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Quality-Assurance Study of the Special-Purpose Finite-Element Program — SPECTROM: I. Thermal, Thermoelastic, and Viscoelastic Problems

Technical Report

December 1980

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This report was prepared by RE/SPEC Inc. under Subcontract E512-02300 with Battelle Project Management Division, Office of Nuclear Waste Isolation under Contract No. DE-AC06-76-RLO1830-ONWI with the U.S. Department of Energy. This contract was administered by the Battelle Office of Nuclear Waste Isolation.

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ABSTRACT

This comparison study involves a preliminary verification of finite element calculations. The methodology of the comparison study consists of solving four example problems with both the SPECTROM finite element program and the MARC-CDC general purpose finite element program. The results show close agreement for all example problems.

FOREWORD

This report was prepared under a subcontract with Battelle Memorial Institute, a DOE contractor. The subcontract was administered by the Office of Nuclear Waste Isolation (ONWI) and is part of the National Waste Terminal Storage (NWTS) Program. The principal objective of the NWTS Program is to provide facilities in various deep geologic formations at multiple locations in the United States which will safely dispose of commerical radioactive waste, which must be delivered to a Federal Repository for terminal storage. Some of the expected wastes produce both heat and radioactivity. This situation leads to many unique problems in rock mechancs. This report addresses a particular problem relative to the Rock Mechanics Program.

The overall objective of the ONWI Rock Mechanics Program is to predict the response of a rock mass hosting a waste repository during its construction and operation, as well as the post-operational phase. The operational phase is expected to be approximately 20 years while the post-operational phase will last until the repository no longer poses any potential hazard to mankind, a period that may last several thousand years. The Rock Mechanics Program is concerned with near field effects on mine stability as well as far field effects relative to the overall integrity of the geologic containment of waste.

To accomplish the objectives of the Rock Mechanics Program, numerical simulation, laboratory (including bench scale), and field studies are in progress. The laboratory and field studies provide input to the numerical simulations and also the opportunity for validation of the predictive capabilities of the computer codes. Ultimately, the computer codes will provide the predicted response of the host rock mass and thereby form an essential part of the overall Rock Mechanics Program.

This study involves the preliminary evaluation of the SPECTROM series of finite element programs. These finite element programs have been developed by RE/SPEC Inc. for the analyses of rock mechanic problems. Four relatively simple example problems were evaluated by both the SPECTROM and the MARC-CDC general purpose finite element programs.

The technical contents of this report have been reviewed by Dr. Arlo F. Fossum, Mr. Joe L. Ratigan, and Mr. Gary D. Callahan. The report was typed by Ms. Judy Hey.

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1. INTRODUCTION

This report compares computed results from the RE/SPEC Inc. finite element programs (SPECTROM-21 and 41) with a commercial finite element program (MARC-CDC General Purpose Finite Element Analysis Program). This comparison considers axisymmetric and two-dimensional geometries in the evaluation of four example problems. These four examples include the analyses of heat transfer, thermoelastic and viscoelastic problems. An exhaustive comparison of all features common to both programs was not intended; but rather a comparison of the basic logic and algorithms used to solve these four fundamental problems. Excluding one situation, eight-noded quadrilateral isoparametric elements were used in the comparison of numerical results from the two finite element programs.

The SPECTROM finite element program series was developed by RE/SPEC Inc. for geotechnical and rock mechanics applications. Specifically, SPECTROM-21 is a finite element program designed to analyze certain types of mechanical behavior including elasticity, viscoelasticity, and thermo/viscoelasticity. The modeling capabilities consist of two-dimensional geometries comprised of eight-noded isoparametric elements. Either plane stress or plane strain assumptions can be considered in the two-dimensional calculations. The initial stress state can be arbitrarily prescribed. Kinematic boundary conditions are allowed in either the global or skewed (inclined) coordinate directions. Multiple excavations within a model may be simulated. The SPECTROM-41 finite element program considers heat transfer analyses with emphasis on the conductive process. Again, two-dimensional geometry, modeled with eight-noded isoparametric elements, is used exclusively in this program. Steady-state or transient solutions are available and heat generation or source terms may be time dependent. The thermal properties may be constant, temperature dependent or anisotropic. The boundary conditions could consist of a combination of isothermal, adiabatic, convective, and/or applied heat flux. The SPECTROM series of specialized finite element programs have not been compared previously with any commercial finite element program. Documentation of the SPECTROM series has been initiated.

The MARC-CDC general purpose finite element program was developed by Dr. Pedro V. Marcel and his associates of the MARC Analysis Research

Corporation (1)*. This program is commercially available through the Control Data Corporation's (CDC) Cybernet Centers. The MARC-CDC program provides elastic, elastic-plastic, creep, large displacement, buckling and heat transfer analysis capabilities. The program derives its broad applicability by providing comprehensive libraries for elements, materials and structural procedures. The element library consists of 50 elements which describe a wide variety of two-and three-dimensional geometries. The material library consists of more than 35 material models which describe most engineering materials in the linear and nonlinear regimes. The 14 structural procedures allow the simulation of various physical phenomena such as temperature cycling, buckling and dynamic behavior. The combination of any number of components from these three libraries allow the solution of many different structural mechanics problems.

The purpose of this study was to provide a comparison of results obtained with RE/SPEC Inc. finite element programs with an accepted commercial finite element program (MARC-CDC). This study enhances the credibility of the RE/SPEC finite element programs by providing a check on the computational procedures used in the programs. Some capabilities of the SPECTROM-21 and 41 programs were not addressed in this study. A comprehensive comparison of all the features associated with these two programs would require a much larger scope of work. The intent of this study was to conduct a preliminary comparison of some simplistic problems. A more comprehensive comparison of the capabilities of the SPECTROM series of finite element programs would be encouraged following the satisfactory comparison of example problems presented herein. Also, a cost comparison between MARC-CDC and SPECTROM was conducted to evaluate the cost effectiveness of the RE/SPEC Inc. programs.

The four example problems represent relatively simple problems to solve with numerical methods. Despite the simplicity involved with the numerical solution, closed form analytical creep solutions are difficult because the state of stress is not constant. Consequently, presentations of the analytical solutions were not considered in the verification of the numerical

*Numbers in parenthesis indicate references at the end of the text.

solutions. The absence of the analytical solutions requires confidence in the accuracy of the solutions obtained from the MARC-CDC finite element program. The MARC-CDC program appears reliable based on verification studies considering a wide assortment of problems (1) and general acceptance by the technical community.

The comparisons between the MARC-CDC results and the SPECTROM results were excellent for the four example problems. The special purpose programs (SPECTROM) offer a significant cost advantage for the example problems considered. The following sections describe each example problem and compare the results obtained.

2. METHOD OF APPROACH

The comparison of results from two finite element programs (MARC-CDC and SPECTROM) was conducted by solving four example problems. The characteristics of the example problems are given in Table 2.1.1. These four example problems provide a cross-section of program usage which will assist the evaluation of the SPECTROM series of finite element programs.

TABLE 2.1.1.

CHARACTERISTICS OF EXAMPLE PROBLEMS

EXAMPLE PROBLEM	TYPE OF COMPARISON	GEOMETRY
A	HEAT TRANSFER	AXISYMMETRIC
	THERMOELASTIC	
B	VISCOELASTIC	PLANE STRAIN
C	VISCOELASTIC	PLANE STRAIN
D	VISCOELASTIC	PLANE STRESS

3. DISCUSSION OF EXAMPLE PROBLEMS

Discussion of the specific example problems is presented in the following sections. The finite element results from both the MARC-CDC and SPECTROM programs are illustrated and explanations are given for the few numerical comparisons in which minor deviations exist.

3.1. Example Problem A

3.1.1. Problem Description

Example problem A, consisting of a thin-walled cylinder as shown in Figure 3.1.1 (a), was used to compare heat transfer and thermoelastic results of the MARC-CDC and SPECTROM programs. This particular example problem has been previously solved in the MARC-CDC program manual (1). The thermal and mechanical representations of this axisymmetric problem are given in Figure 3.1.1 (b). The finite element model used in the numerical analysis of example problem A is shown in Figure 3.1.2. The finite element model consisted of 6 elements and 33 nodal points.

3.1.2. Material Properties

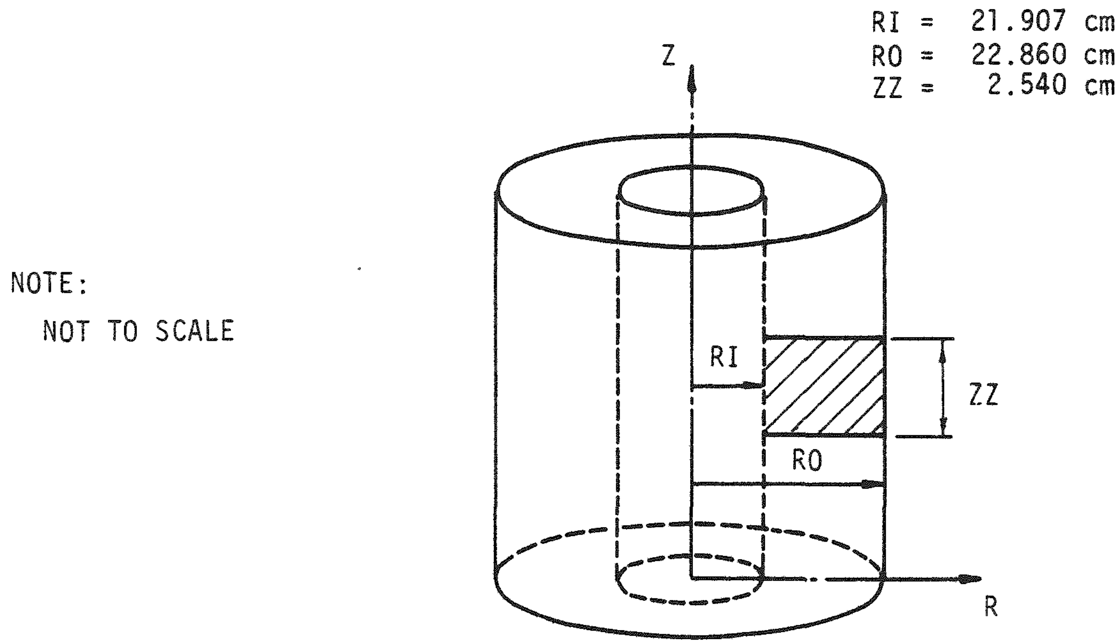
The thermomechanical properties used in example problem A, shown in Table 3.1.1 , were assumed to be independent of temperature.

3.1.3. Heat Transfer Analysis

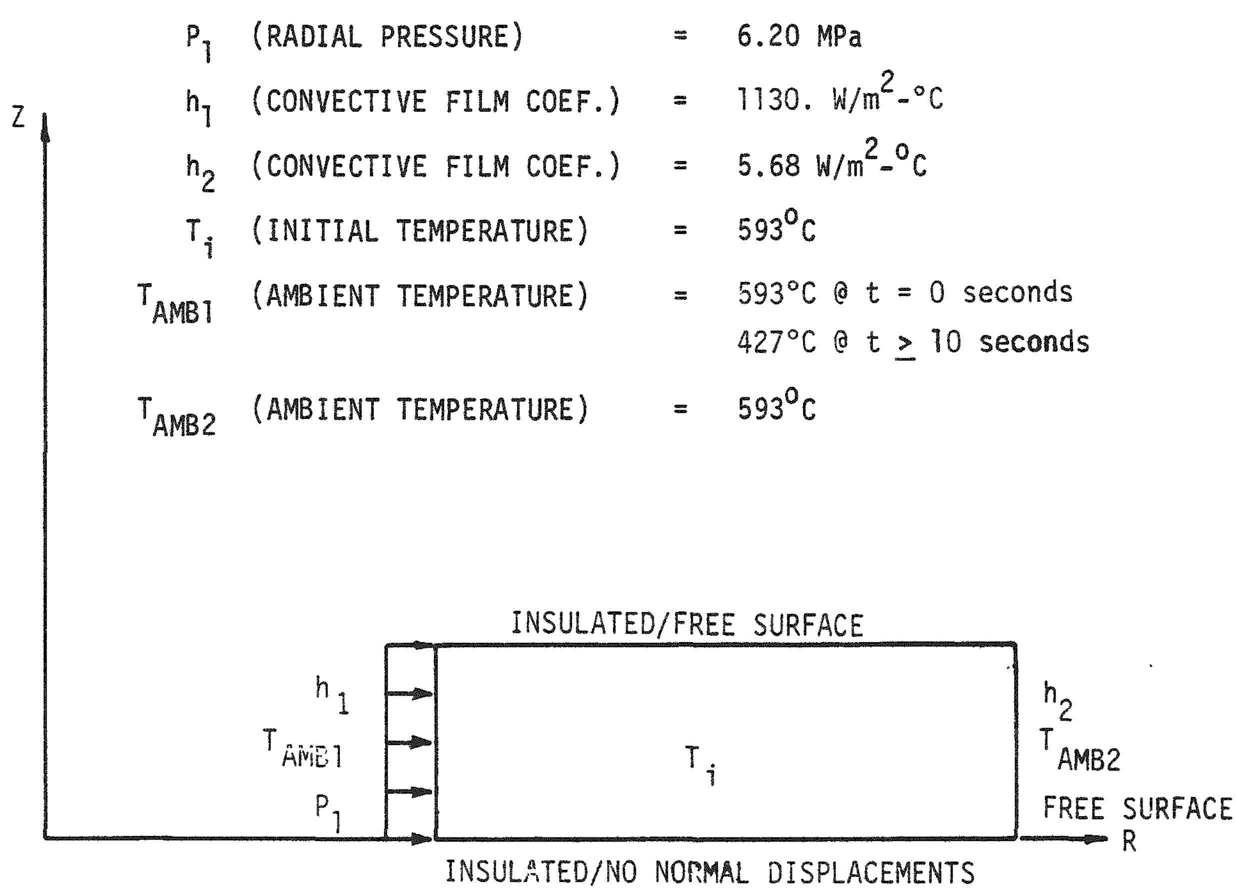
The initial temperature was assumed to be 593°C. The outer ambient temperature was held constant at 593°C while the inner ambient temperature decreased from 593°C to 427°C in 10 seconds and remained constant thereafter. A graphical description of the inner and outer ambient temperature history is given in Figure 3.1.3. A uniform film coefficient for the outside surface was specified as 5.68 W/m^2-°C. The inner surface had a film coefficient of 1130.0 W/m^2-°C to simulate forced convection.

3.1.4. Heat Transfer Results

The transient linear thermal analysis resulted in the temperature distributions along the inner and outer wall as illustrated in Figure 3.1.4. The tabulated thermal results are presented in Table 3.1.2. The temperatures predicted by the two finite element programs appear to converge to a steady state solution. Close agreement throughout the transient temperature regime is apparent.



(a.) Schematic Representation of Example Problem A.



(b.) Thermomechanical Representation of Example Problem A.

Figure 3.1.1. Description of Example Problem A.

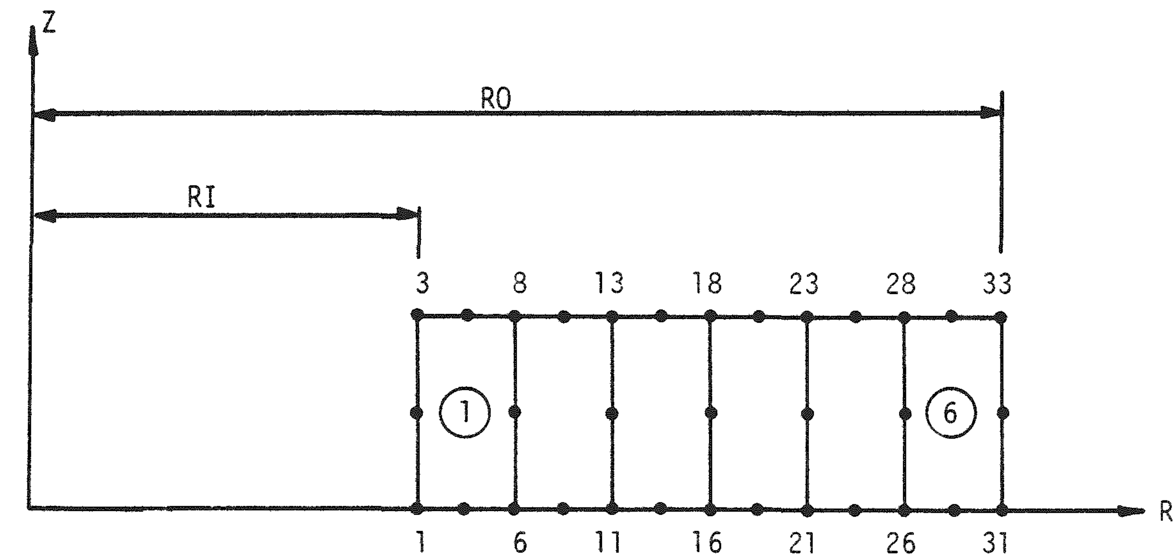


Figure 3.1.2. Axisymmetric Finite Element Model of Example Problem A.

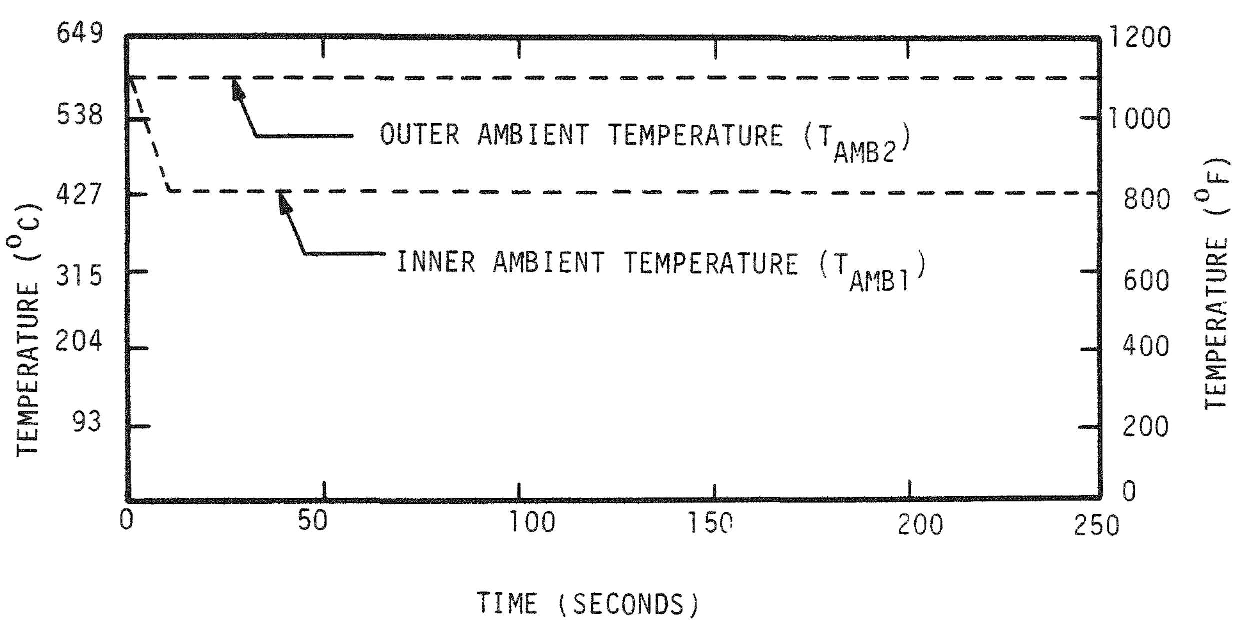


Figure 3.1.3. Time History of Ambient Temperature Conditions for Example Problem A.

TABLE 3.1.1.

THERMOMECHANICAL PROPERTIES FOR EXAMPLE PROBLEM A

PROPERTY	UNITS	VALUE
SPECIFIC HEAT (C_p)	$\frac{J}{kg\text{-}°C}$	486
DENSITY (ρ)	$\frac{kg}{m^3}$	7840
THERMAL CONDUCTIVITY (k)	$\frac{W}{m\text{-}°C}$	36.3
MODULUS OF ELASTICITY (E)	MPa	$1.503(10^5)$
POISSON'S RATIO (ν)	---	0.320
THERMAL EXPANSION (α)	$\frac{1}{°C}$	$2.232(10^{-5})$

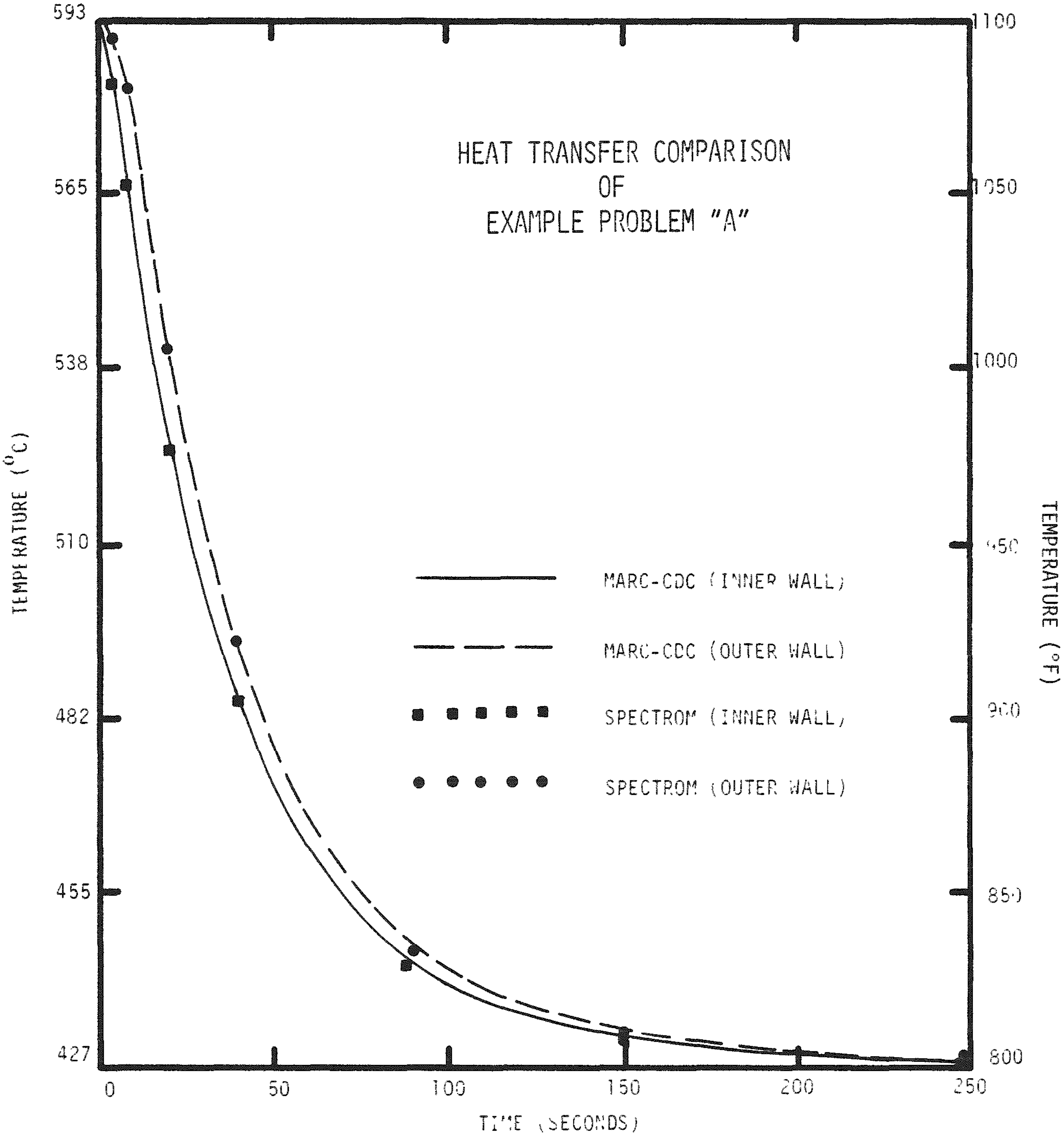


Figure 3.1.4. Comparison of Temperature Distributions for Example Problem A.

TABLE 3.1.2.

TEMPERATURES ASSOCIATED WITH FIGURE 3.1.4. (EXAMPLE PROBLEM A)

MARC-CDC			SPECTROM		
TIME (SECONDS)	INNER WALL °C	OUTER WALL °C	TIME (SECONDS)	INNER WALL °C	OUTER WALL °C
0	593.3	593.3	0.0	593.3	593.3
4.1	583.6	590.7	10.0	557.4	576.0
10.7	554.7	572.7	20.0	526.1	541.5
18.6	529.6	544.7	30.0	503.0	514.5
28.4	505.7	517.7	40.0	485.1	494.1
42.2	481.6	490.0	50.0	471.6	478.4
62.7	459.1	464.1	100.0	439.0	440.9
108.0	438.0	439.8	150.0	430.7	431.4
250.0	428.2	428.6	250.0	427.8	428.1

3.1.5. Thermoelastic Analysis

The thermoelastic analysis, using the boundary conditions illustrated in Figure 3.1.1 , considered both initial (elastic) and transient (thermoelastic) simulations. The statically determinate elastic analysis involved the mechanical loading only. The thermoelastic analysis employs the temperature fields generated from the transient thermal analysis discussed in the previous sections, thus time varying thermal stress fields are produced.

3.1.6. Thermoelastic Results

The thermoelastic stress results predict behavior near the inner and outer radius of the cylinder. Both effective stresses and stress components are used in the comparison. An expression defining effective stress in terms of the stress components for axisymmetric geometry is given in Equation 3-1.

$$\sigma_e = \left[0.5 \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \right]^{1/2} \quad (3\text{-}1)$$

where:

σ_e = Effective Stress

$\sigma_1 = \sigma_2 = \sigma_3$ = Principal Stresses

Figure 3.1.5 shows the time history of effective stress near the inner and outer wall of the cylinder from the results obtained from the MARC-CDC and SPECTROM programs. The exact location of the comparison is at a common integration point near the periphery of the inner and outer wall. Graphically, the agreement between the calculated results from the two programs appears to be excellent. Table 3.1.3 lists the effective stresses at specific times. These tabulated results complement the graphical results shown in Figure 3.1.5. Since many of the times shown in Table 3.1.3 do not agree between the MARC-CDC and SPECTROM programs, the close agreement is not readily observed by inspection of the tabulated results. In addition to the comparison of the effective stress, two of the stress components (radial and tangential) have been tabulated and are given in Tables 3.1.4 and 3.1.5. The third stress component (vertical) was omitted from the tabulated comparison because it was less than one percent of the

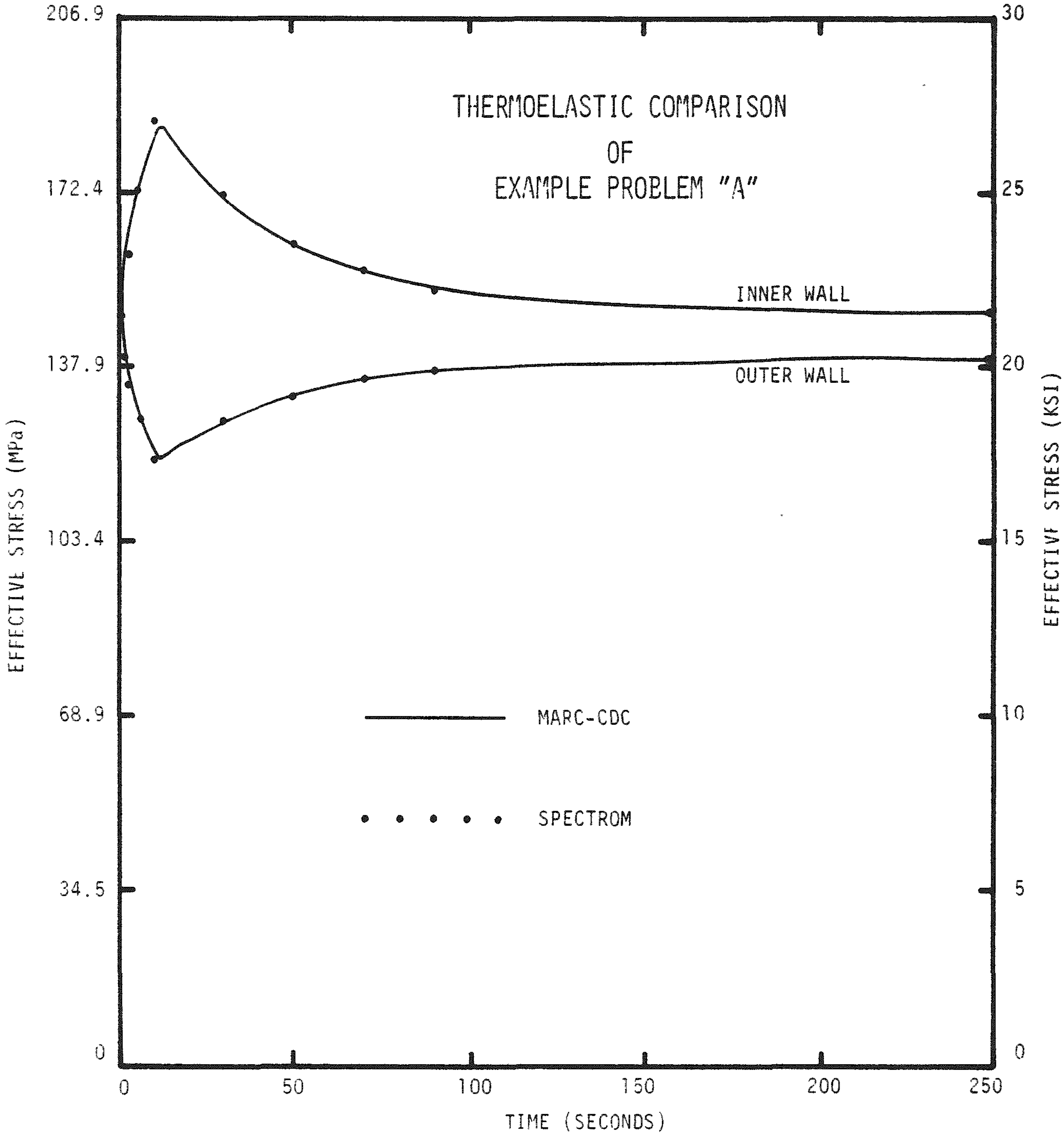


Figure 3.1.5. Comparison of Effective Stresses From the Thermoelastic Analysis of Example Problem A.

TABLE 3.1.3.

EFFECTIVE STRESSES ASSOCIATED WITH FIGURE 3.1.5. (EXAMPLE PROBLEM A)

MARC-CDC			SPECTROM		
TIME (SECONDS)	INNER WALL (MPa)	OUTER WALL (MPa)	TIME (SECONDS)	INNER WALL (MPa)	OUTER WALL (MPa)
0.0	148.8	140.0	0.0	148.7	140.0
7.4	175.9	125.8	3.0	160.0	134.4
12.3	185.0	120.0	6.0	171.9	127.7
18.6	180.0	122.5	10.0	186.4	119.6
25.8	174.4	125.6	30.0	166.1	127.1
46.2	163.8	131.5	50.0	162.4	132.2
62.6	158.7	134.3	90.0	153.6	137.1
92.8	153.7	137.1	150.0	150.1	139.1
250.0	149.3	139.5	250.0	149.2	139.6

TABLE 3.1.4.

RADIAL STRESSES (EXAMPLE PROBLEM A)

MARC-CDC			SPECTROM		
TIME (SECONDS)	INNER WALL (MPa)	OUTER WALL (MPa)	TIME (SECONDS)	INNER WALL (MPa)	OUTER WALL (MPa)
0.0	5.97	0.21	0.0	5.97	0.21
7.4	5.81	0.27	3.0	5.90	0.23
12.3	5.80	0.29	6.0	5.85	0.26
18.6	5.83	0.28	10.0	5.78	0.30
25.8	5.85	0.27	30.0	5.86	0.26
46.2	5.90	0.24	50.0	5.91	0.24
62.6	5.93	0.23	90.0	5.95	0.22
92.8	5.95	0.22	150.0	5.97	0.21
250.0	5.97	0.20	250.0	5.97	0.20

Note: All Radial Stresses are Compressive

TABLE 3.1.5.

TANGENTIAL STRESSES (EXAMPLE PROBLEM A)

MARC-CDC			SPECTROM		
TIME (SECONDS)	INNER WALL (MPa)	OUTER WALL (MPa)	TIME (SECONDS)	INNER WALL (MPa)	OUTER WALL (MPa)
0.0	145.7	139.9	0.0	145.7	139.9
7.4	173.1	126.1	3.0	157.1	134.5
12.3	182.2	120.5	6.0	169.1	128.0
18.6	177.1	123.0	10.0	183.7	120.2
25.8	171.6	125.9	30.0	168.9	127.4
46.2	160.8	131.6	50.0	159.5	132.4
62.6	155.7	134.4	90.0	150.7	137.1
92.8	150.7	137.1	150.0	147.0	139.0
250.0	146.3	139.4	250.0	146.2	139.5

Note: All Tangential Stresses are Tensile tangential stress component. The tabulated results indicate only a slight difference between the results obtained from the MARC-CDC and SPECTROM programs. Once again, these tabulated results are from common integration points near the periphery of the inner and outer wall.

3.2. Example Problem B

3.2.1. Problem Description

Example Problem B involved an unconfined vertically loaded rectangular solid in plane strain. The viscoelastic behavior was predicted for a period of 24 hours. This brief time period should provide a sufficient length of time for comparison of creep results. The initial time step was 24 minutes for this particular problem. Therefore, several time steps are calculated in the 24 hour period. Although the example problem appears to be relatively straightforward, an analytical solution is difficult. The creep strains are quite small in early time, thus the stress state would remain nearly constant. Assuming the stress state remains constant through time simplifies the analytical solution. Since this assumption would produce approximate results, the analytical solution is not presented. The boundary conditions and the finite element discretization are shown in Figure 3.2.1. The finite element model consisted of 6 eight-noded isoparametric quadrilateral elements and 33 nodal points.

Example Problem B considered the transient deformation of a viscoelastic solid. An empirical power law formulation was used to represent the creep behavior of the material. The MARC-CDC program allows the user to provide his own constitutive law in terms of specific variables supplied through a set argument list for the user supplied subroutine. The arguments available for constitutive law representation in the MARC-CDC user supplied subroutine are:

$$\varepsilon_e^c = \sqrt{\frac{2}{3}\,\varepsilon_{ij}^c\,\varepsilon_{ij}^c} = \text{Equivalent Creep Strain}$$

$$\sigma_e = \sqrt{\frac{3}{2}\,S_{ij}\,S_{ij}} = \sqrt{3J_2} = \text{Equivalent (Effective) Stress}$$

$$\Delta t = \text{Time Increment}$$

$$t = \text{Total Time}$$

where:

$$\varepsilon_{ij}^c \quad (i,j = 1,2,3) = \text{Creep Strain}$$

$$S_{ij} \quad (i,j = 1,2,3) = \text{Deviatoric Stress}$$

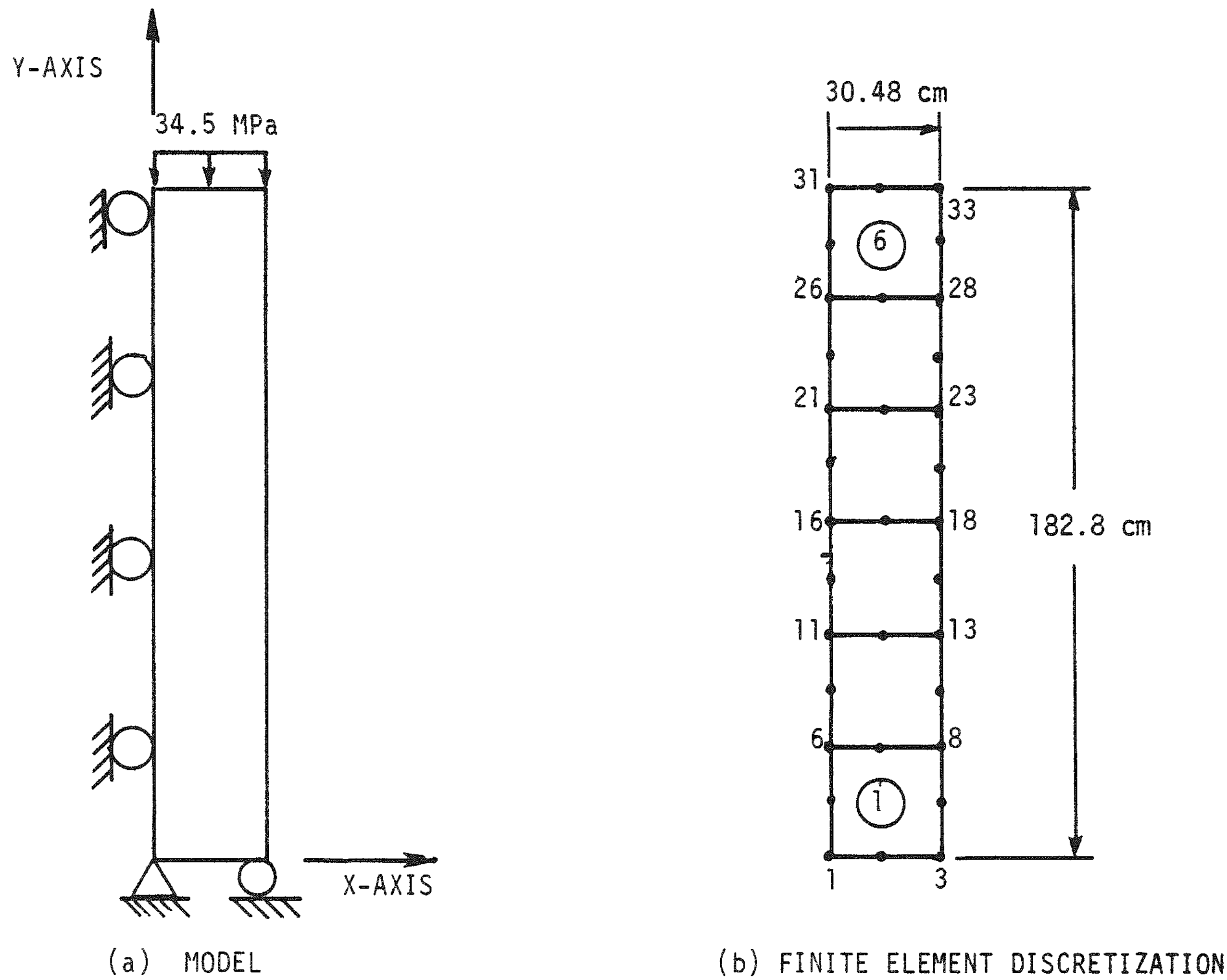


Figure 3.2.1. Two-Dimensional Model and Finite Element Discretization for Example Problem B.

J_2 = Second Invariant of Deviator Stress

The creep form used in MARC-CDC was

$$\varepsilon_e^c = A\, t^m\, \sigma_e^n \qquad (3\text{-}2)$$

where the values of the constants A, m, and n were taken to be

$$A = 2.487 \times 10^{-7}$$

$$m = 0.48$$

$$n = 2.42$$

The form of the creep law in SPECTROM-21 is

$$\varepsilon_{ij}^c = A\, t^m\, \sigma_e^{n-1} \left(\frac{3}{2}\, S_{ij}\right) \qquad (3\text{-}3)$$

Equation (3-3) may be shown to be equivalent to equation (3-2). First rewrite equation (3-3) with

$$\lambda = 3\, A\, t^m\, \sigma_e^{n-1}/2 \qquad (3\text{-}4)$$

which gives

$$\varepsilon_{ij}^c = \lambda\, S_{ij} \qquad (3\text{-}5)$$

Premultiply and post multiply equation (3-5) by ε_{ij}^c and S_{ij}, respectively, giving

$$\varepsilon_{ij}^c\, \varepsilon_{ij}^c\, S_{ij} = \lambda\, \varepsilon_{ij}^c\, S_{ij}\, S_{ij} \qquad (3\text{-}6)$$

By definition of equivalent creep strain and equivalent stress, equation (3-6) becomes

$$\frac{3}{2}\, (\varepsilon_e^c)^2\, S_{ij} = \varepsilon_{ij}^c\, \lambda\, \frac{2}{3}\, \sigma_e^{\,2} \qquad (3\text{-}7)$$

Using equation (3-5) and rearranging equation (3-7), we have

$$\lambda = \frac{3\, \varepsilon_e^c}{2\sigma_e} \qquad (3\text{-}8)$$

Using equations (3-5) and (3-8), we have

$$\varepsilon_e^c = \varepsilon_{ij}^c \frac{2\sigma_e}{3S_{ij}} \tag{3-9}$$

Finally, substituting equation (3-9) into equation (3-3), one obtains equation (3-2) as required.

The method for determining the time increment varies between the MARC-CDC and SPECTROM programs. A numerical interpretation of the logic for predicting time increments by both MARC-CDC and SPECTROM is given in Figure 3.2.2. Basically, the MARC-CDC program (1) predicted time increments based on stress change per stress ($\frac{\Delta\sigma}{\sigma}$) and creep strain change per elastic strain ($\frac{\Delta\varepsilon^c}{\varepsilon^e}$). The greater ratio of these stress and strain relations dictates the magnitude of the next time step. If both the stress and strain ratios are below 80 percent of predefined tolerances, the next time step will be 1.25 of the current time increment. Similarly, if both stress and strain increments are less than 65 percent of their respective tolerances, the time step multiplier becomes 1.5. Because of the relatively small stress and strain changes in this example problem, the 1.5 time step multiplier was always chosen by MARC-CDC. In contrast, the SPECTROM program predicts time increments by using a predetermined initial time step which is increased or accelerated by an increment parameter (2) chosen a priori and dependent on the desired accuracy. Previous studies (including ref. 2) have shown values between 0.1 and 0.2 lead to good and relatively inexpensive results. To maintain conformance, the time stepping procedure in the SPECTROM program was modified to use the 1.5 time step multiplier. This procedure coincides with the MARC-CDC program for most of the example problems involving creep behavior. The original time stepping procedure used in the SPECTROM program was not used in the comparison. The implementation of this procedure would have made the comparison more difficult since the logic associated with the MARC-CDC time-stepping procedure cannot be adjusted. If each program used a unique time stepping procedure, a variation in the results of the two programs could not be directly attributed to the respective solution techniques. Rather, speculation would have to be made as to the degree of error due to either time stepping or solution technique. Again, this preliminary study did not intend to consider all capabilities associated with the SPECTROM series of finite element programs.

<u>MARC-CDC Time Increment Logic</u>

If: $\quad \text{Max} \left| \frac{\Delta\varepsilon^c}{\varepsilon^e} \right| < 0.80\varepsilon_{TOL} > 0.65\ \varepsilon_{TOL}$

AND

$\text{Max} \left| \frac{\Delta\sigma}{\sigma} \right| < 0.80\sigma_{TOL} > 0.65\ \sigma_{TOL}$

Then: $\quad \Delta t_n = 1.25\ \Delta t_{n-1}$

If: $\quad \text{Max} \left| \frac{\Delta\sigma^c}{\varepsilon^e} \right| < 0.65\ \varepsilon_{TOL}$

AND

$\text{Max} \left| \frac{\Delta\sigma}{\sigma} \right| < 0.65\ \sigma_{TOL}$

Then: $\quad \Delta t_n = 1.50\ \Delta t_{n-1}$

Where: $\quad \Delta\varepsilon^c$ = Creep Strain Increment

ε^e = Elastic Strain

ε_{TOL} = Specified Strain Tolerance

$\Delta\sigma$ = Stress Increment

σ = Stress

σ_{TOL} = Specified Stress Tolerance

n = Current Time Interval

Δt = Time Increment

<u>SPECTROM Time Increment Logic</u>

$$\Delta t_n = K \frac{||\varepsilon||}{||\dot{\varepsilon}^c||}$$

where minimum Δt_n of all integration points is chosen.

Where:

K = Constant (Usually less than 0.2)

$||\varepsilon|| = (\varepsilon_1{}^2 + \varepsilon_2{}^2 + \varepsilon_3{}^2)^{1/2}$ = Norm of Total Strain

$||\dot{\varepsilon}^c|| = ((\varepsilon_1^c)^2 + (\dot{\varepsilon}_2^c)^2 + (\dot{\varepsilon}_2^c)^2\ )^{1/2}$ = Norm of Incremental Creep Strain

Figure 3.2.2. Time Increment Logic.

3.2.2. Material Properties

The material properties used in example problem B, are shown in Table 3.2.1. These material properties consist of the modulus of elasticity, Poisson's ratio and the creep law expression. These material properties are representative of salt.

TABLE 3.2.1.

MATERIAL PROPERTIES FOR EXAMPLE PROBLEMS B,C, AND D

PROPERTIES	EXAMPLE PROBLEM B,C,D - PART 1	EXAMPLE PROBLEM D - PART 2
Modulus of Elasticity	3400	690
Poisson's Ratio	0.40	0.25
Creep Law (Eq.3-2)	$\varepsilon_e^c = 2.487\text{x}10^{-7}\ t^{0.48}\sigma_e^{2.42}$	$\varepsilon_e^c = 2.487\text{x}10^{-6}\ t^{0.48}\sigma_e^{2.66}$

3.2.3. Viscoelastic Results

The vertical displacements along the upper surface of the model were used to compare the viscoelastic behavior. Because of the specified boundary conditions, the time history of the stress state was not compared because of the relatively slight change in the out-of-plane stress component for the time period considered. Figure 3.2.3 and Table 3.2.2 compare the results between the MARC-CDC and SPECTROM programs. The agreement is essentially identical for both the elastic and viscoelastic solutions. Since the numerical creep solution procedures are similar for the two programs, the results should compare for later times as well.

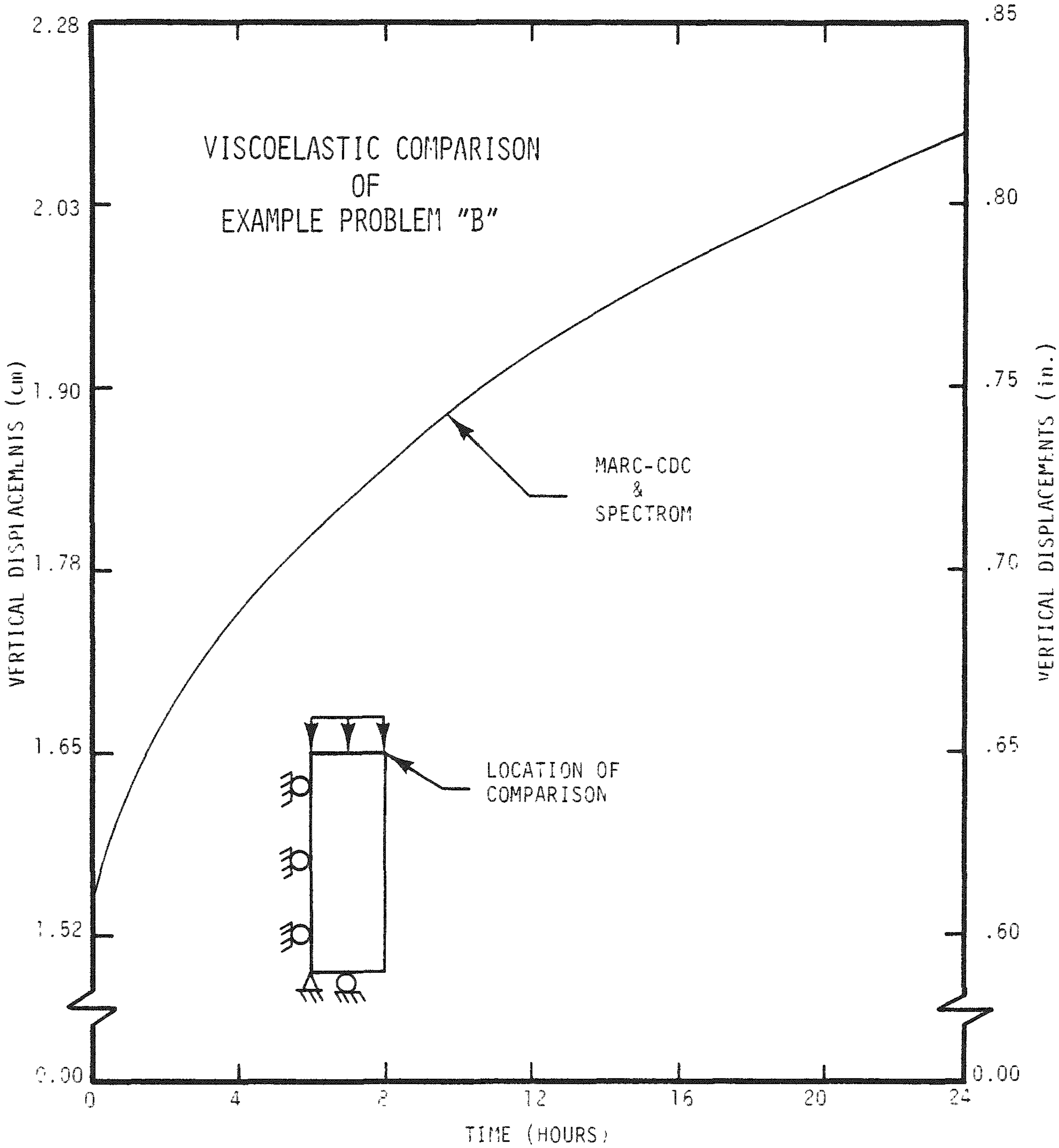


Figure 3.2.3. Comparison of Vertical Displacements Resulting from the Viscoelastic Analysis of Example Problem B.

TABLE 3.2.2.

VERTICAL DISPLACEMENTS ASSOCIATED WITH FIGURE 3.2.3
(EXAMPLE PROBLEM B)

MARC-CDC		SPECTROM	
TIME (HOURS)	VERTICAL DISPLACEMENTS cm	TIME (HOURS)	VERTICAL DISPLACEMENTS cm
0.0	1.537	0.0	1.537
0.4	1.582	0.4	1.582
1.0	1.627	1.0	1.627
1.9	1.671	1.9	1.671
3.3	1.724	3.3	1.724
5.3	1.786	5.3	1.786
8.3	1.857	8.3	1.857
12.9	1.943	12.9	1.943
19.7	2.047	19.7	2.047

3.3. Example Problem C

3.3.1. Problem Description

Example problem C is identical to example problem B except for the kinematic boundary condition as shown in Figure 3.3.1 (a). This restriction provides confinement of the model along the horizontal plane. The initial elastic stress in the x direction is given as $\sigma_x = (1-\nu)\sigma_y$. Subsequently through time, σ_x will approach σ_y according to the creep law. The solid is said to be creeping toward a lithostatic (hydrostatic) state of stress. An example of a linear Maxwell model under these types of conditions may be found in Jaeger and Cook (3).

3.3.2. Material Properties

The material properties used in example problem C are identical to the material properties used in example problem B and can be found in Table 3.2.1.

3.3.3. Viscoelastic Results

A comparison of effective stress (Equation 3-1) and stress components was used to evaluate the agreement obtained from the MARC-CDC and SPECTROM programs. The boundary conditions used in this example create a homogeneous state of stress, thus, the results presented represent any location in the model. Figure 3.3.2 illustrates the effective stresses as a function of time. The agreement between the solutions of the two programs appears to be excellent. Table 3.3.1 provides a list of the stress components and effective stress at specific times. An examination of these stresses indicates exact agreement between the MARC-CDC and SPECTROM programs.

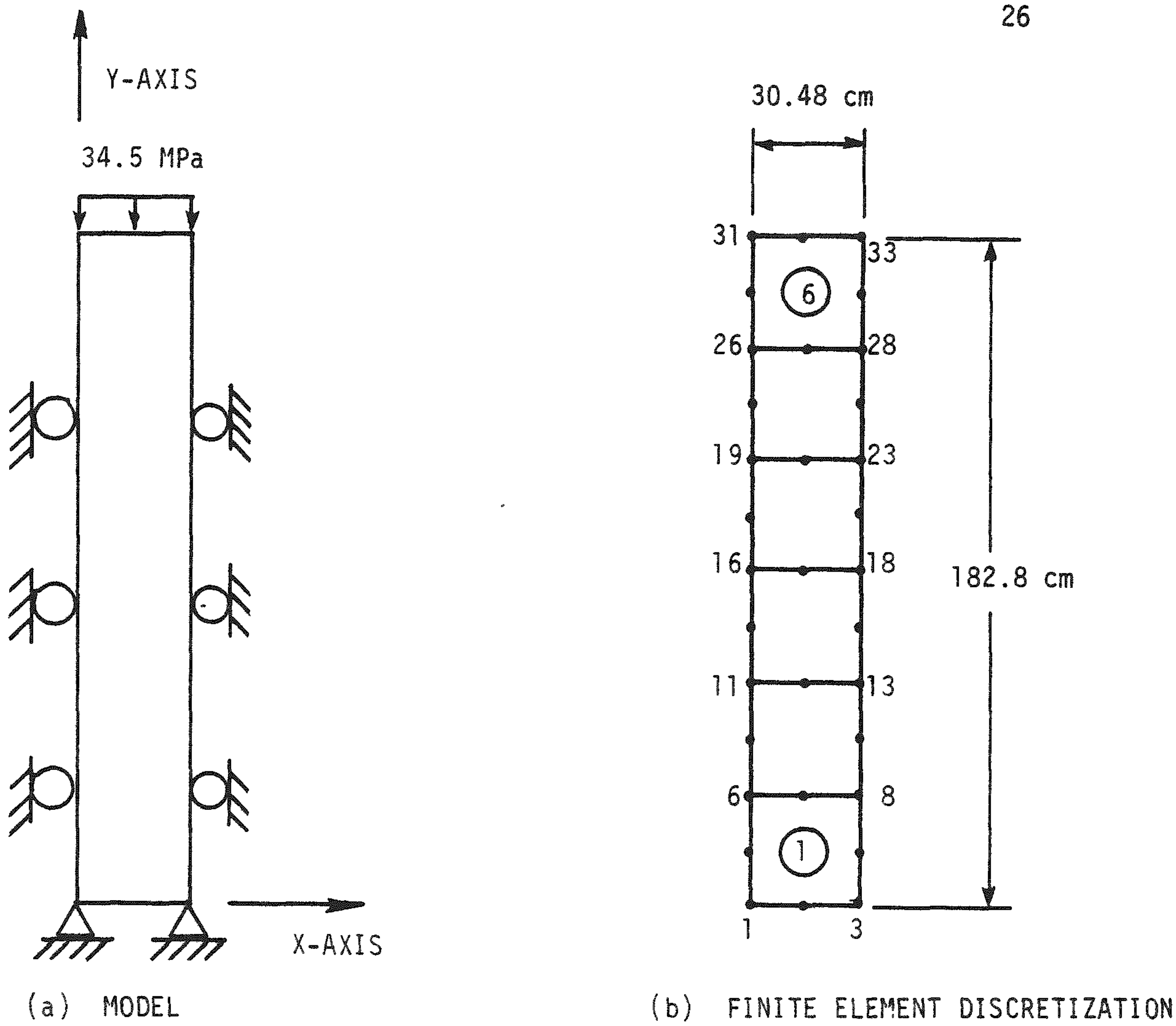


Figure 3.3.1. Two-Dimensional Model and Finite Element Discretization of Example Problem C.

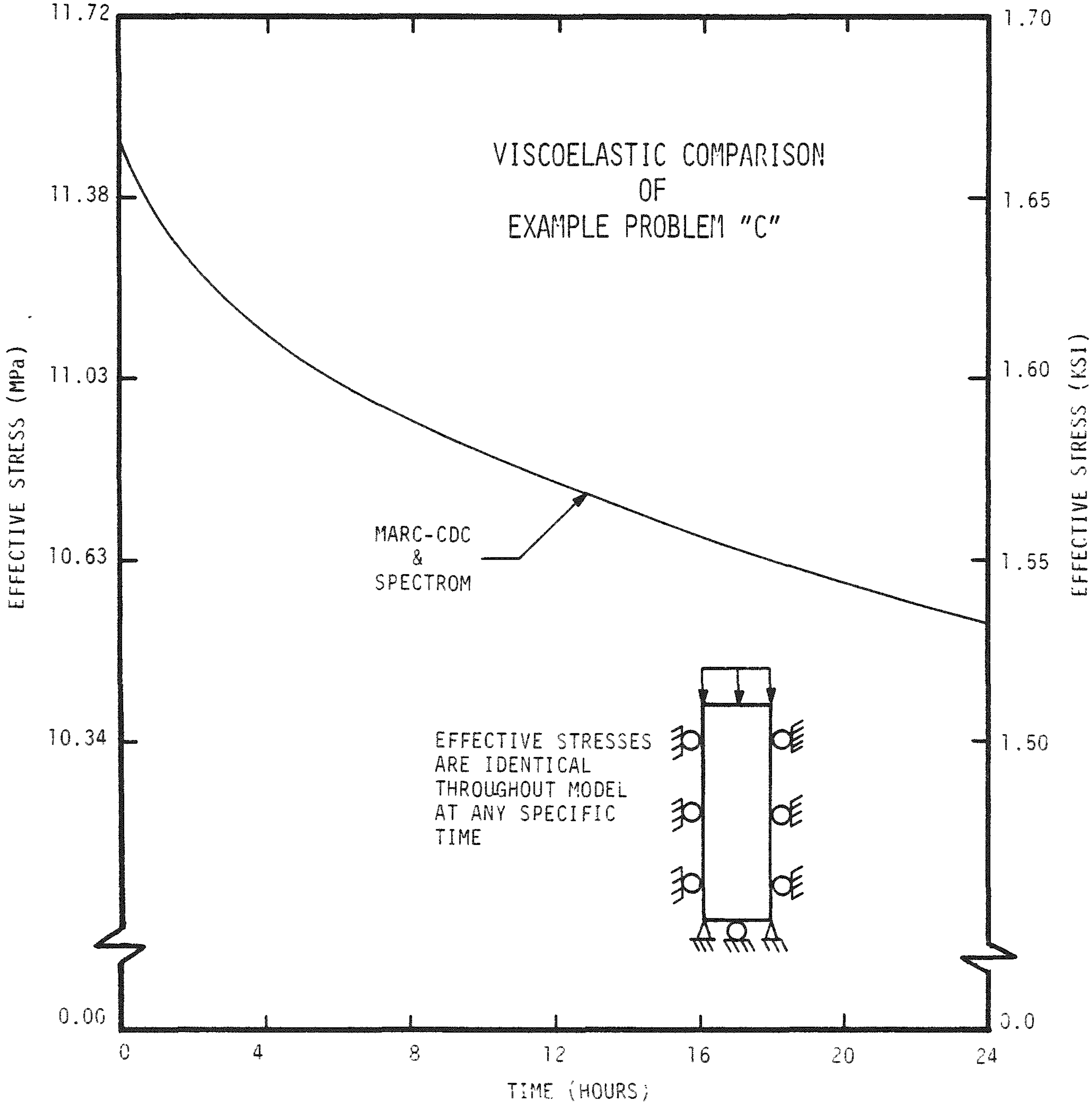


Figure 3.3.2. Comparison of Effective Stresses from the Viscoelastic Analysis of Example Problem C.

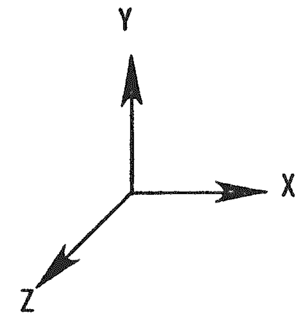


TABLE 3.3.1.

STRESSES-MPa (EXAMPLE PROBLEM C)

TIME (HOURS)	*X-STRESS		*Y-STRESS		*Z-STRESS		EFFECTIVE STRESS	
	MARC-CDC	SPECTROM	MARC-CDC	SPECTROM	MARC-CDC	SPECTROM	MARC-CDC	SPECTROM
0.0	22.99	22.99	34.48	34.48	22.98	22.98	11.49	11.49
1.0	23.14	23.14	34.48	34.48	23.14	23.14	11.34	11.34
5.3	23.41	23.41	34.48	34.48	23.41	23.41	11.07	11.07
8.3	23.53	23.53	34.48	34.48	23.53	23.53	10.95	10.95
12.9	23.66	23.66	34.48	34.48	23.66	23.66	10.82	10.82
19.7	23.82	23.82	34.48	34.48	23.82	23.82	10.66	10.66

*Compressive Stress

3.4. Example Problem D

3.4.1. Problem Description

Example problem D involved a uniaxially loaded square plate with a hole located at the center. A viscoelastic analysis was performed with a creep power law formulation (Eq. 3-2) describing the viscous behavior of the material. The viscoelastic analysis of this problem extended for a 24 hour period. Figure 3.4.1 shows the boundary conditions and finite element discretization. The finite element model consisted of 12 quadrilateral elements (eight-noded and isoparametric) and 51 nodal points. Because of symmetry, only a quarter section of the plate was modeled. This problem assumes plane stress whereas the previous example problems assumed plane strain.

Two sets of different material properties and creep parameters were used in this example problem to compare the solution techniques over a range of material properties.

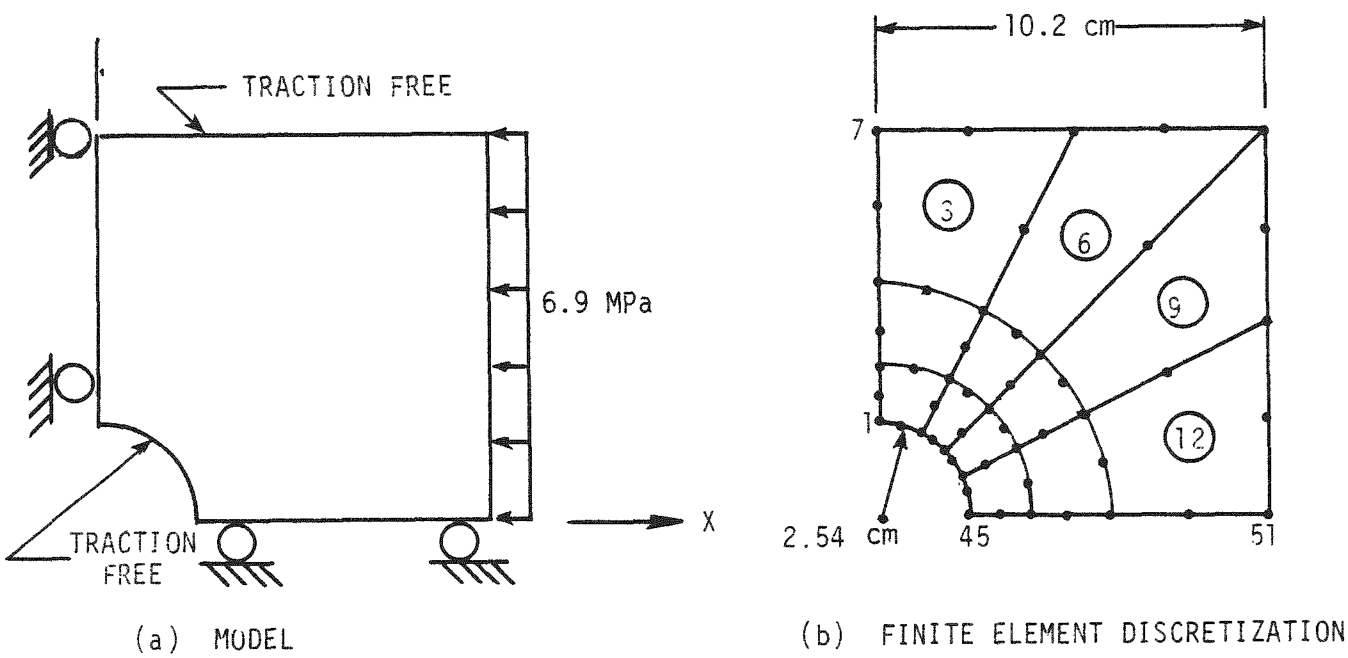


Figure 3.4.1. Two-Dimensional Model and Finite Element Discretization for Example Problem D.

3.4.2. Material Properties

The material properties used in example problem D (Part 1) are identical to the material properties used in example problem B and C. As was stated, the material properties used in example problem D (Part 2) have purposely been adjusted from those in either example problem B, C, or D (Part 1). The two sets of material properties for example problem D are listed in Table 3.2.1.

3.4.3. Viscoelastic Results

The results of example problem D for the MARC-CDC and SPECTROM programs are compared using the displacement results at a point along the periphery of the hole. The time history of the displacements at a specific location was used in the comparison of the results from example problem D. Figure 3.4.2 and Table 3.4.1 illustrates excellent agreement between the MARC-CDC and SPECTROM programs for Part 1. The variation in magnitude of material properties and creep parameters created a slight disagreement in the first 12 hours of Part 2 as shown in Figure 3.4.3. An explanation of this disagreement can be attributed to the time step adjustments made by MARC-CDC because of the increased amount of creep resulting from the changes in material properties. The time step adjustment in the SPECTROM program (section 3.2.1) did not allow for this difference in time step multiplier. Consequently, different time step increments for the two programs were used in early time. Table 3.4.2 shows the vertical displacements at different times for both the MARC-CDC and SPECTROM analyses of example problem D (Part 2). The difference in time increments between MARC-CDC and SPECTROM programs results in a discrepancy of predicted creep strain increments because each time step increment is calculated from a state of stress assumed to remain constant during the time step increment when the stress actually varies (2). It should be noted that the variation is slight and in later time the discrepancy decreases.

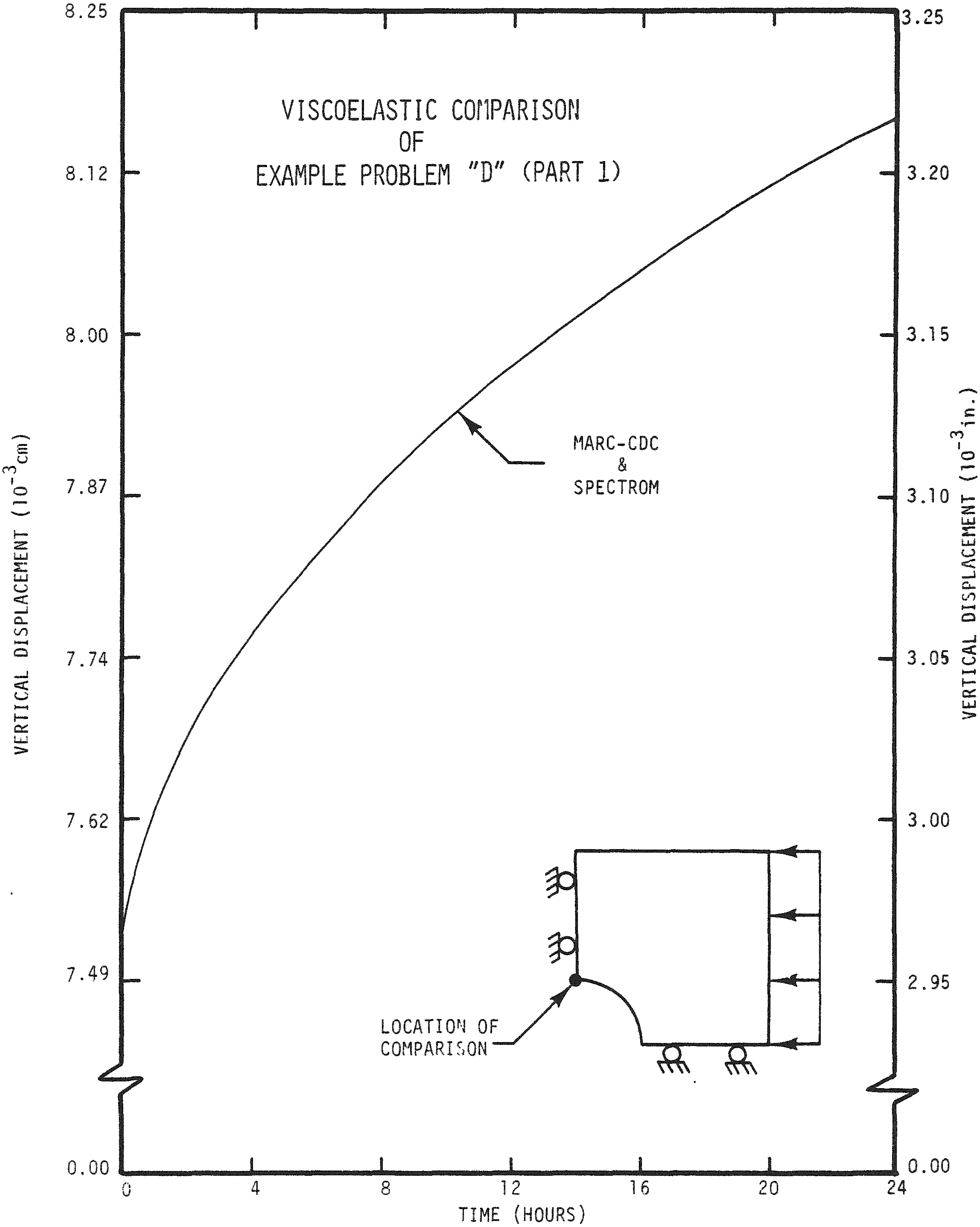


Figure 3.4.2. Comparison of Radial Displacements Along Periphery of Hole Resulting from Viscoelastic Analysis of Example Problem D (Part 1).

TABLE 3.4.1.

VERTICAL DISPLACEMENTS ASSOCIATED WITH FIGURE 3.4.2
(EXAMPLE PROBLEM D - PART 1)

MARC-CDC TIME (HOURS)	MARC-CDC VERTICAL DISPLACEMENTS cm (10^{-3})	SPECTROM TIME (HOURS)	SPECTROM VERTICAL DISPLACEMENTS cm (10^{-3})
0.0	7.521	0.0	7.521
0.4	7.574	0.4	7.574
1.0	7.625	1.0	7.625
1.9	7.678	1.9	7.678
3.3	7.739	3.3	7.739
5.3	7.810	5.3	7.810
8.3	7.894	8.3	7.894
12.9	7.993	12.9	7.993
19.7	8.113	19.7	8.113

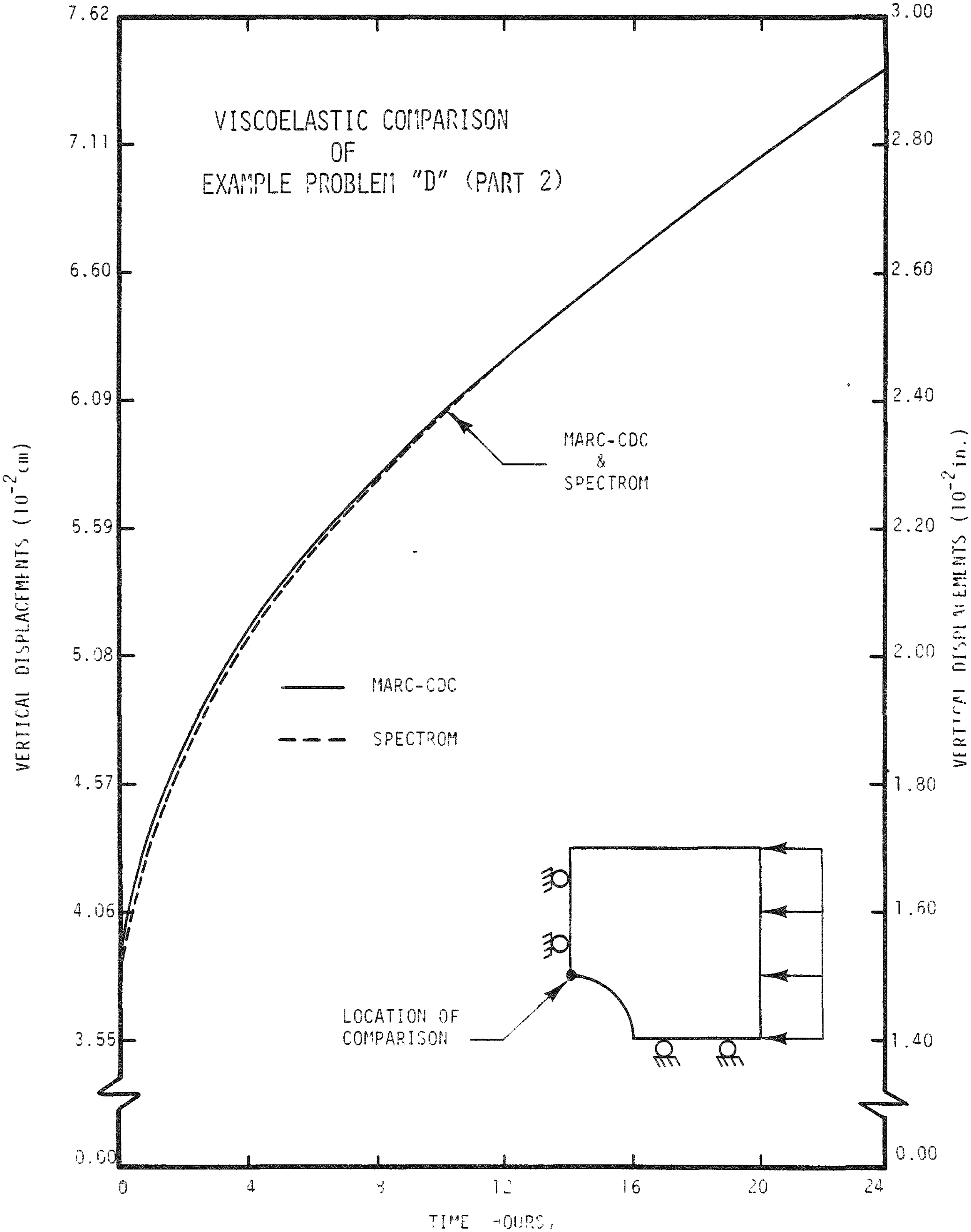


Figure 3.4.3. Comparison of Radial Displacements Along Periphery of Hole Resulting From Viscoelastic Analysis of Example Problem D (Part 2).

TABLE 3.4.2.

VERTICAL DISPLACEMENTS ASSOCIATED WITH FIGURE 3.4.3
(EXAMPLE PROBLEM D - PART 2)

MARC-CDC		SPECTROM	
TIME (HOURS)	VERTICAL DISPLACEMENTS cm (10^{-2})	TIME (HOURS)	VERTICAL DISPLACEMENTS cm (10^{-2})
0.0	3.764	0.0	3.764
0.9	4.320	0.4	4.072
2.8	4.892	1.0	4.356
7.0	5.659	1.9	4.653
16.5	6.736	3.3	4.986
30.0	7.808	5.3	5.372
		8.3	5.827
		12.9	6.363
		19.7	7.003

4. COST COMPARISON

A cost comparison supplements the numerical comparison of MARC-CDC and SPECTROM finite element programs. As expected, the computer processing costs of a general purpose finite element program would be higher than a more specialized finite element program. The general purpose program would include many additional solution techniques not required for a specific individual problem. Table 4.1.1 was developed from the costs associated with the four example problems. The results of the cost comparison are shown with both System Billing Units (SBU) and computer execution time. SBU's represent the CDC Cybernet Centers method of billing. Although the cost/SBU varies with the priority of the program execution, a $1.00/SBU would be a reasonable approximation for this cost comparison. The computer execution time provides another indication of the cost effectiveness of the two programs.

The results of the cost comparison indicate a substantial difference in computer costs between the two finite element programs. These costs are representative of the CDC-Cybernet 7600 Series computer processing system. The results of the cost comparison would vary depending on the computer processing system used in the numerical calculations. But, the relative cost differenece is not anticipated to change signifcantly between the two finite element programs. The relation between the total computer costs and computer execution time is not straightforward. The total computer costs are largely dependent on the amount of storage or core required from the computer processing system. These simple example problems require a limited amount of computer core in comparison to problems associated with the analyses of radioactive waste disposal. Consequently, determining computer costs of large scale rock mechanics problems should not be extrapolated from the costs presented in Table 4.1.1. The cost effectiveness of the SPECTROM series finite element programs indicates a definite advantage of a specialized finite element program over a general purpose finite element program.

TABLE 4.1.1.

COST COMPARISON OF EXAMPLE PROBLEMS COMPUTED WITH MARC-CDC AND SPECTROM PROGRAMS‡

EXAMPLE PROBLEMS	MARC-CDC *SBU'S (WITH)	MARC-CDC *SBU'S (WITHOUT)	MARC-CDC EXECUTION TIME (SECONDS)	SPECTROM *SBU'S (WITH)	SPECTROM *SBU'S (WITHOUT)	SPECTROM EXECUTION TIME (SECONDS)
A (HEAT-TRANSFER)	17	15	3.45	4	1	0.58
A (THERMO-ELASTIC)	34	32	9.20	6	1	1.36
B	13	11	2.37	6	1	0.50
C	13	11	2.37	6	1	0.50
D	16	14	3.57	6	1	0.96

*SBU's Are Given With and Without Compilation Of The Finite Element Program.

Note: The MARC-CDC program is compiled in a manner which prevents updates or changes. Consequently, the compilation costs are reduced.

‡These Costs Are Associated With The Control Data Corporation (CDC) Cybernet-7600 Series Computer Processing System.

5. SUMMARY AND CONCLUSIONS

Comparison of the MARC-CDC and SPECTROM program involved a number of parameters used in numerical solutions by the finite element method. These parameters included axisymmetric and two-dimensional geometry, plane stress, plane strain and a variation in material properties and creep parameters. Four example problems involving heat transfer, thermoelastic and viscoelastic analyses were examined.

The agreement of results calculated from the MARC-CDC and SPECTROM programs was excellent for the four example problems. These simple problems represent an initial step toward the quality assurance program to validate the numerical prediction of rock mechanics problems with finite element programs. The close agreement shown in this comparison of four simple problems is essential. Subsequently, a comparison of more complex problems should be attempted which are representative of the models used for radioactive waste disposal.

Quality assurance studies have been performed and are presently being planned or executed. A quality assurance study is being conducted for plasticity. The plasticity analysis is being performed by another finite element program (SPECTROM-11) from the SPECTROM series. The results of these plasticity analyses will be compared with various analytical and numerical solutions and the MARC-CDC program. Previous comparisons between testing (laboratory or field) and numerical simulations using the SPECTROM series of finite element programs have been performed. These comparisons include Project Salt Vault (4), simulations of the Avery Island heater tests, and a simulation of bench scale laboratory tests (5). Other comparisons related to the disposal of radioactive waste presently being conducted include a thermo/viscoelastic analysis of both a room and pillar configuration and a global model of an entire repository. The results of these two problems analyzed by the SPECTROM series of finite element programs are being compared with the results obtained from either finite difference or boundary element solution techniques. The analyses by these two solution techniques are being conducted by two other agencies. All of these attempts to verify results should strengthen the reliability of the SPECTROM series of finite element programs.

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