	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	TAN	DTAN	23	0	0	0	0	0	0	0	0	0	0	0	0	0	244	444	160	62	37	54
22	TAN	DTAN	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
23	TAN	DTAN	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
24	TAN	DTAN	18	42	91	123	77	37	162	213	119	71	31	19	9	4	1	1	0	0	0	1
25	TAN	DTAN	14	61	86	110	58	60	131	207	149	75	38	10	7	5	3	1	0	0	0	0
26	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1000
27	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
28	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1000
29	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
30	TAN	DTAN	47	49	71	16	14	12	16	14	17	23	13	16	26	22	19	22	14	24	21	569
31	ASIN	DASINT	11	45	72	103	97	146	107	108	122	116	79	5	1	0	0	0	0	0	0	0
32	ASIN	DASINT	11	43	83	83	108	141	107	117	122	114	74	8	1	0	0	0	0	0	0	0
33	ASIN	DASINT	2	0	739	262	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	ASIN	DASINT	2	0	739	262	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	ACOS	DACOST	9	85	116	178	149	92	66	56	91	96	71	0	0	0	0	0	0	0	0	0
36	ACOS	DACOST	7	120	193	182	134	99	106	95	72	0	0	0	0	0	0	0	0	0	0	0
37	ACOS	DACOST	6	677	252	28	21	12	10	1	0	0	0	0	0	0	0	0	0	0	0	0
38	ACOS	DACOST	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1000
39	ACOS	DACOST	1	233	768	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	ATAN	DTAN	1	697	304	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	ATAN	DTAN	1	896	105	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	ATAN	DTAN	1	882	119	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	ATAN	DTAN	1	993	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	ATAN	DTAN	1	995	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE III
(cont)

N	FUNCT1	FUNCT2	MAX	NUMBER OF BITS 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	MORE
45	SQRT	DSQRT	1	539	462	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	SQRT	DSQRT	1	544	457	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	SQRT	DSQRT	1	541	460	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	EXP	DEXP	2	380	542	79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	EXP	DEXP	2	416	482	103	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	EXP	DEXP	2	402	483	116	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	EXP	DEXP	2	424	498	79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	ALOG	DLOG	2	652	339	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	ALOG	DLOG	2	547	445	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	ALOG	DLOG	2	598	394	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	ALOG	DLOG	2	606	389	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	ALOG	DLOG	2	666	331	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	ALOG	DLOG	2	512	478	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	ALOG	DLOG	1	794	207	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	ALOG	DLOG	1	1000	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	ALOG	DLOG	1	1000	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	ALOG10	DLOG10	2	341	523	137	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	ALOG10	DLOG10	2	228	567	206	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	ALOG10	DLOG10	2	273	588	140	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	ALOG10	DLOG10	2	298	571	132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	ALOG10	DLOG10	2	349	567	85	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	ALOG10	DLOG10	2	154	596	251	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	ALOG10	DLOG10	2	295	669	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	ALOG10	DLOG10	2	430	559	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	ALOG10	DLOG10	2	434	557	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	TANH	DTANHQ	1	992	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	TANH	DTANHQ	4	312	374	224	82	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
72	TANH	DTANHQ	5	556	284	91	55	14	1	0	0	0	0	0	0	0	0	0	0	0	0	0
73	TANH	DTANHQ	5	485	262	208	39	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0
74	TANH	DTANHQ	4	954	32	8	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	TANH	DTANHQ	4	945	33	18	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE IV

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CDC 6600 UNDER NOS

TEST PARAMETERS AND STATISTICS ON DIFFERENCES IN THE COEFFICIENTS OF THE RESULTS

N	FUNCT1	FUNCT2	TYP	DIS	SGN	BEGIN	END	NUM	ERS	MIN DIF	MAX DIF	MEAN DIF	MEAN ABS	STAN DEV
1	SIN	DSIN	LIN	EQU		-6.283185	6.283185	1001	0	-3	3	.12E-01	.18E+00	.44E+00
2	SIN	DSIN	LIN	RAN		0.	1.570796	1001	0	-1	1	.14E-01	.15E+00	.38E+00
3	SIN	DSIN	LIN	RAN		3141.593	3147.876	1001	0	-1	1	-.10E-01	.26E+00	.51E+00
4	SIN	DSIN	LIN	RAN		3141593.	3141599.	1001	0	-2	2	-.80E-02	.29E+00	.55E+00
5	SIN	DSIN	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	-2	2	.17E-01	.21E+00	.46E+00
6	SIN	DSIN	LIN	NOR		3.000000	3.300000	1001	0	-2	1	-.54E-01	.37E+00	.62E+00
7	SIN	DSIN	LIN	INC		3.141593	3.141593	1001	0	-1	12	-.20E+00	.38E+00	.69E+00
8	SIN	DSIN	LIN	INC		3.141593	3.141593	1001	0	-1	3	-.72E-01	.37E+00	.61E+00
9	SIN	DSIN	EXP	RAN	NEG	-5.000000	45.00000	1001	0	-3	4	-.20E-02	.28E+00	.55E+00
10	COS	DCOS	LIN	EQU		-6.283185	6.283185	1001	0	-2	2	-.80E-02	.29E+00	.54E+00
11	COS	DCOS	LIN	RAN		0.	1.570796	1001	0	-2	1	-.60E-02	.27E+00	.53E+00
12	COS	DCOS	LIN	RAN		3141.593	3147.876	1001	0	-2	1	.40E-02	.18E+00	.42E+00
13	COS	DCOS	LIN	RAN		3141593.	3141599.	1001	0	-2	1	.35E-01	.29E+00	.54E+00
14	COS	DCOS	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	-2	2	.50E-02	.28E+00	.53E+00
15	COS	DCOS	LIN	NOR		1.500000	1.650000	1001	0	-2	2	-.27E-01	.38E+00	.62E+00
16	COS	DCOS	LIN	INC		1.570796	1.570796	1001	0	-1	12	-.20E+00	.38E+00	.69E+00
17	COS	DCOS	LIN	INC		1.570796	1.570796	1001	0	-1	3	-.72E-01	.37E+00	.61E+00
18	COS	DCOS	EXP	RAN	NEG	-5.000000	45.00000	1001	0	-8	15	.13E-01	.31E+00	.91E+00
19	TAN	DTAN	LIN	EQU		-6.283185	6.283185	1001	0	-13160756743433	131924754458871	.23E+12	.27E+12	.50E+13
20	TAN	DTAN	LIN	RAN		0.	1.570796	1001	0	-7	3	-.28E+00	.75E+00	.10E+01
21	TAN	DTAN	LIN	RAN		3141.593	3147.876	1001	0	0	5466595	.51E+05	.51E+05	.26E+06
22	TAN	DTAN	LIN	RAN		3141593.	3141599.	1001	0	0	2346032904	.16E+08	.16E+08	.85E+08
23	TAN	DTAN	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	0	1516616752882	.16E+11	.16E+11	.70E+11
24	TAN	DTAN	LIN	NOR		1.500000	1.650000	1001	0	-139151	3159	-.15E+03	.24E+03	.44E+04
25	TAN	DTAN	LIN	NOR		3.000000	3.300000	1001	0	-7679	8705	.14E+02	.11E+03	.49E+03
26	TAN	DTAN	LIN	INC		1.570796	1.570796	1001	1	***************	48128195293719	-.14E+12	.86E+12	.55E+13
27	TAN	DTAN	LIN	INC		1.570796	1.570796	1001	0	-70846690961574	0	-.89E+11	.89E+11	.22E+13
28	TAN	DTAN	LIN	INC		3.141593	3.141593	1001	1	***************	373875555E0445	-.16E+12	.83E+12	.52E+13
29	TAN	DTAN	LIN	INC		3.141593	3.141593	1001	0	***************	0	-.15E+12	.15E+12	.42E+13
30	TAN	DTAN	EXP	RAN	NEG	-5.000000	45.00000	1001	23	***************	7	-.29E+13	.29E+13	.15E+14
31	ASIN	DASINT	LIN	EQU		0.	1.000000	1001	0	-1	2	.18E+00	.33E+00	.56E+00
32	ASIN	DASINT	LIN	EQU		-1.000000	0.	1001	0	-2	1	-.14E+00	.30E+00	.55E+00
33	ASIN	DASINT	LIN	INC		-1.000000	-1.000000	1001	0	0	1	.28E+00	.28E+00	.45E+00
34	ASIN	DASINT	LIN	INC		1.000000	1.000000	1001	0	-1	0	-.28E+00	.28E+00	.45E+00
35	ACOS	DACOST	LIN	EQU		0.	1.000000	1001	0	-1	1	-.19E+00	.23E+00	.44E+00
36	ACOS	DACOST	LIN	EQU		-1.000000	0.	1001	0	-1	1	-.12E+00	.12E+00	.33E+00
37	ACOS	DACOST	LIN	NOR		-.1000000	.1000000	1001	0	-1	0	-.24E+00	.24E+00	.43E+00
38	ACOS	DACOST	LIN	INC		1.000000	1.000000	1001	0	-2	2	-.30E-01	.52E+00	.77E+00
39	ACOS	DACOST	LIN	INC		-1.000000	-1.000000	1001	0	-1	0	-.50E-02	.50E-02	.71E-01
40	ATAN	DTAN	LIN	RAN		-1.000000	1.000000	1001	0	-1	1	.18E-01	.25E+00	.50E+00
41	ATAN	DTAN	EXP	RAN	POS	0.	20.00000	1001	0	-1	1	.20E+00	.24E+00	.45E+00
42	ATAN	DTAN	EXP	RAN	NEG	0.	20.00000	1001	0	-1	1	-.22E+00	.25E+00	.46E+00
43	ATAN	DTAN	EXP	RAN	POS	0.	1069.000	1001	0	0	1	.17E-01	.17E-01	.13E+00
44	ATAN	DTAN	EXP	RAN	NEG	0.	1069.000	1001	0	-1	1	-.13E-01	.15E-01	.12E+00

TABLE IV
(cont)

N	FUNCT1	FUNCT2	TYP	DIS	SGN	BEGIN	END	NUM	ERS	MIN DIF	MAX DIF	MEAN DIF	MEAN ABS	STAN DEV
45	SQRT	DSQRT	LIN	EQU		1.000000	2.000000	1001	0	-1	1	.15E-01	.20E+00	.45E+00
46	SQRT	DSQRT	EXP	RAN	POS	0.	1069.000	1001	0	-1	1	.20E-01	.96E-01	.31E+00
47	SQRT	DSQRT	EXP	RAN	POS	-974.0000	0.	1001	0	-1	1	.19E-01	.87E-01	.29E+00
48	EXP	DEXP	LIN	EQU		-1.000000	0.	1001	0	-1	1	-.10E+00	.32E+00	.55E+00
49	EXP	DEXP	LIN	RAN		0.	1.000000	1001	0	-1	1	-.26E-01	.25E+00	.50E+00
50	EXP	DEXP	LIN	RAN		0.	741.0000	1001	0	-1	1	-.27E-01	.26E+00	.51E+00
51	EXP	DEXP	LIN	RAN		-675.0000	0.	1001	0	-2	1	-.11E+00	.33E+00	.57E+00
52	ALOG	DLOG	LIN	EQU		1.000000	2.000000	1001	0	-1	1	.42E-01	.27E+00	.51E+00
53	ALOG	DLOG	LIN	NOR		1.000000	1.500000	1001	0	-1	1	-.19E-01	.25E+00	.50E+00
54	ALOG	DLOG	LIN	NOR		.5000000	1.500000	1001	0	-1	2	.10E-01	.44E+00	.67E+00
55	ALOG	DLOG	LIN	NOR		.5000000	1.000000	1001	0	-1	1	-.58E-01	.20E+00	.44E+00
56	ALOG	DLOG	LIN	RAN		.5000000	1.000000	1001	0	-1	2	.90E-02	.29E+00	.55E+00
57	ALOG	DLOG	LIN	INC		1.000000	1.000000	1001	0	-1	1	-.12E+00	.39E+00	.61E+00
58	ALOG	DLOG	LIN	INC		1.000000	1.000000	1001	0	0	1	.46E+00	.46E+00	.50E+00
59	ALOG	DLOG	EXP	RAN	POS	0.	1069.000	1001	0	-1	1	.18E+00	.25E+00	.46E+00
60	ALOG	DLOG	EXP	EQU	POS	-974.0000	0.	1001	0	-1	1	-.17E+00	.25E+00	.47E+00
61	ALOG10	DLOG10	LIN	EQU		1.000000	2.000000	1001	0	-2	1	-.52E+00	.57E+00	.64E+00
62	ALOG10	DLOG10	LIN	NOR		1.000000	1.500000	1001	0	-2	1	-.67E+00	.67E+00	.59E+00
63	ALOG10	DLOG10	LIN	NOR		.5000000	1.500000	1001	0	-3	3	.97E-01	.77E+00	.10E+01
64	ALOG10	DLOG10	LIN	NOR		.5000000	1.000000	1001	0	-1	3	.56E+00	.58E+00	.59E+00
65	ALOG10	DLOG10	LIN	RAN		.5000000	1.000000	1001	0	-1	3	.56E+00	.61E+00	.66E+00
66	ALOG10	DLOG10	LIN	INC		1.000000	1.000000	1001	0	-3	1	-.75E+00	.76E+00	.66E+00
67	ALOG10	DLOG10	LIN	INC		1.000000	1.000000	1001	0	0	3	.99E+00	.99E+00	.52E+00
68	ALOG10	DLOG10	EXP	RAN	POS	0.	1069.000	1001	0	-2	1	-.42E+00	.48E+00	.59E+00
69	ALOG10	DLOG10	EXP	EQU	POS	-974.0000	0.	1001	0	-1	2	.43E+00	.46E+00	.58E+00
70	TANH	DTANHQ	LIN	RAN		-.1000000E-01	.1000000E-01	1001	0	-1	1	-.10E-02	.90E-02	.95E-01
71	TANH	DTANHQ	LIN	RAN		-1.000000	1.000000	1001	0	-9	10	-.36E-01	.12E+01	.19E+01
72	TANH	DTANHQ	EXP	RAN	POS	-5.000000	5.000000	1001	0	-12	9	-.30E+00	.75E+00	.15E+01
73	TANH	DTANHQ	EXP	RAN	NEG	-5.000000	5.000000	1001	0	-5	12	.35E+00	.96E+00	.16E+01
74	TANH	DTANHQ	EXP	RAN	POS	-48.00000	48.00000	1001	0	-9	3	-.30E-01	.78E-01	.48E+00
75	TANH	DTANHQ	EXP	RAN	NEG	-48.00000	48.00000	1001	0	-3	7	.22E-01	.96E-01	.51E+00

TABLE V

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CDC 6600 UNDER NOS

NUMBER OF COMPARISONS WITH A GIVEN DIFFERENCE IN THE COEFFICIENTS OF THE RESULTS

									DIFFERENCES IN THE COEFFICIENTS											
N	FUNCT1	FUNCT2	LESS	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8 MORE
1	SIN	DSIN	0	0	0	0	0	0	1	0	83	821	95	0	1	0	0	0	0	0 0
2	SIN	DSIN	0	0	0	0	0	0	0	0	67	853	81	0	0	0	0	0	0	0 0
3	SIN	DSIN	0	0	0	0	0	0	0	0	133	745	123	0	0	0	0	0	0	0 0
4	SIN	DSIN	0	0	0	0	0	0	0	1	148	712	138	2	0	0	0	0	0	0 0
5	SIN	DSIN	0	0	0	0	0	0	0	1	93	796	110	1	0	0	0	0	0	0 0
6	SIN	DSIN	0	0	0	0	0	0	0	8	194	643	156	0	0	0	0	0	0	0 0
7	SIN	DSIN	0	0	0	0	0	0	0	0	291	629	80	0	0	0	0	0	0	0 1
8	SIN	DSIN	0	0	0	0	0	0	0	0	221	634	144	1	1	0	0	0	0	0 0
9	SIN	DSIN	0	0	0	0	0	0	1	0	140	723	135	1	0	1	0	0	0	0 0
10	COS	DCOS	0	0	0	0	0	0	0	1	146	716	136	2	0	0	0	0	0	0 0
11	COS	DCOS	0	0	0	0	0	0	0	1	138	728	134	0	0	0	0	0	0	0 0
12	COS	DCOS	0	0	0	0	0	0	0	1	84	826	90	0	0	0	0	0	0	0 0
13	COS	DCOS	0	0	0	0	0	0	0	1	128	707	165	0	0	0	0	0	0	0 0
14	COS	DCOS	0	0	0	0	0	0	0	2	133	725	140	1	0	0	0	0	0	0 0
15	COS	DCOS	0	0	0	0	0	0	0	3	197	628	170	3	0	0	0	0	0	0 0
16	COS	DCOS	0	0	0	0	0	0	0	0	291	629	80	0	0	0	0	0	0	0 1
17	COS	DCOS	0	0	0	0	0	0	0	0	221	634	144	1	1	0	0	0	0	0 0
18	COS	DCOS	0	1	0	0	0	2	2	1	123	745	120	5	0	0	0	0	0	0 2
19	TAN	DTAN	30	6	5	4	20	17	40	78	180	234	179	83	39	25	12	4	8	4 33
20	TAN	DTAN	0	0	1	0	2	2	11	75	309	409	154	31	7	0	0	0	0	0 0
21	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1001
22	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1001
23	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1001
24	TAN	DTAN	368	7	11	14	22	30	40	80	83	42	8	3	0	0	0	0	0	0 293
25	TAN	DTAN	341	12	25	5	3	25	47	63	69	61	17	0	0	0	0	0	0	0 333
26	TAN	DTAN	501	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 499
27	TAN	DTAN	1001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
28	TAN	DTAN	501	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 499
29	TAN	DTAN	1001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
30	TAN	DTAN	823	5	3	1	2	0	2	4	27	49	44	10	0	4	3	0	1	0 0
31	ASIN	DASINT	0	0	0	0	0	0	0	0	74	680	239	8	0	0	0	0	0	0 0
32	ASIN	DASINT	0	0	0	0	0	0	0	10	200	713	78	0	0	0	0	0	0	0 0
33	ASIN	DASINT	0	0	0	0	0	0	0	0	0	721	280	0	0	0	0	0	0	0 0
34	ASIN	DASINT	0	0	0	0	0	0	0	0	280	721	0	0	0	0	0	0	0	0 0
35	ACOS	DACOST	0	0	0	0	0	0	0	0	208	774	19	0	0	0	0	0	0	0 0
36	ACOS	DACOST	0	0	0	0	0	0	0	0	122	876	3	0	0	0	0	0	0	0 0
37	ACOS	DACOST	0	0	0	0	0	0	0	0	240	761	0	0	0	0	0	0	0	0 0
38	ACOS	DACOST	0	0	0	0	0	0	0	18	239	515	213	16	0	0	0	0	0	0 0
39	ACOS	DACOST	0	0	0	0	0	0	0	0	5	996	0	0	0	0	0	0	0	0 0
40	ATAN	DTAN	0	0	0	0	0	0	0	0	116	751	134	0	0	0	0	0	0	0 0
41	ATAN	DTAN	0	0	0	0	0	0	0	0	21	758	222	0	0	0	0	0	0	0 0
42	ATAN	DTAN	0	0	0	0	0	0	0	0	235	747	19	0	0	0	0	0	0	0 0
43	ATAN	DTAN	0	0	0	0	0	0	0	0	0	984	17	0	0	0	0	0	0	0 0
44	ATAN	DTAN	0	0	0	0	0	0	0	0	14	986	1	0	0	0	0	0	0	0 0

TABLE V
(cont)

N	FUNCT1	FUNCT2	LESS	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	MORE
			DIFFERENCES IN THE COEFFICIENTS																		
45	SQRT	DSQRT	0	0	0	0	0	0	0	0	93	800	108	0	0	0	0	0	0	0	0
46	SQRT	DSQRT	0	0	0	0	0	0	0	0	38	905	58	0	0	0	0	0	0	0	0
47	SQRT	DSQRT	0	0	0	0	0	0	0	0	34	914	53	0	0	0	0	0	0	0	0
48	EXP	DEXP	0	0	0	0	0	0	0	0	209	683	109	0	0	0	0	0	0	0	0
49	EXP	DEXP	0	0	0	0	0	0	0	0	139	749	113	0	0	0	0	0	0	0	0
50	EXP	DEXP	0	0	0	0	0	0	0	0	143	742	116	0	0	0	0	0	0	0	0
51	EXP	DEXP	0	0	0	0	0	0	0	1	220	670	110	0	0	0	0	0	0	0	0
52	ALOG	DLOG	0	0	0	0	0	0	0	0	112	735	154	0	0	0	0	0	0	0	0
53	ALOG	DLOG	0	0	0	0	0	0	0	0	137	746	118	0	0	0	0	0	0	0	0
54	ALOG	DLOG	0	0	0	0	0	0	0	0	216	562	220	3	0	0	0	0	0	0	0
55	ALOG	DLOG	0	0	0	0	0	0	0	0	127	805	69	0	0	0	0	0	0	0	0
56	ALOG	DLOG	0	0	0	0	0	0	0	0	143	710	144	4	0	0	0	0	0	0	0
57	ALOG	DLOG	0	0	0	0	0	0	0	0	258	609	134	0	0	0	0	0	0	0	0
58	ALOG	DLOG	0	0	0	0	0	0	0	0	0	536	465	0	0	0	0	0	0	0	0
59	ALOG	DLOG	0	0	0	0	0	0	0	0	35	754	212	0	0	0	0	0	0	0	0
60	ALOG	DLOG	0	0	0	0	0	0	0	0	208	754	39	0	0	0	0	0	0	0	0
61	ALOG10	DLOG10	0	0	0	0	0	0	0	52	445	478	26	0	0	0	0	0	0	0	0
62	ALOG10	DLOG10	0	0	0	0	0	0	0	63	547	390	1	0	0	0	0	0	0	0	0
63	ALOG10	DLOG10	0	0	0	0	0	0	1	35	262	361	259	76	7	0	0	0	0	0	0
64	ALOG10	DLOG10	0	0	0	0	0	0	0	0	11	453	501	35	1	0	0	0	0	0	0
65	ALOG10	DLOG10	0	0	0	0	0	0	0	0	28	448	466	56	3	0	0	0	0	0	0
66	ALOG10	DLOG10	0	0	0	0	0	0	4	107	530	353	7	0	0	0	0	0	0	0	0
67	ALOG10	DLOG10	0	0	0	0	0	0	0	0	0	140	730	130	1	0	0	0	0	0	0
68	ALOG10	DLOG10	0	0	0	0	0	0	0	28	394	553	26	0	0	0	0	0	0	0	0
69	ALOG10	DLOG10	0	0	0	0	0	0	0	0	19	563	392	27	0	0	0	0	0	0	0
70	TANH	DTANHQ	0	0	0	0	0	0	0	0	5	992	4	0	0	0	0	0	0	0	0
71	TANH	DTANHQ	1	1	6	6	7	8	42	69	217	332	172	71	33	16	10	2	2	2	4
72	TANH	DTANHQ	3	3	4	4	8	10	28	34	182	572	116	17	10	6	3	0	0	0	1
73	TANH	DTANHQ	0	0	0	0	1	2	8	79	113	486	151	87	32	14	14	6	2	2	4
74	TANH	DTANHQ	1	0	0	1	0	2	1	5	18	955	14	2	2	0	0	0	0	0	0
75	TANH	DTANHQ	0	0	0	0	0	0	3	7	14	946	19	6	2	2	0	0	2	0	0

TABLE VI

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CDC 6600 UNDER NOS

NUMBER OF COMPARISONS FOR WHICH THE DIFFERENCE IN THE COEFFICIENTS OF THE RESULTS REQUIRES A GIVEN NUMBER OF BITS

				NUMBER OF BITS																	
N	FUNCT1	FUNCT2	MAX	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17 MORE
1	SIN	DSIN	2	821	178	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
2	SIN	DSIN	1	853	148	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
3	SIN	DSIN	1	745	256	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
4	SIN	DSIN	2	712	286	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
5	SIN	DSIN	2	796	203	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
6	SIN	DSIN	2	643	350	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
7	SIN	DSIN	4	629	371	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0 0
8	SIN	DSIN	2	634	365	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
9	SIN	DSIN	3	723	275	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
10	COS	DCOS	2	716	282	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
11	COS	DCOS	2	728	272	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
12	COS	DCOS	2	826	174	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
13	COS	DCOS	2	707	293	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
14	COS	DCOS	2	725	273	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
15	COS	DCOS	2	628	367	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
16	COS	DCOS	4	629	371	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0 0
17	COS	DCOS	2	634	365	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
18	COS	DCOS	4	745	243	8	2	3	0	0	0	0	0	0	0	0	0	0	0	0	0 0
19	TAN	DTAN	47	234	359	240	95	37	18	7	2	1	0	0	0	0	0	0	0	0	0 8
20	TAN	DTAN	3	409	463	124	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
21	TAN	DTAN	23	0	0	0	0	0	0	0	0	0	0	0	0	0	244	444	160	62	37 54
22	TAN	DTAN	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1001
23	TAN	DTAN	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1001
24	TAN	DTAN	18	42	91	123	77	37	162	213	119	71	31	19	9	4	1	1	0	0	0 1
25	TAN	DTAN	14	61	86	110	58	60	131	207	149	75	38	10	7	5	3	1	0	0	0 0
26	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1000
27	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1001
28	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1000
29	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 1001
30	TAN	DTAN	47	49	71	16	14	12	16	14	17	23	13	16	26	22	19	22	14	24	21 569
31	ASIN	DASINT	2	680	313	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
32	ASIN	DASINT	2	713	278	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
33	ASIN	DASINT	1	721	280	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
34	ASIN	DASINT	1	721	280	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
35	ACOS	DACOST	1	774	227	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
36	ACOS	DACOST	1	876	125	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
37	ACOS	DACOST	1	761	240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
38	ACOS	DACOST	2	515	452	34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
39	ACOS	DACOST	1	996	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
40	ATAN	DTAN	1	751	250	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
41	ATAN	DTAN	1	758	243	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
42	ATAN	DTAN	1	747	254	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
43	ATAN	DTAN	1	984	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0
44	ATAN	DTAN	1	986	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0

TABLE VI
(cont)

N	FUNCT1	FUNCT2	MAX	NUMBER OF BITS 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	MORE
45	SQRT	DSQRT	1	800	201	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	SQRT	DSQRT	1	905	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	SQRT	DSQRT	1	914	87	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	EXP	DEXP	1	683	318	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	EXP	DEXP	1	749	252	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	EXP	DEXP	1	742	259	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	EXP	DEXP	2	670	330	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	ALOG	DLOG	1	735	266	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	ALOG	DLOG	1	746	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	ALOG	DLOG	2	562	436	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	ALOG	DLOG	1	805	196	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	ALOG	DLOG	2	710	287	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	ALOG	DLOG	1	609	392	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	ALOG	DLOG	1	536	465	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	ALOG	DLOG	1	754	247	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	ALOG	DLOG	1	754	247	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	ALOG10	DLOG10	2	478	471	52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	ALOG10	DLOG10	2	390	548	63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	ALOG10	DLOG10	2	361	521	119	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	ALOG10	DLOG10	2	453	512	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	ALOG10	DLOG10	2	448	494	59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	ALOG10	DLOG10	2	353	537	111	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	ALOG10	DLOG10	2	140	730	131	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	ALOG10	DLOG10	2	553	420	28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	ALOG10	DLOG10	2	563	411	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	TANH	DTANHQ	1	992	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	TANH	DTANHQ	4	332	389	215	57	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
72	TANH	DTANHQ	4	572	298	89	35	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
73	TANH	DTANHQ	4	486	264	206	39	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	TANH	DTANHQ	4	955	32	10	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	TANH	DTANHQ	3	946	33	18	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE VII

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CONTROL DATA 7600

TEST PARAMETERS AND STATISTICS ON DIFFERENCES IN THE COEFFICIENTS OF THE RESULTS

N	FUNCT1	FUNCT2	TYP	DIS	SGN	BEGIN	END	NUM	ERS	MIN DIF	MAX DIF	MEAN DIF	MEAN ABS	STAN DEV
1	SIN	DSIN	LIN	EQU		-6.283185	6.283185	1001	0	-3	3	.12E-01	.18E+00	.44E+00
2	SIN	DSIN	LIN	RAN		0.	1.570796	1001	0	-1	1	.14E-01	.15E+00	.38E+00
3	SIN	DSIN	LIN	RAN		3141.593	3147.876	1001	0	-1	1	-.10E-01	.26E+00	.51E+00
4	SIN	DSIN	LIN	RAN		3141593.	3141599.	1001	0	-2	2	-.80E-02	.29E+00	.55E+00
5	SIN	DSIN	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	-2	2	.17E-01	.21E+00	.46E+00
6	SIN	DSIN	LIN	NOR		3.000000	3.300000	1001	0	-2	1	-.54E-01	.37E+00	.62E+00
7	SIN	DSIN	LIN	INC		3.141593	3.141593	1001	0	-1	12	-.20E+00	.38E+00	.69E+00
8	SIN	DSIN	LIN	INC		3.141593	3.141593	1001	0	-1	3	-.72E-01	.37E+00	.61E+00
9	SIN	DSIN	EXP	RAN	NEG	-5.000000	45.00000	1001	0	-3	4	-.20E-02	.28E+00	.55E+00
10	COS	DCOS	LIN	EQU		-6.283185	6.283185	1001	0	-2	2	-.80E-02	.29E+00	.54E+00
11	COS	DCOS	LIN	RAN		0.	1.570796	1001	0	-2	1	-.60E-02	.27E+00	.53E+00
12	COS	DCOS	LIN	RAN		3141.593	3147.876	1001	0	-2	1	.40E-02	.18E+00	.42E+00
13	COS	DCOS	LIN	RAN		3141593.	3141599.	1001	0	-2	1	.35E-01	.29E+00	.54E+00
14	COS	DCOS	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	-2	2	.50E-02	.28E+00	.53E+00
15	COS	DCOS	LIN	NOR		1.500000	1.650000	1001	0	-2	2	-.27E-01	.38E+00	.62E+00
16	COS	DCOS	LIN	INC		1.570796	1.570796	1001	0	-1	12	-.20E+00	.38E+00	.69E+00
17	COS	DCOS	LIN	INC		1.570796	1.570796	1001	0	-1	3	-.72E-01	.37E+00	.61E+00
18	COS	DCOS	EXP	RAN	NEG	-5.000000	45.00000	1001	0	-8	15	.13E-01	.31E+00	.91E+00
19	TAN	DTAN	LIN	EQU		-6.283185	6.283185	1001	0	-13160756743433	131924754468871	.23E+12	.27E+12	.50E+13
20	TAN	DTAN	LIN	RAN		0.	1.570796	1001	0	-7	3	-.10E+00	.76E+00	.11E+01
21	TAN	DTAN	LIN	RAN		3141.593	3147.876	1001	0	0	5466596	.51E+05	.51E+05	.26E+06
22	TAN	DTAN	LIN	RAN		3141593.	3141599.	1001	0	0	2346032904	.16E+08	.16E+08	.85E+08
23	TAN	DTAN	LIN	RAN		.3141593E+10	.3141593E+10	1001	0	0	1516616752882	.16E+11	.16E+11	.70E+11
24	TAN	DTAN	LIN	NOR		1.500000	1.650000	1001	0	-139151	3158	-.15E+03	.24E+03	.44E+04
25	TAN	DTAN	LIN	NOR		3.000000	3.300000	1001	0	-7679	8705	.14E+02	.11E+03	.49E+03
26	TAN	DTAN	LIN	INC		1.570796	1.570796	1001	1	***************	48128195293719	-.14E+12	.86E+12	.55E+13
27	TAN	DTAN	LIN	INC		1.570796	1.570796	1001	0	-70846690961573	0	-.89E+11	.89E+11	.22E+13
28	TAN	DTAN	LIN	INC		3.141593	3.141593	1001	1	***************	37387555560445	-.16E+12	.83E+12	.52E+13
29	TAN	DTAN	LIN	INC		3.141593	3.141593	1001	0	***************	0	-.15E+12	.15E+12	.42E+13
30	TAN	DTAN	EXP	RAN	NEG	-5.000000	45.00000	1001	23	***************	7	-.29E+13	.29E+13	.15E+14
31	ASIN	DASINT	LIN	EQU		0.	1.000000	1001	0	-1	2	.18E+00	.33E+00	.56E+00
32	ASIN	DASINT	LIN	EQU		-1.000000	0.	1001	0	-2	1	-.14E+00	.30E+00	.55E+00
33	ASIN	DASINT	LIN	INC		-1.000000	-1.000000	1001	0	0	1	.28E+00	.28E+00	.45E+00
34	ASIN	DASINT	LIN	INC		1.000000	1.000000	1001	0	-1	0	-.28E+00	.28E+00	.45E+00
35	ACOS	DACOST	LIN	EQU		0.	1.000000	1001	0	-1	1	-.19E+00	.23E+00	.44E+00
36	ACOS	DACOST	LIN	EQU		-1.000000	0.	1001	0	-1	1	-.12E+00	.12E+00	.33E+00
37	ACOS	DACOST	LIN	NOR		-.1000000	.1000000	1001	0	-1	0	-.24E+00	.24E+00	.43E+00
38	ACOS	DACOST	LIN	INC		1.000000	1.000000	1001	0	-2	2	-.30E-01	.52E+00	.77E+00
39	ACOS	DACOST	LIN	INC		-1.000000	-1.000000	1001	0	-1	0	-.50E-02	.50E-02	.71E-01
40	ATAN	DTAN	LIN	RAN		-1.000000	1.000000	1001	0	-1	1	.18E-01	.25E+00	.50E+00
41	ATAN	DTAN	EXP	RAN	POS	0.	20.00000	1001	0	-1	1	.18E+00	.24E+00	.45E+00
42	ATAN	DTAN	EXP	RAN	NEG	0.	20.00000	1001	0	-1	1	-.20E+00	.25E+00	.46E+00
43	ATAN	DTAN	EXP	RAN	POS	0.	1069.000	1001	0	0	1	.17E-01	.17E-01	.13E+00
44	ATAN	DTAN	EXP	RAN	NEG	0.	1069.000	1001	0	-1	1	-.12E-01	.14E-01	.12E+00

TABLE VII
(cont)

N	FUNCT1	FUNCT2	TYP	DIS	SGN	BEGIN	END	NUM	ERS	MIN DIF	MAX DIF	MEAN DIF	MEAN ABS	STAN DEV
45	SQRT	DSQRT	LIN	EQU		1.000000	2.000000	1001	0	-1	1	.15E-01	.20E+00	.45E+00
46	SQRT	DSQRT	EXP	RAN	POS	0.	1069.000	1001	0	-1	1	.20E-01	.96E-01	.31E+00
47	SQRT	DSQRT	EXP	RAN	POS	-974.0000	0.	1001	0	-1	1	.19E-01	.87E-01	.29E+00
48	EXP	DEXP	LIN	EQU		-1.000000	0.	1001	0	-1	1	-.10E+00	.32E+00	.55E+00
49	EXP	DEXP	LIN	RAN		0.	1.000000	1001	0	-1	1	-.26E-01	.25E+00	.50E+00
50	EXP	DEXP	LIN	RAN		0.	741.0000	1001	0	-1	1	-.27E-01	.26E+00	.51E+00
51	EXP	DEXP	LIN	RAN		-675.0000	0.	1001	0	-2	1	-.11E+00	.33E+00	.57E+00
52	ALOG	DLOG	LIN	EQU		1.000000	2.000000	1001	0	-1	1	.42E-01	.27E+00	.51E+00
53	ALOG	DLOG	LIN	NOR		1.000000	1.500000	1001	0	-1	1	-.19E-01	.25E+00	.50E+00
54	ALOG	DLOG	LIN	NOR		.5000000	1.500000	1001	0	-1	2	.10E-01	.44E+00	.67E+00
55	ALOG	DLOG	LIN	NOR		.5000000	1.000000	1001	0	-1	1	-.58E-01	.20E+00	.44E+00
56	ALOG	DLOG	LIN	RAN		.5000000	1.000000	1001	0	-1	2	.90E-02	.29E+00	.55E+00
57	ALOG	DLOG	LIN	INC		1.000000	1.000000	1001	0	-1	1	-.12E+00	.39E+00	.61E+00
58	ALOG	DLOG	LIN	INC		1.000000	1.000000	1001	0	0	1	.46E+00	.46E+00	.50E+00
59	ALOG	DLOG	EXP	RAN	POS	0.	1069.000	1001	0	-1	1	.18E+00	.25E+00	.46E+00
60	ALOG	DLOG	EXP	EQU	POS	-974.0000	0.	1001	0	-1	1	-.17E+00	.25E+00	.47E+00
61	ALOG10	DLOG10	LIN	EQU		1.000000	2.000000	1001	0	-2	1	-.52E+00	.57E+00	.64E+00
62	ALOG10	DLOG10	LIN	NOR		1.000000	1.500000	1001	0	-2	1	-.67E+00	.67E+00	.59E+00
63	ALOG10	DLOG10	LIN	NOR		.5000000	1.500000	1001	0	-3	3	.97E-01	.77E+00	.10E+01
64	ALOG10	DLOG10	LIN	NOR		.5000000	1.000000	1001	0	-1	3	.56E+00	.58E+00	.59E+00
65	ALOG10	DLOG10	LIN	RAN		.5000000	1.000000	1001	0	-1	3	.56E+00	.61E+00	.66E+00
66	ALOG10	DLOG10	LIN	INC		1.000000	1.000000	1001	0	-3	1	-.75E+00	.76E+00	.66E+00
67	ALOG10	DLOG10	LIN	INC		1.000000	1.000000	1001	0	0	3	.99E+00	.99E+00	.52E+00
68	ALOG10	DLOG10	EXP	RAN	POS	0.	1069.000	1001	0	-2	1	-.42E+00	.48E+00	.59E+00
69	ALOG10	DLOG10	EXP	EQU	POS	-974.0000	0.	1001	0	-1	2	.43E+00	.46E+00	.58E+00
70	TANH	DTANHQ	LIN	RAN		-.1000000E-01	.1000000E-01	1001	0	-1	1	-.10E-02	.90E-02	.95E-01
71	TANH	DTANHQ	LIN	RAN		-1.000000	1.000000	1001	0	-9	10	-.64E+00	.13E+01	.19E+01
72	TANH	DTANHQ	EXP	RAN	POS	-5.000000	5.000000	1001	0	-16	5	-.53E+00	.84E+00	.17E+01
73	TANH	DTANHQ	EXP	RAN	NEG	-5.000000	5.000000	1001	0	-7	10	-.35E+00	.10E+01	.16E+01
74	TANH	DTANHQ	EXP	RAN	POS	-48.00000	48.00000	1001	0	-9	3	-.44E-01	.80E-01	.49E+00
75	TANH	DTANHQ	EXP	RAN	NEG	-48.00000	48.00000	1001	0	-4	7	-.38E-01	.12E+00	.59E+00

TABLE VIII

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CONTROL DATA 7600

NUMBER OF COMPARISONS WITH A GIVEN DIFFERENCE IN THE COEFFICIENTS OF THE RESULTS

				DIFFERENCES IN THE COEFFICIENTS																		
N	FUNCT1	FUNCT2	LESS	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	MORE	
1	SIN	DSIN	0	0	0	0	0	0	1	0	83	821	95	0	1	0	0	0	0	0	0	
2	SIN	DSIN	0	0	0	0	0	0	0	0	67	853	81	0	0	0	0	0	0	0	0	
3	SIN	DSIN	0	0	0	0	0	0	0	0	133	745	123	0	0	0	0	0	0	0	0	
4	SIN	DSIN	0	0	0	0	0	0	0	1	148	712	138	2	0	0	0	0	0	0	0	
5	SIN	DSIN	0	0	0	0	0	0	0	1	93	796	110	1	0	0	0	0	0	0	0	
6	SIN	DSIN	0	0	0	0	0	0	0	8	194	643	156	0	0	0	0	0	0	0	0	
7	SIN	DSIN	0	0	0	0	0	0	0	0	291	629	80	0	0	0	0	0	0	0	1	
8	SIN	DSIN	0	0	0	0	0	0	0	0	221	634	144	1	1	0	0	0	0	0	0	
9	SIN	DSIN	0	0	0	0	0	0	1	0	140	723	135	1	0	1	0	0	0	0	0	
10	COS	DCOS	0	0	0	0	0	0	0	1	146	716	136	2	0	0	0	0	0	0	0	
11	COS	DCOS	0	0	0	0	0	0	0	1	138	728	134	0	0	0	0	0	0	0	0	
12	COS	DCOS	0	0	0	0	0	0	0	1	84	826	90	0	0	0	0	0	0	0	0	
13	COS	DCOS	0	0	0	0	0	0	0	1	128	707	165	0	0	0	0	0	0	0	0	
14	COS	DCOS	0	0	0	0	0	0	0	2	133	725	140	1	0	0	0	0	0	0	0	
15	COS	DCOS	0	0	0	0	0	0	0	3	197	628	170	3	0	0	0	0	0	0	0	
16	COS	DCOS	0	0	0	0	0	0	0	0	291	629	80	0	0	0	0	0	0	0	1	
17	COS	DCOS	0	0	0	0	0	0	0	0	221	634	144	1	1	0	0	0	0	0	0	
18	COS	DCOS	0	1	0	0	0	2	2	1	123	745	120	5	0	0	0	0	0	0	2	
19	TAN	DTAN	28	8	5	4	19	20	37	75	178	252	173	81	37	23	12	5	6	5	33	
20	TAN	DTAN	0	0	1	0	2	1	9	57	268	399	207	49	8	0	0	0	0	0	0	
21	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001	
22	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001	
23	TAN	DTAN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001	
24	TAN	DTAN	368	6	11	15	17	31	36	77	82	50	11	4	0	0	0	0	0	0	293	
25	TAN	DTAN	343	14	22	5	2	31	53	63	72	49	14	0	0	0	0	0	0	0	333	
26	TAN	DTAN	501	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	499	
27	TAN	DTAN	1001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
28	TAN	DTAN	501	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	499	
29	TAN	DTAN	1001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
30	TAN	DTAN	324	5	2	1	2	0	2	2	32	54	37	8	1	5	2	0	1	0	0	
31	ASIN	DASINT	0	0	0	0	0	0	0	0	74	680	239	8	0	0	0	0	0	0	0	
32	ASIN	DASINT	0	0	0	0	0	0	0	10	200	713	78	0	0	0	0	0	0	0	0	
33	ASIN	DASINT	0	0	0	0	0	0	0	0	0	721	280	0	0	0	0	0	0	0	0	
34	ASIN	DASINT	0	0	0	0	0	0	0	0	280	721	0	0	0	0	0	0	0	0	0	
35	ACOS	DACOST	0	0	0	0	0	0	0	0	208	774	19	0	0	0	0	0	0	0	0	
36	ACOS	DACOST	0	0	0	0	0	0	0	0	122	876	3	0	0	0	0	0	0	0	0	
37	ACOS	DACOST	0	0	0	0	0	0	0	0	240	761	0	0	0	0	0	0	0	0	0	
38	ACOS	DACOST	0	0	0	0	0	0	0	18	239	515	213	16	0	0	0	0	0	0	0	
39	ACOS	DACOST	0	0	0	0	0	0	0	0	5	996	0	0	0	0	0	0	0	0	0	
40	ATAN	DTAN	0	0	0	0	0	0	0	0	116	751	134	0	0	0	0	0	0	0	0	
41	ATAN	DTAN	0	0	0	0	0	0	0	0	27	763	211	0	0	0	0	0	0	0	0	
42	ATAN	DTAN	0	0	0	0	0	0	0	0	228	748	25	0	0	0	0	0	0	0	0	
43	ATAN	DTAN	0	0	0	0	0	0	0	0	0	984	17	0	0	0	0	0	0	0	0	
44	ATAN	DTAN	0	0	0	0	0	0	0	0	13	987	1	0	0	0	0	0	0	0	0	

TABLE VIII
(cont)

N	FUNCT1	FUNCT2	LESS	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	MORE
									DIFFERENCES IN THE COEFFICIENTS												
45	SQRT	DSQRT	0	0	0	0	0	0	0	0	93	800	108	0	0	0	0	0	0	0	0
46	SQRT	DSQRT	0	0	0	0	0	0	0	0	38	905	58	0	0	0	0	0	0	0	0
47	SQRT	DSQRT	0	0	0	0	0	0	0	0	34	914	53	0	0	0	0	0	0	0	0
48	EXP	DEXP	0	0	0	0	0	0	0	0	209	683	109	0	0	0	0	0	0	0	0
49	EXP	DEXP	0	0	0	0	0	0	0	0	139	749	113	0	0	0	0	0	0	0	0
50	EXP	DEXP	0	0	0	0	0	0	0	0	143	742	116	0	0	0	0	0	0	0	0
51	EXP	DEXP	0	0	0	0	0	0	0	1	220	670	110	0	0	0	0	0	0	0	0
52	ALOG	DLOG	0	0	0	0	0	0	0	0	112	735	154	0	0	0	0	0	0	0	0
53	ALOG	DLOG	0	0	0	0	0	0	0	0	137	746	118	0	0	0	0	0	0	0	0
54	ALOG	DLOG	0	0	0	0	0	0	0	0	216	562	220	3	0	0	0	0	0	0	0
55	ALOG	DLOG	0	0	0	0	0	0	0	0	127	805	69	0	0	0	0	0	0	0	0
56	ALOG	DLOG	0	0	0	0	0	0	0	0	143	710	144	4	0	0	0	0	0	0	0
57	ALOG	DLOG	0	0	0	0	0	0	0	0	258	609	134	0	0	0	0	0	0	0	0
58	ALOG	DLOG	0	0	0	0	0	0	0	0	0	536	465	0	0	0	0	0	0	0	0
59	ALOG	DLOG	0	0	0	0	0	0	0	0	35	754	212	0	0	0	0	0	0	0	0
60	ALOG	DLOG	0	0	0	0	0	0	0	0	208	754	39	0	0	0	0	0	0	0	0
61	ALOG10	DLOG10	0	0	0	0	0	0	0	52	445	478	26	0	0	0	0	0	0	0	0
62	ALOG10	DLOG10	0	0	0	0	0	0	0	63	547	390	1	0	0	0	0	0	0	0	0
63	ALOG10	DLOG10	0	0	0	0	0	0	1	35	262	361	259	76	7	0	0	0	0	0	0
64	ALOG10	DLOG10	0	0	0	0	0	0	0	0	11	453	501	35	1	0	0	0	0	0	0
65	ALOG10	DLOG10	0	0	0	0	0	0	0	0	28	448	466	56	3	0	0	0	0	0	0
66	ALOG10	DLOG10	0	0	0	0	0	0	4	107	530	353	7	0	0	0	0	0	0	0	0
67	ALOG10	DLOG10	0	0	0	0	0	0	0	0	0	140	730	130	1	0	0	0	0	0	0
68	ALOG10	DLOG10	0	0	0	0	0	0	0	28	394	553	26	0	0	0	0	0	0	0	0
69	ALOG10	DLOG10	0	0	0	0	0	0	0	0	19	563	392	27	0	0	0	0	0	0	0
70	TANH	DTANHQ	0	0	0	0	0	0	0	0	5	992	4	0	0	0	0	0	0	0	0
71	TANH	DTANHQ	1	2	8	10	15	19	69	107	255	326	121	35	18	6	4	2	1	1	1
72	TANH	DTANHQ	7	3	5	10	13	17	30	39	189	565	105	9	5	2	2	0	0	0	0
73	TANH	DTANHQ	0	0	1	0	7	23	61	128	121	482	101	34	23	7	8	3	0	0	2
74	TANH	DTANHQ	1	0	0	1	0	3	1	6	20	954	13	1	1	0	0	0	0	0	0
75	TANH	DTANHQ	0	0	0	0	0	3	10	10	16	944	11	2	1	2	0	0	2	0	0

TABLE IX

COMPARISON OF THE RESULTS OF ALTERNATIVE COMPUTATIONS ON THE CONTROL DATA 7600

NUMBER OF COMPARISONS FOR WHICH THE DIFFERENCE IN THE COEFFICIENTS OF THE RESULTS REQUIRES A GIVEN NUMBER OF BITS

				NUMBER OF BITS																		
N	FUNCT1	FUNCT2	MAX	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	MORE
1	SIN	DSIN	2	821	178	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	SIN	DSIN	1	853	148	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	SIN	DSIN	1	745	256	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	SIN	DSIN	2	712	286	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	SIN	DSIN	2	796	203	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	SIN	DSIN	2	643	350	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	SIN	DSIN	4	629	371	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	SIN	DSIN	2	634	365	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	SIN	DSIN	3	723	275	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	COS	DCOS	2	716	282	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	COS	DCOS	2	728	272	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	COS	DCOS	2	826	174	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	COS	DCOS	2	707	293	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	COS	DCOS	2	725	273	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	COS	DCOS	2	628	367	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	COS	DCOS	4	629	371	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	COS	DCOS	2	634	365	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	COS	DCOS	4	745	243	8	2	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	TAN	DTAN	47	252	351	230	94	38	18	7	2	1	0	0	0	0	0	0	0	0	0	8
20	TAN	DTAN	3	399	475	123	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	TAN	DTAN	23	0	0	0	0	0	0	0	0	0	0	0	0	0	244	444	160	62	37	54
22	TAN	DTAN	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
23	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
24	TAN	DTAN	18	50	93	117	74	36	163	212	119	71	31	19	9	4	1	1	0	0	0	1
25	TAN	DTAN	14	49	86	116	60	66	133	203	149	75	38	10	7	5	3	1	0	0	0	0
26	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1000
27	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
28	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1000
29	TAN	DTAN	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1001
30	TAN	DTAN	47	54	69	13	13	13	16	14	17	23	13	16	26	22	19	22	14	24	21	569
31	ASIN	DASINT	2	680	313	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	ASIN	DASINT	2	713	278	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	ASIN	DASINT	1	721	280	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	ASIN	DASINT	1	721	280	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	ACOS	DACOST	1	774	227	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	ACOS	DACOST	1	876	125	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	ACOS	DACOST	1	761	240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	ACOS	DACOST	2	515	452	34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	ACOS	DACOST	1	996	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	ATAN	DTAN	1	751	250	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	ATAN	DTAN	1	763	238	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	ATAN	DTAN	1	748	253	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	ATAN	DTAN	1	984	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	ATAN	DTAN	1	987	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE IX
(cont)

				NUMBER OF BITS																		
N	FUNCT1	FUNCT2	MAX	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	MORE
45	SQRT	DSQRT	1	800	201	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	SQRT	DSQRT	1	905	96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	SQRT	DSQRT	1	914	87	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	EXP	DEXP	1	683	318	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	EXP	DEXP	1	749	252	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	EXP	DEXP	1	742	259	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	EXP	DEXP	2	670	330	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	ALOG	DLOG	1	735	266	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	ALOG	DLOG	1	746	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	ALOG	DLOG	2	562	436	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	ALOG	DLOG	1	805	196	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	ALOG	DLOG	2	710	287	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	ALOG	DLOG	1	609	392	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	ALOG	DLOG	1	536	465	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	ALOG	DLOG	1	754	247	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	ALOG	DLOG	1	754	247	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	ALOG10	DLOG10	2	478	471	52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	ALOG10	DLOG10	2	390	548	63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	ALOG10	DLOG10	2	361	521	119	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	ALOG10	DLOG10	2	453	512	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	ALOG10	DLOG10	2	448	494	59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	ALOG10	DLOG10	2	353	537	111	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	ALOG10	DLOG10	2	140	730	131	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	ALOG10	DLOG10	2	553	420	28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	ALOG10	DLOG10	2	563	411	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	TANH	DTANHQ	1	992	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	TANH	DTANHQ	4	326	376	229	65	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
72	TANH	DTANHQ	5	565	294	83	49	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0
73	TANH	DTANHQ	4	482	222	246	49	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	TANH	DTANHQ	4	954	33	9	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	TANH	DTANHQ	3	944	27	23	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE X

RESULTS WHICH HAVE EITHER DIFFERENT EXPONENTS OR DIFFERENT SIGNS ON THE COEFFICIENTS

TEST 38, A COMPARISON OF ACOS AND DACOST

NUM	ARGUMENT OF ACOS	REAL ACOS VALUE	ARGUMENT OF DACOST	SINGLE DACOST VALUE	DOUBLE PRECISION DACOST VALUE	
1	17204000000000000000	16414000000000000000	17204000000000000000	00000000000000000000	0000000000000000000000000000000000000000	4

TABLE XI

RESULTS WHICH HAVE THE LARGEST OBSERVED DIFFERENCES IN THE COEFFICIENTS

TEST 19, A COMPARISON OF TAN AND DTAN

NUM	ARGUMENT OF TAN	REAL TAN VALUE	DOUBLE PRECISION DTAN VALUE		DIFFERENCE
1	6055155700452735 7227	16426134743306206552	16426043230461461213	15620000000000000010	3961408105183
247	60561473423051606451	60541437014270151572	60641437014270151534	61446645220011370503	30
627	17206252564077305035	60513015517254035530	60513015517254035444	61311372140376565433	53
251	60561557004527357227	16424721416234674511	16416043230461461213	15610000000000000002	131924754468871
250	60561542110214025074	60661441501400267063	60661441501400237273	61460440104175260465	-135
129	60553246436062572573	60531276233170637745	60531276233170637716	61330202273326272172	24
128	60553240077715015515	60523016035167756425	60523016035167756375	61322435556403600325	25
127	60553231541547240437	60513015517254036651	60513015517254036571	61312351632164613423	49
126	60553223203401463361	60022165360511743644	60020760360743600655	60625473266556367001	4432402217 5223
874	17224537700062762261	17254761742610021045	17254761742610021074	16455756100727713210	-24
753	17216252564077305036	17126336710731611561	17126336710731611655	16325553641615041604	-61
376	60571557004527357227	60011366473302151721	57772564264552640502	60604416566161230271	70846690961574
375	60571525213700472737	17264762260523741742	17264762260523741662	16467527466126031271	47
374	60571473423051606450	17254761742610021661	17254761742610021712	16452441112350513167	-25
752	17216235667563752702	17116336276377507714	17116336276377510104	16317072207445256254	-121
875	17224546236230537337	17264762260523740311	17264762260523740372	16467055731252045426	-50
249	60561525213700472740	60651441067046165616	60651441067046165722	61451510700221637311	-67
873	17224531341715205204	17246501544607140032	17246501544607140061	16447575504451505605	-24
248	60561510317365140605	60643130134065631522	60643130134065631465	61440630605001761765	30
755	17216304354726171325	17136340763507626205	17136340763507626142	16337703206731310516	34
626	17206220773250420546	17755612417266034133	17755631464666054174	17155121427116536533	-1035825324066
754	17216267460412637171	17134647643712146255	17134647643712146212	16336362244623575020	34
751	17216220773250420547	61350721416234674511	61350756263547147272	62153777777777777765	-1980704052592
1001	17226220773250420547	61340721416234674511	61340756263547147272	62143777777777777707	-1980704052592
876	17224554574376314415	17744563100760570232	17745062510177376643	17140701366755672332	-1316075674343 3

6. Summary

We have defined the "accuracy" of a single precision function relative to a corresponding double precision function. Using this definition, we have measured the error in 11 Fortran functions (SIN, COS, TAN, ASIN, ACOS, ATAN, SQRT, EXP, ALOG, ALOG10, and TANH) in the CDC Fortran libraries for three different computer operating systems--CDC 6600 SCOPE, CDC 6600 NOS, and CDC 7600. The error in each function was measured for several distributions of arguments. The observed errors were presented in the form of various tables and printer plots.

The accuracy of some functions varies significantly from one system to another. For some functions, the accuracy also varies drastically from one value of the argument to another.

We have not in any sense proved that the actual errors are as small as we have observed. Indeed, to be on the conservative side, the prudent reader may well assume that the actual errors are at least as large as we have observed.

The program which we used to measure the errors was carefully designed, based on previous experience, with machine-dependent characteristics being parameterized so that the program could easily be transferred to run on different computer systems; in fact, it has been run on at least five different systems. Of especial interest is the machine-independent method used to compute the error in terms of units in the least significant digit position. Also of interest are the several forms of argument distributions, including exponential forms for ranges covering many values of the exponent and incremental forms for detailed examination of a function in the neighborhood of a zero or pole.

In addition to measuring the accuracy of Fortran library functions, the program has been used in various other applications including, for example:

- Measuring the error in a spline approximation fitted to a known function
- Comparing the relative accuracy of alternative algorithms for evaluating a polynomial, especially near a zero
- Comparing unrounded versus rounded computations
- Showing the accumulation of error in a sequence of values computed by incrementation.

Persons interested in using the program should contact one of the authors.

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