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ALTERNATIVE SCENARIOS FOR FEDERAL TRANSPORTATION POLICY

VOLUME III

AN INTEGRATED POLICY MODEL FOR THE TRANSPORTATION INDUSTRIES

FIRST YEAR FINAL REPORT

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16. Abstract

The research evaluates the economic effects of existing and prospective federal policies governing intercity and international freight and passenger transportation enterprises in the economy of the United States. The analysis encompasses all modes of transportation, including rail, motor, water, air and intermodal coordinative institutions, and focuses upon the impact of alternative regulatory policies. However, other federal policies including subsidy, taxation, procurement, government ownership and investment, special programs for particular transportation industry problems and impacts of general national policies on transportation will be included when relevant.

Economic evaluation includes the study of efficient resource allocation and distributional effects of alternative policies together with consideration of both partial and general equilibrium effects. The research is interdisciplinary in scope, drawing upon engineering, economics, statistics, law and administration.

There are four volumes included in this report:

Volume I - Summary of First Year Report
Volume II - Policy Review and Scenario Development
Volume III - An Integrated Policy Model for the Transportation Industries
Volume IV - Network Models for Transportation Policy Analysis

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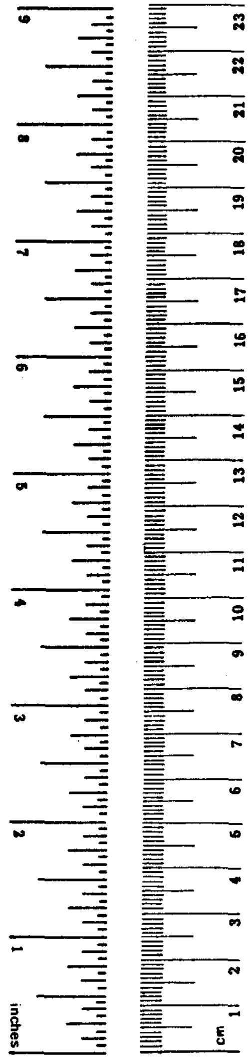
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

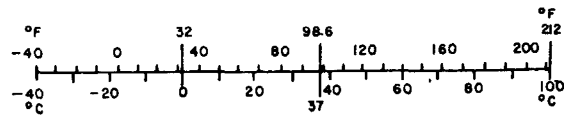
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in ≡ 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



ALTERNATIVE SCENARIOS FOR FEDERAL TRANSPORTATION POLICY

Volume III

An Integrated Policy Model for the Transportation Industries*

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January 1977

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Center for Transportation Studies

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Chapter One

Introduction and Overview

Federal transportation policies have wide ranging impacts upon the transportation industries, and, through them, upon the allocation of economic activity among industries and regions throughout the nation. Federal regulatory policy directly affects rates, entry, routes, etc. in the intercity transportation industries: rail, highway, water, and air. Federal promotional policies directly affect the infrastructure and thus the costs of these various modes, as do federal policies with respect to use charges, subsidies, safety, energy, loan guarantees, environmental impacts, etc.

Clearly, a change in any given federal transportation policy with respect to any given mode will have a direct impact upon the costs and/or demands facing the firms in that mode, and thus upon the equilibrium configuration of rates, traffic allocations, service levels, etc. within that mode; but it will also affect the rates, traffic allocations and service levels of the competing modes by changing the relative prices of the various transport services. Moreover, since transportation is used as an intermediate good in virtually all industries in all regions of the country, changes in the costs of transportation relative to those of other inputs will alter the allocation of economic activity and consequently the level of incomes and employment among regions, among industries, among different kinds of labor and capital, and among cities of different sizes.

When viewed in this context, it is clear that most studies of transportation policy have had an excessively narrow focus. Economic studies have tended to look at the question from the point of view of economic efficiency alone, and have thus concentrated upon providing global measure of user savings, resource savings, or welfare losses.[1/]

[1/] See, for example, Keeler (1972), Moore (1973), Douglas and Miller (1974).

While informative, these studies have tended to ignore questions of the income distribution as well as broader questions of efficiency concerned with full employment and transfer costs. Thus, what happens to employment and wages in a given transportation industry; what happens to regional income levels and the regional allocation of economic activity; what happens to the level of service to given communities have been questions that economists have generally not raised, much less answered.

Clearly, however, if one looks at legislative or regulatory proceedings, issues of the income distribution have tended to dominate the discussion. Whether service will be curtailed to a given city or class of cities; whether labor income and/or employment will fall within a given transportation industry or a given region; whether industry incomes and outputs will rise or fall; are all questions that the policy maker has tended to weigh more heavily than questions of aggregative economic efficiency. Thus, if economic analysis is to be used to help evaluate changes in transportation policy, it must not only provide answers concerning aggregative efficiency impacts, but also provide answers relating to a whole host of distributional questions. Consequently, one of the major goals of this research is to provide analytical models that can be used to quantify the magnitude of the various distributional effects as well as to quantify the magnitude of the efficiency effects of a given change in transportation policy.

This volume describes such a modeling effort. The basic premise of the analysis is that relative prices matter. Thus any change in transportation policy should lead to a change in the transportation rate structure, which in turn will affect a wide range of regional and national variables concerning income, output, employment, etc. Since these, however, can influence transportation costs and/or demands, the entire system is interrelated and simultaneously determined.

These propositions are illustrated in Figure 1, which depicts four linked models:

- A regional transportation model that determines costs, revenues, profits, outputs, shipment characteristics, rates and factor demands by firm, by mode, by broad commodity type and by region.
- A national interindustry model that determines interindustry coefficients, commodity prices, commodity outputs and factor employment by broad commodity type.
- A regional income model that determines factor prices, consumer prices, increases, outputs and employment by broad commodity type.
- A small-scale national macroeconometric model that determines factor prices, final demands and consumer prices.

With the exception of the exogenous variables in the national macroeconometric sub-model, every variable that is exogenous to a given sub-model is endogenous to another sub-model. Hence, the entire system is interrelated and interactive; a full solution to the model must be simultaneously determined.

An Integrated Policy Model for the Transportation Industries

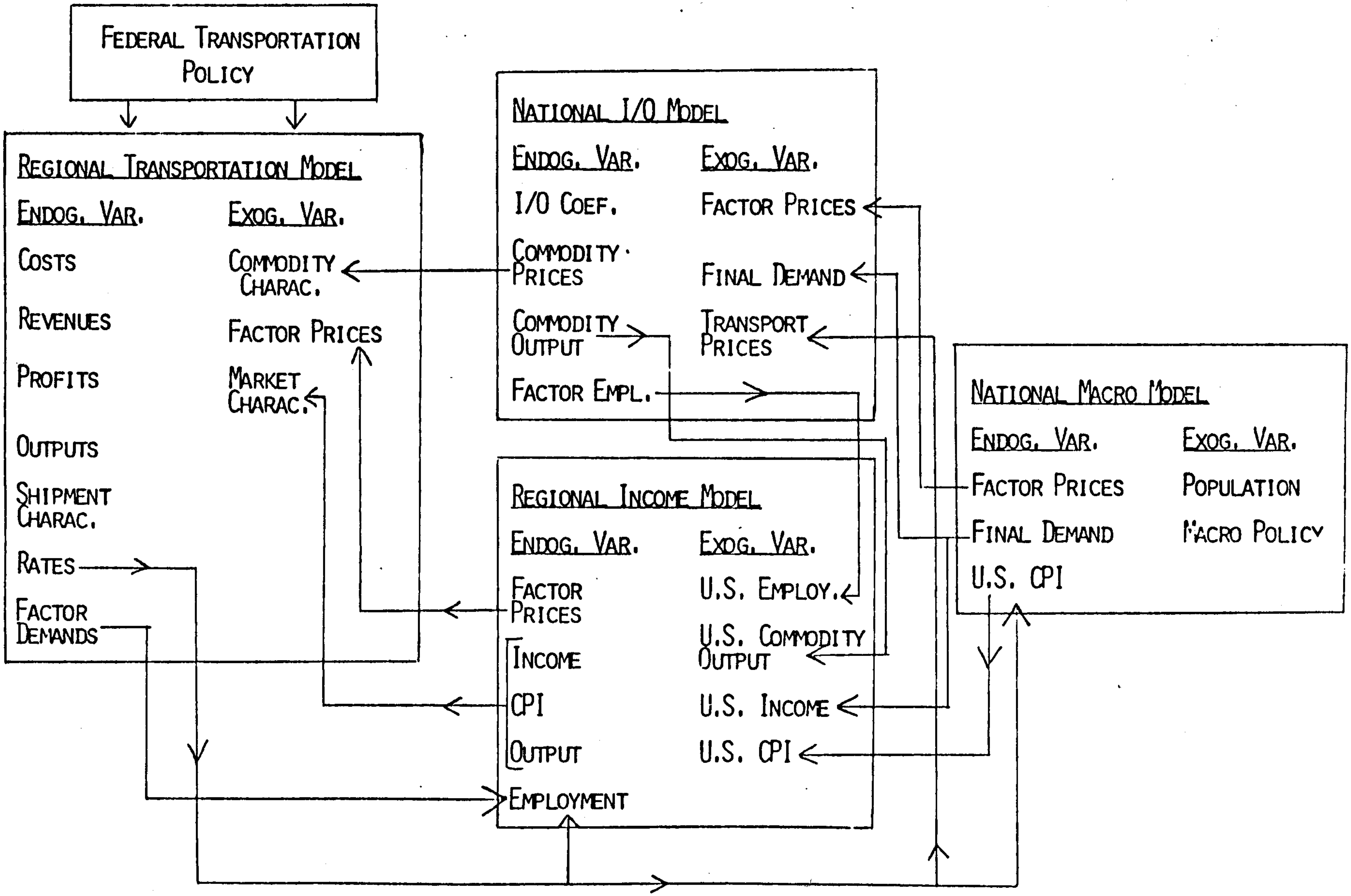

Figure 1

In terms of policy analysis, we can postulate a change in transportation policy that affects costs, demands or the nature of market equilibrium in the transportation industries in a given region or the nation as a whole.[2/] These in turn affect transportation rates and factor employment, which, in turn, affect regional and national outputs, employment, factor prices and so forth. However, these also affect the nature of the equilibrium in the transportation industries. Thus by using these interrelated models, we can analyze the impact of a wide range of transportation and related policies upon a wide range of variables that measure distributional as well as efficiency impacts.

To make the problem tractable, our initial efforts will be quite aggregative and deal with broad categories with respect to modes, regions, commodities, and factors. We thus plan to consider the following:

Modes. Initially we plan to focus upon the rail and trucking industries. Because of data limitations, we will probably have to confine our analysis to regulated trucking, although it would obviously be desirable to extend it to private and exempt carriage.[3/]

Regions. A wealth of regional data exist from the Census of Transportation, which makes it possible to perform a regional analysis on a fairly fine level of detail. At this time, however, we are primarily interested in developing an

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[2/] For a full discussion of the proposed policy analysis see Friedlaender et al. (1977).

[3/] Insofar as data and resources permit, we will also analyze the water and pipeline industries.

integrated model that can be used for aggregative policy analysis. Consequently we plan to limit ourselves to the five ICC regions: The Official, Southern, Southwestern, Mountain-Pacific and Western Territories. Once we have developed working models of these five regions, we can always extend the analysis to more regions.

Commodities. Similarly, a wealth of commodity detail exists. Nevertheless, for reasons of tractability, we plan to limit our initial analysis to the following broad commodity groups: durable manufacturers; nondurable manufacturers; feed grains; other agricultural commodities; coal; petroleum and petroleum products; minerals, chemicals and others.

Factors. The regional transportation models will consider labor, fuel, equipment and track (for the railroads) as the relevant factors of production, while the regional models will only consider labor. The national interindustry model will treat transportation as a factor of production as well as labor, capital, energy, and materials.

Our basic approach is one of comparative statistics, with transportation policy as the primary exogenous variable. We thus determine an initial equilibrium and postulate a change in transportation policy. After determining the new equilibrium as well as its time path, we can then assess the impact of the policy change.

As indicated above, our main analytical tools are changes in the cost functions, changes in the demand functions, and changes in the nature of the market equilibrium in the transportation industries. These, however, permit us to consider a wide range of policies under a wide range of situations.

With respect to industry behavior, we can analyze the nature of the equilibrium if the transportation industries operate under joint monopoly profit maximization, workable competition, or oligopoly or monopolistic competition. We can also analyze how the equilibrium would change if transportation firms maximized objective functions other than profits.

Within this framework, it is relatively straightforward to analyze policies that affect demands or costs. In particular, we should be able to assess the impact of the following: deregulation under different market structures; marginal cost pricing; abandonment and capital adjustments; work force adjustments; entry restrictions; and abolition of rate bureaus. Moreover, in so far as energy policies affect final costs, our analysis should be able to assess their impact. Finally, this framework can also be used to assess the impact of user charges and investment or promotional policies that affect carrier costs.

To recapitulate briefly, these interrelated models permit us to analyze the impact of a wide range of transportation and related policies upon a wide range of variables that measure distributional as well as efficiency impacts. Specifically, by utilizing this framework, it should be possible to consider the following:

Transportation Policies

- Setting rate levels or rate bands in the regulated transportation industries.
- Total deregulation of rates.
- Elimination of rate bureaus or other cartelization in the regulated transportation industries.
- Relaxation or tightening up of restrictions concerning entry in the regulated transportation modes.

- Relaxation or tightening up of restrictions concerning mergers in the regulated transport modes.
- Relaxation of restrictions concerning abandonment and capital adjustments in the transportation industries.
- Relaxation of restrictions upon the utilization of labor in the regulated transportation industries.
- Construction and maintenance of transportation infrastructure and its related user charges.
- Explicit subsidies for specific kinds of transportation services.
- Energy policy in so far as it affects relative fuel costs in the transportation industries.

Efficiency Variables

- Long-run and short-run marginal costs of different outputs by different modes.
- Price-marginal cost ratios by different outputs and different modes.
- Resource cost savings from "optimal" adjustments in capacity and labor utilization.
- Resource savings (or costs) associated with traffic allocations resulting from competitive, monopolistic, or oligopolistic market structures as opposed to the present regulatory environment.
- Measures of productivity by transport mode.
- Measures of industrial concentration by transport mode.
- Measures of profitability, costs, and revenues by firm and by transport mode.
- Measures of factor utilization (employment) by firm and by transport mode.
- Measures of aggregate level of service by mode.

Distributional Variables

- Traffic allocations and profitability by firm and by mode.
- Employment and wages by firm and by mode.
- Employment and wages by national industry, regional industry, and by broad geographical regions.
- Price-marginal cost ratios by class of user and by geographical region.
- Income levels by broad geographical regions and by national industry.
- Producers' prices by broad industry category.

The remaining chapters of this volume discuss the specification of the various models and the preliminary econometric results in some detail. Chapter Two outlines the regional transportation model while Chapters Three and Four respectively discuss the econometric estimation of cost functions in the transportation industries and its application to the trucking industry. Chapter Five describes the interindustry model, while Chpater Six discusses the determination of regional and national income in the context of these policy models. Chapter Seven provides a brief summary and conclusions.

Chapter 2

The Regional Transportation Model

The heart of the analysis lies in the model of the regional transportation market. Conceptually, this is quite straightforward, and is illustrated in Figure 2. Thus we postulate that there is a known industry or firm cost function, which relates costs to outputs, factor prices, and (in the case of the short-run cost function) the amounts of the fixed factors. Similarly, we assume that there is a known firm or industry demand function relating shipments to market characteristics, commodity characteristics, shipment characteristics of own and competing modes, and rates of own and competing modes. Given these cost and demand functions, and assuming profit maximizing behavior as the part of the firms in the industry,[1/] we can determine the equilibrium level of rates, shipments, profits, costs, revenues, shipment characteristics, and factor demands in the short-run and the long run under a number of different market structures: perfect competition, joint profit maximization, rate regulation, oligopoly, and monopolistic competition.

Let us now discuss the specification of the cost and demand functions, and how we plan to utilize them for policy analysis.

[1/] We could also make different assumptions about the firms' objective functions such as sales maximization subject to a profit constraint or profit maximization subject to a rate of return constraint.

COSTS = C [OUTPUTS (CARLOADS) BY COMMODITY; FACTOR PRICES; FIXED FACTORS]

DEMAND (TONS OR TON MILES, BY COMMODITY) = D [MARKET CHARACTERISTICS; COMMODITY CHARACTERISTICS; SHIPMENT CHARACTERISTICS, OWN MODE; SHIPMENT CHARACTERISTICS, COMPETING MODES; RATES, OWN MODE; RATES, COMPETING MODES]

→ FIRM MAXIMIZING BEHAVIOR (PROFIT MAXIMIZATION) ←

EQUILIBRIUM CONDITIONS

↓

COSTS
REVENUES
PROFITS
SHIPMENT CHARACTERISTICS
OUTPUTS
RATES
FACTOR DEMANDS

FIGURE 2

I. Cost Functions

The validity of econometric estimates of the costs of the various transportation modes remains an issue surrounded by controversy. While there have been numerous econometric studies of rail, trucking, and airline cost,[2/] no one has yet developed a costing methodology that has yielded results that are generally accepted as valid. This inability to obtain a consensus concerning costing methodology and/or the validity of the empirical results arises not so much from a lack of effort, but rather from the failure to specify the cost functions that appropriately characterize the structure of technology.

Specifically, there appear to be three fundamental problems that one must address in specifying and estimating cost functions for the transportation industries.

First, the output of a transportation firm, whatever the mode, is multi-dimensional by its very nature. Not only does the firm produce different types of transportation services for different users at different origins and destinations, but also at different levels of quality. Consequently, the mix of output can have a major impact upon the costs of any given firm. For example, railroads specializing in coal traffic have very different cost characteristics than those specializing in general manufactured commodities.

Since the mix of output affects the firm's costs, it is clearly inappropriate to estimate cost functions by using a single aggregate measure

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[2/] For a review of the literature, see Kneafsey (1975) for rail, Oramas (1975), for trucking, and Douglas and Miller (1974) for air.

of output such as ton miles or passenger miles. To the extent that the mix of traffic and quality levels affect costs, a vector of outputs and quality levels that characterize the range of activities undertaken by the firms in a given transportation mode should be incorporated into the analysis. While it is unlikely that the available data will permit the fully desired degree of output disaggregation, it is clear that considerably more disaggregation is possible that has been undertaken in existing studies of transportation costs.

Second, it is generally agreed that the activities of each of the transportation modes are characterized by joint and common costs, implying that their technology is characterized by joint production. Although Hall (1973) has shown that a separable technology will always imply joint production, he has also shown that the converse is not true. We cannot assume, therefore, that cost functions based on a separable Cobb-Douglas technology are good representations of reality.[3/] Instead, a flexible form is needed that will permit the determination of the underlying structure of technology from its estimated coefficients.

Third, to the extent that regulatory or other constraints prevent the firms in each mode from making optimal adjustments in capacity, they are not generally in a position of long-run equilibrium operating along their long-run cost function. Consequently, efforts to estimate long-run cost functions directly from cross-sectional data will yield seriously biased coefficients and resulting measures of marginal costs. The sign of this bias will depend upton the degree of excess capacity.[4/] Since, however, this relationship is not

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3/ See, for example, Keeler (1974), Kneafsey (1975) and Eads, Nerlove, and Raduchel (1969).

4/ See Friedlaender (1969) for a discussion of this point.

generally known, it is impossible to make any adjustment to correct for this bias.

This implies that one should estimate short-run functions when one suspects that an industry may be in long-run disequilibrium with chronic excess capacity. Since the long-run cost function is merely the envelope of the short-run cost function, it is always possible to derive the unobserved long-run cost function from the observed short-run cost function.[5/] Thus, to the extent that the short-run cost function has been correctly specified, and its coefficients are therefore unbiased, the coefficients of the derived long-run cost function will also be unbiased and the long-run marginal costs obtained from the derived long-run total cost curve will also be unbiased.

These arguments imply that in estimating cost functions for the transportation industries, one should specify a multiple-output cost function in a sufficiently flexible form to permit the testing of a number of hypotheses concerning the separability, homogeneity, and jointness of the underlying production function. Moreover, if there is reason to believe that regulatory or other institutional constraints prevent "optimal" capacity adjustment, one should estimate a short-run variable cost function, which can be used to derive the associated long-run cost function and the underlying production function.

This analysis will use a translog cost function that meets the objections raised with respect to most cost functions: it permits multiple outputs and quality levels; it is of a sufficiently flexible form to test hypotheses concerning the underlying structure of production;

[5/] This approach has been utilized by Keeler (1974) and Kneafsey (1975) in the railroad industry and by Eads, Nerlove, and Raduchel (1969) in the airline industry.

and it can be used in either its short-run or long-run form.[6]

Since the methodology we use to estimate cost functions is entirely general, we will apply it to all of the relevant modes. Thus, by using cross-sectional and time series data, we plan to estimate a short-run variable cost function of the folowing general form:

$$\tilde{C} = C(y,\tilde{x},w) \qquad (2.1)$$

where C = short-run variable cost

y = a (1xN) vector of outputs

x = a (1xH) vector of fixed factors

w = a (1xJ) vector of prices of the variable factors.

We can then establish the short-run total cost function:

$$C = \tilde{C}(y,\tilde{x},w) + \sum_{h=1}^{H} \tilde{w}_h x_h \qquad (2.2)$$

where $\tilde{w}_h$ represents the price of fixed factor h. Differentiating the short-run total cost function with respect to each fixed factor and setting the resulting expression equal to zero we thus obtain:

$$\frac{\partial\tilde{C}(y,\tilde{x},w)}{\partial X_h} = - \tilde{w}_h \quad h=1,\ldots,H \qquad (2.3)$$

---

[6] For a full discussion of the costing methodology used see Chapter 3, below.

Solving the above system of equations for each $\tilde{x}_h$ we obtain:

$$\tilde{x}_h = \tilde{x}_h(y,w,\tilde{w}) \tag{2.4}$$

By substituting the above into the short-run total cost function we can then obtain the long-run total cost function:

$$C^* = C^*(y,q) \tag{2.5}$$

where q represents the vector of all factor prices; hence $q = (w_1,\ldots, w_J; \tilde{w}_1,\ldots,\tilde{w}_H)$

Moreover, we can obtain short-run and long-run factor demand equations by utilizing the well-known relationships that

$$\frac{\partial \tilde{C}(y,\tilde{x},w)}{\partial w_j} = x_j \quad j=1,\ldots,J \tag{2.6a}$$

$$\frac{\partial C^*(y,q)}{\partial q_h} = x_h \quad h=1,\ldots J+H \tag{2.6b}$$

where x_j represents the demand for the j^{th} variable factor and x_h represents the demand for the h^{th} factor (variable or fixed).

Finally, by differentiating the short-run and long-run cost functions with respect to output, we can obtain the marginal costs of different shipments; thus

$$\frac{\partial \tilde{C}(y,\tilde{x},w)}{\partial y_i} = \tilde{m}c_i \quad i=1,\ldots,N \tag{2.7a}$$

$$\frac{\partial C^*(y,q)}{\partial y_i} = mc_i^* \quad i=1,\ldots,N \tag{2.7b}$$

where $\tilde{m}c_i$ and mc_i^* respectively represent short-run and long-run marginal costs.

The derivation of the long-run cost function from the short-run cost function is fairly straightforward in the case of a specific functional form.[7] In the case of second-order approximations it is considerably more difficult, since we are not only interested in the numerical value of costs in the short-run and in the long-run around a particular point, but also in the specification of the entire function. Nevertheless, using the translog approximation it is possible not only to derive the value of long-run costs around a particular point, but also to derive short-run and long-run cost functions over the relevant range. Consequently, because it enables us to determine much more about the nature of the underlying technology than a specific functional form, we plan to use the following translog approximation in analyzing the short-run cost variable functions of the transportation industries.[8]

$$
\begin{aligned}
\ell n \tilde{C}(y,\tilde{x},w) = {} & \alpha_0 + \sum_{i=1}^{N}\alpha_i \ \ell n\ y_i + \sum_{h=1}^{H}\beta_h \ \ell n\ \tilde{x}_h + \sum_{j=1}^{J}\gamma_j \ \ell n\ w_j \\
& + \tfrac{1}{2}\Big\{\sum_{i}^{N}\sum_{h}^{N} A_{ik}\ \ell n\ y_i\ \ell n\ y_k + \sum_{h}^{H}\sum_{\ell}^{H} B_{h\ell}\ \ell n\ \tilde{x}_h\ \ell n\ \tilde{x}_\ell \\
& + \sum_{i}^{J}\sum_{r}^{J}\left(C_{jr} + C'_{rj}\right)\ell n\ w_j\ \ell n\ w_r \\
& + \sum_{i}^{N}\sum_{j}^{J}\left(D_{ij} + D'_{ji}\right)\ell n\ w_j\ \ell n\ w_j \qquad (2.8) \\
& + \sum_{i}^{N}\sum_{h}^{H}\left(E_{ih} + E'_{.ni}\right)\ell n\ y_i\ \ell n\ \tilde{x}_h \\
& + \sum_{h}^{H}\sum_{j}^{J}\left(F_{hj} + F'_{jh}\right)\ell n\ \tilde{x}_h\ \ell n\ w_j\Big\}
\end{aligned}
$$

7/ See, for example Keeler (1974), Kneafsey (1975).

8/ This approach has also been used by Caves and Christensen (1976). Our work differs from theirs, however, in that we utilize multiple outputs and adjust for quality.

As indicated in Chapter Three, below, by using this specification of the cost function and by imposing various restrictions on the coefficients, we can test for homogeneity in factor prices, homogeneity in outputs, separability, and jointness in production. Moreover, it is also possible to derive long-run total costs, long-run and short-run marginal costs and long-run and short-run factor demands over the relevant range from eq. (2.8). Finally, by adjusting the vector of outputs to reflect differences in the quality of service,[9/] we can take service differentials into account.

For notational simplicity, we now revert to the general specification and write the short-run variable cost function for mode m in region d as:

$$\tilde{C}_m^d = \tilde{C}_m^d(y_m^d, \tilde{x}_m^d, \tilde{w}_m^d) \qquad (2.9a)$$

where the d's range over the ICC territories and the m's range over rail, truck, water, and (possibly) pipelines.

The long-run cost function derived from this is given by:

$$C_m^d = C_m^d(y_m^d, q_m^d) \qquad (2.9b)$$

and the respective marginal costs are denoted by $\tilde{C}_{im}^d$ and C_{im}^d, where $C_{im}^d = \partial C_m^d/\partial y_i$.

Finally, since the cost functions are derived from cross-sectional and time series data, as long as all firms in a given mode face the same technology, we can derive firm-specific cost functions for each mode and write:

9/ This can be done by deflating the output measures directly by use of hedonic price equations or by estimating hedonic cost functions directly, in which the output vector y is replaced by a function $\Phi(y,Q)$, where Q represents the vector of qualities. For a full discussion of the use of hedonic adjustments in this context see Chapters 3 and 4, below.

$$\tilde{C}^d_{mf} = \tilde{C}^d_{mf}(y^d_{mf}, \tilde{x}^d_{mf}, w^d_{mf}) \quad (2.10a)$$

$$C^d_{mf} = C^d_{mf}(y^d_{mf}, q^d_{mf}) \quad (2.10b)$$

where the variables have their previous meaning and f ranges over the firms in the mode.

We can similarly obtain the firm's marginal cost curves and write $\tilde{C}^d_{imf}$ and C^d_{imf} as the respective short-run and long-run marginal cost curve associated with shipment type i by firm f in mode m in region d.

II. Demand

Since freight transportation is used as an intermediate input, it seems reasonable to specify its demand in terms of a derived demand for a factor of production. Hence following the approach outlined above, let us characterize the costs (C_i^r) in a given industry i in region r as depending upon the level of output (Q_i^r), capital and labor costs in that region (w_{iL}^r, w_{iK}^r) and the costs of rail and trucking transportation between that and other regions $(w_{iR}^{rd}, w_{iT}^{rd})$.[10/] If we assume that the producing industry only bears the costs of final goods shipments, then we can write the industry's general cost function as:

$$C_i^r = C_i^r(w_{iL}^r, w_{iK}^r, w_{iR}^{rd}, w_{iT}^{rd}, Q_i^r) \tag{2.11}$$

where the d's range over all regions of destination.

By differentiating this cost function with respect to the appropriate transportation cost, we can then derive the demand for truck or rail transportation from regions r to region d as

$$R_i^{rd} = \frac{\partial C_i^r}{\partial w_{iR}^{rd}} \tag{2.12a}$$

$$T_i^{rd} = \frac{\partial C_i^r}{\partial w_{iT}^{rd}} \tag{2.12b}$$

10/ We omit other modes for notational simplicity.

Since we assume that the industry of origin ships its final products to all other regions, the market demand in region r for each mode is respectively given by

$$R_i^r = \sum_d R_i^{rd} \qquad (2.14a)$$

$$T_i^r = \sum_d T_i^{rd} \qquad (2.14b)$$

The specification of the actual demand function, is complicated, however, by two problems. First, we must utilize a specific functional form for the cost function given is Equation (2.11); and second, we must recognize that transport costs reflect inventory costs as well as rates.

Following our approach used to estimate cost functions, it would be natural to specify a general translog cost function and estimate its factor share equation to derive the demand for transportation. Thus let the translog approximation of an industry's cost function be given by:

$$\ln C_i^r = \alpha_0 + \sum_f \alpha_{if} \ln w_{if}^r + \beta_i \ln Q_i^r$$

$$+ 1/2 \sum_f \sum_m a_{if} \ln w_{i_f}^r \ln w_{im}^r$$

$$+ 1/2 \sum_f b_{if} \ln Q_i^r \ln w_{if}^r + 1/2\, C_{ii} \ln Q_i^{r^2} \qquad (2.15)$$

Thus the factor share equation for, say, rail transportation is given by

$$\frac{\partial \ \ell n \ C_i^r}{\partial \ \ell n \ w_{iR}^{rd}} = \frac{Rev_{iR}^{rd}}{Costs_i^r}$$

$$= a_{iR} \ \ell n \ w_{iR}^{rd} + 1/2 \ [a_{iL} \ \ell n \ w_{iL}^r + a_{iK} \ \ell n \ w_{iK}^r +$$

$$+ a_{iR}^{rd} \ell n \, w_{iR}^{rd} + a_{iT}^{rd} \ell n \, w_{iT}^{rd} + b_i \ \ell n \ Q_i^r] \qquad (2.16)$$

where the superscripts range over the region of destination as necessary. The trucking factor share equation would take a similar form. As indicated in Berndt and Wood (1974) it is straightforward to derive the Allen-Uzawa elasticities of substitution and then own and cross price elasticities for the transportation modes from these equations.

Because a translog approximation makes no restrictive assumptions about the nature of technology and permits variable elasticities of substitution among the relevant factors, it is attractive. Indeed, if we were solely interested in estimating the own and cross elasticities for each mode, it would make sense to estimate the factor share equations (2.16) for each mode and derive the relevant elasticities from them. In fact, however, we must have the specific demand function to determine the equilibrium in the transportation industries under various assumptions concerning market structure. Because the demand function generated by the translog factor share equation is of a particularly intractable function form,[11/] it is preferable to use a simpler, if more restrictive cost function to derive the transportation demand functions.

Consequently, let us therefore assume that the technology of each industry can be characterized by a Cobb-Douglas production function. The cost function of each industry is then given by the following expression:

$$C_i^r = A_i^r \left(\frac{w_{iK}^r}{\alpha_{iK}^r}\right)^{\alpha_{iK}^r/\mu_i^r} \cdot \left(\frac{w_{iL}^r}{\alpha_{iL}^r}\right)^{\alpha_{iL}^r/\mu_i^r} \cdot \left(\frac{w_{iR}^{rd}}{\partial_{iR}^{rd}}\right)^{\alpha_{iR}^{rd}/\mu_i^r} \cdot$$

$$\cdot \left(\frac{w_{iT}^{rd}}{\alpha_{iT}^{rd}}\right)^{\alpha_{iT}^{rd}/\mu_i^r} \cdot Q_i^{r^{1/\mu_i^r}} \qquad (2.17)$$

where $\mu_i^r = \sum_f \alpha_{if}^r$.

The demand function for, say, rail transportation can be obtained by differentiating Equation (2.17), collecting terms, and taking logs

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11/ Specifically, if T_{iT}^{rd} represents the demand for truck transportation of commodity i between regions r and d, T_{iT}^{rd} can be obtained from the translog share equation by the following expression:

$$T_{iT}^{rd} = \frac{\partial \ln C_i^r}{\partial \ln w_{iT}^{rd}} \cdot \frac{C_i^r}{w_{iT}^{rd}}$$

where C_i^r is obtained from the estimated translog cost function for the industry producing commodity i.

to obtain

$$\ln R_i^{rd} = \ln B + \frac{\alpha_{iK}^r}{\mu_i^r} \ln w_{iK}^r + \frac{a_{iL}^r}{\mu_i^r} \ln w_{iL}^r + \frac{a_{iT}^{rd}}{\mu_i^r} \ln w_{iT}^{rd} +$$

$$+ \frac{\alpha_{iR}^{rd}}{\mu_i^r} - 1 \quad \ln w_{iR}^{rd} + \frac{1}{\mu_i^r} \ln Q_i^r \qquad (2.18)$$

where B represents a constant term containing the relevant coefficients of production. The demand functions for the other modes would take a similar form.

The transport cost variables (w_{iT}^{rd}, w_{iR}^{rd}) are not directly observable, but depend upon the rates(P_{iT}^{rd}, P_{iK}^{rd}) and the inventory costs associated with goods in transit, which depend in turn upon the quality of service as measured by such variables as reliability, size of shipment, length of haul, loss and damage, and value of the commodity. If we denote the vector of the quality-of-service variable that affects inventory costs as q, then we can relate the cost of transport of mode on to the rate and inventory costs by the following expression:

$$\ln w_{im}^{rd} = \gamma_o + \gamma_m \ln P_{im}^{rd} + \sum_s \gamma_s \ln q_{ims}^{rd} +$$

$$+ 1/2 \; (\sum_s \sum_v g_{sv} \ln q_{ims}^{rd} \ln q_{imv}^{rd} + \sum_s h_{sm} \ln P_{im}^{rd} \ln q_{ims}^{rd} +$$

$$+ k_{mm} \ln P_{im}^{rd^2}) \qquad (2.19)$$

In the case of rail and truck transportation, let us specifically denote the quality-of-service variable as:

$$q_{iR}^{rd} = (S_{iR}^{rd} , M_{iR}^{rd} , D_i^{rd} , V_i)$$

$$q_{iT}^{rd} = (S_{iT}^{rd} , M_{iT}^{rd} , D_i^{rd} , V_i)$$

where S_{iR}^{rd} , S_{iT}^{rd} = average size of shipment of commodity i between r and d on rail or truck

M_{iR}^{rd} , M_{iT}^{rd} = average length of haul of commodity i between r and d on rail and truck

D_i^{rd} = differential loss and damage costs between rail and truck borne by commodity i between r and d

V_i = value of commodity i.

Thus by substituting Equation (2.19) into Equation (2.18), collecting terms, and jointly estimating the modal demand functions, we can obtain consistent estimates of the market demand for each mode.[12/]

If we let the symbol T_{im}^{rd} represent the ton-miles carried of commodity i between regions r and d by mode m, we can readily derive

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[12/] We will generally have to use cross sectional data to estimate these demand functions, which requires us to assume that all industries in the sample have the same technology. While restrictive, this is probably acceptable if we utilize reasonably similar industries in the sample (e.g., all industries are nondurable manufacturing). It would obviously be preferable to utilize time series data on one commodity. Unfortunately, however, this is not generally available.

the total revenue function facing that mode by multiplying the rates and the volume and summing appropriately. Thus:

$$R_m^d = \sum_i \sum_d P_{im}^{rd} \, T_{im}^{rd} \qquad (2.20)$$

where the subscript m ranges over the relevant modes and the superscript r ranges over the relevant regions.

We assume that each firm's demand function is some proportion of the market demand function and write:

$$T_{imf}^r = \mu_{mf}^r \, T_{im}^r \qquad (2.21)$$

where $\mu_{mf}^r = \dfrac{T_{mf}^r}{\sum_i \sum_d T_{im}^{rd}}$

Thus, μ_{mf}^r represents the share of the total ton-miles carried in region r by mode m accruing to firm f. If data permit, we can, of course, disaggregate this market-share variable into commodities and regions of origin.

Since service is a major competitive weapon in the transportation industries, it is quite likely that a firm's share of total freight shipments also depends upon its level of service relative to other firms. In the airline industries where data on flight frequency are readily available, frequency is generally taken to measure levels of service.[13/] In the surface freight industries, however, such data

13/ See, for example, Douglas and Miller (1974).

do not exist. Hence, we must find another proxy for level of service.

In so far as firms with large amounts of rolling stock are able to meet shipper demands more quickly than firms with small amounts of rolling stock, it is likely that the level of service offered by the former firms is greater than that of the latter. Hence, as a first approximation we can postulate that

$$\mu^r_{mf} = \mu^r_{mf}[E^r_{mf}/E^r_m] \tag{2.22}$$

where E^r_{mf} represents the rolling stock of firm f in mode m in region r, and E^r_m represents the total rolling stock of mode m in region r.

III. Market Equilibrium

Having specified the industry and firm cost and demand functions within a given region, we are now in a position to analyze the nature of equilibrium in the regional transportation market under a number of different assumptions concerning the competitive structure of the industry. Note that since we are dealing with a number of regions and modes, a partial-equilibrium analysis of a given mode within a given region will not in general be sufficient.

A. Perfect Competition

Under perfect competition, equilibrium is given when the supply price equals the demand price. The market demand for commodity i in region d for mode m is given by:

$$T^d_{im} = \sum_r T^{rd}_{im}(P^{rd}_{im}, P^{rd}_{ic}, A^d_m) \tag{2.23}$$

where P_{im}^{rd}, P_{ic}^{rd} refer to the own and competitive price of shipping the commodity and A_m^d refers to the other variables in the demand function; see Equations (2.18) and (2.19). In perfectly competitive equilibrium, the market must clear at the common price. Hence, there can be no regional price discrimination and

$$T_{im}^d = T_{im}^d(P_{im}^d, P_{ic}^d, A_m^d) \tag{2.24}$$

The long-run total cost function for firm f in mode m in region d is given by:

$$C_{mf}^d = C_{mf}^d(y_{imf}^d, \ldots, y_{Nmf}^d, q_{mf}^d) \tag{2.25}$$

where the y's represent shipment carried by the firm and the q's represent the vector of factor prices facing the firm. Note that since we will estimate the short-run cost function directly, we will also undertake an analysis of market equilibrium using the relevant short-run cost functions. Hence, our use of the long-run cost function is purely for expositional and notational simplicity.

The firm's marginal cost function for commodity i is similarly given by

$$mc_{imf}^d = mc_{imf}^d(y_{imf}^d, \ldots, y_{Nmf}^d, q_{mf}^d) \tag{2.26}$$

In equilibrium, the firm equates its marginal cost with its price. Hence:

$$P_{im}^d = mc_{imf}^d(y_{imf}^d, \ldots, y_{Nmf}^d, q_{mf}^d) \tag{2.27}$$

Note that in this formulation, the marginal costs of shipment i not only depend upon its own level of output, but also upon the levels of output of all other commodities. Therefore, we must solve the system of equations given in (2.27) for all of the output levels and thus obtain the firm's supply function in terms of all price. Thus:

$$y^d_{imf} = S^d_{imf}(P^d_{im},\ldots,P^d_{Nm}, q^d_{mf}) \tag{2.28}$$

Having obtained each firm's supply functions, we can then obtain the market supply function by summing over all firms.

$$y^d_{im} = \sum_f \; {}^d_{imf}(P^d_{im},\ldots,P^d_{Nm}, q^d_{mf}) \tag{2.29}$$

Equilibrium requires that the quantity supplied equals the equantity demanded. If we take the prices of the competing modes as given, then equilibrium of any given transportation mode is given by the following expression:

$$y^d_{im}(P^d_{im},\ldots P^d_{Nm}, q^d_m) = T^d_{im}(P^d_{im}, P^d_{ic}, A_m) \tag{2.30}$$

This yields a set of N equations that can be used to solve for the N equilibrium rates, and thus the equilibrium levels of output for the industry as a whole as well as for each firm.

Of course, the problem is considerably more complicated than this because we cannot analyze the equilibrium of a transportation industry apart from the equilibrium of its competitiors. Hence, instead of solving eq. (2.30) on the assumption that P^d_{ic} is constant, we must also analyze the full general equilibrium solution of the transportation industries. This, however, is a realtively straightforward, if computationally complex, problem. Hence, we simply extend our system of equations in (2.30) to

$$y^d_{im}(P^d_{im},\ldots,P^d_{Nm},\ q^d_m) = Y^d_{im}(P^d_{im},\ P^d_{ic},\ A_m) \tag{2.31a}$$

$$y^d_{ic}(P^d_{ic},\ldots,P^d_{Nc},\ q^d_c) = Y^d_{ic}(P^d_{im},\ P^d_{ic},\ A_c) \tag{2.31b}$$

where c ranges over the relevant competing modes. We thus obtain a system of MN equations to obtain the full competitive equilibrium of the rates in each mode. The traffic allocations in each mode, and the traffic allocations in each firm.

B. Joint Monopoly Profit Maximization

We now turn to the other extreme and assume that the firms in a given transportation industry collude to maximize joint monopoly profits. While akin to the usual text-book case of the profit-maximizing monopolist, our analysis is somewhat more complex because: (1) The "monopolist" can practice regional and commodity price discrimination; (2) the "monopolist's" marginal costs of any given output depend on all levels of output; (3) the "monopolist" has many plants, each corresponding to a given firm. Thus, the "monopolist" not only has to decide what price to charge in each market, but how much traffic of each type to allocate to each firm.

The problem can be stated formally as follows. The monopolist in region d faces the following demand function for any given commodity[14/]

[14/] In the case of the monopolist, it is more convenient to utilize the inverse demand functions.

$$P_{im}^{rd} = P_{im}^{rd}(Y_{im}^{rd},\ Y_{ic}^{rd},\ A_m^d) \tag{2.32}$$

where P_{im}^{rd} represents the price of commodity c shipped by mode m to region d from region r; y_{im}^{rd}, y_{ic}^{r} represent the quantities shipped on mode m and its competing mode(s) c, and A_m^d represents the other variables in the demand function.

The revenues derived from commodity i are given by:

$$R_{im}^{rd} = y_{im}^{rd} \cdot P_{im}^{rd}(y_{im}^{rd},\ y_{ic}^{rd},\ A_m^d) \tag{2.33}$$

Thus,

$$R_{im}^{rd} = R_{im}^{rd}(y_{im}^{rd},\ y_{ic}^{rd},\ A_m^d) \tag{2.34}$$

Hence, total revenues equal:

$$R_{im}^{d} = \sum_i \sum_r R_{im}^{rd}(y_{im}^{rd},\ y_{ic}^{rd},\ A_m^d) \tag{2.35}$$

The monopolist's total costs are the sum of the costs in each of its plants. Thus, if there are F firms in the industry, total costs are given by:

$$C_m^d(y_{1m}^d,\ldots,y_{Nm}^d,\ q_m^d) = \sum_f C_{mf}^d(y_{1mf}^d,\ldots,y_{Nmf}^d,\ q_{mf}^d) \tag{2.36}$$

where y_{imf}^d represents the traffic carried of commodity i by firm f in mode m and q_{mf}^d represents the factor price vector facing firm f in mode m.

Thus, the firm's profits are given by:

$$\pi = \sum_i \sum_r R_{im}^{rd}(y_{im}^{rd},\ y_{ic}^{rd},\ A_m^d) - \sum_f C_f^d(y_{1mf}^d,\ldots,y_{Nmf}^d,\ q_{mf}^d) \tag{2.37}$$

The monopolist has to decide how much of commodity i should be carried by firm f from region r. If there are F firms, N commodities, and D regions, it then has FND control variables, each given by y^{rd}_{imf}. We then differentiate eq. (2.37) with respect to y^{rd}_{imf} and obtain:

$$\frac{\partial R^{rd}_{im}(y^{rd}_{im},\ y^{rd}_{ic},\ A^{d}_{m})}{\partial y^{rd}_{im}} \cdot \frac{\partial y^{rd}_{im}}{\partial y^{rL}_{imf}} \tag{2.38}$$

$$= \frac{\partial C^{d}_{mf}(y^{d}_{imf},\ldots,y^{f}_{Nmf},\ q^{d}_{f})}{\partial y^{d}_{imf}} \cdot \frac{\partial y^{d}_{imf}}{\partial y^{rd}_{imf}}$$

i=1,...,N

f=1,...,F

r=1,...D.

Since:

$$y^{rd}_{im} = \sum_{f} y^{rd}_{imf} \tag{2.39a}$$

and

$$y^{d}_{imf} = \sum_{r} y^{rd}_{imf}, \tag{2.39b}$$

we readily see that the above conditions for profit maximization reduce to:

$$mr^{rd}_{im} = mc^{d}_{imf} \tag{2.40}$$

r=1,...,D

i=1,...,N

f=1,...,F.

Thus, joint profit maximization requires that: (1) the marginal revenue derived from shipping commodity i from region r must be the same for all regions and commodities; (2) the marginal cost of carrying commodity i by firm f must be the same for all firms; and (3) the common marginal revenue must equal the common marginal cost.

The conditions for joint profit-maximization, given in eq.(2.39) yield NFD equations which can be used to solve for the NFD variables, y^{rd}_{imf}. From this, we can obtain the equilibrium rates, and traffic allocations of commodities among firms, under the assumption that the quantities shipped (and implicitly, the prices) of the competing modes are constant.

Of course, a full general equilibrium solution requires that we recognize that rate determination is interdependent. If we also assume that the competing mode(s) is (are) also pursuing a joint profit-maximizing solution, the problem is formally equivalent to one of duopoly or oligopoly. In this case we are faced with the familiar problem of determining the duopolists' behavior, and we can adopt a Cournot, Stackelberg, or even a grand joint profit maximization solution. Because, however, it is likely that the firms in any given transportation mode are somewhat myopic, the Cournot solution seems the most plausible. In this case, each mode pursues its joint-maximizing behavior on the assumption that the output or rates of its competing mode are constant. To solve this problem we simply let the modal index, m, range over the relevant modes, with the understanding that the relevant firms and commodities vary by mode. Thus, the profit-maximization conditions are formally identical to those given in eq. (2.37) and can be written as:

$$\frac{\partial R_{im}^{rd}(y_{im}^{rd}, y_{ic}^{rd}, A_m^d)}{\partial y_{imf}^{rd}} = \frac{\partial C_{mf}^d(y_{imf}^d,\ldots,y_{Nmf}^d, q_{mf}^d)}{\partial y_{imf}^{rd}}$$

$$\frac{\partial R_{ic}^d(y_{im}^{rd}, y_{ic}^{rd}, A_c^d)}{\partial y_{icf}^{rd}} = \frac{\partial C_{cf}^d(y_{icf}^d,\ldots,y_{Ncf}^d, q_{mf}^d)}{\partial y_{icf}^{rd}} \qquad (2.41)$$

where for mode m

$i=1,\ldots,N_m$

$f=1,\ldots,F_m$

$r=1,\ldots,D$

and for mode c

$i=1,\ldots,N_c$

$f=1,\ldots,F_c$

$r=1,\ldots,D$

Hence the above equations yield $(N_m)(F_m)(D)+(N_c)(F_c)(D)$ equations to determine the variables y_{imf}^{rd} and y_{icf}^{rd}.

We could also obtain equilibrium on the assumption that mode m pursued joint profit maximization, while mode c experienced the perfectly competitive solution. In this case we would utilize the equilibrium conditions given in eqs. (2.30) and (2.38) to obtain a full market equilibrium.

Finally, we should also note that in addition to controlling price or outputs, the firm typically controls the level of service as measured by size of shipment and length of haul. Thus in addition to maximizing profi[t] with respect to output, the firm can also maximize with respect to level o[f] service. Thus in addition to the marginal-revenue equals marginal-cost conditions with respect to output given in eq. (2.40), we must utilize add[i]tional marginal-revenue equals marginal cost conditions with respect to level of service.

C. Oligopoly

An oligopolistic market is one in which there are a sufficiently small number of sellers so that the members of the market take the behavior of their competitors into account in setting prices and quantities. Since there are a wide range of postulates concerning the behavior of the oligopolists, there are an equally wide range of solutions to the determination of equilibrium in an oligopolistic market.

In considering oligopolistic behavior, however, it is useful to distinguish between traditional oligopoly theory, which is based on some sort of profit-maximizing behavior, and modern oligopoly theory, which assumes that the firm maximizes a general objective function that contains many arguments other than profits.[15/]

In assuming that all firms maximize profits, traditional oligopoly theory postulates a number of different behavioral responses on the part of the firms in the industry.

Fellner (1949) has argued that oligopolists will tend to collude implicitly and adopt joint profit-maximizing behavior. In this case, the analysis would simply follow that of the previous section.

The Cournot solution was discussed at the end of the previous section. As indicated above, this assumes that although the revenue of each oligopolist depends upon both its own output and that of its

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[15/] For a good summary of oligopoly theory see Baumol (1967).

rivals, each oligopolist is myopic and assumes that its rivals keep their prices and quantities constant. While this assumption may seem reasonable for multi-modal equilibrium, it is probably not very reasonable for intra-modal equilibrium. Hence it is probably unrealistic to adopt a Cournot solution in determining the equilibrium of any given model.

The Stackelberg solution assumes that one member of the oligopoly maximizes its profits given the responses of its competitors, while the remaining members of the oligopoly ignore the behavioral responses of its competitors. Since it is unlikely that firms in an oligopolistic industry actually act according to the Stackelberg solution, a formal analysis of this solution does not seem warranted.

Another well-known variant of oligopoly theory is price leadership. If the leaders set the joint profit-maximizing monopoly price, this analysis merely follows that of the previous section. If the leader is sufficiently dominant, however, it may set its price to maximize its own profits, with its followers acting as price takers. An analysis of market equilibrium in this case would combine elements of the monopoly and perfectly competitive solution. Specifically, the dominant firm would determine its profit maximizing price, and the remaining firms would equate their marginal costs with this price. The equilibrium price would then be determined by the sum of the output of the dominant firm (acting as a monopolist) and the remaining firms in the market (acting as price takers).

In recent years, a number of people have questioned the relevance of traditional oligopoly theory.[16/] Basically, they argue that the

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[16/] See, for example, Baumol (1967), Marris (1964), Cyert and Marsh (1963) behavior of the large modern corporation is much more complicated than that implied by the simple behavioral postulates of profit maximization. Thus instead of maximizing profits, the modern corporation probably maximizes a complex objective function that includes variables such as market share, sales, and net workth, as well as profits. In this case, the conventional analysis that follows the profit maximizing postulate has to be modified accordingly.

Although this research has not yet developed a formal framework that could be used to analyze the market equilibrium tht would occur under broader objective functions than that implied by simple profit maximization, such an extention is planned. Thus our present analysis is limited to examples of traditional oligopolistic behavior. Nevertheless, it is hoped to extend this to other forms of oligopolistic behavior in the near future.

D. Monopolistic Competition

With the exception of the railroads and the airlines, transportation industries tend to be characterized by large numbers of carriers rather than a few. Hence models of monopolistic competition may give a better description of the behavior of their industries than models of oligopoly. We thus now analyze the the determination of industry equilibrium under a market structure characterized by monopolistic competition.

As explained above (see eqs.(2.21), (2.22)), we assume that each firm's demand function is some proportion of the market demand function. Thus:

$$T^{d}_{imf} = \mu^{d}_{mf}\ T^{d}_{im}$$

where μ^{d}_{mf} represents the share of ton-miles carried in region d by firm f in mode m.

Since price is determined by the market demand function, each firm's revenue must also be a proportion of total revenue. Hence:

$$R^{rd}_{imf} = \mu^{d}_{mf} \cdot R^{rd}_{im}(P^{rd}_{im},\ P^{rd}_{ic},\ A^{d}_{m}) \tag{2.42}$$

Consequently, each firm's profits are given by:

$$\Pi_f = \mu^{d}_{mf} \sum_i \sum_r R^{rd}_{im}(P^{rd}_{im},\ P^{rd}_{ic},\ A^{d}_{m}) - C^{d}_{mf}(y^{d}_{imf},\ldots,y^{d}_{Nmf},\ q^{d}_{mf}) \tag{2.43}$$

Profit maximization thus requires that

$$\mu^{d}_{mf} \sum_i \sum_r \frac{\partial R^{rd}_{im}}{\partial y^{rd}_{im}} \ \frac{\partial y^{rd}_{im}}{\partial y^{rd}_{imf}} = \frac{\partial C^{d}_{mf}}{\partial y^{d}_{imf}} \tag{2.44}$$

By using the relationship that $\sum_f y^{rd}_{imf} = y^{rd}_{im}$, we thus obtain a system of NDF simultaneous equations that can be used to solve for each y^{rd}_{imf} and yield equilibrium values of P^{rd}_{im}.[17/]

[17/] Since μ^{d}_{mf} depends on the equipment utilized by firm f in mode m, we may also want to treat this as a control variable.

As in the previous cases, a full market equilibrium occurs when all transportation markets are in equilibrium. This can be obtained by making the appropriate assumptions about the market behavior of the competing mode(s) and solving the expanded set of equations for y^{rd}_{imf} and y^{rd}_{icf} to obtain P^{rd}_{im} and P^{rd}_{ic}.

This analysis deviates from the traditional Chamberlinian formulation of the problem, which assumes that it is possible to estimate demand functions for each firm that depend on the outputs of the competing firms as well as its own output. Since this formulation is more conventional, it is useful to present it, although it is important to realize that data limitations will probably make its implementation impossible.

In the case of Chamberlinian monopolistic competition, each firm's demand function can be written as:

$$P^{rd}_{imf} = P^{rd}_{imf}(y^{rd}_{imf}, \sum_{\substack{h=1 \\ h\neq f}}^{F} y^{rd}_{imh}, y^{rd}_{ic}, A^{d}_{mf}) \tag{2.45}$$

where P^{rd}_{imf} = the demand price facing firm f in mode m for commodity i shipped from region r.

y^{rd}_{imf} = amount of commodity i shipped by firm f in mode m from region r.

y^{rd}_{imh} = amount of commodity i shipped by firm h in mode m from region d.

y^{rd}_{ic} = amount of commodity i shipped by mode c from region r.

A^{d}_{mf} = other factors affecting the demand schedule of firm f in mode m.

The costs of firm f depend upon its outputs and factor prices and thus are given by:

$$C^{d}_{mf} = C^{d}_{mf}({}^{d}_{imf}, \ldots, {}^{d}_{Nmf}, q^{d}_{f}) \qquad (2.46)$$

Thus, the firm's profits are given by:

$$\sum_{r} \sum_{i} R^{rd}_{imf}(y^{rd}_{imf}, \sum_{\substack{h=1 \\ h \neq f}}^{F} y^{rd}_{imh}, y^{rd}_{ic}, A^{d}_{f}) \qquad (2.47)$$

Equilibrium is obtained by differentiating the above expression with respect to y^{rd}_{imf} and solving the resulting system of simultaneous equations. We can also obtain a full market equilibrium by assuming that the competing modes can be described by perfect competition, joint monopoly profit maximization, or monopolistic competition. Thus, if we can determine firm demand functions that utilize the output of other firms as arguments, it should be possible to utilize a conventional Chamberlinian analysis to determine market equilibrium. Otherwise, we will utilize a similar analysis, with the firm demand functions based on market shares.

IV. Policy Analysis

Having specified the nature of the cost and demand functions and the determination of the modal and industry equilibrium under alternative market structures, it is useful to consider briefly how alternative transportation policies could be evaluated within this modeling and framework. A full discussion of the policy analysis is contained in Volume II of this report.

The methodological approach to the evaluation of transportation policies with respect to the surface freight industries is comparative statistics. We thus derive an initial equilibrium under a set of initial conditions concerning the cost functions, demand functions, and the competitive behavior of the firms in the transportation industries. We then postulate a change in transportation policy that affects these initial conditions and determine the new equilibrium resulting from these changes. The differences in the relevant variables between the initial and new equilibrium then measures the impact of a given policy.

As indicated above, we analyze changes in transportation policies by relating them to changes in the cost functions, the demand functions, or the competitive structure of the affected transportation industry. Within this framework, however, it is possible to evaluate a wide range of transportation policies.

Figure 3 indicates whether various transportation policies affect the cost or demand functions or the competitive structure of the industry. This indicates that policies generally fall into one of the following categories:

- Those that affect the demand function alone
  - Permissible price discrimination
  - Setting rate levels
- Those that affect both the demand function and the market structure
  - Elimination of rate bureaus
  - Total deregulation of rates
  - Entry controls
- Those that affect the cost function through factor prices
  - Wage settlements
  - Energy policy
  - User charges and subsidies
- Those that affect the cost function through factor utilization
  - Abandonment
  - Union work rules
  - Provision of infrastructure
  - Weight and size limitations
  - Nationalization of the roadbed
- Those that affect cost functions, demand functions, and market struct
  - Mergers and consolidation

FIGURE 3

	Demand Function	Market Structure	Cost Function
Permissible Price Discrimination	X	-	-
Setting Rate Levels	X	-	-
Total Rate Deregulation	X	X	-
Elimination of Rate Bureaus	X	X	-
Entry Controls	X	X	-
Subsidies	-	-	X
Energy Policy	-	-	X
User Charges	-	-	X
Abandonment	-	-	X
Union Work Rules	-	-	X
Provision of Infrastructure	-	-	X
Weight and Size Limitations	-	-	X
Roadbed Nationalization	-	-	X
Mergers and Consolidation	X	X	X

Thus by translating changes in specific transportation policies into changes in cost functions, demand functions, the competitive structure of the industry and comparing the resulting equilibria with respect to rates, traffic, profits, etc., it should be possible to obtain quantitative estimates of the impact of alternative transportation policies. It is important to stress, however, that this analysis requires a careful translation of the specific policy into a specific change in the appropriate function. While some policies, such as changes in the level of user fuel taxes, can be analyzed fairly simply, other policies, such as the nationalization of the railroad roadbed, require a major research effort to translate them into appropriate changes in the relevan functions. Thus any specific policy will generally require considerable effort to ensure that the specified change in the cost or demand function or market structure accurately reflects the direct impact of the proposed policy. Nevertheless, such an analysis should be feasible and provide a valuable aid in the decision making process of policy makers.

Chapter Three

Econometric Estimation of Cost Functions in the Transportation Industries

The previous chapter argued that it is desirable to use a general translog approximation when estimating cost functions in the transportation industries. Briefly stated, unlike other cost functions the translog cost function permits multiple outputs and quality levels; it is of a sufficiently flexible form to test hypotheses concerning the underlying structure of production; and it can be used in either its short-run or long-run form.

This chapter discusses the translog cost functions in some detail and shows how various kinds of information concerning the structure of costs and technology can be obtained from it. Section I presents a translog approximation to a general function and explores the relationship between the translog approximation and its underlying function. Section II then discusses the relationships between the short-run cost function, the long-run cost function, the underlying production functions, and their translog approximations. Section III considers the problems posed by the need for aggregation and the existence of different quality levels. Section IV presents the restrictions that can be imposed in the short-run and long-run cost functions to test for jointness, separability, and homogeneity in the underlying production function and discusses the econometric specification of these equations.

I. The Translog Function

In recent years increasing attention has been paid to a number of general cost and production functions that are second-order approximation to any given cost or production function. These functions can therefore deal with multiple outputs, variable elasticities of substitution among factors and variable elasticities of transformation among outputs, etc. As such, they offer substantial gains in flexibility, and enable the researcher to test important hypotheses concerning the underlying structure of production.[1/] The best known functional forms of second-order approximations are the transcendental logarithmic functions proposed by Christensen, Jorgenson and Lau (1973), the generalized Leontief function proposed by Diewert (1971), and the Hall joint cost function (1973). Because, however, the translog function explicitly permits multiple outputs, and enables us to test for separability, homogeneity and nonjoint production, we shall concentrate upon it.[2/]

Since any translog function is a second-order approximation about an arbitrary point of expansion, it is useful to consider its specific construction and its accuracy as the point of observation diverges from the point of approximation. We will then consider the relationship between the coefficients of the translog function and the derivatives of its underlying function.

A. The Translog Approximation

The conventional translog function can be interpreted as a Taylor's

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[1/] For an example of this, see Christensen, Jorgenson and Lau (1973).

[2/] The Hall joint cost function also permits tests of separability and jointness with multiple outputs, but assumes constant returns to scale in production.

series approximation to the function $\ell n\ g(x)$ about the unit point. To see this, we recall that any continuous function $f(z)$ obeys

$$f(z) = f(z_o) + (z-z_o)\ f_i(z_o) + \frac{1}{2}\ (z-z_o)[f_{ij}(z_o)](z-z_o)' \quad (3.1)$$
$$+ \text{ higher order terms}$$

Where z is the vector of arguments of $f(z)$; z_o is the vector of arbitrary points of evaluations; and f_i and f_{ij} respectively represent the first and second derivatives of $f(z)$ with respect to its arguments. The second-order Taylor's approximation $\hat{f}(z)$ is simply given by

$$\hat{f}(z) = f(z_o) + (z-z_o)\ f_i(z_o) + \frac{1}{2}(z-z_o)[f_{ij}(z_o)](z-z_o)' \quad (3.1a)$$

Suppose we now want to derive the translog approximation to $g(x)$, where x is a vector of positive numbers. We do this in two steps. First, we construct an exact function f, satisfying $f(\ell nx) = \ell n\ g(x)$; and second, we write $z = \ell n\ x$ and form the Taylor's approximation $\hat{f}(z)$ to $f(z)$ given in eq. (3.1a). The first step is carried out by replacing each x_i in the function $g(x)$ by $e^{\ell nx_i}$ and then by taking the log of the resulting function. This yields $f(\ell n\ x) = f(z)$. We then obtain $\hat{f}(z) \equiv \ell n\ \hat{g}(x)$ by applying eq. (3.1a) to $f(z)$; thus:

$$\ell n\ \hat{g}(x) = f(\ell nx_o)+(\ell nx-\ell nx_o)f_i(\ell nx_o) + \frac{1}{2}(\ell nx-\ell nx_o) [f_{ij}(\ell nx_o)](\ell nx-\ nx_o)' \quad (3.2)$$

If the function is evaluated at the unit point so that $x_o = (1,\ldots,1)$, $\ell nx_o = (0,\ldots,0)$ and eq. (3.2) reduces to

$$\ell n\ \hat{g}(x) = f(0) + \ell nx[f_i(0)] + \frac{1}{2}\ \ell nx[f_{ij}(0)]\ell nx' \quad (3.2a)$$

This, of course, is precisely the conventional translog function, which is written as

$$\ell n\ g(x) = \alpha_o + \sum \alpha_i \ell nx_i + \frac{1}{2} \sum_i \sum_j \beta_{ij}\ \ell n\ x_i\ \ell n\ x_j \quad (3.2b)$$

where $\alpha_0 = f(0)$, $\alpha_i = f_i(0)$, and $\beta_{ij} = f_{ij}(0)$. Thus, in the conventional translog function, the constant term, the α_i's, and the β_{ij}'s can be respectively interpreted as the value, the first derivatives, and the second derivatives of the log of the underlying function whose arguments are evaluated at the unit point. Note that from the symmetry of the second derivatives $\beta_{ij} = \beta_{ji}$.

Because the conventional translog approximation represents an expansion around the unit point, it may be a rather poor approximation if the actual values of the variables are far removed from the unit point.[3/] This implies that it may be desirable to utilize the general translog function, given in eq. (3.2), which represents the Taylor's approximating function $\ell n\ \hat{g}(x)$ to the function $\ell n\ g(x)$ about an arbitrary point of expansion x_o. Thus in utilizing translog approximations, instead of using the unit point as the point of approximation it may be preferable to use either the sample mean or the current value of the variable as the point of approximation and to use eq. (3.2) instead of eq. (3.2b) as the specific form of the translog function. In this case the estimating equation would take the following form:

$$\ell n\ \hat{g}(x) = a_o + \sum_i a_i(\ell n x_i - \ell n x_{oi}) + \frac{1}{2}\left(\sum_i \sum_j \beta_{ij}(\ell n x_i - \ell n x_{oi})(\ell n x_j - \ell n x_{oj})\right) \qquad (3.3)$$

[3/] Burgess (1975) has argued that the translog functions are not self dual in the sense that the production functions cannot be obtained from the cost functions and vice versa. Moreover, because the approximation error depends upon the divergence between the point of evaluation and the point of approximation, different points of approximation used in the cost and production functions can lead to different estimates of the underlying technology. Consequently, estimates of technology based on cost and production functions may be inconsistent. However, these problems may occur because the point of approximation is far removed from the actual point of evaluation; hence they may be resolved if the point of approximation were closer to the point of evaluation.

where $a_o = f(\ell n x_o)$

$a_i = f_i(\ell n x_o)$

$\beta_{ij} = f_{ij}(\ell n x_o)$

Thus in eq. (3.3) the constant, a_i coefficients and the β_{ij} coefficients can be interpreted as the value, first, and second derivatives (with respect to ℓn x) of the log of the underlying function whose arguments are evaluated at the arbitrary point of expansion x_o.[4/]

B. The Translog Function and its Underlying Function

Because the coefficients of the translog function can be interpreted as the logarithmic value, logarithmic gradient, and logarithmic Hessian of the underlying function, evaluated at the point of expansion (the vector (1,...,1) in case of the conventional translog function, and the vector (x_o) in the case of the general translog function), we cannot directly infer anything about the value, gradient or Hessian of the underlying function at its point of expansion by simple inspection of the coefficients. Since, however, we are interested in the total costs, marginal costs, and the second derivatives of the cost function at the point of output, we must find a way to translate the coefficients of the translog function into the value, gradient, and Hessian of the underlying function.

A function $\hat{g}(x)$ is said to provide a second-order numerical approximation to a given function g(x), if its value, gradient, and Hessian at the point of approximation equals the value, gradient, and Hessian of g(x) at that point. That is,

$$\hat{g}(x_o) = g(x_o) \qquad (3.4a)$$

$$\hat{g}_i(x_o) = g_i(x_o) \qquad (3.4b)$$

[4/] While it is true that the application of OLS to (3.3) and (3.2b) gives identical results (in the sense that the coefficient estimates of (3.3) will be a nonsingular transformation, uniquely determined by x^o, of the coefficient estimates of (3.2b)) when there are no restrictions on the coefficients, this is not generally true when there are restrictions, as is usually the case.

$$\hat{g}_{ij}(x_o) = g_{ij}(x_o) \tag{3.4c}$$

Since a translog approximation is a second-order numerical approximation, by differentiating the translog function given above in eq. (3.3) and by utilizing the equalities given in eq. (3.4), we can determine the relationship between the coefficients of the translog function and the derivatives of the underlying function.

Since eq. (3.3) is a logarithmic function, we know that

$$\frac{\partial g(x)}{\partial x_i} = \frac{\partial \log \hat{g}(x)}{\partial \log x_i} \cdot \frac{\hat{g}(x)}{x_i}$$

Hence, by differentiating eq. (3.3) we obtain

$$\hat{g}_i = [\alpha_i + \sum_h \beta_{ih}(\ell n x_h - \ell n x_{oh})] \frac{\hat{g}}{x_i} \tag{3.5a}$$

$$\hat{g}_{ij} = \frac{\hat{g}_i \hat{g}_j}{\hat{g}} + \frac{\hat{g}\,\beta_{ij}}{x_i x_j} \tag{3.5b}$$

$$\hat{g}_{ii} = \frac{-\hat{g}_i}{x_i} + \frac{\hat{g}_i^{\,2}}{\hat{g}} + \frac{g\beta_{ii}}{x_i^{\,2}} \tag{3.5c}$$

Using the equalities given in eq. (3.4) and solving for α_i, β_{ij}, and β_{ii}, we therefore obtain

$$\alpha_i = \frac{x_i g_i}{g} - \sum_h \beta_{ih}(\ell n x_h - \ell n x_{oh}) \tag{3.6a}$$

$$\beta_{ij} = (g_{ij} - \frac{g_i g_j}{g}) \frac{x_i x_j}{g} \tag{3.6b}$$

$$\beta_{ii} = [g_{ii} + \frac{g_i}{x_i} - \frac{g_i}{g}] \frac{x_i^{\,2}}{g} \tag{3.6c}$$

Finally. since

$$\alpha_o = \ell n g\,(x_o) - \sum \alpha_i(\ell n x_i - \ell n x_{oi}) - \frac{1}{2} \sum_i \sum_j \beta_{ij}\,(\ell n x_i - \ell n x_{oi})(\ell n x_j - \ell n x_{oj}) \tag{3.6d}$$

we can obtain $\ell n\hat{g}(x_0)$ directly and from that, $\ell ng(x_0)$ and thus $g(x_0)$. Thus using the translog approximation, we can obtain the value, gradient, and Hessian of the underlying function at the point of expansion x_0. This property will prove to be extremely useful when we attempt to derive marginal costs and the structure of the underlying production function from the estimated translog cost functions.

II. The Long-Run Cost Function and Production Function

Let us assume that we have estimated a short-run variable cost function using the translog approximation. From this function, we can derive estimates of short-run total costs, the short-run variable costs, the short-run marginal costs and the change in the short-run marginal costs, all around the relevant point of expansion, using the relationships given in Section I. For policy purposes, however, it is desirable to know the nature of the long-run equilibrium total costs, marginal costs, factor demands, and the underlying structure of production. In this section, we therefore show how the long-run cost function can be determined from the estimated short-run cost function and how the production function can be obtained from the long-run cost function.

A. The Long-Run Cost Function

We assume that the firm produces m outputs, (denoted by the vector y) and utilizes n inputs (denoted by the vector x) at given prices (denoted by the vector w). Then the firm's long-run total cost function, $C(y,w)$ gives the minimum cost of producing outputs y at factor prices w. In the short-run the firm cannot adjust all of its factors, and without loss of generality we assume that the first factor, $\bar{x}_1$, is fixed. Then its short-run variable cost function $C(y,\bar{x}_1,w_2,\ldots,w_n)$ represents the minimum costs of producing a given output y, exclusive of the costs associated with the fixed factor. The short-run total costs represent the sum of the fixed and variable costs and are given by $C(y,\bar{x}_1,w_2,\ldots,w_n) + w_1\bar{x}_1$.

Let the vector $q = (y, w_2, w_3,\ldots, w_n)$. Then $C(q,w_1)$ represents the long-run cost function, and $\tilde{C}(q,\bar{x}_1)$ represents the short-run variable cost function. The problem at hand is stated as follows: Given that we have a translog approximation of $\tilde{C}(q,\bar{x}_1)$ can we derive the long-run cost function

$C(q,w_1)$? The answer to this question is clearly "yes" if we have an observed point of long-run equilibrium; the answer is tentatively "yes" if we do not have an observed point of long-run equilibrium.

The problem arises because the translog function only represents an approximation around a given point of expansion and may not be a good representation of the underlying function over the entire range. Let us proceed, however, as if the translog function were a true representation of the underlying function and then discuss how our procedure must be modified to take approximating errors into account.

1. No Approximating Errors

We have shown in Section I.B, above, that there is a unique relationship between the value, gradient, and Hessian of the underlying cost functions and the coefficients of the translog cost function. Therefore, given the estimated translog short-run variable cost function, we can derive the value, first derivative, and second derivative of the under-cost function. If we can then derive the value, gradient, and Hessian of the long-run cost function from their short-run counterparts at a given point, we can also construct a translog long-run cost function. Given this, we can then determine the long-run total cost, marginal costs, and factor demands for any specified levels of output or factor prices (assuming, of course, that the translog approximation is valid over the relevant range).

Let us begin by assuming that we have derived the appropriate short-run variable cost function $\tilde{C}(q,\bar{x}_1)$ from its translog approximation.[5/] Since the short-run total cost function is given by

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[5/] If we treat the translog approximation as an exact function, then

$$\frac{\partial \tilde{C}(q,x_1)}{\partial q} = \frac{\partial \ell n \tilde{C}(q,x_1)}{\partial \ell n q} \cdot \frac{\tilde{C}}{q}$$

The precise expression for this is given in eq. (3.5a) above.

$$\bar{C}(q,\bar{x}_1) = \tilde{C}(q,\bar{x}_1) + w_1 x_1 \tag{3.7}$$

The equilibrium demand $\bar{x}_1$ is given by solving the relationship

$$\frac{\partial \tilde{C}(q,\bar{x}_1)}{\partial x} = -w_1 \tag{3.8}$$

We also know that in equilibrium, long-run marginal costs must equal short-run marginal costs and that variable factor demands must be identical:

$$\frac{\partial \tilde{C}(q,x_1)}{\partial q} = \frac{\partial C(q,w_1)}{\partial q} \tag{3.9}$$

Finally, from Shephard's Lemma, we know that the long-run equilibrium quantity of the fixed factor is given by

$$\frac{\partial C(q,w_1)}{\partial w_i} = x_1 \tag{3.10}$$

Using the relationships given in eq. (3.8) we can therefore write that

$$\frac{\partial C(q,w_1)}{\partial w_i} = \frac{-\partial \tilde{C}(q,x_1)}{\partial x_i} \cdot \frac{x_1}{w_1} \tag{3.11}$$

It is useful to rewrite equations (3.9) - (3.11) in compact notation as a system of q+2 simultaneous equations in the following variables: C_q, C_{w_1}, and x_1.

$$\tilde{C}_q - C_q = 0 \tag{3.9a}$$

$$\tilde{C}_{x_1}(x_1/w_1) + C_{w_1} = 0 \tag{3.10a}$$

$$x_1 - C_{w_1} = 0 \tag{3.11a}$$

By solving this system we can thus obtain the long-run equilibrium value of the fixed factor x_1, and the long-run marginal costs with respect to outputs and the price of the variable factors, C_q and the long-run marginal costs with respect to the price of the fixed factor, C_{w_1}.

To construct the long-run cost function, we also need to know its second derivatives. We can obtain these by performing a comparative statics experiment in which we treat C_q, C_{w_1}, and x_1 as endogenous and q and w_1 as exogeneous and then determine how C_q and C_{w_1} change in response to changes in q and w_1; that is, we determine C_{qq}, C_{qw_1}, and $C_{w_1w_1}$.[6] Since eq. (3.11a) relates x_1 to C_{w_1}, we can substitute for x_1 to obtain the following expressions.[7]

$$\tilde{C}_q(q,C_{w_1}) - C_q = 0 \tag{3.12a}$$

$$C_{x_1}(q,C_{w_1})\frac{C_{w_1}}{w_1} + C_{w_1} = 0 \tag{3.12b}$$

By implicitly differentiating eq. (3.12) with respect to q and w_1 we obtain

$$\begin{bmatrix} C_{qq} & C_{qw_1} \\ C_{w_1q} & C_{w_1w_1} \end{bmatrix} = \begin{bmatrix} -I & C_{qx} \\ 0 & Q \end{bmatrix}^{-1} \begin{bmatrix} C_{qq} & 0 \\ C_{x_1q}\frac{C_{w_1}}{w_1} & -\frac{C_{x_1}C_{w_1}}{w_1^2} \end{bmatrix} \tag{3.13}$$

where $Q = C_{x_1x_1}\frac{C_{w_1}}{w_1}$ (since $C_{x_1}/w_1 + 1 = 0$).

6/ See Appendix 3.A for a brief discussion of comparative statics.

7/ Note that $\tilde{C}_q = \tilde{C}_q(q,x_1)$ and that $\tilde{C}_{x_1}(q,x_1)$. But since $x_1 = C_{w_1}$, $\tilde{C}_q = \tilde{C}_q(q,C_{w_1})$ and $\tilde{C}_{x_1} = \tilde{C}_{x_1}(q,C_{w_1})$.

Solving, we obtain

$$C_{qq} = \tilde{C}_{qq} + [\tilde{C}_{x_1x_1}]^{-1}[C_{qx_1}][C_{x_1q}] \quad (3.14a)$$

$$C_{qw_1} = -[\tilde{C}_{x_1x_1}]^{-1}[\tilde{C}_{qx_1}] \quad (3.14b)$$

$$C_{w_1q} = -[\tilde{C}_{x_1q}][\tilde{C}_{x_1x_1}]^{-1} \quad (3.14c)$$

$$C_{w_1w_1} = -[\tilde{C}_{x_1x_1}]^{-1} \quad (3.14d)$$

Having obtained the first and second derivatives of the long-run cost function, it is possible to construct a general translog long-run cost function of the form given in eq. (3.3). If we take the solution value of x_1 and the associated values of the input prices and outputs, we can obtain the coefficients of the general translog cost function from eq. (3.6) at that point of equilibrium. Given these, we can readily obtain a general translog long-run cost function that can be interpreted as a second-order approximation to the underlying long-run cost functions. This can be used to estimate industry and firm costs in long-run equilibrium under different output levels and factor prices.

2. Approximating Error

Up to now we have assumed that the translog approximation is a good representation of the true cost function and proceeded as if the translog approximation was an exact function. Because, however, the translog approximation is only valid around a point of expansion, it is likely that then it will not be valid over the entire range. Specifically, if we use the sample mean as the point of expansion, the translog function may not be a good approximation around the unobserved point of long-run equilibrium.

Since approximation error grows as the distance between the point of expansion and point of evaluation grows, approximation error will be minimized if we choose the point of long-run equilibrium as the point of expansion.

Unfortunately, we have by hypothesis not actually observed any point of long-run equilibrium. Thus, we are required to estimate one of the infinitely many possible. Under the weak regularity conditions usually assumed of technologies, we know that there exists a vector of prices for the fixed factors such that, at the observed mean values of $y, \bar{x}$, and w, equilibrium would obtain. This is the vector of fixed factor prices that would lead a cost-minimizing firm to voluntarily use the mean values of the fixed factor quantities $\bar{x}$ to produce the mean outputs y at mean variable factor prices w. Denote this vector of fixed factor prices $\bar{w}^*$, and the mean values of $y, \bar{x}$, and w by y^*, $\bar{x}^*$, and w^*, respectively. Then from eq. (3.8) we know that $\tilde{C}_{xr} = -\bar{w}_r^*$. At y^*, $\bar{x}^*$, and w^*, we have:

$$\tilde{C}_{\bar{x}_r} = b_r \frac{C(y^*, w^*, \bar{x}^*)}{\bar{x}_r^*} = -\bar{w}_r^* \tag{3.15}$$

But $\tilde{C}(y^*, w^*, \bar{x}^*) = e^{a_0}$, since a_0, the constant term in the translog $\tilde{C}(y,w,\bar{x})$, is the value of $\ln \tilde{C}(y,w,\bar{x})$ at $y = y^*$, $w = w^*$, and $\bar{x} = x^*$, by the properties of second-order numerical approximation. Thus

$$b_r = \frac{-\bar{w}_r^* \bar{x}_r^*}{e^{a_0}} \qquad r=1,\ldots,f \tag{3.16}$$

This requires no constraints in the estimation of $\tilde{C}(y,w,\bar{x})$, except that $b_r \leq 0$ for $\bar{x}_r^*$ being marginally productive (i.e. cost-lowering), if one is satisfied to approximate the technology around $\bar{x}_r^*$. On the other hand, if $\bar{x}_r$ has been, say, in chronic excess supply (due to regulation, perhaps), then one may wish to estimate $\bar{x}_r$ corresponding to a given $\bar{w}_r$, as this point of expansion (and, therefore, evaluation) would give estimates that would better approximate the technology after the chronic excess supply of $\bar{x}_r$ were relieved -- for example, after deregulation. In this case, eq. (3.16) would require direct implementation by substitution in the estimating equations, and $\ell n\ \bar{x}_r - \ell n\ \bar{x}_r^o$ would be replaced as a variable by $\ell n\ \bar{x}_r - \ell n\ \bar{x}_r^*$, where $\bar{x}_r^*$ was not the mean of $\bar{x}_r$ but a parameter to be estimated.

B. The Production Function

Just as it is possible to derive a translog approximation to the long-run cost function from the value, gradient, and Hessian of the short-run cost function, it is possible to derive a translog approximation to the underlying production function has the value, gradient and Hessian of the long-run cost function. Thus, using the derived long-run cost function, we can construct its associated production function.

Let us begin by writing the production function as

$$\ell = F(y,x) \qquad (3.17)$$

where y represents the m component vector of outputs, x represents the n-1 component of inputs and ℓ represents the n^{th} factor, labor. Thus, ℓ represents the amount of labor required to produce y with factor inputs x. Writing the production function in this form implies separability between the outputs and the factor ℓ, but not separability between the output and the remaining factors x. Consequently, writing the production function in the form given in eq. (3.17) imposes some restrictions, although they do not appear to be too severe.[8/]

Since $C(y,w)$ is homogeneous of degree one in w, we can divide both sides of the long-run cost function by w to obtain a normalized long-run cost function with the following form:

$$C^* = C^*(y,w^*) \tag{3.18}$$

where $C^* = C/w_\ell$ and $w^* = (w_1/w_\ell,\ldots,w_{n-1}/w_\ell)$.

In equilibrium, we know that the production function is related to the cost function by the following three relationships:

$$F_y(y,x) - C^*_y(y,w^*) = 0 \tag{3.19}$$

$$F_x(y,x) + w^* = 0 \tag{3.20}$$

$$C^*_{w^*}(y,w^*) - x = 0 \tag{3.21}$$

Equation (3.19) states the equilibrium condition that the marginal rate of transformation equals the ratio of marginal costs, with labor as the numeraire.[9/] Equation (3.20) is the familiar condition of cost minimization that the ratio of the marginal products of the factors equals the ratio of the factor prices, with labor again being treated as the numeraire. Equation (3.21) is simply Shephard's Lemma, which states that in equilibrium the factor demand is given by the partial derivative of the cost function with respect to factor price.

Substituting eq. (3.21) into eqs. (3.20) and (3.19) we obtain

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[8/] The full implications of writing the production function in this form are described in Jorgenson and Lau (1974, 1975) and in Lau (1976). Note that it is not necessary that labor be the normalizing factor, although it is often more convenient to have labor act in this capacity.

[9/] Since both the production function and the cost function are normalized on labor, eq. (3.19) can be interpreted in terms of marginal ratio of transformation and ratios of marginal costs. While we could clearly normalize on any factor, it seems natural to treat labor as the numeraire factor. For a full discussion of the "Generalized Hotellings Lemma", see Lau (1976).

$$F_y[y, C^*_{w^*}(y,w^*)] - C^*_y(y,w^*) = 0 \quad (3.19a$$

$$F_x[y, C^*_{w^*}(y,w^*)] + w^* = 0 \quad (3.20a$$

Equations (3.19a) and (3.20a) yield a system of m+n-1 equations that can be used to solve for the m marginal rates of transformation, F_y, and the n-1 marginal products of the factors, F_x.

We are now in a position to perform a comparative statics experiment in which we treat y and w* as exogenous variables and F_y, F_x as the endogenous variables. Thus differentiating eq. (3.20a) and (3.19a) with respect to w* and y respectively yields

$$F_{xx}C^*_{w^*w^*} + I = 0 \quad (3.22a)$$

$$F_{xy} + F_{xx}C^*_{w^*y} = 0 \quad (3.22b)$$

$$F_{yx}C^*_{w^*w^*} - C^*_{yw^*} = 0 \quad (3.22c)$$

$$F_{yy} + F_{yx}C^*_{w^*y} - C^*_{yy} = 0 \quad (3.22d)$$

Solving for F_{xx}, F_{xy}, F_{yx}, F_{yy} we readily obtain

$$F_{xx} = -[C^*_{w^*w^*}]^{-1} \quad (3.23a)$$

$$F_{xy} = - F_{xx}C^*_{w^*y} \quad (3.23b)$$

$$F_{yx} = - C^*_{yw^*}F_{xx} \quad (3.23c)$$

$$F_{yy} = C^*_{yy} - F_{yx}C^*_{w^*y} \quad (3.23d)$$

Equations (3.23a) - (3.23d) completely characterize the relationship between the Hessians of F(y,x) and the Hessians of C*(y,w*), while equations (3.19) - (3.22) completely characterize the gradient. Therefore, we can construct a Taylor's approximation (at an arbitrary point) to the production function that is dual to the long-run cost function. If the translog cost function is derived around a point of long-run equilibrium, it should be a reasonable approximation of the underlying cost function, and the production function derived from it should be a reasonable approximation of the underlying technology.

To recapitulate briefly, having estimated a translog approximation to the short-run variable cost functions around a long-run equilibrium, it is possible to determine the short-run marginal costs and factor demands at that point. From this translog approximation to the short-run variable cost function, we can then derive the value, gradient, and Hessian of the underlying short-run variable cost function and from these, the value, gradient and Hessian of the underlying long-run cost function. Given these, we can then not only construct the translog approximation to the cost function, which can be used to estimate long-run marginal costs and factor demands at points other than one of long-run equilibrium, but also the value, gradient, and Hessian of the underlying production function; and these latter variables can be used to construct a translog approximation to the underlying production function. Consequently, estimating a translog approximation to a short-run variable cost function around an arbitrary point, it is possible to derive estimates of short-run marginal costs and factor demands; long-run marginal costs and factor demands; and the underlying structure of technology.

III. Aggregation and Quality Differentials

Most analyses of transportation cost functions use gross measures of outputs and inputs such as ton-miles and total rolling stock. Since however, the output mix as well as the way the good is carried (size of shipment, length of haul, etc.) can affect carrier costs, it is necessary to disaggregate ton-miles in such a way that reflects these differences. Similarly, because the mix of rolling stock and the compositio of that work force can also affect carrier costs, differences in the composi tion of the various types of factors must be taken into account. Nevertheless, because transportation firms normally produce a wide range of outputs at different levels of quality and also utilize a wide range of inputs, also with different levels of quality, it is virtually impossible to introduce specific variables in the cost function for each type of output and each type of specific factor. Thus aggregation of factors and outputs is necessary. This section, therefore, discusses various proposed approaches to aggregation which ensure that, under certain conditions, the underlying cost function will not be mis-specified by the aggregation procedure.

A. Attributes of Aggregation Functions

Suppose that a given firm has N "micro" factors, denoted by x's, which it uses to produce M micro outputs, denoted by y's, according to the general transformation function

$$t(y_1, \ldots, y_M; x_1, \ldots, x_N) = 0 \tag{3.24}$$

Let us stratify the outputs and factors into the mutually exclusive and exhaustive output category vectors $y^1, \ldots, y^m$ and input category vectors $x^1, \ldots, x^n$, each of possibly different lengths, so that each micro output or input is included in one (and only one) category.

Let us furthermore define an aggregate output or factor measure as

$$Y^i = H^i(y^i) \tag{3.25a}$$

$$X^j = G^j(x^j) \tag{3.25b}$$

Then the functions $H^i(y^i)$ and $G^j(x^j)$ are acceptable aggregations if and only if a transformation equivalence is satisfied such that:

$$t(Y^1,\ldots, Y^m; X^1,\ldots,X^n) = 0 \tag{3.26a}$$

is equivalent to

$$t(y_1,\ldots, y_M; x_1,\ldots,x_N) = 0 \tag{3.26b}$$

We can similarly express aggregation in terms of cost functions. Thus let us define an aggregate factor price measure as

$$W^j = E^j(w^j) \tag{3.26c}$$

where w^j represents the vector of factor prices corresponding to the factor vector x^j. Then the functions $H^i(y^i)$ and $E^j(w^j)$ are acceptable aggregations if and only if the cost equivalence is such that

$$C(y, w) = C(y_1,\ldots, y_M; w_1,\ldots, w_N) \tag{3.27a}$$

$$= C(Y^1,\ldots, Y^m; W^1,\ldots, W^n) \tag{3.27b}$$

where $Y^i = H^i(y^i)$ and $W^j = E^j(w^j)$

Thus the problem at hand is to determine the functional form of H^i and E^j that will ensure that the transformation and cost equivalencies are satisfied.

Clearly simple addition to obtain output measures such as ton-miles or input measures such as freight-cars will not in general satisfy these conditions.[10/] Even if two firms produce the identical number of ton miles, one would expect their costs to be different if (for rail) one was specialized in coal traffic and the other was specialized in TOFC; or if (for truck) one was specialized in truckload traffic and the other was specialized in less-than-truckload traffic. Similarly, one would expect costs to vary among firms as the composition the fleet or rolling stock varied. Consequently, any acceptable aggregat must be able to take these differences into account.

In recent years, a considerable amount of work has been done on aggregation functions and index numbers,[11/] and it has been shown that if the aggregation functions, H^i, G^j, and E^j are homothetic, it is possible to construct index numbers of the aggregation of y^i, x^j, and w^j which act in well behaved manner.[12/] Homothetic functions are formally defined as a monotonic transformation of a homogeneous function. Thus

$$H^i = h^i[f^i_y\ (y^i)] \qquad (3.28a)$$

$$G^j = g^j[f^j_x\ (x^j)] \qquad (3.28b)$$

$$E^j = e^j[f^j_w\ (w^j)] \qquad (3.28c)$$

[10/] To utilize these measures, one must implicitly assume that the conditions for employing the Hicks composite good theorem obtain, that is: (a) each firm must utilize the same proportion of micro factors and produce the same proportion of micro outputs; and (b) these proportions are invariant to the scale of output.

[11/] See, for example, Samuelson and Swamy (1974), Fisher (1969) and the references cited in these papers.

[12/] That is, they satisfy the following: if prices double, the index doubles; the index between two dates is invariant to the base period; the index is invariant to the unit of the goods (tons or pounds) or the unit of money (dollars or $1,000 dollars).

where h^i, g^j and e^j are monotonically increasing functions with $h^i(0) = g^j(0) = e^j(0) = 0$; and f_y^i, f_x^j, and f_w^j are homogeneous functions of degree one. Since h^i, g^j, and e^j take scalars to scalars, we can estimate then the relevant cost or production functions by substituting f_y^i, f_x^j, or f_w^j for H^i, G^j, or E^j.

We can interpret this procedure as follows. For the production function, the N micro factors are combined to produce n abstract inputs, which are then utilized according to eq. (3.26a) to produce m abstract outputs, which are then divided so that eq. (3.25a) is satisfied in each output subcategory. Similarly, on the cost side, the optimal (cost-minimizing) method of producing the given bundles $H^i(y^i)$ from the abstract inputs $G^j(x^j)$ is determined via the production function, and then each abstract output $H^i(y^i)$ is produced via the cost function, eq. (3.27b), for the components of the abstract fixed factors $G^j(\bar{x}^j)$ and the abstract prices $E^j(w^j)$ of the variable factors. Note that the assumption that the aggregation functions are homothetic does not imply that the overall production process is homothetic.

Although the assumption of homothetic aggregation functions may appear restrictive, it is not really so. All it states is that the index number generated by the aggregation function is a monotonically increasing transformation of a linear homogeneous function relating micro inputs or outputs to aggregate inputs or outputs. In any event, there is really no alternative. If one rejects homothetic aggregation, it has been shown that there is not generally any aggregation function that exists with the desirable properties with respect to measurement scale and so forth. Thus in the absence of homothetic aggregation, one must utilize totally disaggregate data, which, of course, is generally infeasible in view of the large number of different inputs and outputs associated with transportation firms. On the other hand, if one ignores the restrictions imposed by homothetic aggregation and simply adds together ton-miles or freight cars, it is likely that extreme biases will result in the estimated cost or production functions. Consequently, there seems to be relatively little alternative to the use of homothetic aggregation functions, which can, in fact, be quite flexible and general.

B. Approaches to Homothetic Aggregation

In the most abstract theory of the multiple output firm, in which outputs are choice variables of the firm, output prices are given to the firm, and, as a consequence of profit maximization, price equals marginal cost, there is no fundamental difference between the theory of output aggregation and the theory of input aggregation. In fact, in the pure theory of the perfectly competitive firm, inputs and outputs are treated symmetrically.

In the transportation industries, however, it probably does not make sense to treat factors and outputs symmetrically for two reasons. First, even though these industries are regulated, price is typically different from marginal cost and the firms have the ability to excercise some monopoly power; and second, while micro factors are usually available in discreet units, micro outputs are usually available in a continuum of goods of a different quality, as measured by size of shipment and length of haul. Consequently, there are some operational differences in approaching aggregation of factors and outputs in the transportation industries.

We will first discuss the general approach to aggregation in the context of factors and then show how the analysis may be modified for outputs.

1. Factor Aggregation[13/]

The general approach to homothetic aggregation consists of specifying a convenient form for f_x^j or f_w^j, the homogeneous quantity and price aggregations respectively, and respectively estimating the resulting factor share equations as functions of quantities or prices. From the resulting quantity or price index (denoted by Q_0 and P_0) the corresponding price and quantity indices are implicitly defined by

$$\tilde{P}_0(t) = \frac{\text{Expenditures in } t}{Q_0(t)} \tag{3.29a}$$

$$\tilde{Q}_0(t) = \frac{\text{Expenditures in } t}{P_0(t)} \tag{3.29b}$$

Samuelson and Swamy (1974) have shown that the indices $(Q_0, \tilde{P}_0)$ and $(\tilde{Q}_0, P_0)$ have the desirable properties concerning invariance with respect to units and time. Since, however, it is generally true that $P_0 \neq \tilde{P}_0$ and $Q_0 \neq \tilde{Q}_0$, one has a choice of constructing either a price or a quantity index explicitly and defining the other implicitly. Let us consider the construction of a price index first.

As indicated above, we assume that it is possible to group the firm's inputs into categories $x^j (j=1,\ldots,m)$. The problem then is to determine a single measure x^j corresponding to the components of each vector x^j. Conceptually, this problem can be handled by assuming that the firm has a sub-production problem, to transform the micro factors $x_1^j,\ldots,x_h^j$, into a simple aggregate factor x^j. Thus, we assume that the firm has a production function relating x^j to $x_1^j,\ldots,x_h^j$, which is seper-

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[13/] This section relies on the work by Diewert (1974).

able.[14] Corresponding to this, we furthermore assume that the firm has a sub-production function, relating the costs of producing each abstract factor to the factor prices of its associated micro factors. From eqs. (3.25b) and (3.28b) we know that $x^j = G^j(x^j)$ and $G^j = g^j[f_x^j(x^j)]$. Because, however, h^j is a montonic transformation of f_x^j, it simply takes a scalar into a scalar and can consequently be ignored. Thus by the assumption of separability of the sub-production function we can write

$$C^j(x^j,w^j) = f_x^j \cdot \phi^j(w^j) \tag{3.30}$$

where f_x^j is the aggregation function for factor subcategory j, w^j is a vector of the factor prices associated with subcategory j; and ϕ^j is the unit cost function of the j^{th} subcategory. Suppose, for example, that ϕ^j is translog; then

$$\phi^j = \exp[d_o + \sum \alpha_i \ell nw_i^j + \frac{1}{2} \sum \sum \beta_{ik} \ell nw_i^j \ell nw_k^j] \tag{3.31}$$

We can therefore write the share of the i^{th} component of the j^{th} subcategory as

$$\frac{w_i^j x_i^j}{w^j \cdot x^j} = \frac{\partial \ell n \phi^j}{\partial \ell nw_i^j} = \alpha_i + \sum \beta_{ik} \ell nw_k^j \tag{3.32}$$

After imposing the conditions of symmetry and homogeneity, we can estimate the factor share equations for each component of the j^{th} subcategory and obtain estimates of each α_i and each β_{ik}. Using an arbitrary normalization to obtain d_{oj}, we can then obtain the function ϕ^j from eq. (3.31). Then eq. (3.29b) is used to obtain an impli-

[14] Note, however, that this contains no assumption concerning the separability of the total production function that relates all outputs to all inputs.

cit quantity index

$$\tilde{Q}_o \equiv X^j = f_x^j = \frac{C^j(x^j, w^j)}{\phi^j(w^j)} = \frac{w^j \cdot x^j}{\phi^j(w^j)} = \frac{w^j \cdot x^j}{P_o(w^j)} \tag{3.33}$$

To calculate an explicit price index, we adopt a similar approach, and assume a specific functional form for f_x^j and estimate its associated factor demand equations and combine these estimates (along with an arbitrary normalization to obtain the constant term) to obtain a quantitative estimate of f_x^j. For instance, suppose f_x^j is translog; then its factor demand equation is given by

$$\frac{w_i^j x_i^j}{w^j \cdot x^j} = a_i + \beta_{ik} \ln x_k^j \tag{3.34}$$

After imposing the necessary symmetry and homogeneity conditions, the estimates of the equations corresponding to eq. (3.31) can be combined with an arbitrary normalization to completely specify f_x^j. The aggregate factor price (W^j) is then obtained by the relationship

$$\tilde{P}_o \equiv W^j = \frac{w^j \cdot x^j}{X^j} = \frac{w^j \cdot x^j}{f_x^j} = \frac{w^j \cdot x^j}{Q_o(x^j)} \tag{3.35}$$

A related approach to aggregation is considered by Diewert (1974). Under this formulation, we choose a functional form for f_x^j which is consistent with an index number formulation that depends only upon observable prices and quantities. In this approach, it is unnecessary to estimate specific factor share equations.

In particular, Diewert calls a quantity index Q exact for f_x^j if it satisfies

$$\frac{f_x^j(x_t^j)}{f_x^j(x_o^j)} = Q(P_o^j, P_t^j, x_o^j, x_t^j) \tag{3.36a}$$

where f is a homogeneous aggregation and the subscripts o and t refer to the base year quantities and prices and the current year quantities and prices. Thus a quantity index is exact if the ratio of its present value to its base value is a function of the prices and quantities in the current and base time periods. Similarly, an exact price index P for the unit cost function, satisfies the relationship

$$\frac{\phi(w_t^j)}{\phi(w_o^j)} = P\,(p_o^j, p_t^j, x_o^j, x_t^j) \tag{3.36b}$$

Then, Diewert defines an index number to be superlative if the function for which it is exact is capable of providing a second-order approximation to a homogeneous function.

Diewert (1974) and others[15/] have argued that the following index number has these desirable properties

$$\frac{g(z_t)}{g(z_o)} = \prod_{i=1}^{h} \left(\frac{z_{it}}{z_{io}}\right)^{\frac{1}{2}(S_{it} + S_{io})} \tag{3.37}$$

where S_t^i is the cost share of the i^{th} micro factor in period t and S_{io} is the cost share in the base period and the z's represent factor prices or quantities, depending upon whether we use a price or quantity index. In particular, this index provides a discrete approximation to the Divisia index, is exact for translog f_x^j or f_w^j, and, therefore, is superlative. Furthermore, its use does not require the estimation of factor

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[15/] See I. Fisher (1922), Tornquist (1936), Theil (1967), Christensen and Jorsenson (1973) and Starr and Hall (1976). In the latter two papers, it was also used to provide a discrete approximation to a Divisia index.

share equations; numerically, it differs from the estimated translog index, given by equation (3.31), only because of the stochastic specification implicit in the latter.

For these reasons, we will concentrate on (3.37) for factor aggregation. Nevertheless, some analysis with all types of aggregator functions will be useful to test for consistency. Thus we plan to explore the application of each of the various aggregator indices described and use eqs. (3.33), (3.35), and (3.37) to determine the best aggregator index.

2. Output Aggregation

The usual theory of aggregation assumes perfectly competitive markets, that is price equals marginal cost in product and factor markets, and that the micro units (factors or outputs) are available in a finite number of different qualities (or that each firm has the same proportion of each quality type within each generic category). Since, however, transportation firms typically produce at prices that differ from marginal cost and since they can produce a continuum of different outputs by varying the length of haul and the size of shipment, neither of these assumptions will usually be met with respect to output. Thus, we need to modify the usual approach to aggregation.

To fix ideas concerning the problems caused by the divergence of price from marginal cost, let us first consider the usual case. Thus suppose the firm faces the production function

$$t(H'(y_1,y_2),\ldots,y_M, x_1,\ldots,x_N) = 0 \tag{3.38}$$

where y_i represents the micro outputs; x_j represents the micro inputs, and $H'(y_1,y_2)$ represents some aggregator function on the first and second micro outputs. The cost function is defined by

$$C(y,w) = \min_{x} w \cdot x \quad s \cdot t \quad t(y,x) = 0 \tag{3.39}$$

Consider two different output vectors, $y' = (y_1', y_2', \bar{y}_3, \ldots, \bar{y}_M)$ and $y'' = (y_1'', y_2'', \bar{y}_3, \ldots, \bar{y}_M)$, which are identical except for their first two components. Furthermore, let the aggregator functions for y_1 and y_2 be such that

$$H'(y_1', y_2') = H'(y_1'', y_2'') = Y^1 \tag{3.40}$$

Clearly, $C(y';w) = C(y'';w)$, since from the cost function, eq. (3.39), we know that identical x's are required to produce y' and y". Thus if the firm is instructed to cost-minimize subject to $Y^1 = H^1(y_1', y_2') = H^1(y_1'', y_2'')$, the precise output configuration is indeterminate.

If, however, the firm is a price taker and profit maximizer, th the y_1, y_2 combination it chooses will solve the problem

$$\max_{y_1, y_2} \; p_1 y_1 + p_2 y_2 \quad s \cdot t \quad H'(y_1, y_2) = Y^1 \tag{3.41}$$

for some Y^1. Thus the problem is determinate; and it is upon this fac that the usual theory of aggregation is based.

When the government (or a rate bureau) sets output prices and r quires that firms satisfy all demand forthcoming at that price, the firm is no longer free to adjust quantity in a profit maximizing way. Only by chance, will the quantity produced be the profit-maximizing quantity, and price may either be above marginal cost (implying that t firm would like to sell more at the regulated price) or below marginal cost (implying that it would like to sell less). In either case, we cannot determine the equilibrium quantity y_1 and y_2 by solving the profit-maximizing problem given by eq. (3.41), since y_1 and y_2 are no longer control variables of the firm. In effect, the firm has an output restriction imposed upon it such that the quantity supplied equals the quantity demanded at the regulated price.

To see the implications of this for aggregation let us assume that the micro output of y_3 is fixed at $\bar{y}_3$ and that we want to determine the appropriate aggregate for y_1, y_2, and y_3. We thus write the transformation function as

$$t(H^2(y_1,y_2,\bar{y}_3),y_4,\ldots,y_M;\ x_1,\ldots,x_M) = 0 \tag{3.42}$$

Then the (y_1,y_2) configuration chosen by the firm solves the problem

$$\max_{y_1,y_2} \quad p_1y_1 + p_2y_2 \qquad \text{s·t} \quad H^1(y_1,y_2,\bar{y}) = Y^1;\ y_3=\bar{y}_3. \tag{3.43}$$

This can be written equivalently as:

$$\max_{y_1,y_2} \quad p_1y_1 + p_2y_2 \qquad \text{s·t} \quad \tilde{H}^1(y_1,y_2) = Y^1 \tag{3.43a}$$

where $\tilde{H}^1$ is a function that depends upon the constant $\bar{y}_3$, which has been subsumed. If $\tilde{H}^1$ is homogeneous of degree that is independent of $\bar{y}_3$, then a suitable aggregate measure for y_1 and y_2 can be found, since $\tilde{H}^1$ can then be written as a monotonic transformation of a linear homogeneous function. In general, however, such a function is extremely difficult to find.[16/]

This implies that unless we can find an aggregator function $\tilde{H}^j$ that is homogeneous of degree that is independent of the restricted

[16/] The Cobb-Douglas function has this property, but it implies a transformation locus that is convex to the origin. The translog function and other usual aggregator functions do not have this property.

outputs, we should never include a micro output for which $p \neq mc$ with micro outputs where $p = mc$ in any given aggregate category. This gives us an approximate rule to follow for output aggregation: a category (aggregate) can only be formed when no micro output in that category is more severely restricted than the others.

Suppose, however, regulation is such that the price levels of t broad aggregates are regulated, but that some freedom is permitted with respect to the micro outputs. This may actually be the case, sin the ICC appears to have certain notions about the rate levels for broa classes of commodities, but permits considerable lattitude for specifi commodities. Certainly, the value of service rate structure has thes characteristics. In this case, let us define an aggregate $Y^j = H^j(y^j, \ldots, y^j_k)$, whose price is directly regulated and whose quantity is implicitly regulated by the constraint that the firm must satisfy the existing demand at that price. Then the firm's sub-problem is to

$$\max_{y^j_1, \ldots, y^j_h} \; p_1 y^j_1 + \ldots + p_h y^j_h \quad \text{s.t} \quad H^j(y^j_1, \ldots, y^j_h) = Y^1 \;, \; Y^1 = \overline{Y}^1 \qquad (3.4$$

Thus the firm is free to set its micro quantities in a second-best profit-maximizing manner. Since the firm will set prices proportional to marginal cost in this case, this means that aggregates can be founc for micro outputs whose price bear the same relationship to marginal cost.

As a practical matter, this means that aggregates should be formed from outputs which are subjected to the same regulatory constraints, which have similar demand functions, and which impose simila marginal costs upon the firms. For example, this means that grains, coal, and manufactured commodities should not be aggregated into a single output measure, but that it may be acceptable to aggregate different types of grains into a single output measure, and similarly for coal and manufactured commodities.

C. Quality Adjustments

The second problem with output aggregation arises from the continuum of quality levels (e.g., size of shipment). Ideally, we would like to treat a discrete number of quality levels as separate goods and proceed in the manner outlined for factor aggregation. Lacking that, even if the continuum of quality levels bunched around a discrete number of values, we could still proceed in the conventional manner. Since, however, there is no a priori reason to expect such bunching to occur in the case of transportation outputs, we must accordingly modify the aggregation theory outlined above.

To state the problem formally, let us assume that each firm produces n vectors of output, $y^1,\ldots,y^n$, each of which has an associated vector of quality levels, $q^1,\ldots,q^n$, where q^i is of arbitrary dimension.[17/] Let us furthermore assume that each output price is a function of quality and that the firm's technology is such that production is readily separable, that is the components of each output category can be written in terms of an aggregate index. Thus, the firm's problem is to choose output quantities and qualities to maximize profits

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17/ Thus if y^i represents the i^{th} category of output, its micro components can be described by $y^i_1,\ldots,y^i_{h_i}$. Th quality vector q^i would then have components $q^i_1,\ldots,q^i_{h}$, where q^i_h would also be a vector of qualities associated with the micro output y^i_h.

$$\max_{q^i, y^i} \quad \pi = \sum p^i(q^i)y^i - C[\psi^1(y^1 q^1), \ldots, \psi^n(y^n, q^n), \bar{x}, w] \tag{3.45}$$

where $\bar{x}$ is a vector of fixed inputs and w is a vector of prices of the variable factors, and $\psi^i(y^i, q^i)$ represents a function relating the micro components and their related qualities to some aggregate measure.

Analogous to the usual aggregator functions, it seems desirable to assume the following three properties of the ψ^i functions for all i

(i) $\psi^i(0, q^i) = 0$; ψ^i is defined for all $y^i > 0$, $q^i > 0$; and $\psi^i(y^i, q^i) > 0$ for $y^i > 0$, $q^i > 0$.

(ii) $\psi^i(y^i, q^i)$ is homogeneous of degree 1 in y^i for fixed q^i.

(iii) $\psi^i(y^i, q^i)$ is monotonically non-decreasing in q.

Assumption (i) is straightforward and simply state that the aggregate output measure must be zero if its micro output components are zero and that as long as one micro output is positive, the aggregate output measure must also be positive.

Assumption (ii) states that doubling the level of micro outputs will double the level of the aggregate output measure, but "doubling" the quality measures for fixed y need not double the aggregate output measure. Thus the definition of the output measure (tons, ton-miles,etc can have important implications for the measurement of aggregate output.

Assumption (iii) specifies that output with higher qualities is <u>ceteris paribus</u> at least as difficult (in terms of input requirements to produce as the same generic output with lower qualities. One way of interpreting this is to view qualities as freely disposable. This assumption is entirely plausible when output qualities are separable from specific inputs, as we have assumed here. On the other hand, no assumption about the concavity of $\psi^i(y^i, q^i)$ in q^i for fixed y^i is made since measures of quality may have no natural units of expression: $\sqrt{q^i}$ may serve as well as q^i in the sense that $\psi^i(y^i,\sqrt{q^i})$ could represent the "ψ-technology" as well as $\psi^i(y^i,q^i)$, but such a transformation could affect the concavity-convexity behavior of $\psi^i(\cdot)$.

The problem at hand is to estimate a cost function of the following general form:

$$C[\psi^1(y^1,q^1),\ldots,\psi^n(y^n,q^n),\ \bar{x},\ w] \tag{3.46}$$

where $\psi^i(y^i,q^i)$ is some aggregate output measure that is controlled for quality; x represents the vector of fixed inputs; and w represents the prices of the variable inputs.

There are basically two approaches to estimating this cost function. The first approach is more familiar and involves deriving a standardized output measure where quality differentials have been taken into account. The second is to estimate the cost function directly, making the appropriate substitution for $\psi^i(y^i,q^i)$.

1. Hedonic Adjustments for Quality

The usual approach to accounting for quality differentials is to determine a "standard" price for a "standard" level of quality, and then deflate revenues to obtain a "standard" level of output, which we can use to form aggregate output measures in the usual way, taking into account differentials between prices and marginal cost.

Specifically, let us consider a specific firm's output category i, which is composed of micro outputs $y^i,\ldots,y^i_h$ that have been chosen to ensure that their price/marginal-cost ratios are comparable. By assumption, however, the quality levels of $y^i_s \neq y^i_r$. Let R^i_{rf} represent the revenues from y^i_r and p^i_r represent the price of y^i_r, which we obtain by dividing revenues by the relevant output measure (presumably tons or ton-miles). We then estimate a hedonic regression,[18] relating the price of the micro output to its quality characteristics and some specific attribute of the firm. If each output has H quality dimensions, we can postulate the following general relationship

$$P_r^i = P_r^i\,(q_{r_i}^i - \bar{q}_{r_i}^i, \ldots, q_{rH}^i - \bar{q}_{rH}^i, A_f) \tag{3.47}$$

where q_{ri}^i and $\bar{q}_{ri}^i$ respectively represent the actual and mean quality dimention i associated with micro output y_r^i and A_F^f represents specific attributes of firm f. If, for example, we had a time-series, cross-section sample of output y_r^j, we could estimate the following regression:[19]

$$P_{rft}^i = \alpha_o + \sum_{t=1}^{T} \mu_t d_t + \sum_{f=2}^{M} \beta_f F_f + \sum_{r=2}^{n} \gamma_r C_r^i + \sum_{s=1}^{H} \alpha_s\,(q_{rst}^i - \bar{q}_s^i) \tag{3.47a}$$

where P_{rft}^i represents the price of commodity r in category i at time t for firm f; d_t represents a time dummy (1 if period t; 0 otherwise); F_f represents a firm dummy (1 if firm F; 0 otherwise); C_r^i represents a commodity dummy (1 if commodity r; 0 otherwise); q_{rst}^i represents the quality dimension s associated with commodity r at time t; and $\bar{q}_s^i$ represents the mean quality dimension s, over all firms, time periods

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[18] See Rosen (1974) for a good discussion of the use of hedonic regressions.

[19] Note that we standardize the base period (t=0), the first firm (f=1) and the first commodity (r=1). Moreover, there is no reason why this relationship needs to be linear. In fact, since a linear formulation could result in negative standardized prices, it is probably desirable to estimate a hedonic regression in a logarithmic form that includes interaction terms.

and commodities.

We then define P^{*i}_{rt} as the price of the micro-output y^i_r at a standard quality level at time t and estimate it by assuming that each firm's quality is at the mean quality level. Thus, for firm f

$$P^{*i}_{rft} = \alpha_o + \mu_t d_t + \beta_f F_f + \gamma_r C^i_r$$

We now are in a position to construct a price or a quantity index. To obtain an aggregate output measure, it is desirable to construct an aggregate price index and then obtain an aggregate output measure by dividing revenue by the price index. The aggregate price index for each firm is constructed by the following expression

$$IP^i_{ft} = \prod_{r=1}^{h} \left(\frac{P^{*i}_{rft}}{P^{*}_{rfo}} \right)^{\sqrt{s^i_{rft} + s^i_{rfo}}} \qquad (3.48)$$

where P^{*i}_{rft} and P^{*i}_{rfo} represent the standardized prices of commodity r for firm f in the base and current periods; and s^i_{rft} and s^i_{rfo} represent the revenue shares of commodity r from category i in the base and current periods for firm f. The aggregate output measure of category i for firm f in time t (Y^i_t) is then simply defined as

$$Y^i_{ft} = R^i_{ft} / IP^i_{ft} \qquad (3.49)$$

where R^i_{ft} represents revenues from category i for firm f in time t.

A similar analysis which constructs a Divisia quantity index can be performed by deflating each micro revenue by its P^{*i}_{rft}:

$$y^{*i}_{rft} = \frac{R^i_{rft}}{P^{*i}_{rft}}$$

to obtain the quality-adjusted micro output measure y^{*i}_{rft}. The micro output quantity can then be used in a Divisia index number formula analgous to eq. (3.48) to provide an index of aggregate output quantit This index, when divided into aggregate revenues, yields an alternativ aggregate price index.

While attractive, this approach has a number of problems associ ated with it. Specifically, it requires that a very special relationship must exist between the technology as embodied in the ψ^i's and the market functions, $p^i(q^i)$, and it does not seem that in general such a relationship should obtain except under perfect competition.

Let us consider the assumptions about ψ^i that are made when deflation by an estimated hedonic price index is performed. Basically, there are three steps involved in the procedure.

(i) Estimate $p^i(q^i)$ directly by means of a "hedonic" regressi analogous to eq. (3.47).

(ii) Define $p^{*i}(q^{*i})$ by setting the value of each level of quality equal to its mean and implicitly define the "deflated" value of output as

$$\psi^{*i} = \frac{p^i(q^i) \cdot y^i}{p^{*i}(q^{*i})}$$

(iii) Estimate the value of $c(\psi^{*i},\overline{x},w)$.

While the estimation of $p^i(q^i)$ involves some subtle issues,[20/] we shall concentrate upon the implications of the determination of ψ^{*i}. In particular, we want to explore the question of what assumptions are being made about ψ^{*i} when deflation by an estimated hedonic

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[20/] See Sherwin Rosen (1974) for a good discussion of this problem.

price index is performed.

To answer this question, note that any solution to the profit maximization problem also solves the i=1,...,n sub-problems of the allocation of micro outputs and their associated quality levels.

$$\max_{y^i, q^i} \quad p^i(q^i) \cdot y^i \qquad \text{s.t.} \quad \psi^i(y^i, q^i) = \bar{\psi}^i \tag{3.50}$$

or

$$\mathcal{L} = p^i(q^i) \cdot y^i + \lambda(\psi^i(y^i, q^i) - \bar{\psi}^i) \tag{3.50a}$$

This follows from the form of the separability of the cost function and the assumption that each firm is a competitive price taker. Thus once the $\bar{\psi}^i$'s are chosen, the allocation of the y^i's and q^i's must solve the sub-problem given in eq. (3.50) or else, revenue (for identical cost) would not be maximized, since the y^i, q^i decision given the $\bar{\psi}^i$'s does not affect any other output market.

Thus let $p^i(q^i) \cdot y^i$ denote the revenue accruing from the i^{th} output aggregate for the quality bundle actually chosen by the firm; and let q^{*i} be the "standard" quality bundle for category i. From the maximization sub-problem given in eq. (3.50), we know that

$$p^i(q^i) \cdot y^i \geq p^i(q^{*i}) \cdot y^{*i} \tag{3.51}$$

where y^{*i} is chosen to satisfy $\psi^i(y^{*i}, q^{*i}) = \bar{\psi}^i$, that is, the maximum output that could have been produced at the standard quality q^{*i} would have realized not more revenue than the bundle that was actually chosen did.

Hedonic deflation defines

$$\psi^{*i} = \frac{p^i(q^i) \quad y^i}{p^{*i}(q^{*i})} \geq y^{*i} \tag{3.52}$$

with the last inequality holding by virtue of eq. (5.28). Since $\psi^i(y^i,q^{*i})$ is homogeneous of degree one in y^i, an arbitrary choice of numeraire allows the above, eq. (3.52) to be written

$$\psi^{*i} = \frac{p^i(q^i) \cdot y^i}{p^i(q^{*i})} \geq y^{*i} = \psi^i$$

as

$$\psi^i(y^i,q^{*i}) \equiv y^i \tag{3.53}$$

From this we see that hedonic deflation never underestimates aggregat output quantities, and will typically overestimate them except in the special case when

$$p^i(q^i) \cdot y^i = p^i(q^{*i}) \cdot y^{*i} \qquad \text{for } \psi^i = \bar{\psi}^i \; ; \tag{3.54}$$

that is, when revenues from different breakdowns of a given ψ^i are <u>always</u> the same. Suppose, for example, that ψ^i is fixed at $\bar{\psi}^i$; then all (q^i,y^i) that satisfy $\psi^i(y^i,q^i) = \bar{\psi}^i$ generate the same revenu the level of which k^* is a function of $\bar{\psi}^i$. Thus we have

$$p^i(q^i) \cdot y^i = p^i(q^{*i}) \cdot y^{*i} = k^*(\bar{\psi}^i) \tag{3.55a}$$

$$\psi^i(y^i,q^i) = \psi^i(y^{*i},q^{*i}) = \bar{\psi}^i \tag{3.55b}$$

$$y^i = y^i(\bar{\psi}^i,q^i) \tag{3.55c}$$

$$y^{*i} = y^{*i}(\bar{\psi}^i,q^{*i}) \tag{3.55d}$$

Since $\psi^i(y^i,q^{*i})$ is homogeneous of degree one in y, $k^*(\bar{\psi}^i)$ is also homogeneous of degree one in $\bar{\psi}^i$. Fixing $\bar{\psi}^i$, we therefore obtain

$$p^i(q^i) \cdot y^i(q^i) = k^{i*}, \forall y^i, q^i \ni \psi^i(y^i,q^i) = \overline{\psi}^i \qquad (3.56)$$

It follows directly from this that

$$y^i(q^i) = \frac{k^*(\overline{\psi}^i)}{p^i(q^i)} \qquad (3.57)$$

or that

$$\psi^i(y^i,q^i) \cdot p^i(q^i) = k^* \qquad (3.58)$$

where k^* is chosen so that the normalization

$$\psi^i(y^i,q^{*i}) = \frac{k^*}{p^i(q^{*i})} = y^i \qquad (3.59)$$

is satisfied.

From the forgoing analysis, we can therefore see the following: Hedonic aggregation using estimated hedonic price frontiers is exact if and only if the aggregator function $\psi^i(y^i,q^i)$ is proportional to the inverse of the "frontier" price of q^i. This apparently indicates that a very special relationship must obtain between the technology as embodied in the ψ^i's and the market price function $p^i(q^i)$; and it does not seem that such a condition should generally obtain except under perfect competition. Furthermore, hedonic aggregation, if correct, implies that firms motivelessly place themselves in the quality space, since any efficient production bundle will realize the same revenue for identical costs. Note, however, that the distribution of the firm's production of the ψ^i's as aggregates is not motiveless, but depends upon differences in $\overline{x}$ and w.

2. Hedonic Cost Functions

As an alternative to hedonic deflation, we can incorporate the

ψ^i function into the cost function and estimate it directly. Briefly, this approach requires us to approximate ψ^i by

$$\begin{aligned} \ell n\, \psi^i(y^i,q^i) &= \alpha_o + \sum \alpha_j\ \ell n y^i_j + \sum b_k\ \ell n q^i_k \\ &+ \frac{1}{2} \sum \sum A_{j\ell}\ \ell n y^i_j\ \ell n y^i_\ell \\ &+ \frac{1}{2} \sum \sum B_{jk}\ \ell n y^i_j\ \ell n q^i_k \\ &+ \frac{1}{2} \sum \sum C_{kr}\ \ell n q^i_k\ \ell n q^i_r \end{aligned} \quad (3.60)$$

where we impose the relevant coefficient restrictions to ensure that ψ^i is linearly homogeneous in y^i.[21/] Thus in estimating the usual translog cost function and its associated factor demand equations, we substitute for y^i the expression for ψ^i, given above. The result is a system of non-linear equations with the number of parameters equal to

$$\frac{1}{2}(N^2 + 2N + 3) + \frac{1}{2}\sum_{\ell=1}^{m}(Q^2_\ell + Q_\ell + 2)$$

where Q = 1 + the number of quality dimensions of the ℓ^{th} output and N = (m + n), that is, the number of aggregate outputs (m) plus the number of fixed and variable factors (n). The model is apparently identified.[22/]

Since direct estimation of hedonic regressions does not involve the restrictive assumptions involved in hedonic deflation, it seems to be a theoretically superior approach to quality adjustment. Against this theoretical advantage, however, must be weighed the increase in variables if some previous aggregation is not made over commodities.[23/]

21/ See Section IV, below.

22/ This is aided by the assumption of weak separability in the cost function and the homogeneity of ψ^i in y^i.

23/ This approach is used in estimating trucking equations, discussed in Chapter Four, below.

IV. Estimation of the Translog Cost Function

We begin by specifying a general long-run translog cost function and show how we can test for homogeneity, separability, and non-joint production using this function. We then show how the long-run translog cost function and its associated tests must be modified in the context of a short-run cost function. Finally, since the general translog formulation is not in suitable form for estimation, we show how the specific estimating equations are derived.

A. The Long-Run Cost Function

Let us being by writing a general translog function as[24/]

$$\ell n\ C(y) = \alpha_0 + \alpha \ell nq + \frac{1}{2}\ \ell nq'\ \beta \ell nq \tag{3.61a}$$

where q represents a (Nx1) vector of outputs and factor prices, α_0 represents a scalar; α represents a (1xN) vector of coefficients associated with the first-order terms; and β represents a (NxN) matrix. of coefficients associated with the second-order terms.

This general expression can be given more intuitive meaning if

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[24/] Since we will generally take the sample mean as the point of expansion we can interpret the q's as deviations from the sample mean and can similarly interpret the coefficients as the value, first derivative, and second derivative (with respect to ℓnq) of the log of the underlying function, evaluated as the sample mean.

we partition the vector q into factor prices and commodity outputs and partition the vector and matrix of parameters (α and β) into the relevant interaction terms between factor prices and commodity outputs as follows:

$$q \equiv (w \;\vdots\; y)$$

where w represents an (nx1) vector of factor prices and y represents a (mx1) vector of commodity outputs and n + m = N.

$$\beta \equiv \left[\begin{array}{c:c} A & B \\ \hdashline B' & C \end{array}\right]$$

where A represents an (nxn) symmetric matrix of parameters associated with factor price interactions; B represents a (nxm) matrix of parameters associated with the interactions among factor prices and commodity outputs (note that (B') = B); and C represents a (mxm) symmetric matrix of parameters associated with commodity output interactions.

$$\alpha \equiv (a \;\vdots\; c)$$

where a represents a (nx1) vector of parameters associated with factor prices and c represents a (mx1) vector of parameters associated with commodity outputs.

In matrix notation, we can then write the translog cost functions as

$$\ell n\ C(y,w) = a_o + (a \mid c)'\ \ell n\ (w \mid y)$$

$$\frac{1}{2}\ \ell n\ (w \mid y) \begin{bmatrix} A & \vdots & B \\ \hline B' & \vdots & C \end{bmatrix} \ell n\ (w \mid y) \qquad (3.61b)$$

Performing the indicated multiplication, we thus obtain

$$\ell n\ C(y,w) = a_o + \sum_{i=1}^{m} a_i \ell n w_i + \sum_{\ell=1}^{m} c_\ell \ell n y_\ell$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij}\ \ell n w_i\ \ell n w_j$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{\ell=1}^{m} (\ B_{i\ell} + B'_{\ell i}) \ell n w_i \ell n y_\ell$$

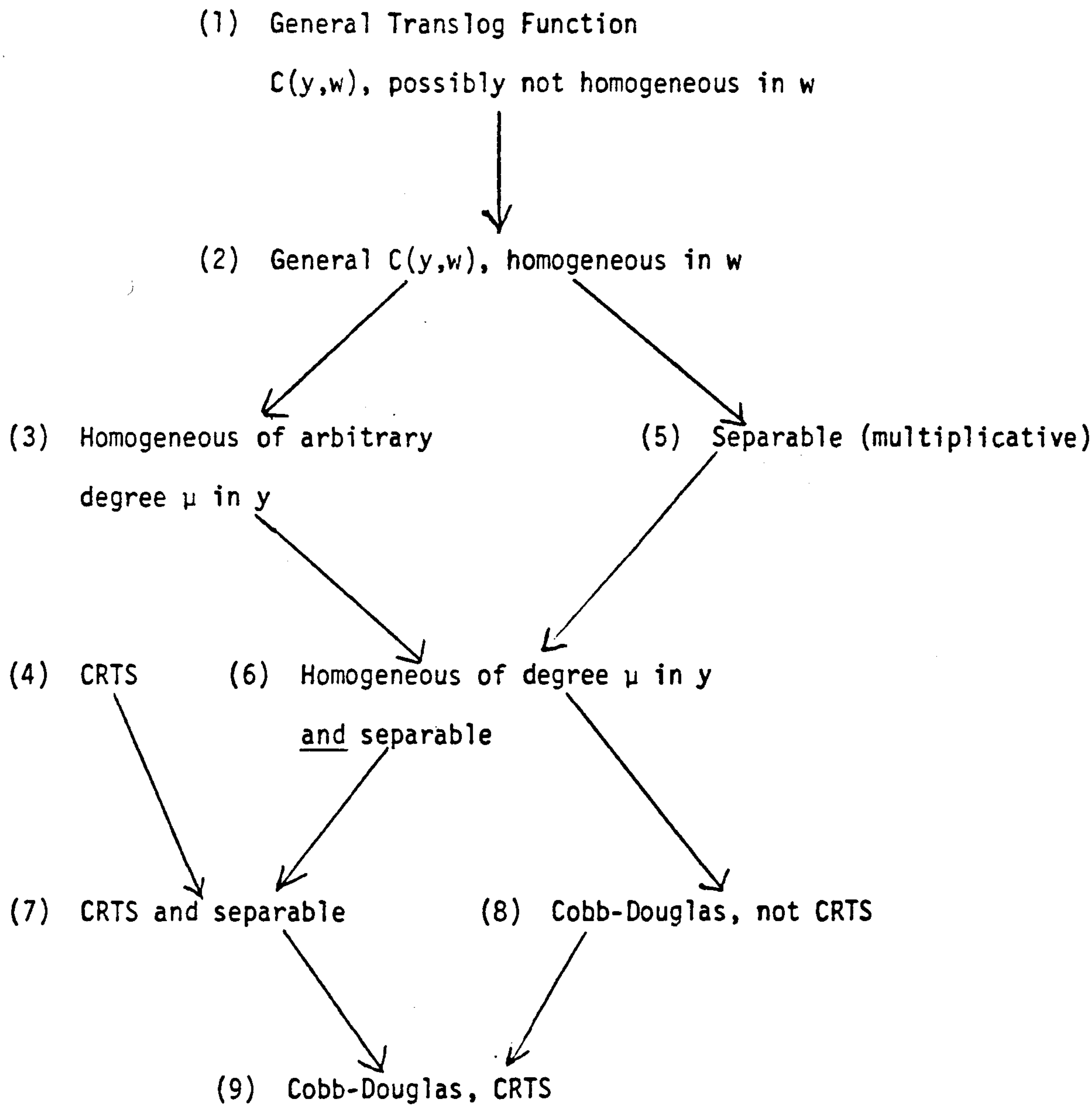
$$+ \frac{1}{2} \sum_{\ell=1}^{m} \sum_{h=1}^{m} C_{\ell h}\ \ell n y_\ell\ \ell n y_h \qquad (3.61c)$$

The function described in eq. (3.61c) is entirely general and does not require homogeneity, separability or non-joint production. Nevertheless, using this translog cost function, we can test a series of nested hypotheses concerning questions of homogeneity and separability as well as hypotheses concerning non-joint production.

1. Homogeneity and Separability

Figure 1 illustrates the structure of the nested hypotheses concerning homogeneity and separability. We thus being with a general trans-

Figure 1. Tests of Underlying Production Structure Using Translog Cost Functions



log function (1) and derive the restrictions necessary for cost-minimization (2). We then derive the necessary restrictions for production to be homogeneous of an arbitrary degree in output (3), and for production to be subject to constant returns to scale in output (CRTS) (4). We then derive the restrictions necessary for production to be separable (5), and separable and homogeneous in output (6). Finally, we indicate the combinations of restrictions needed if production is subject to CRTS and is separable (7); if production can be described by a Cobb-Douglas technology of arbitrary degree in y (8); and production can be described by a CRTS, Cobb-Douglas technology (9).

The basic theory of cost-minimization requires that the cost function be homogeneous of degree one in factor prices (w). We thus develop the necessary restrictions on the parameters to ensure that C(y,w) be homogeneous of degree one in w.[25/] Note that since C(y,w) is symmetric in w and y, we can easily extend the analysis to derive the necessary restrictions for homogeneity in output (y) of an arbitrary degree.

Homogeneity of degree one in w for fixed y requires[26/]

$$C(y,\lambda w) = \lambda C(y,w) \tag{3.25}$$

Thus

$$\ell n\ C(y,\lambda w) = \ell n[\lambda C(y,w)] = \ell n\lambda + \ell n\ C(y,w) \tag{3.63}$$

By direct calculation, we therefore obtain

[25/] See Spady and Friedlaender (1976) for a proof that a translog approximation to a homogeneous function of degree k must also be homogeneous of degree k.

[26/] This is only a demonstration of sufficiency. For a rigorous proof of necessity and sufficiency, see Spady and Friedlaender (1976).

$$\ln C(y,\lambda w) = a_0 + \sum_{i=1}^{n} a_i(\ln\lambda + \ln w_i) + \sum_{\ell=1}^{m} c_\ell \ln y_\ell$$

$$+ \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} A_{ij}(\ln\lambda + \ln w_i)(\ln\lambda + \ln w_j)$$

$$+ \frac{1}{2} \sum_{i=1}^{m} \sum_{\ell=1}^{m} (B_{i\ell} + B'_{\ell i})(\ln\lambda + \lambda w_i)(\ln y_\ell)$$

$$+ \frac{1}{2} \sum_{\ell=1}^{m} \sum_{h=1}^{m} C_{\ell h} \ln y_\ell \ln y_h \qquad (3.64)$$

Thus

$$\ln C(y,\lambda w) = \ln C(y,w) + \sum_{i=1}^{n} a_i \ln\lambda$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij}[(\ln\lambda)^2 + \ln\lambda(\ln w_j)]$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{\ell=1}^{m} (B_{i\ell} + B'_{\ell i})\ln\lambda \ln y_\ell \qquad (3.65)$$

Examination of the last two terms of equation (3.65) indicates that their magnitudes depend upon the $\ln w_i$'s and the $\ln y_\ell$'s. Since we do not want to impose any restrictions upon magnitudes of these variables, we want to impose restrictions on the parameters that will ensure that these terms equal zero.

Examining the last term first, since $B_{i\ell} = B'_{\ell i}$, we have

$$\ln \sum_{i}^{n} \sum_{\ell}^{n} B_{i\ell} \ln y_\ell = 0 \qquad (3.66)$$

Clearly, this term will equal zero if

$$\sum_{i=1}^{n} B_{i\ell} = 0 \qquad \ell=1,\ldots,m \qquad (3.67)$$

Thus, for the cost function to be homogeneous of degree one in factor prices, we must have the factor price - commodity output interactions sum to zero for each output.

Similarly, (after multiplying by 2) the third term can be written as

$$(\ln\lambda)^2 \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} + \ln\lambda \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} \ln w_i$$

$$+ \ln\lambda \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} \ln w_j$$

For the last two terms of this expression to equal zero, we must have

$$\sum_{j=1}^{n} A_{ij} = 0 \qquad i=1,\ldots,n \tag{3.68a}$$

$$\sum_{i=1}^{n} A_{ij} = 0 \qquad j=1,\ldots,n \tag{3.68b}$$

But, since A is a symmetric matrix, this implies that $\sum_{i=1}^{n} A_{ij} = \sum_{j=1}^{n} A_{ij}$ and that

$$\sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} = 0 \tag{3.68c}$$

Thus the third term of eq. (3.65) vanishes. Then eq. (3.65) can be written as

$$\ln C(\lambda w,y) = \ln C(w,y) + \ln\lambda \sum_{i=1}^{n} a_i$$

Therefore, to ensure that the cost function is homogeneous of degree one in factor prices, we must also require that

$$\sum_{i=1}^{n} a_i = 1 \tag{3.69}$$

To recapitulate briefly, homogeneity of degree one in w requires the following parameter restrictions

$$\sum_{i=1}^{n} a_i = 1$$

$$\sum_{i=1}^{n} A_{ij} = 0 \quad j=1,\ldots,n \tag{3.70}$$

$$\sum_{i=1}^{n} B_{i\ell} = 0 \quad \ell=1,\ldots,m$$

By a similar argument, homogeneity of degree k in y requires that

$$\sum_{\ell=1}^{m} c_\ell = k$$

$$\sum_{h=1}^{m} C_{\ell h} = 0 \quad \ell=1,\ldots,m \tag{3.71}$$

$$\sum_{\ell=1}^{m} B_{i\ell} = 0 \quad i=1,\ldots,n$$

Thus, to test for homogeneity of degree one in w, we first estimate the unconstrained cost function (3.61c) and then estimate it subject to the restrictions given in eq. (3.70) and then perform the usual F test to see if the equations are "significantly" different.

To test for homogeneity of an arbitrary degree k in y, we estimate the cost function subject to the constraints given by (3.70) and (3.71) and then utilize the F-test. To test for CRTS, we can impose the additional constraint that $\sum_{\ell=1}^{m} c_\ell = 1$ in (3.71) and compare the estimated cost function to the one that did not impose this constraint.

Since the restrictions (3.70) are implicit in all cost functions, we now assume that they hold and discuss how we can test for separability in the translog framework.

A separable function is one in which we can express the transformation frontier $F(y,x) = 0$ as $\psi(y) = f(x)$, where x represents the factor inputs and y represents the commodity outputs. The cost function associated with a separable production function can therefore be written as

$$C(w,y) = C^*(w,\psi(y)) \tag{3.72}$$

If $F(y,x)$ is homothetic (i.e. all its "isoquants" or level sets are are scalar multiples of each other; see Jacobsen (1970)),then $C^*(w,\psi(y))$ can be written as

$$C(w,y) = \phi(w)\cdot\psi(y) \tag{3.73}$$

or in logarithmic form

$$\ln C(w,y) = \ln \phi(w) + \ln \psi(y) \tag{3.74}$$

Using eq. (3.74) and the translog form of the cost function, we can readily obtain that the separable translog cost function is written as

$$\begin{aligned}\ln C(w,y) = a_0 &+ \sum_{i=1}^{n} a_i \ln w_i + \sum_{\ell=1}^{m} c_\ell \ln y_\ell \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} A_{ij} \ln w_i \ln w_j \\ &+ \frac{1}{2}\sum_{\ell=1}^{m}\sum_{h=1}^{m} C_{\ell h} \ln y_\ell \ln y_h\end{aligned} \tag{3.75}$$

Thus, if the production function is separable, the translog cost function has the added restriction that

$$B_{i\ell} = 0 \qquad \begin{matrix} i=1,\dots,n; \\ \ell=1,\dots,m \end{matrix} \tag{3.76}$$

That is, all interaction terms between w_i and y_ℓ must equal zero if the production function is separable.

If we want to impose separability and homogeneity of an arbitrary degree in y, we add the further restrictions given in eq. (3.71).

Summarizing, we can rewrite Figure 1 with the required restrictions. Thus, we begin (1) with the arbitrary function given in eq. (3.61). By imposing the restrictions given in eqs. (3.70) as can test for the underlying assumption of cost minimization (2), which we shall subsequently assume to hold. Thus the ensuing analysis assumes that restrictions (3.70_ hold, as well as those needed for homogeneity and separability. Homogeneity of an arbitrary degree in y (3) requires that eqs. (3.71) hold; while constant returns to scale (4) imposes the additional restriction that $\sum_{\ell=1}^{m} c_\ell = 1$. Separability (5) requires that eq. (3.76) holds and that $B_{ij} = 0$. The restrictions implied by eqs. (3.71) and (3.76) can then be combined to test for separability and homogeneity (6). By further requiring that eqs. (3.71) and (3.76) hold and that $\sum_{\ell=1}^{m} c_\ell = 1$, we can test for separability and CRTS (7). Moreover, by requiring that eqs. (3.71) and (3.76) hold and that $A_{ij} = 0$, we can test for Cobb-Douglas technology, with no CRTS restriction (8). Finally, by requiring that eqs (3.71) and (3.76) hold and that $A_{ij} = 0$, and $\sum_{\ell=1}^{m} c_\ell = 1$, we can test for CRTS, Cobb-Douglas technology (9).

2. <u>Non-Joint Production</u>

If production is non-joint, the cost function can be written as the sum of the cost functions of each of the separate outputs; thus

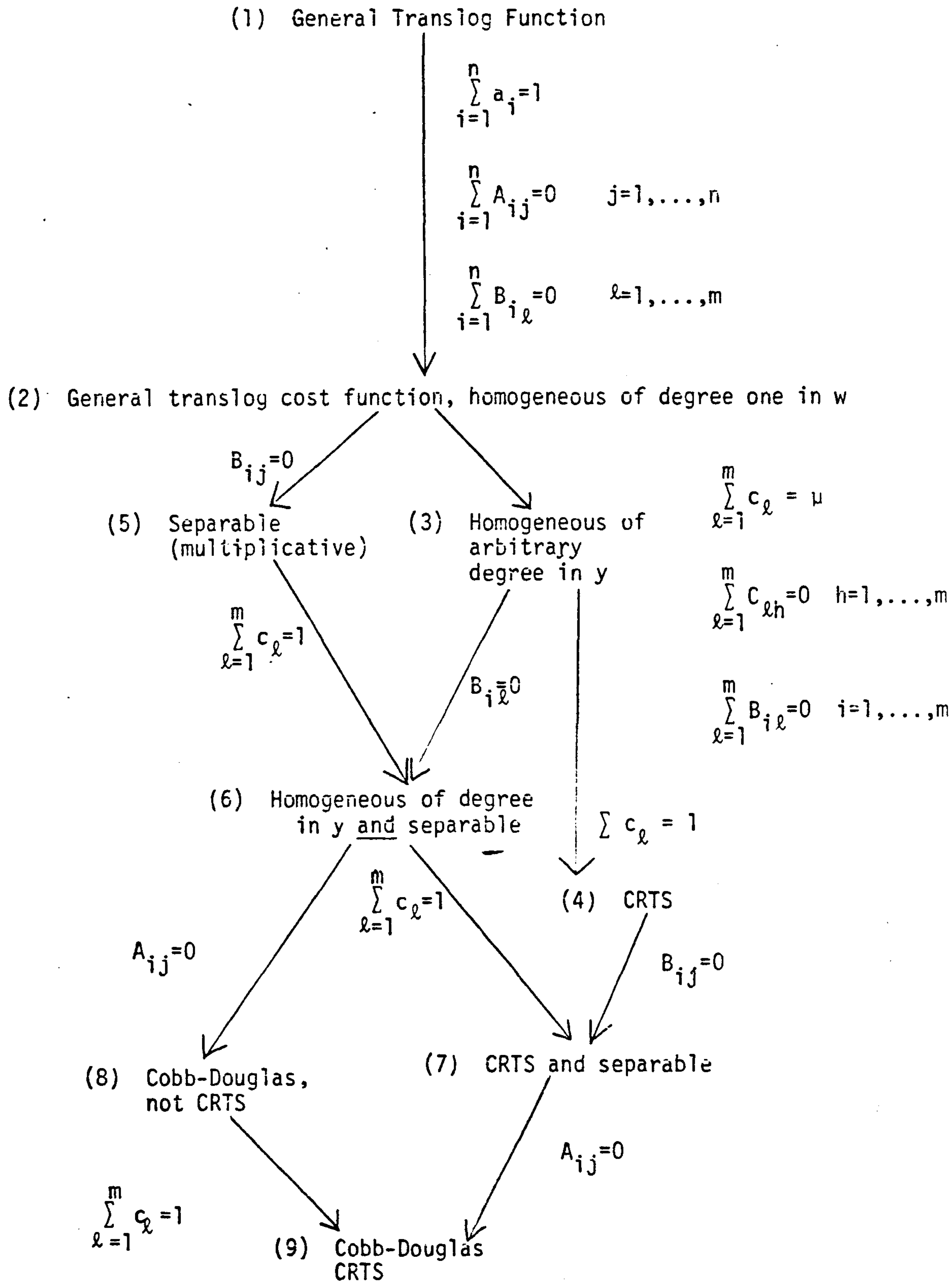
$$C(y_1,\ldots,y_n;w_1,\ldots,w_n) = \sum_{\ell} C^{\ell}(y_\ell, w_1,\ldots,w_n) \tag{3.77}$$

This implies, of course, that

$$\frac{\partial^2 C}{\partial y_\ell \partial y_h} = 0 \text{ for } \ell \neq h \tag{3.78}$$

Consequently, if we can relate the coefficients of the translog cost function to the second derivative of the underlying cost function and impose

Figure 2. Parameter Restrictions Needed to Test The Underlying Production Structure Using a Translog Cost Function



the condition that $C_{ij} = 0$, we can then determine the restrictions on the coefficients of the translog functions to ensure that production is non-joint.

Let us write the log of the non-joint cost function (3.77) as

$$\ell n\ C(y,w) = \ell n(\sum_{\ell} C^{\ell}(y_{\ell},w)) = \ell n\ f(x) \tag{3.79}$$

Thus its translog approximation can be written as

$$\ell n\ \hat{f}(x) = \alpha_0 + \sum_i \alpha_i(\ell nx_i - \ell nx_{oi}) + \frac{1}{2}\sum_i \sum_j \beta_{ij}(\ell nx_i - \ell nx_{oi})(\ell nx_j - \ell nx_{oj}) \tag{3.80}$$

when x_o represents the point of expansion.

From eq. (3.6b), we know that if $f_{ij} = \hat{f}_{ij} = 0$, then

$$\beta_{ij} = -\left(\frac{\hat{f}_i \hat{f}_j}{\hat{f}}\right)\left(\frac{x_i x_j}{\hat{f}}\right)$$

Substituting eq. (3.6a) for $\hat{f}_i$ into the above expression and collecting terms, we thus obtain

$$\beta_{ij} = -[\alpha_i + \sum_h \beta_{ih}(\ell nx_h - \ell nx_{oh})][\alpha_j + \sum_h \beta_{jh}(\ell nx_h - \ell nx_{oh})] \tag{3.81}$$

If the point of expansion x_o, also equals the point of evaluation, x, then $\ell n\ x_h = \ell nx_{oh}$ and

$$\beta_{ij} + \alpha_i\alpha_j = 0 \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,m;\ j \neq i \end{array} \tag{3.82a}$$

Since the i and j indices in expression (3.82a) refer to outputs, this expression can be translated into the coefficients of the translog cost function as

$$C_{\ell h} + c_{\ell}c_h = 0 \qquad \begin{array}{l} \ell=1,\ldots,m \\ h=1,\ldots,m\ ;\ h \neq \ell \end{array} \tag{3.82b}$$

Thus by using these non-linear restrictions on the interaction coefficients of the output variables, we can test for non-joint production.

B. The Short-Run Variable Cost Function

Instead of estimating a short-run total cost function, it is more convenient to estimate a short-run variable cost function. We obtain this by subtracting the fixed costs from the total costs and by replacing the prices of the fixed factors with their quantities. Thus, let us denote the short-run variable costs as $\tilde{C}$; the vector of prices of the variable factors by $w(w = w_1,\ldots,w_v)$; and the vector of fixed factors by $\bar{x}(\bar{x} = \bar{x}_1,\ldots,\bar{x}_f)$ where $f + v = n$, the total number of factors. Then the translog short-run variable cost function can be written as

$$\begin{aligned}\ell n\ \tilde{C}(y,w,\bar{x}) = {} & \alpha_o + \sum_{i=1}^{v} a_i\ \ell n w_i + \sum_{r=1}^{f} b_r \ell n\bar{x}_r + \sum_{\ell=1}^{m} c_\ell\ \ell n y_\ell \\ & + \frac{1}{2}\{\sum_i^v \sum_j^v A_{ij}\ell n w_i \ell n w_j + \sum_\ell^m \sum_h^m C_{\ell h}\ell n y_\ell \ell n y_h \\ & + \sum_r^f \sum_s^f D_{rs}\ell n\bar{x}_r \ell n\bar{x}_s + \sum_i^v \sum_\ell^m (B_{i\ell} + B'_{\ell i})\ \ell n w_i \ell n y_\ell \\ & + \sum_i^v \sum_r^f (E_{ir} + E'_{ri})\ell n w_i \ell n x_r + \sum_\ell^m \sum_r^f (F_{\ell r} + F'_{r\ell})\ell n y_\ell \ell n\bar{x}_r\}\end{aligned} \quad (3.83)$$

1. Homogeneity in Factor Prices

As long as a firm minimizes costs, its cost function must be homogeneous of degree one in factor prices. Thus even if the firm is subject to regulatory or institutional constraints that prevent it from adjusting its factors in an optimal fashion, it should still minimize costs with respect to its unrestricted or variable factors. Therefore the cost function should always be homogeneous of degree one in variable factor prices, whether the firm is in a point of long-run equilibrium or not.

Elementary economic theory requires that short-run (variable) cost functions be homogeneous of degree one in the variable factor prices $\tilde{w}$. Thus, by arguments analogous to those above, the translog approximation to $\tilde{C}(y,\bar{x},\tilde{w})$ must also be homogeneous of degree one in $\tilde{w}$. This implies that the following restrictions must be satisfied:

$$\sum_{i=1}^{v} \alpha_i = 1 \tag{3.84a}$$

$$\sum_{i=1}^{v} A_{ij} = 0 \qquad j=1,\ldots,v \tag{3.84b}$$

$$\sum_{i=1}^{v} B_{i\ell} = 0 \qquad \ell=1,\ldots,m \tag{3.84c}$$

$$\sum_{i=1}^{v} E_{ir} = 0 \qquad r=1,\ldots,f \tag{3.84d}$$

2. Homogeneity in Output

It is important to realize that homogeneity of degree k in output in the long-run cost function does not imply homogeneity of degree k in the short-run cost function. Nevertheless, the long-run cost function will be homogeneous of degree k in output if the following restrictions are satisfied as the short-run cost function:[27/]

[27/] For a full discussion of this point see Spady and Friedlaender (1976).

$$\sum_{\ell=1}^{m} c_\ell + k \sum_{r=1}^{f} b_r = k \tag{3.85a}$$

$$\sum_{\ell=1}^{m} B_{i\ell} + k\sum_{r=1}^{f} E_{ir} = 0 \qquad i=1,\ldots,n \tag{3.85b}$$

$$\sum_{\ell=1}^{m} C_{\ell n} + k \sum_{r=1}^{f} F_{rh} = 0 \qquad h=1,\ldots,m \tag{3.85c}$$

$$\sum_{\ell=1}^{m} F_{\ell s} + k \sum_{r=1}^{f} D_{rs} = 0 \qquad s=1,\ldots,f \tag{3.85d}$$

3. Separability and Non-Joint Production

If the long-run cost function is multiplicitively separable in outputs and all inputs, it will also be separable in the outputs and variable inputs and the outputs and fixed inputs. Hence separability requires

$$B_{i\ell} = 0 \qquad \begin{array}{l} i=1,\ldots,v \\ \ell=1,\ldots,m \end{array} \tag{3.86a}$$

$$F_{r\ell} = 0 \qquad \begin{array}{l} r=1,\ldots,f \\ \ell=1,\ldots,m \end{array} \tag{3.86b}$$

Finally, since the restrictions for non-joint production only apply to outputs, they are identical in the case of the short-run cost function to those of the long-run cost function, and are given by eq. (3.82b).

C. The Factor Demand Equations

From Shephard's Lemma we know that the derivative of the cost function with respect to a given factor price equals the demand for that factor, that is

$$\frac{\partial C(y,w)}{\partial w_i} = x_i$$

Since logarithmic differentiation of the translog cost function yields

$$\frac{\partial \ln C(y,w)}{\partial \ln w_i} = \frac{\partial C(y,w)}{\partial w_i} \cdot \frac{w_i}{C} \tag{3.87a}$$

we can readily obtain that

$$\frac{\partial \ln C(y,w)}{\partial \ell w_i} = \sigma_i \tag{3.87b}$$

where $\sigma_i = x_i w_i/C$, the share of the total costs accruing to factor i.

Consequently, in addition to estimating the translog cost function, it is useful to estimate its associated factor demand (or factor share) equations. In the long-run cost function, these equations take the following form

$$\sigma_i = a_i + \frac{1}{2}\left[\sum_{j=1}^{n} (A_{ij} + A'_{ji}) \ln w_j + \sum_{\ell=1}^{m} (B_{i\ell} + B'_{\ell i}) \ln y_\ell\right] \qquad i=1,\ldots,m \tag{3.88a}$$

where σ_i represents the share of the total costs attributed to factor i. However, in the case of the short-run variable cost function, these equations take the following form

$$\tilde{\sigma}_i = a_i + \frac{1}{2}\,[\sum_{j=1}(A_{ij} + A'_{ji})\ell n w_j + \sum_{\ell=1}^{m}(B_{i\ell} + B'_{\ell i})\ell n y_\ell$$

$$+ \sum_{r=1}^{f}(E_{ir} + E'_{ri})\ell n \bar{x}_r] \qquad (3.88b)$$

$$i=1,\ldots,v$$

where $\tilde{\sigma}_i$ represents the share of the variable costs attributed to factor i. Because the factor demand equations provide more information than the cost functions alone, it is desirable to estimate the cost function and the factor demand equations jointly and apply the appropriate restrictions over all equations. This will be discussed below when we describe our estimation procedures and empirical findings.

Neither the long-run translog cost function, given in eq. (3.61c), nor the short-run translog cost function, given in eq. (3.83), and their associated factor demand equations are suitable for estimation in their present form since similar terms have not been combined in them (i.e., $A_{ij}\ell nw_i\ell nw_j$ and $A_{ji}\ell nw_j\ell nw_i$ have not been combined into a single term $(A_{ij} + A_{ji})\ell nw_i\ell nw_j$). We therefore must rewrite these cost functions and modify the coefficient restrictions for homogeneity, separability, and non-joint production accordingly. We perform this transformation for the short-run cost function and its associated factor demand equations. The extension to the long-run cost function is apparent.

We thus rewrite the short-run cost function, given by eq. (3.83), as:

$$\ell n\tilde{C}(q,w,x) = a_o + \sum_i^v a_i \ell nw_i + \sum_r^f b_r \ell nx_r + \sum_\ell^m c_\ell \ell ny_\ell$$
$$+ \sum_{i=1}^{v} \sum_{j=1}^{v} \tilde{A}_{ij} \ell nw_i \ell nw_j + \sum_{\ell=1}^{m} \sum_{h=\ell}^{m} \tilde{C}_{\ell h} \ell ny_\ell \ell ny_r$$
$$+ \sum_{r=1}^{f} \sum_{s=r}^{f} \tilde{D}_{rs} \ell nx_r \ell nx_s + \sum_{i=1}^{v} \sum_{\ell=1}^{m} \tilde{B}_{i\ell} \ell nw_i \ell ny_\ell$$
$$+ \sum_{i=1}^{v} \sum_{r=1}^{f} \tilde{E}_{ir} \ell nw_i \ell nx_r + \sum_{r=1}^{f} \sum_{\ell=1}^{m} \tilde{F}_{r\ell} \ell ny_\ell \ell nx_r \qquad (3.89)$$

where

$$A_{ij} = \begin{bmatrix} A_{ij}/2 \text{ for } i=j \\ (A_{ij} + A_{ji})/2 \text{ for } i \neq j \end{bmatrix}$$

$$\tilde{C}_{\ell h} = \begin{bmatrix} C_{\ell h}/2 \text{ for } \ell=h \\ (C_{\ell h} + C_{h\ell})/2 \text{ for } \ell \neq h \end{bmatrix}$$

$$\tilde{D}_{rs} = \begin{bmatrix} D_{rs}/2 \text{ for } r=s \\ (D_{rs} + D_{sr})/2 \text{ for } r \neq s \end{bmatrix}$$

$$B_i = (B_{i\ell} + B'_{\ell i})/2 \text{ for all } i,\ell$$

$$\tilde{E}_{ir} = (E_{ir} + E'_{ri})/2 \text{ for all } i,r$$

$$\tilde{F}_{r\ell} = (F_{r\ell} + F'_{\ell r})/2 \text{ for all } r,\ell$$

Using equation (3.89) we can therefore include each pair of variables only once and ensure that the symmetry conditions in the relevant interaction coefficients are enforced.

The factor demand conditions must be similarly rewritten as

$$\sigma_i = a_i + \sum_j \bar{A}_{ij} \ell n\ w_i + \sum_\ell B_{i\ell} \ell n\ y_\ell + \sum_r \tilde{E}_{ir}\ \ell n\ x_r \qquad (3.90)$$

Equations (3.89) and (3.90) are the most general formulation of the cost and factor share equations since they contain no restrictions concerning homogeneity of factor prices, outputs, separability and so forth. Since however, we want to test for these restrictions, we must modify the equations accordingly.

Cost minimization requires that the cost function be homogeneous of degree one in w. As shown above in eqs. (3.84a) to (3.84d), this implies that

$$a_v = 1 - \sum_{i=1}^{v=1} a_i \tag{3.91a}$$

$$A_{vj} = - \sum_{i=1}^{v-1} A_{ij} \qquad j=1,\ldots,v \tag{3.94b}$$

$$B_{v\ell} = - \sum_{i=1}^{v-1} B_{i\ell} \qquad \ell=1,\ldots,m \tag{3.94c}$$

$$E_{vr} = - \sum_{i=1}^{v-1} E_{ir} \qquad r=1,\ldots,f \tag{3.94d}$$

Therefore,

$$\tilde{A}_{vj} = - \sum_{i=1}^{v-1} (A_{ij} + A_{ji})/2 = - [\sum_{i\neq 1} \tilde{A}_{ij} + 2\tilde{A}_{ii}] \tag{3.92a}$$

$$\tilde{B}_{v\ell} = - \sum_{i=1}^{v-1} (B_{i\ell} + B_{\ell i})/2 = - \sum \tilde{B}_{i\ell} \tag{3.94b}$$

$$\tilde{E}_{vr} = - \sum_{i=1}^{v-1} (E_{ir} + E_{ri})/2 = - \sum \tilde{E}_{ir} \tag{3.94c}$$

Thus, by substituting for a_v, A_{vj}, B_v and E_{vs} in eqs. (3.88) and (3.89) and by imposing the relevant cross equation restrictions between the cost and factor share equations we can estimate the set of equations that assumes cost-minimization.

The restrictions for homogeneity in outputs of an aribtrary degree k can similarly be incorporated into the estimating equations by using the following relationships to substitute for c_m, $\tilde{C}_{mh}$, $\tilde{B}_{mi}$, and $\tilde{F}_{mr}$, in eqs. (3.88) and (3.89).

$$C_m = \mu - \sum_{\ell=1}^{m-1} C_\ell - k \sum_{r=1}^{f} b_r \tag{3.93a}$$

$$\tilde{B}_{im} = - k \sum_{r=1}^{f} \tilde{E}_{ir} - \sum_{\ell=1}^{m-1} \tilde{B}_{i\ell} \qquad i=1,\ldots,n \tag{3.93b}$$

$$\tilde{C}_{mh} = - [\sum_{\ell \neq h}^{m-1} \tilde{C}_{\ell h} + 2\tilde{C}_{hh}] - k \sum_{r=1}^{f} \tilde{F}'_{rh} \quad h=1,\ldots,m \tag{3.93c}$$

$$\tilde{F}_{ms} = - \sum_{\ell=1}^{m-1} \tilde{F}_{\ell s} - k[\sum_{r \neq s}^{f} \tilde{D}_{rs} + 2\tilde{D}_{ss}] \quad s=1,\ldots,f \tag{3.93d}$$

Separability requires that there be no interactions among the inputs and the outputs and hence implies that

$$\tilde{B}_{i\ell} = 0 \qquad \begin{matrix} i=1,\ldots,v \\ \ell=1,\ldots,m \end{matrix} \tag{3.94a}$$

$$\tilde{F}_{r\ell} = 0 \qquad \begin{matrix} r=1,\ldots,f \\ \ell=1,\ldots,m \end{matrix} \tag{3.94b}$$

Finally, non-joint production implies that $C_{\ell h} + c_\ell c_h = 0$ (see eq. (3.82b). Since $C_{\ell h} = C_{h\ell}$ and $C_{\ell n} = (C_{\ell h} + C_{h\ell})/2$, we readily see that non-joint production implies

$$\tilde{C}_{\ell h} = - c_\ell c_h \qquad \begin{matrix} \ell=1,\ldots,m \\ h=1,\ldots,m;\ \ell \neq h \end{matrix} \tag{3.95}$$

Consequently, by making the successive substitutions indicated by eqs. (3.91) - (3.95), we can simultaneously estimate the cost and factor share equations under the following restrictions: cost-minimization, homogeneity of degree k in output; separability; and non-joint production.

Appendix 3.A

Comparative Statics

x=n - component vector of endogenous variables

a=k - component vector of exogenous or shift variables

We have structural equations:

(1) $f(x,\alpha) = 0$

Suppose there exists locally a function g such that $x=g(\alpha)$.

Then we can rewrite (1) as

(2) $F(g(\alpha),\alpha) = 0$

We want to find $\frac{\partial g}{\partial \alpha}$, i.e., how the x's locally change with the α's, with the effects of system (1) taken into account.

Differentiating (2) with respect to α:

(3) $\frac{\partial F}{\partial g} \frac{\partial g}{\partial \alpha} + \frac{\partial f}{\partial \alpha} = 0;$ but $\frac{\partial F}{\partial g} \equiv \frac{\partial F}{\partial x}$,

Rearranging

(4) $\frac{\partial g}{\partial \alpha} = - \left|\frac{\partial f}{\partial x}\right|^{-1} \frac{\partial f}{\partial \alpha}$

(nxk) (nxn) (nxk)

If $[\frac{\partial f}{\partial}]$ is singular, then $g(\alpha) =$ does not exist, even locally.

Appendix 3.B

Aggregation in the Railroad Industry

The railroad industry poses difficult problems of aggregation because of the large number of different types of track, rolling stock, and labor employed and because of the large numbers of different commodities carried in different shipment sizes and lengths of haul. In this Appendix we therefore consider procedures for factor and output aggregation that will capture the relevant differences, while still maintaining a tractable number of variables.

I. Factor Aggregation

Consistent with traditional economic theory, we plan to utilize three (or possibly four) broad factor aggregates: capital, labor, and materials (and possibly fuel as a separate category). Since, however, capital and labor consist of disparate categories, we will utilize a number of subcategories for capital and labor.

Expenditures are broken down into six broad categories which themselves consist of expenditures on the various factors. The table on the following page presents this schematically.

Thus expenditures on maintenance of way and structures will be allocated to maintain track, "other" track, and the labor needed to maintain them. Expenditures on maintenance of equipment will be allocated to the various kinds of rolling stock and the labor needed to maintain them. Expenditures on traffic and transportation will be allocated to the train, yard and supervisory labor and materials and fuel. Finally the remaining expenditure categories, general and miscellaneous, will be allocated among labor (executives, staff, professionals and clerical) and materials.

Expenditure Category	Factor		
	Capital	Labor	Materials
Maintenance of way and structures (Tracks)	Different types of track	MWS labor	None
Maintenance of Equipment (Rolling Stock)	Different type of rolling stock	ME labor	None
Traffic and Transportation	None	Three grades of labor	Fuel, other traffic and trans.
General and Miscellaneous	None	Executives,staff professionals, clerical	Materials and. Misc.

Note that the factor called "materials" is really a residual category that acts as a catch all for unallocated expenditures.

For each factor (and its subcategory), we need data in quantities, prices, and expenditures. Since, however, any two of these can generate the third, we really only need data on two of these three items. We now consider how they can be obtained for each factor and its components.

A. Capital

1. Track

Each railroad's track is divided into a number of different types of track of varying grades or qualities. The problem at hand, therefore, is to aggregate these various types of track into a measure of abstract track.

Expenditures on track are readily available for each railroad and are defined as expenditures on maintenance of way and structures less their labor components (EMWST).[1/] We thus want to create an aggregate price index for track, which can then be used to generate measure of abstract track for each railroad.

The ICC's Transport Statistics gives data on the average cost of repairing mainline and "other" track.[2/] If we assume that the price of each type of track is proportional to its cost of repair, we can construct an aggregate price index as follows. First, we obtain measure of the price of each type of track from the following identity:

$$EMWST_t = k_t(P_1 T_{1t} + P_{2t} T_{2t}) \qquad (3B.1)$$

where $EMWST_t$ represents the total expenditures for all railroads on maintenance of way and structures less their labor component; P_{1t} and P_{2t} represent the costs of repairing mainline and "other" track in year t; T_{1t} and T_{2t} represent the miles of mainline and "other" track for all railroads; and k_t represents a constant, which is determined by solving eq. (3B.1) for k_t. Note that in estimating the railroad cost functions, we plan to use cross-sectional and time series data for the years, 1961-1974. For each year then, "prices" of each type of track are defined as $k_{1t}P_{1t}T_{1t}$ and $k_{2t}P_{2t}T_{2t}$.

We now form a Divisia price index for the aggregate of all railroads using the following price index, given in eq. (3.37), above.

1/ Note that since maintenance is often deferred, these expenditures will reflect the fact that a mile of un-maintained track is not equivalent to a mile of maintained track.

2/ See ICC, Transport Statistics, Part 1, second release (TS.I.2), Table 92.

$$IPT_t = \left(\frac{PI_{1t}}{PT_{1o}}\right)^{\sqrt{s_{1t} + s_{1o}}} \cdot \left(\frac{PT_{2t}}{PT_{2o}}\right)^{\sqrt{s_{2t} + s_{2o}}} \qquad (3B.2)$$

where IPT_t represents the aggregate price index for all track in year t; PT_{it} and PT_{io} represent the prices of track type i in the base period and in period t (i.e. $PT_{1t} = k_t P_{1t} T_{1t}$); and S_{it} and S_{io} represent the share of each type of track in the base period and in period t (i.e., $S_{it} = PT_{it}/EMWST_t$).

Having obtained an aggregate price index for all track for all railroads, we can then divide the above index into each railroad's expenditures on maintenance of way and structures less its labor component to obtain its quantity of abstract track. Thus

$$T_{rt} = \frac{EMWST_{rt}}{IPT_t} \qquad (3B.3)$$

where T_{rt} represent the quantity of abstract track utilized by railroad r in year t; $EMWST_{rt}$ represents the expenditures on track by railroad r in period t; and IPT_t represents the aggregate price index of all track in period t.

2. Rolling Stock

Since each railroad uses different kinds of engines and cars, we must aggregate these into two categories, engines and cars.

The price of a new engine is related to its type (diesel, electric, steam, etc.) and its horsepower. We thus adjust its price to take differences of tractive power into account and estimate the following regression.

$$P_{Eit} = do + \sum_{j=2} \gamma_j e_j + \sum_{\tau=1} \mu_\tau d_\tau + \alpha_1 (HP_{it} - \overline{HP}_t) \qquad (3B.4)$$

where P_{Eit} represents the price of a new engine of type i in year t; e_j represents a dummy variable for each engine type (1 if engine of type j; 0 otherwise); $d\tau$ represents a time dummy (1 if observation in year t; 0 otherwise); HP_{it} represents the horsepower of engine i in period t; and HP represents the average horsepower of all engines. With three types of engines and data for 14 years, we will thus have 42 observations to estimate 17 parameters.

The "price" of a standard engine of type i in year t is thus estimated as

$$P^*_{Eit} = do + \mu_t d_t + \gamma_i e_i \qquad (3B.5)$$

where the dummies take on values of 1 or 0 as appropriate.

Data are available on the costs of new freight cars, but there do not seem to be any data available on their qualities. Hence we cannot perform a hedonic adjustment on freight cars, similar to that employed on engines.

The prices of new cars and engines do not reflect the prices of their services, which we assume to be proportional to these purchase prices. We thus relate expenditures on maintenance of equipment less their labor component to the expenditures on engines and cars as follows:

$$MERS_t \equiv k_t \left[\sum P^*_{Eit} E_{it} + \sum P_{C_{it}} C_{it} \right] \qquad (3B.6)$$

where $MERS_t$ represents expenditures on maintenance of equipment less labor; P^*_{Eit} represents the price of a standard engine of type i in year t; E_{it} represents the number of engines of type i in year t; $P_{C_{it}}$ represents the price of car type i in year t; and C_{it} represents the number of cars in year t. Solving for k_t, we thus defined the price of services of a standard engine of typ i in year t as

$$\rho E_{it} = k_t P^*_{Eit} \tag{3B.7a}$$

We similarly define the price of the service of car type i in year t as

$$\rho_{Cit} = k_t P_{Cit} \tag{3B.7b}$$

We now divide expenditures on maintenance of equipment less labor into expenditures on engines and cars and then define

$$MERSE_t = \sum k_t P^*_{Eit} E_{it} \equiv \sum \rho_{Eit} E_{it} \tag{3B.8a}$$

$$MERSC_t = k_t \sum P_{Cit} C_{it} \equiv \sum \rho_{Cit} C_{it} \tag{3B.8b}$$

Given these, we can then define two Divisia indices, one for engines and one for cars. Specifically, we calculate

$$IPE_t = \prod_{i=1} \left(\frac{\rho_{Eit}}{\rho_{Eio}}\right)^{\sqrt{S_{Eit} + S_{Eio}}} \tag{3B.9a}$$

where the i's range over engine types and S_{Eit} represents the share of $MERSE_t$ spent on locomotive type i in year t.

Similarly, we define the index of the price of car services as

$$IPC_t = \prod_{i=1} \left(\frac{\rho_{Cit}}{\rho_{Cio}}\right)^{\sqrt{S_{Cit} + S_{Cio}}} \tag{3B.9b}$$

where the i's range over car types and S_{Cit} represents the share of $MERSC_t$ spent on cars of type i in year t.

By dividing these indices into the appropriate measure of expenditures on maintenance of equipment, we can then obtain quantity measures of engines and cars.

Alternatively, we could construct Divisia quantity indices, using the physical quantities of engines and cars of each type instead of their prices. We could then divide these indices into the appropriate measure of expenditures on maintenance of equipment to obtain measure of abstract prices of engines and cars.

B. Labor

The ICC's A-300 wage statistics have detailed information on numerous types of labor, giving the number of individuals employed and their total compensation. We plan to aggregate these into three categories of labor: train labor, yard labor, and other labor by constructing Divisia quantity indices. We can then obtain aggregate price measures by dividing the relevant labor expenditure category by its Divisia quantity index.

C. Materials

Expenditures on materials will be treated as a residual category and defined as total expenditures less expenditures on capital and labor. There are no direct price measures available for materials. However, using national or regional price indices for materials or energy would probably be acceptable and thus enable us to obtain quantity indices if desired.

II. Output

The ICC's quarterly commodity statistics give annual data on numerous commodity types for each railroad. We plan to aggregate these into the following broad categories:

Grains

Other agricultural commodities

Coal

Other raw materials

Non-durable manufactures (including petroleum)

Durable manufactures

Forwarder and Related Traffic

Since these are composed of a number of two-digit STCC codes, it probably makes sense to aggregate by these codes. Thus our procedure will be to aggregate by 2-digit STCC where appropriate, and to perform no aggregation where the broad aggregate does not comprise more than one STCC code. The procedure followed will be the same for all output categories.

Since the size of shipment and length of travel vary by commodity type, it is desirable to adjust for these differences directly. Thus we estimate a hedonic regression of the following type[3/]

$$P^i_{jft} = do + \sum_{t=1}^{T} \gamma_\tau \, d_\tau + \sum_{j=1}^{h} \mu_j y^i_j + \sum_{f=1}^{F} \beta_f F_f$$

$$+ \alpha_1 \, (M^i_{jft} - \overline{M}^i) + \alpha_2 \, (S^i_{jft} - \overline{S}^i) \qquad (3B.10)$$

where P^i_{jft} represents the revenues per ton-mile for commodity j in category i for firm f in time t; $d\tau$ represents a time dummy (1 if in year t; otherwise 0); y^i_j represents a commodity dummy (1 if commodity j; otherwise 0); R_f represents a firm dummy (1 if firm f; otherwise 0); M^i_{jft} represents the length of haul of commodity j for firm f in year t; S^i_{jft} represents the

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[3/] This can also be in logarithmic form and include interaction terms.

average size of shipment ~f commodity (for firm f in year t); $\overline{M}^i$ represents the average size of shipment for commodity; over all years and over all firms; and $\overline{M}^j$ represents the average length of haul over all firms and all years.

The "price" of a standard shipment of commodity j for firm f in year t is therefore given by

$$P^{*i}_{jt} = do + \gamma_\varepsilon d_\varepsilon + \mu_j y^i_j + \beta_f R_f \tag{3B.11}$$

Given this, we can therefore define a Divisia price index as

$$IP^i_{ft} = \prod_{j=1}^{h} \left(\frac{P^{*i}_{jt}}{P^{*i}_{jo}}\right)^{\sqrt{S^i_{jft} + S^i_{jfo}}} \tag{3B.12}$$

where S^i_{jft} and S^i_{jfo} represent the revenue shares of commodity; for firm f in year t and the base year 0.

Finally, we define the "abstract" quantity of Y^i_{ft} carried by firm f in year t as

$$Y^i_{ft} = \frac{Rev^i_{ft}}{IP^i_{ft}} \tag{3B.13}$$

Thus Y^i_{ft} is the variable that enters the estimated cost functions.

Instead of adjusting the quantities of each broad commodity type carried by hedonic regressions, we could estimate a hedonic cost function directly, in which case instead of using Y^i in the cost functions, we would introduce $\psi(Y^i, M^i - \overline{M}^i, S^i - \overline{S}^i)$ for each aggregate output category as explained in the text (see eq. (3.60) above). While this approach has definite theoretical advantages since it does not assume the existence of perfectly competitive markets, which are assumed by hedonic regressions, it has the disadvantage of not considering quality differences among the component of each category i.[4/] Thus we will probably estimate cost functions using direct hedonic adjustments and indirect hedonic adjustments via prices and determine which approach gives the best results.

[4/]Although these could be introduced in principle, in practice, they would make the regressions too unwieldy.

Chapter Four

Hedonic Cost Functions for the Trucking Industry

The previous chapter outlined a general methodology that can be used to estimate cost function in the transportation industries. This chapter presents the application of this methodology to the trucking industry and shows that failure to take quality of output explicitly into account may lead to seriously biased estimates of costs and therefore to poor policy conclusions.

I. Introduction and Overview

During the past decade, there have been a number of econometric studies of the costs of regulated trucking.[1/] While they have varied in detail, they have generally utilized a cross section of firms to estimate costs as a functions of output (usually measured by ton-mile or revenue ton-miles) and a number of other variables to reflect regional differences or technical change.

Because, however, trucking output is highly heterogeneous, it is questionable whether a single output measure, such as ton-miles, is appropriate to use in estimating trucking costs. Not only do different firms carry different commodities; but also, different firms utilize widely different shipment sizes and lengths of haul. Moreover, firms vary widely in the share of less-than-truckload (LTL) traffic they carry. Thus, two firms, each carrying an equal number of ton-miles over a year can have

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[1/] See Oramas (1975) for a good summary of these.

very different types of output. One could concentrate on short-haul, small-load, LTL traffic, while the other could concentrate on long-haul, large-load, truckload traffic. In view of the differences in the composition of their output, it would be highly unlikely that they would have the same costs, although this would be predicted by conventional econometric studies of the trucking industry.

Basically, there are two sources in differences in output for any given measure of ton-miles. First, the nature of the commodities carried may differ; and second, the way in which the commodities are carried with respect to length of haul and size of shipment may differ. Ideally, econometric estimates of trucking costs should take both of these factors into account.

By limiting our analysis to regulated common carriers of general freight we are largely able to take the first factor into account. These firms typically carry manufactured commodities whose characteristics with respect to handling, etc. should be similar.[2/]

Within regulated carriers of general freight there are significant inter-firm differences with respect to size of haul, length of haul, and the share of LTL traffic. Fortunately, data are available to take these factors into account, and this chapter reports on efforts to relate costs to differences in the composition of output with respect to length of

[2/] Nevertheless, to the extent that firms specialize with respect to certain types of manufactured commodities, biases may still exist. If, for example, one firm specialized in computer components and another specialized in fabricated steel products, it is likely that their costs would differ for any given number of ton-miles. Unfortunately, however, data are unavailable to take these differences into account.

haul, size of shipment, and the share of LTL traffic. An understanding of the cost effects of these differences is important for evaluating alternative policies, not only because some alternatives (such as the relaxation of backhauling prohibitions) directly affect shipment sizes and haul lengths, but also because these factors affect firms' responses to factor-price (particularly fuel price) increases and the economies (or diseconomies) of increased firm size. Clearly, this last question is important to merger policy and policies affecting the distribution of freight across modes: economies from the expansion of trucking activity depend not only on the present level of activity (in ton-miles) but also on its distribution by shipment size, length of haul, and share of LTL traffic.

Briefly, then, this chapter takes the following form. Part II presents a general econometric specification of technology for the trucking industry, while Part III presents a number of estimates under alternative assumptions concerning the importance of quality differentials and the structure of the industry. Part IV presents a brief summary and outlines areas for further research.

II. Econometric Specification of Technology

Duality theory indicates that every specification of a cost strucuture corresponds to a specification of a production structure. One can therefore interchangeably specify a cost function or a production function.[3/] Because, however, it is possible to specify more econometrically testable hypotheses concerning the structure of technology by using cost functions than by using production functions, it is generally agreed that econometric estimation of cost functions is more useful than econometric estimation of production functions.[4/]

The simplest specification that might reasonably be expected to take account of shipment size, length of haul, and share of LTL traffic is the hedonic cost function given by:

$$\text{Cost} = C[\psi(y,q_1,q_2,q_3);\ w_1,w_2,w_3,w_4] \qquad (4.1)$$

where $\psi(y,q_1,q_2,q_3)$ is a function that measures output, with y = ton-miles, q_1 = average size of shipment, q_2 = average length of haul, q_3 = percentage of tons shipped in LTL lots; and w_1,w_2,w_3,w_4 represent the respective price of labor, fuel, capital, and purchased transportation (primarily rental vehicles).

3/ See Shephard (1970) for the theoretical equivalence between costs and production and the regularity conditions that are needed to ensure that duality holds.

4/ See Varian (1975) for a discussion of the econometric problems associated with estimating production functions.

We call this cost function "quality-seperable" because the effect of quality variations upon the output measure ψ, and therefore on costs, is independent of relative factor prices. The technology implied by such a specification can be envisioned as combining the four input factors to produce one abstract output, "trucking capacity", measured by $\overline{\Psi}$, which can then be divided into any (y,q_1,q_2,q_3) combination which satisfies $\overline{\Psi} = \psi(y,q_1,q_2,q_3)$. This specification is moderately restrictive for it implies, for example, that the price of fuel does not affect the combinations of ton-miles and average size of shipment that can be produced at equal cost with equal haul lengths and LTL ratios.[5]

As indicated above, the value of the function of (y,q_1,q_2,q_3) serves as the output measure in this specification of the cost function. This assumes that a continuum of different "quality" ton-miles exists, which can be consistently aggregated by the function $\psi(\cdot)$. By analogy with conventional theory of aggregation,[6] it is natural to require that $\psi(\cdot)$ is separable into ton-miles and qualities. Thus:

$$\psi(y,q_1,q_2,q_3) = y\cdot\phi(q_1,q_2,q_3) \tag{4.2}$$

This implies that a doubling of ton-miles at a given quality level doubles ψ the measure of output. No restrictions need be placed on $\phi(\cdot)$.

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[5] In fact, our econometric results indicate that there may be some interaction among fuel prices and average size of shipment and average length of haul. We are presently trying to develop a more general specification that would relax this separability restriction.

[6] See Diewert (1974) and Samuelson and Swamy (1974) on aggregation theory, and Chapter Three, above, for the details of the specification of the $\psi(\cdot)$ function.

Because the translog approximation to a cost function permits us to test a wide range of hypotheses concerning the structure of technology,[7] we use it here. Since the trucking firms presumably are able to adjust capacity easily, either by selling trucks or by rental agreements, it seems sensible to estimate a long-run cost function, which takes the following general form:[8]

$$\begin{aligned} \ln C(\psi,w) = \alpha_0 &+ \alpha_y(\ln\psi-\ln\bar{\psi}) + \sum_{i=1}^{4}\alpha_i(\ln w_i-\ln\bar{w}_i) \\ &+ 1/2 \sum_{i}^{4}\sum_{j}^{4} \beta_{ij}(\ln w_i-\ln\bar{w}_i)(\ln w_j-\ln\bar{w}_j) \\ &+ 1/2\ \beta_{\psi\psi}(\ln\psi-\ln\bar{\psi})^2 \\ &+ \sum_{i=1}^{4}\beta_{\psi i}(\ln\psi-\ln\bar{\psi})(\ln w_i-\ln\bar{w}_i) \end{aligned} \qquad (4.3)$$

In addition, we estimate the factor share equations, which take the following form:[9]

$$\frac{w_i x_i}{C} = \alpha_0 + \sum_j \beta_{ij}(\ln w_j-\ln\bar{w}_j) + \beta_{\psi i}(\ln\psi-\ln\bar{\psi}) \qquad (4.4)$$

$$i = 1,\ldots,3$$

---

[7] See Chapter Three, above, for a full discussion of these tests.

[8] Note that we take the sample mean as the point of approximation.

[9] Note that we only need to estimate three factor share equations explicitly, since the fourth is implied by the previous three. The results are invariant to the equation dropped. See Barten (1969) or Berndt and Savin (1975).

From equation (2) we know that:

$$\ell n\psi = \ell ny + \ell n\ \phi(q_1,q_2,q_3). \tag{4.5}$$

We thus utilize a translog approximation of $\phi(\cdot)$ and write:[10/]

$$\begin{aligned} \ell n\phi(q_1,q_2,q_3) = a_o &+ a_1(\ell nq_1-\ell n\bar{q}_1) + a_2(\ell nq_2-\ell n\bar{q}_2) + a_3(\ell nq_3-\ell n\bar{q}_3) \\ &+ 1/2\ b_{11}(\ell nq_1-\ell n\bar{q}_1)^2 + b_{12}(\ell nq_1-\ell n\bar{q}_1)(\ell nq_2-\ell n\bar{q}_2) \\ &+ b_{13}\ \ell n(q_1-\ell n\bar{q}_1)(\ell nq_3-\ell n\bar{q}_3) \\ &+ 1/2\ b_{22}(\ell nq_2-\ell n\bar{q}_2)^2 + b_{23}/\ell nq_2-\ell n\bar{q}_2)(\ell nq_3-\ell n\bar{q}_3) \\ &+ 1/2\ b_{33}(\ell nq_3-\ell n\bar{q}_3)^2 \end{aligned} \tag{4.6}$$

In the most general case, therefore, we substitute eq. (4.6) into eqs. (4.3) and (4.4) and jointly estimate these equations, subject to the following constraints, which ensure linear homogeneity of $C(\psi,w)$ in w and the symmetry restrictions implied by cost minimization.[11/]

[10/] Note that we have collected similar terms in this expression and thus imposed the necessary symmetry conditions.

[11/] The LSQ procedure in TSP was used for all regressions reported here; it provides a minimum distance estimation whose properties are discussed in Berndt, Hall, Hall, and Hausman (1974).
Estimating the factor share equations jointly with the cost functions improves the efficiency of the resulting estimates; see Christensen and Greene (1976) on this and related points concerning returns to scale estimation to be covered below. For a development of the homogeneity and symmetry restrictions, and a number of other restrictions useful in testing hypotheses concerning the technology represented by $C(\psi,w)$ see Chapter Three, above.

$$\sum_{i=1}^{4} \alpha_i = 1$$

$$\sum_{j=1}^{4} \beta_{ij} = 0 \quad i = 1,\ldots,4; \quad \beta_{ij} = \beta_{ji} \tag{4.7}$$

$$\sum_{i=1}^{4} \beta_{\psi i} = 0$$

III. Econometric Estimation of Trucking Costs

Having specified a general quality-separable hedonic cost function, with the appropriate restrictions needed to ensure cost-minimization, we now consider a number of alternative specifications and their associated restrictions, which have definite implications concerning the role of quality differentials and the competitive structure of the industry. We thus begin by presenting the estimation associated with the general quality-separable hedonic cost function presented above, and then consider the restrictions implied by ignoring quality differentials and the assumptions concerning separability and homogeneity.

A. Data

The sample used in this study consists of 171 firms in 1972, located in the Central, Middle Atlantic, and New England trucking regions, as defined by the ICC, which roughly corresponds to the ICC's Official Railroad Territory.[12/] As indicated above, we use the following variables in the cost functions:

y = ton-miles

q_1 = average size of shipment (tons/shipment)

q_2 = average length of haul

q_3 = 1 + percentage of tons shipped in LTL lots[13/]

w_1 = price of labor

w_2 = price of fuel

w_3 = price of capital

w_4 = price of fuel

12/ This regional aggregation was performed to ensure rough regional similarity between trucking and rail costs. This similarity will prove useful when intermodal competition is analyzed. For a discussion of the analysis of intermodal competition see Chapter Two, above.

13/ The variable q_3 was defined as 1+% LTL since some firms had no LTL shipments.

C = total costs

$w_i x_i / C$ = share of factor i

All of these data were taken from Trinc's Blue Book (1972), which summarizes the individual firm reports to the ICC. The firms' total costs were divided into labor costs, fuel expenditures and fuel taxes, purchased transportation, and other. "Other" expenditures (which included depreciation) were assumed to be payments for capital services; each firm's "carrier operating property - net" was taken as a measure of the quantity of capital (and thus of capital services), so that "other expenditures" divided by "carrier operating property net" gave a firm-specific price of capital. A firm specific price of labor was obtained by dividing labor expenditures by the average number of employees. Since direct quantity measures of purchased transportation and fuel were not available, regional prices for these commodities were estimated by a method whose assumptions and results are given in Appendix 4. [14]

The sample of 171 firms included all firms without missing data in five regions (Central States East, Central States West, Middle Atalantic, North Middle Atlantic, New England) that met the following conditions:

1. They purchased some of all four factors; but no more than 10 percent of their costs were for purchased transportation. (If a firm does not purchase any of a particular factor, this indicates a corner solution which the specification is incapable of modelling. Firms which rent most of their vehicles do so from subsidiaries set up for tax and regulatory purposes, due to an ICC ruling which allows the deduction of such expenses as current costs, which has the effect of artificially lowering their operating ratio, which is a primary regulatory target.

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[14] For similar translog models which use some firm-specific prices and some regional prices, see Christensen and Greene (1976) and Nerlove (1963).

2. They reported an average salary of $8000/year or more per employee. (Some firms implicitly reported salaries as low as $2000, presumably because they counted owner/operators whose trucks they rented as employees, even though they did not directly pay them any wages).

3. They had a calculated price of capital of less than 10. (Due to reasons related to (1) above, a few carriers report almost no operating property, as it is (presumably) owned by subsidiaries. (Note that carrier operating property is the value of the property that the firm owns, not its equity in that property.)) The mean price of capital in the sample is 2.725 with a standard deviation of 1.287.

4. They had no other "obvious" error in the data. (For instance, one firm reported an average load of 92 tons.)

B. Econometric Estimate

In a cross-firm estimate of the cost function, as long as each firm faces the same $\psi(y,q)$ function of the form $\psi(y,q) = y\cdot\phi(q)$, we can estimate a cost function given by equation (4.3) and its associated factor share equations (4.4), with the appropriate substitution of $\ell n\ \psi(y,q)$, given in eq. (4.6).

Table I gives the joint estimates of the cost and factor share equations under varying assumptions concerning the nature of technology and the hedonic cost function. Equations (1) - (3) estimate the hedonic cost function under the following assumptions: (1) no restrictions concerning separability and homogeneity in output; (2) separable technology; (3) homogeneity in output of degree 1.

Since the estimates of the hedonic function $\phi(q)$ are quite similar in each of these equations, we will concentrate on the estimate of this function given by the unrestricted hedonic cost function eq. (1). These are given by the coefficients above the dotted line. A constant does not appear in the hedonic function $\psi(q)$ since its effect in this specification would be merely to change units of measurement of $\psi(q)$. The estimates of this function accord well

Table 1
Joint Estimates of Cost and Factor Share Equations

		Hedonic Cost Function							
		(1) Unrestricted		(2) Separability		(3) Homogeneity in Output		(4) Nonhedonic Cost Function	
Coefficient	Variable	Value	Standard Error	Value	Standard Error	Value	Standard Error	Value	Standard Error
a_1	(Size) q_1	-.1501	-.04205	-.1267	.04186	-.1441	.0397	--	--
a_2	(Haul) q_2	-.7070	.05497	-.6810	.05496	-.6880	.0537	--	--
a_3	(LTL) q_3	1.2029	.23339	1.1757	.23374	1.2181	.2166	--	--
b_{11}	$1/2\ q_1^2$	.1286	.05663	.1337	.05681	.1222	.0560	--	--
b_{12}	q_1q_2	.0749	.06147	.0879	.06152	.0635	.0607	--	--
b_{13}	q_1q_3	.7418	.18522	.7408	.18570	.7575	.1799	--	--
b_{22}	$1/2\ q_2^2$	.1121	.09211	.1259	.09253	.1376	.0915	--	--
b_{23}	q_2q_3	-.2816	.3417	.2884	.34213	-.3492	.3365	--	--
b_{33}	$1/2\ q_3^2$	7.5114	1.8130	7.6940	1.81239	7.6999	1.7016	--	--
α_0	1	8.6639	.04789	8.6684	.04755	8.6830	na	8.6858	.0457
α_ψ	(Output)	1.0408	.02753	1.0345	.02568	1.0000	na	.7665	.0352
α_1	(Lab) w_1	.5928	.00458	.5870	.00416	.5929	.0045	.5870	.0049
α_2	(Fuel) w_2	.0397	.00124	.0409	.00111	.0399	.0012	.0416	.0013
α_3	(Cap) w_3	.3323	.00374	.3376	.00340	.3319	.0037	.3341	.0040
α_4	(Purch. Trans.) w_4	.0352†	na†	.0342	na	.0353	na	.0373	na
β_{11}	w_1w_1	.0324	.01671	.0372	.0168	.0326	.0167	.0229	.0149
β_{12}	w_1w_2	-.0235	.00725	-.0254	.00716	-.0237	.0072	-.0149	.0062
β_{13}	w_1w_3	-.0147	.00872	-.0171	.00892	-.0148	.0087	-.0109	.0085
β_{14}	w_1w_4	.0058†	na†	.0053	na	.0059	na	.0028	na
β_{22}	w_2w_2	.0296	.00749	.0280	.00741	.0272	.0075	.0176	.0066
β_{23}	w_2w_3	-.00732	.00235	-.0068	.00231	.0073	.0023	-.0068	.0020
β_{24}	w_2w_4	.0012†	na†	.0095	na	..0038	na	.0041	na
β_{33}	w_3w_3	.0159	.00735	.0185	.00754	.0158	.0073	.0118	.0076
β_{34}	w_3w_4	.0061†	na†	.0054	na	.0063	na	.0059	na
β_{44}	w_4w_4	-.0070†	na†	-.0202	na	-.0160	na	-.0127	na
$\beta_{\psi 1}$	ψw_1	.0128	.00445	--	--	.0130	.0041	-.0005	.0035
$\beta_{\psi 2}$	ψw_2	-.0027	.00119	--	--	-.0023	.0011	.0008	.0009
$\beta_{\psi 3}$	ψw_3	-.0115	.00361	--	--	-.0122	.0035	-.0045	.0028
$\beta_{\psi 4}$	ψw_4	.0014†	na†	--	--	-.0015	na	.0082	na
$\beta_{\psi\psi}$	$\psi\psi$	.0644	.03896	.0634	.03887	--	--	.1117	.0319

†Coefficient value is implied by symmetry and homogeneity restrictions. Therefore, standard errors are not available.

Table 2

Summary Statistics

	Hedonic Cost Function (1) Unrestricted	(2) Separability	(3) Homogeneity	Nonhedonic Cost Function (4)
R^2				
Cost Eq.	.9286	.9265	.9264	.7596
Labor Eq.	.0847	.0419	.0821	.0315
Fuel Eq.	.0418	.0111	.0393	.0313
Capital Eq.	.0778	.0227	.0760	.0366
SSR				
Cost Eq.	11.1824	11.5071	11.5264	37.6612
Labor Eq.	.4708	.49278	.47210	.49809
Fuel Eq.	.0337	.0348	.03382	.03410
Capital Eq.	.3101	.3287	.31077	.32400
Log of Likelihood Function	1157.08	1150.69	1155.46	1075.80

with a priori expectations. The significant negative signs in the linear size and haul terms indicate that ton-miles characterized by larger loads and longer lengths of haul are easier to produce than ton-miles characterized by smaller loads and shorter lengths of haul.[15/] Conversely, the significantly positive coefficient in the linear LTL term indicates that ton-miles characterized by a larger percentage of LTL shipments are harder to produce than those characterized by small percentage of LTL shipments. Stated alternatively these findings indicate that LTL shipments, small loads, and short hauls are more costly to produce than TL shipments, large loads and long hauls, for any given amount of ton-miles.

Equation (1) indicates that the squares and interaction terms of the $\psi(q)$ function are marginally significant, with the exception of those containing the LTL variable, which are all highly significant. This indicates that LTL shipments interact with size of shipments and length of haul. Specifically, for any given size of shipment, increase in the share of LTL will increase costs ($b_{13} > 0$) while for any given length of haul, increases in the share of LTL will reduce costs ($b_{23} < 0$). Finally, the significantly positive sign in the squared LTL term indicates that costs will rise at an increasing rate as the share of LTL increases. Thus from Equation (1), we can

[13/] Specifically, the linear coefficients can be taken to represent the change in output occasioned by a change in quality. Thus a positive sign in the linear terms implies that cet.par., an increase in the quality will increase the effective output, and thus increase costs. Similarly, a negative sign in a linear coefficient implies an increase in the quality will reduce effective output and hence costs.

The signs in the interaction terms are somewhat harder to interpret. Basically, they represent the impact of a given quality in the rate of change in output. For example, a positive b_{13} coefficient implies that for any given size of load, an increase in the share of LTL will lead to greater increase in effective output and thus higher costs. Thus, for any given size of load, costs increase with the share of LTL.

infer that although size of shipment, length of haul, and share of LTL shipments all affect effective output and thus costs, the share of LTL shipments probably has a greater impact upon effective output and costs than does either of the other quality variables.

The figures below the dotted line in Equations (1) - (3) represent the joint estimate of the cost and factor share equations of the hedonic cost functions.

It is difficult to interpret directly the estimates of the translog cost function's coefficients, since the elasticities of substitution and returns to scale generally depend upon the output level and factor prices at which they are calculated. In particular, nonzero $\beta_{\psi i}$'s, which we generally estimated in Table 1, indicate that the cost function is not separable,[16/] and therefore, that the structure of production is nonhomothetic.

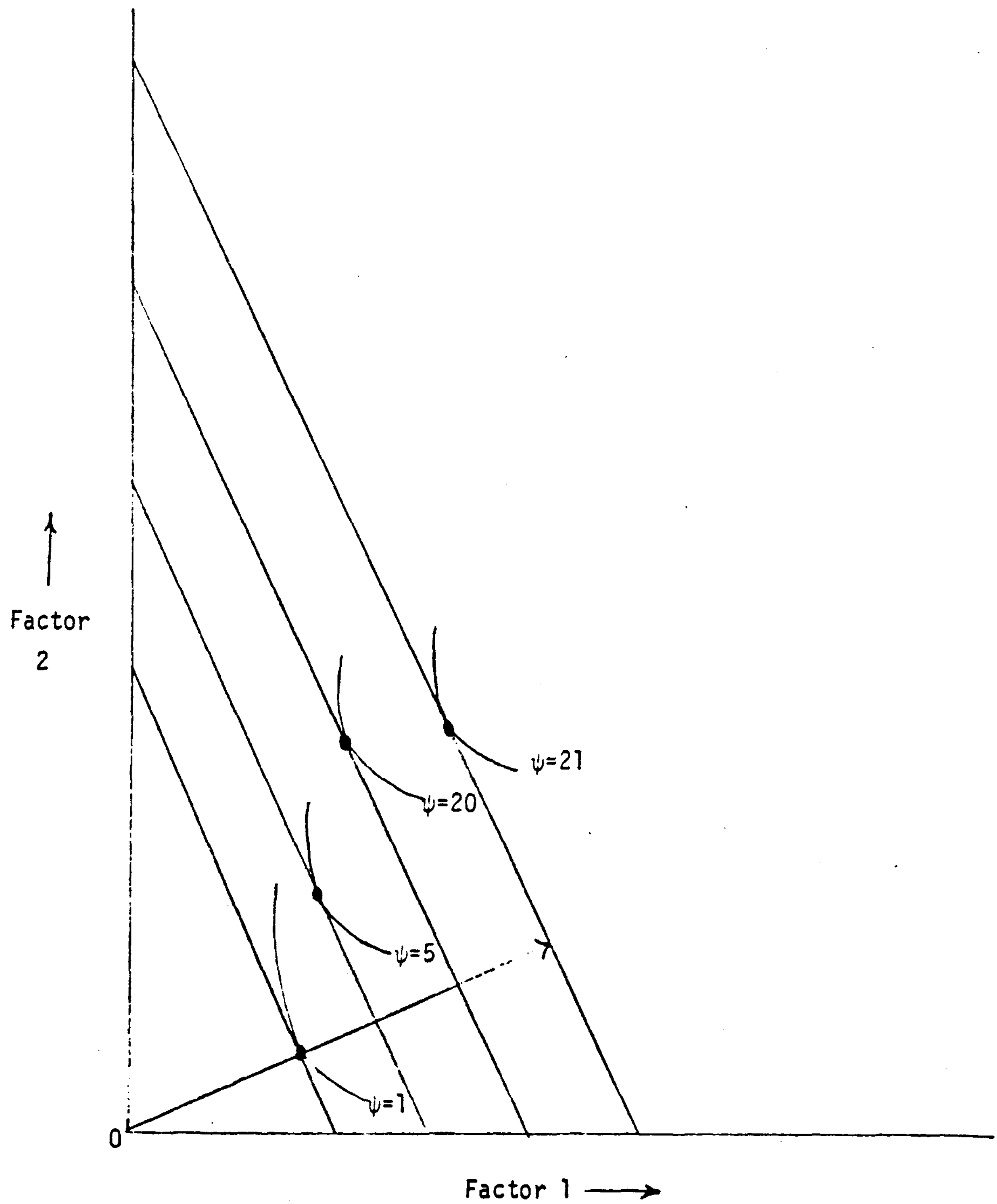
A production structure is nonhomothetic if the cost-minimizing factor intensities are not independent of the output level for fixed relative factor prices. Another way of characterizing this is to say that (factor) isoquants are not radial "blow-ups" of a unit isoquant or that they change shape as output increases. All of these effects are illustrated for the two-factor one-output case in Figure 1.

While it is not possible to strictly characterize returns to scale for a nonhomothetic production structure, it is possible to gain some intuition concerning this issue if we limit the analysis to situations where relative factor prices are constant, since in this case we can infer the shape and

16/ A separable cost function can be written $C(\psi,w) = f(\psi)\cdot\phi(w)$, and corresponds to a production function that can be written $f(\psi) = g(x)$, where x is a vector of factor quantities.

Figure 1

NONHOMOTHETIC PRODUCTION WITH VARIABLE RETURNS TO SCALE



location of the average cost curve from the $\beta_{\psi\psi}$ and α_{ψ} coefficients. Specifically, at mean factor prices, a positive $\beta_{\psi\psi}$ indicates that the firm faces a U-shaped average cost curve (a negative $\beta_{\psi\psi}$ would indicate an inverted-U average cost curve; $\beta_{\psi\psi} = 0$ indicates an average cost curve which is either exponentially falling, rising, or constant, depending on α_{ψ}); if $\alpha_{\psi} = 1$, then the bottom of the U, the point of minimum average cost, occurs at $\psi = \overline{\psi}$, the mean output level. If $\alpha_{\psi} < 1$, the point of minimum average cost occurs at $\psi > \overline{\psi}$, since expanding output beyond $\overline{\psi}$ would steadily decrease average costs if $\beta_{\psi\psi}$ were 0, but for $\beta_{\psi\psi} > 0$ additional costs grow with $(\ell n\psi - \ell n\overline{\psi})^2$ until they dominate the effects of the α_{ψ} term. Similarly, if $\alpha_{\psi} > 1$, then the point of minimum average cost occurs at $\psi < \overline{\psi}$.

Examination of Equation 1 indicates that the $\beta_{\psi i}$'s are generally significantly different from zero, indicating that technology is not separable. This is corroborated by Equation 2, which restricts these coefficients to be equal to zero. A comparison of the log of the likelihood functions in these two equations indicates that they are significantly different at the .005 percent level, indicating that we can reject the hypothesis of separability. Stated alternatively, Equations (1) and (2) clearly indicate that technology is <u>not separable</u>.

A comparison of Equations (1) and (3) permits us to examine the question of homogeneity of output and the existence of economies of scale. In particular, Equation (1) indicates that α_{ψ} is not significantly different from one and that $\beta_{\psi\psi}$ is not significantly different from zero. Taken together, these imply that production is subject to constant returns to scale or that output is homogeneous of degree 1. Equation (3) imposes these restrictions, and a comparison of the log of the likelihood functions of Equations (1) and (3) indicates that their values are not significantly different. Thus the equations indicate that production is characterized by constant returns to scale.

Equation (4) presents a conventional translog cost function that makes no adjustment for quality. We thus set $\phi(q_1,q_2,q_3) = 1$ and specify our output measure as:

$$\phi(y,q_1,q_2,q_3) = y\cdot\phi(q_1,q_2,q_3) = y \qquad (4.8)$$

In this case, all ton-miles are treated equally, regardless of their quality characteristics. Thus, all hedonic coefficients are constrained to have a value of zero.

The coefficients of Equation (4) clearly indicate that the ignoring of quality variables (or, equivalently constraining the hedonic coefficients to be zero) leads to serious misspecification. The R^2 of the overall equatic given in Table 2, is substantially less than that of the hedonic cost functior and the likelihood ratio test clearly indicates that the constrained equation differs from the unconstrained. From this we can readily infer that quality variables such as length of haul, size of shipment, and share of LTL traffic clearly do affect costs.

Because costs are typically estimated in terms of ton-miles alone, it is interesting to note the implications of such a misspecification.

With regard to separability, Equation (4) indicates that the $\beta_{\psi i}$ coefficients are generally statistically insignificant. Thus if we did not make a hedonic adjustment we would incorrectly assume that technology was separable.

With respect to homogeneity of output, Equation (4) indicates that α_ψ is significantly less than one and that $\beta_{\psi\psi}$ is significantly greater than zero. Since this implies that production is subject to increasing returns to scale, the nonhedonic cost function implies that the trucking industry is subject to rather substantial economies of scale.

Since this finding has rather important policy implications, it is desirable to consider it further. Table 3 gives the costs per ton-mile implied by Equations (1) and (4), which respectively represent the general hedonic and nonhedonic cost functions. Thus the hedonic cost functions indicate that trucking firms exhaust their economies of scale at a very small level of output of 10,587,000 ton-miles, and that average costs begin to climb substantially by the time a firm has reached a size of 300,000,000 ton-miles, which respresents a medium-sized firm in the trucking industry. In contrast, the nonhedonic cost function indicates that firms only exhaust their economies of scale by the time they have reached 161,377,000 ton-miles. While the average costs of firms of 300,000,000 ton-miles are also above their minimum levels, the differentials between these costs and the minimum costs are substantially smaller than those implied by the nonhedonic cost function.

Since the trucking industry has been characterized by a large number of mergers in recent years, it is useful to reconcile this fact with the apparent lack of economies of scale implied by the hedonic cost function, which appears to be a superior specification to the nonhedonic cost function. The hedonic cost function indicates that there are virtually no economies of scale beyond a low level of output, when output is adjusted for quality differentials. Stated alternatively, it indicates that if all firms had equal lengths of haul, equal sizes of shipment, and equal shares of LTL traffic, there would be no economies of scale and thus incentives for merger. In contrast, the nonhedonic cost function indicates that there are substantial economies of scale when measured in terms of ordinary ton-miles.

Table 3

Costs per Ton-Mile, Evaluated at Mean Factor Prices

Ton-Miles (1,000's)	Hedonic Cost Function	Nonhedonic Cost Function
10,000	20.51	25.78
10,587	20.50	na
19,947	20,80	21.27
50,000	22.20	18.10
161,337	na	16.67
300,000	24.60	17.16

These differences can be reconciled when one realizes that ton-miles are not equal and that larger firms typically have larger lengths of haul, larger loads, and smaller properties of LTL traffic. Thus, the actual ton-miles of large firms are in some sense less costly to produce than the actual ton-miles of small firms. Consequently, firms have a clear incentive to merge if by so doing they can increase the efficiency of their operations by increasing their shipment size or length of haul or by reducing their share of LTL traffic. This, in large part, explains why many of the mergers have consisted of large firms merging with smaller ones that fill in missing portions of their operating rights.

Thus, insofar as larger firms can achieve greater economies of density and utilization than smaller firms, we can understand the large number of mergers that have taken place in the trucking industry in recent years. Nevertheless, it is important to realize that these are not economies of scale in the conventional sense, but rather economies of density and utilization. If smaller firms could operate with the same loads, lengths of haul, and share of LTL traffic as larger firms, there would be little incentive to merge.

In addition to yielding information about separability and economies of scale, the cost functions given in Table 1 also enable us to estimate the elasticities of substitution among factors and thus own price elasticities of the factors. Specifically, the Allen-Uzawa elasticities of substitution can be estimated from the following expression:

$$\sigma_{ij} = CC_{ij}/C_iC_j \qquad (4.9a)$$

where the subscripts on the cost function (C) denote differentiation with respect to a factor price. The price elasticities are readily obtained from the elasticity of substitution by the relationship:

$$E_{ij} = M_j \sigma_{ij} \tag{4.90}$$

where M_j represents the cost share of factor j.

Table 4 presents the own price elasticities and elasticities of substitution implied by the general hedonic and own hedonic cost functions given in Equations (1) and (4). The results are generally similar and imply that while there is relatively little substitutability among labor, capital and fuel, there is substantial substitutability among these three factors and purchased transportation. Thus a small increase in the costs of labor, fuel or capital will cause firms to shift to purchased transportation. Moreover, there is some substitutability among labor and capital indicating that as labor costs rise, firms will tend to use larger vehicles that tend to have lower labor costs per vehicle-mile.

The own price elasticities implied by the hedonic and nonhedonic regression are also similar and indicate that the demand for fuel, labor and capital are all quite inelastic, while the demand for purchased transportation is quite elastic. These findings indicate that trucking firms tend to view the truck, driver and fuel as being in quais fixed proportions. Thus, instead of substituting among these three factors when any of their prices change, firms will tend to treat them together and substitute toward or away from purchased transportation.

Table 4

Elasticities of Substitution

Estimation of substitution between: (a)	Hedonic Cost Function (Equation (1))		Nonhedonic Cost Function (Equation (4))	
	Value	Stand.Error	Value	Stand. Error
Labor-fuel	.0005	.3098	.3911	.2562
Labor-Capital	.9254	.0443	.9442	.0432
Labor-purch.trans	1.2799	.4853	1.1299	.4290
Fuel-capital	.4454	.1824	.5130	.1486
Fuel-purch.Trans.	.37925	1.5279	3.6168	2.3882
Capital-purch.Trans.	1.5279	.4076	1.4705	.3762
Own Price Elasticity				
Labor	-.3526	.0282	-.3738	.0255
Fuel	-.2818	.1899	-.5859	.1605
Capital	-.6200	.0230	-.6305	.0237
Purch. Trans.	-1.4171	.2513	-1.3050	.2342

(a) Note that $\sigma_{ij} = \sigma_{ij}$

IV. Summary and Conclusions

This chapter has highlighted the need for hedonic adjustment for qualit in transportation cost functions and the importance of a general specificati that will not impose unnecessary restrictions upon technology. In particula it has illustrated that conventional econometric estimates of trucking cost functions are not very reliable and hence not very useful for policy purpose for two fundamental reasons: First, the output of the trucking firm is heterogeneous by its very nature. Hence, simple measures of output such as ton-miles will fail to capture the true relationships between cost and output. Second, it is likely that the trucking firm is subject to joint production. Hence, efforts to describe technology by a simple homothetic production function, such as the Cobb-Doublas or the CES production function may lead to serious biases of estimation.

To test these hypotheses, we developed a general quality-separable hedonic cost function that permitted nonhomothetic production and quality adjustments, and estimated it using a cross-section of 171 firms in the Eastern United States in 1972. This (and similar) hedonic regressions indicated the following results, which have important policy implications.

1. The level of service in terms of length of haul, size of shipment, and share of LTL traffic does affect costs. In particular, evidence of increasing returns to scale exists when ton-miles is used as an output measure, but fails to exist when output is adjusted for quality differentials. This implies that any economies that might exist are economies of density or of service, not economies of scale of output *per se*.

2. When measured in terms of quality-adjusted output, trucking firms face U-shaped marginal and average cost curves for a _very_ wide range of factor prices. Over a wide range of outputs, however, these curves are close to being flat. Nevertheless, firms can be found on both sides of the point of minimum average costs. The distribution of firms along this curve has not been calculated, so it only is possible to say at this point that the very largest firms should be discouraged from further expansion, _ceteris paribus_. It is quite certain, however, that the larger firms are facing declining returns to scale.

3. There are substantial nonhomotheticities in the structure of trucking firms' production. Consequently, any attempt to model their technology using a homothetic cost or production function (such as the Cobb-Douglas or the CES) is a serious misspecification. The non-homotheticities make global generalizations about returns to scale impossible, though they are not so large that the general character of scale returns is seriously altered for reasonable (with an order of magnitude of the mean) relative prices. As scale expands, factor shares change: large firms spend proportionately less on fuel and capital, and more on labor and purchased transportation; but these effects are small.

Of course, the preliminary nature of the findings must be stressed. At the very least, we must extend the sample to other regions and other years to see if our findings are robust. In addition, we should extend the hedonic output function to incorporate the effects of traffic density and the composition of output to obtain more information about the extent of economies of scale in the trucking industry.

Nevertheless, these results clearly indicate the perils of conventional econometric estimates of trucking costs. If production is joint and if output is heterogeneous, we clearly want to take these facts into account in specifying cost functions. Otherwise, we may make the wrong policy decisions based on biased estimates of misspecified cost functions.

APPENDIX

THE ESTIMATION OF REGIONAL FACTOR PRICES

The basic problem in establishing prices for both purchased transportation and fuel is that while each firm's total expenditures on these goods is observed, the quantities purchased are not. Instead, an indirect measure of quantity purchased is available.

For fuel, for instance, we know the firm's vehicle miles with firm-owned trucks, and the number of vehicle miles rented with and without drivers. Since vehicles rented with drivers typically include fuel within the rental price,[1/] these miles are subtracted from the total to obtain the vehicle miles for which the firm provided fuel.

Using this mileage figure, a fuel cost per vehicle mile can be calculated for each firm; this would be an appropriate fuel price measure if every vehicle got the same mileage per gallon. An inspection of these figures, however, reveals that if this were true, fuel prices varied between firms by a factor exceeding ten. It is clear that a constant miles per gallon assumption is inappropriate.

The factors that would appear to most directly affect fuel mileage are vehicle size, and the percentage of miles driven on interstate highways. Reasonable proxies for these variables are average size of shipment and average length of haul, respectively. Thus, we can write:

$$\frac{\text{FUEL\$}}{\text{VEH. MILE}_{i,r}} = \text{\$/Fuel Gallon} \cdot \frac{\text{Fuel Gallons}}{\text{Mile}}_i = p_r \cdot \phi(q_1^i, q_2^i) \qquad \text{(A.1)}$$

[1/] This is borne out in the model below: when a parameter measuring the percentage of rented with driver vehicle miles whose fuel is paid for by the firm, this parameter is <u>negative</u>, small, and not significantly different from zero.

where p_r is the price of fuel in region r and i subscripts denote firm-specific variables. A stochastic specification of (A.1) might be[2/]

$$\frac{FUEL\$}{VEH.MILE} = [p_r e^{u_r}][\phi(q_1,q_2)e^{u_\phi}]e^{\eta} \tag{A.2}$$

where $u_r \sim N(0,\sigma^2_{u_r})$, $u_\phi \sim N(0,\sigma^2_{u\phi})$, and $\eta \sim N(0,\sigma^2_\eta)$. Taking logs of both sides and specifying a translog function for $\phi(\cdot)$, we have:

$$\begin{aligned} \ell n[\frac{FUEL\$}{VEH.MILE}] &= \ell np_r + \alpha_1[\ell nq_1-\ell n\bar{q}_1] + \alpha_2[\ell nq_2-\ell n\bar{q}_2] \\ &+ \frac{1}{2}\beta_{11}[\ell nq_1-\ell n\bar{q}_1]^2 + \beta_{12}[\ell nq_1-\ell n\bar{q}_1][\ell nq_2-\ell n\bar{q}_2] \\ &+ \frac{1}{2}\beta_{22}[\ell nq_2-\ell n\bar{q}_2]^2 + u_r + u_\phi + \eta. \end{aligned} \tag{A.3}$$

In this model the three separate variance components u_r, u_ϕ, and η cannot be identified,[3/] although the sums $\sigma^2_{u_r} + \sigma^2_{u_\phi} + \sigma^2_\eta$ can be estimated for each region. Since these estimates do not lead one to reject the hypotheses of homoscedasticity across regions, this assumption is made here.

The estimate of equation (A.3) is given in Table A1; the "natural terms" are direct estimates of the regional prices, whereas the coefficients of the logarithmic terms correspond to the α's and β's in (A.3).

The estimates of the cross-terms (β's) are all insignificant; thus the gallons/mile function $\phi(q_1,q_2)$ can be effectively interpreted by examining the linear terms only. If we take average size of shipment and average length of haul as proxies for truck size and interstate highway mileage respectively, then both variables have the right sign, though

2/ Alternative stochastic specifications, such as additive errors in a variance-component model, give inferior results.

3/ Appropriate assumptions in a time series cross-section model would lead to identification, however.

Table A.1

ESTIMATES OF REGIONAL FUEL PRICES

COEFFICIENT	VALUE	STANDARD ERROR	
PRICE, NEW ENGLAND	.06359	.01451	NATURAL TERMS
PRICE, NORTH MID ATLANTIC	.07065	.01125	
PRICE, MIDDLE ATLANTIC	.07694	.01355	
PRICE, CENTRAL STATES EAST	.06794	.01271	
PRICE, CENTRAL STATES WEST	.07231	.01332	
AVERAGE LENGTH OF HAUL	-.2699	.2051	LOGARITHMIC TERMS
AVERAGE SHIPMENT SIZE	.4285	.2275	
$(HAUL)^2$	-.1192	.7025	
HAUL•SIZE	-.0791	.5232	
$(SIZE)^2$	.3171	.5011	

DEPENDENT VARIABLE IS LOG(FUEL EXPENDITURES/VEHICLE-MILE)

R^2 = .3094
SSR = 12.173
LOG LIKELIHOOD FUNCTION = -21.86
OBSERVATIONS = 171

their standard errors are somewhat larger than is desirable. In the absence of a priori expectations for the regional prices, it is difficult to evaluate these estimates. In interpreting these estimates, it must be remembered that they include fuel taxes, which do differ by region, and that they are for 1972, before the August 1973 Arab oil boycott, which raised prices more in New England than in other regions.

A similar model was used to estimate regional prices for purchased transportation:

$$\ln \frac{\text{PURCH\$}}{\text{RENTED MILES}} = p_r \ln p_r + a_1[\ln \text{AVHAUL} - \ln\overline{\text{AVHAUL}}] \qquad (A.4)$$

$$+ a_2[\ln \text{AVSIZE} - \ln \overline{\text{AVSIZE}}] + a_3[\ln(1+\text{DRIVER}) - \ln(1+\overline{\text{DRIVER}})$$

$$+ a_4[\ln(1+\text{DRIVER}) - \ln(1+\overline{\text{DRIVER}})][\ln \text{AVHAUL} - \ln\overline{\text{AVHAUL}}]$$

$$+ a_5[\ln(1+\text{DRIVER}) - \ln(1+\overline{\text{DRIVER}})][\ln\text{AVSIZE} - \ln \overline{\text{AVSIZE}}]$$

where DRIVER is the percentage of rented vehicle miles rented with driver.

The sample for the estimation for (A.4) was reduced to 101 observations by eliminating the 70 firms that reported purchasing rail, air, or water transportation, since separate expenditures on these categories were not available from TRINC's, though they are reported to the ICC. In addition, separate figures for average haul and average shipment size in rented vehicles are not available, so overall firm averages are used. Again, these variables serve as proxies for interstate highway mileage and vehicle size respectively.

The results are reported in Table A.2 Once again, the cross terms are insignificant, and the linear non-price terms have the correct sign; they are also statistically significant by the usual tests. Since the

Table A.2

ESTIMATES OF REGIONAL PURCHASED TRANSPORTATION PRICES

COEFFICIENT	VALUE	STANDARD ERROR
LOG PRICE, NEW ENGLAND	-.031135	.629987
LOG PRICE, NORTH MID ATLANTIC	-.436373	.552856
LOG PRICE, MIDDLE ATLANTIC	-.265030	.524977
LOG PRICE, CENTRAL STATES EAST	-.752058	.508918
LOG PRICE, CENTRAL STATES WEST	-.209309	.607896
SIZE	.970295	.393755
HAUL	-1.04077	.330888
% RENTED WITH DRIVER (RWD)+1	1.66230	.698720
SIZE•% RWD+1	.570943	1.21665
HAUL•% RWD+1	-.961638	1.10031
$(\% \text{ RWD}+1)^2$	2.55511	4.55707
ANTILOGS OF ESTIMATED LOG PRICES:		
PRICE, NEW ENGLAND	.969345	
PRICE, NORTH MID ATLANTIC	.646377	
PRICE, MIDDLE ATLANTIC	.767183	
PRICE, CENTRAL STATES EAST	.471395	
PRICE, CENTRAL STATES WEST	.811145	

DEPENDENT VARIABLE IS LOG(PURCH. TRANSP. EXPENDITURES/RENTED VEHICLE-MILES)

R^2 = .2666
SSR = 144.952
LOG OF LIKELIHOOD FUNCTION = -167.381
OBSERVATIONS = 101

price terms were not estimated directly (the nonlinear algorithm failed to converge properly in our initial attempts), we take the antilog of the estimated price logarithms as our regional price estimates; the standard errors of these estimated were not calculated. However, from Table A.2, there is reason to believe that they are comparatively large.

Chapter Five

Interindustry Relations in the Surface Freight Transportation Industries

Federal transportation policy not only influences transportation rates and the allocation of shipments among the various modes, but also affects the allocation of economic activity among industries. By causing changes in transportation rates, changes in federal transportation policy cause changes in the price of transportation services relative to other commodities or services. These, in turn, lead to changes in the allocation of economic activity among industries and the producer prices of these industries, which in turn can affect the demand for transportation services. Consequently, it is desirable to develop a general equilibrium framework that can be used to analyze the impact of transportation rates upon the allocation of economic activity among industries. This can then be used to feed back into the models of equilibrium in the transportation industries.

This chapter discusses our initial efforts to develop such a general equilibrium analysis, using an interindustry model with variable coefficients that reflects the impact of federal transportation policy upon all sectors of the economy. Section I briefly discusses the general approach used, while Section II outlines the specific methodology used to estimate variable interindustry coefficients. Section III presents our empirical results and discusses why they are unsatisfactory. Section IV then discusses how changes in federal transportation policy can be analyzed within the context of the conventional input-output framework.

I. An Overview of Interindustry Analysis

The usual approach followed to evaluate interindustry effects is the familiar input-output analysis, first developed by Leontief. According to this approach, the basic relationships for production and prices are respectively given by the following equations:

$$X = AX + Y \qquad (5.1)$$

or

$$X = (I - A)^{-1} Y \qquad (5.1a)$$

$$P' = a_o w + P'A \qquad (5.2)$$

or

$$P' = a_o w(I-A)^{-1} \qquad (5.2a)$$

where X = Vector of total output

A = Matrix of input-output coefficients

Y = Vector of final demand

P = Vector of prices

a_o = Matrix of direct primary factor input

w = Vector of prices of the primary factor.

Thus, AX represents the intermediate demand of all industries. In the original Leontief formulation, the matrix A was given by the pure technological requirements of the industry. No substitutions between factor inputs were permitted.[1/] This is clearly a very restrictive

[1/] However, subsequent formulations have shown that if there is only one primary factor, relative prices are independent of final demand and no substitution among factors is desirable. For a full discussion of this and related points, see Samuelson (1966).

requirement and runs counter to usual views of producers' optimizing behavior. Thus, it is desirable to modify the traditional input-output framework of Leontief to relax the assumption of fixed production coefficients and the lack of substitutability among primary factors and other inputs.

Hudson and Jorgenson (1974) have recently developed a model that attempts to yield interindustry coefficients from the profit-maximizing behavior of the firm rather than from pure technological requirments. In this case, the matrix (A) of input-output coefficients is made endogenous and determined by the relative prices of factor inputs. Thus, instead of the rigid utilization of factors and materials on the part of firms, the Hudson-Jorgenson formulation of the problem permits substitution among factors and materials in response to changes in relative factor prices. Therefore, instead of eqs. (5.1) and (5.2), we write the production and pricing relationships in the economy as

$$X = (I - A(w))^{-1}Y \tag{5.3}$$

$$P' = a_0 w(I - A(w))^{-1} \tag{5.4}$$

In this flexible input-output analysis, a change in the prices of primary factors not only has a direct impact upon the vector of producer prices through the vector of primary factors (w), but it also has an indirect impact through the input-output matrix, A(w). Moreover, in contrast to conventional input-output analyses, even if the vector of final demand remains constant, the vector of total output will generally change as the utilization of inputs changes in response to the change in the prices of primary factors.

Although transportation is clearly a produced activity, since the prices of its services are determined by the regional transportation models described above, transportation can be taken as a primary factor of production for the purposes of this interindustry analysis. Consequently, by using a flexible input-output analysis, we can determine how changes in the transportation rate structure affect interindustry coefficients, commodity prices, industry outputs, and factor demands.

II. Structure of the Analysis

Because we are interested in analyzing the impact of transportation policy upon fairly broad aggregates, we incorporate a model of producer behavior for the following nine broad categories of commodities:

durable manufactures

nondurable manufactures

feed grains

other agriculture

construction

services

non-food materials and non-food mining

coal

petroleum and petroleum products.

The first seven industries comprise a materials aggregate, while the last two comprise an energy aggregate.

Instead of estimating specific factor demand equations for each produced aggregate and primary input, we make use of the familiar duality theorems of Samuelson (1966) and Shephard (1970), which assert that production relations can be completely described either by physical quantities, as in production functions, or by price relations, as in cost functions. These theorems assume, of course, perfect competition and constant returns to scale. Therefore, as long as we assume that these conditions are met for each of the nine industry groups, a price possibility frontier can be estimated which relates the price of each commodity to the prices of its inputs.

Thus, the relative price relationships take the following form:

$$PI = PI(AI, PK, PL, PE, PM, PT) \quad I = 1...9 \tag{5.5}$$

where PI = the price of the output of industry I

AI = a measure of Hicks neutral technical change in industry I

PK = the price of capital

PL = the wage rate

PE = a price index for aggregate energy inputs

PM = a price index for aggregate materials

PT = a price index for transportation services.

This price possibility frontier is defined on the prices of the component aggregate inputs if the overall price possibility frontier (characterizing the whole economy) is separable and homogeneous in the elements of the aggregate. Leontief (1947) has shown that this requires that the relative shares of elements within an aggregate are independent of prices of goods outside the aggregate. Thus, in undertaking this analysis we must assume separability between the aggregates, materials and energy, although we do not need to assume that separability exists among the components of each aggregate.

Given this price possibility frontier, input-output coefficients can be obtained by a two-step procedure. First, the shares of the aggregate inputs are found by the identity:[2/]

$$\frac{\partial \ell nPI}{\partial \ell nPJ} = \frac{PJ \cdot XJI}{PI \cdot XI} = \frac{PJ}{PI} a_{ji} \qquad (5.6)$$

where PJ is the price of input J and XJI is the amount of J needed to produce I, whose total output is XI. Therefore, PJ•XJI/PI•XI is the factor share of input J; and XJI/XI (or a_{ji}) is the technical coefficient of production denoting the elemnent of the input-output matrix at row J, column I. The complete input-output matrix is obtained by finding the price possibility frontier for both the materials and energy aggregates defined on their elements. This enables us to separate out all the interindustry relations by disaggregating the factor shares

[2/] For a full discussion of this and related points, see Hudson and Jorgenson (1974).

of the energy and material input. For instance, if PD is the price of durable manufactures, PM the price of material inputs and XD and XM are outputs defined analogously, the input-output coefficient for the durable manufacture input to output I is:

$$\frac{PM\cdot MI}{PI\cdot XI} \cdot \frac{XD\cdot PD}{MI\cdot PM} \cdot \frac{PI}{PD} = \frac{XD}{XI} = a_{di} \tag{5.7}$$

The first term is obtained from estimating the aggregate price possibility frontier ($\partial \ell nPK/\partial \ell nPM$); the second term is obtained from estimating the materials price frontier; the price ratio of commodity I to durable manufactures is given. Having obtained the input-output coefficients, it is possible to carry out the usual calculations to obtain intermediate factor demands, total output requirements and the prices of all intermediate and final goods under the assumption that prices are independent of the composition of final demand.

A transcendental logarithmic (translog) function was used to estimate the price frontiers. This provides a local, second order approximation around a given point of expansion (usually taken to be the sample mean) to an arbitrary function. Thus, the frontier defined on the prices of aggregate inputs is given by the following expression:

$$\ell nPI + \ell nAI = \alpha_{OI} + \sum_J \alpha_{IJ}\, \ell nPJ + 1/2 \sum_J \sum_F \beta_{IJF}\, \ell nPJ\, \ell nPF \tag{5.8}$$

$I = 1...9$

$J,F = K,L,E,M,T.$

The prices of energy and materials are considered to be endogenous; the prices of the primary factors capital and labor are determined by a macroeconomic model that is independent of the interindustry analysis; and the price of the "primary" factor transportation is determined by a regional transportation model.

Similarly, the price frontiers for energy and materials are given by

$$\ell nPM = \alpha_{om} + \sum_{r} \alpha_{m} \ell nP_{r} + 1/2 \sum_{r} \sum_{q} \beta_{mrq} \ell nP_{r} \ell nP_{q} \qquad (5.9)$$

r,q = (seven material inputs)

$$\ell nPE = \alpha_{oE} + \sum_{r} \alpha_{rE} \ell nA + 1/2 \sum_{r} \sum_{q} \beta_{mrq} \ell nP_{r} \ell nP_{q} \qquad (5.10)$$

r,q = (two energy inputs).

From the relationships given in equations (5.8) - (5.10), the price possibility frontier and input-output coefficients can be obtained by the method outlined above. Alternatively, factor share equations which are formally equivalent to equations (5.8) - (5.10) may be estimated as was done by Hudson and Jorgenson (1974).

$$\frac{PK \cdot KI}{PI \cdot XI} = \alpha_{IK} + \beta_{IKK} \ell nPK + \beta_{IKL} \ell nPL + \beta_{IKE} \ell nPE + \beta_{IKM} \ell nPM + \beta_{IKT} \ell nPT \qquad (5.11a)$$

$$\frac{PL \cdot LI}{PI \cdot XI} = \alpha_{IL} + \beta_{ILK} \ell nPK + \beta_{ILL} \ell nPL + \beta_{ILE} \ell nPE + \beta_{ILM} \ell nPM + \beta_{ILT} \ell nPT \qquad (5.11b)$$

$$\frac{PE \cdot EI}{PI \cdot XT} = \alpha_{IE} + \beta_{IEK}\ln PK + \beta_{IEL}\ln PL + \beta_{IEE}\ln PE + \beta_{IEM}\ln PM + \beta_{IET}\ln PT \qquad (5.11c)$$

$$\frac{PM \cdot MI}{PI \cdot XI} = \alpha_{IM} + \beta_{IMK}\ln PK + \beta_{IML}\ln PL + \beta_{IME}\ln PE + \beta_{IMM}\ln PM + \beta_{IMT}\ln PT \qquad (5.11d)$$

$$\frac{PT \cdot TI}{PI \cdot XI} = \alpha_{IT} + \beta_{ITK}\ln PK + \beta_{ITL}\ln PL + \beta_{ITE}\ln PE + \beta_{ITM}\ln PM + \beta_{ITT}\ln PT \qquad (5.11e)$$

While the relationships given in eq. (5.11) were estimated by Hudson-Jorgenson (1974) and are likely to yield more stable estimates, they require the factor shares of all aggregate inputs to each industry group. Since these data were not available to us, we estimated equations (5.8) - (5.10) using readily available price data.[3/]

III. Criticisms and Results

A. Criticisms

The Hudson-Jorgenson Study has stimulated considerable controversy since its publication and has engendered a variety of criticisms.[4/] While many of its flaws specifically deal with its applicability to the energy sector, there are more fundamental problems, which prevent

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[3/] In fact, they were apparently not available to Hudson and Jorgenson either, who used data constructed from three input-output tables formulated over the sample period. The remaining years' data were found by some interpolation techniques not specified by the authors. The estimation of fourteen parameters with three effective data points cannot be expected to yield valid results as one simply cannot span a higher dimensional space from a lower one. For a good discussion of this and related points, see Khazzoom (1967).

[4/] See, in particular, the criticisms contained in the Workshop of the Electric Power Research Institute (1976).

the proposed modification to the transportation sector from being valid. Indeed, it is the basic theoretical flaws that interfere with our proposed approach since the criticisms of the method as it is applied to energy do not hold much force in the transportation sector.[5/]

The fundamental problem with the approach presented by Hudson and Jorgenson and followed here is the lack of linkages between the vector of final demand and the vector of prices. Thus, according to this approach, the prices of primary factors and the level of final demands are determined by a macroeconomic model which is independent of the interindustry analysis. The interindustry relationships have no effect upon macroeconomic activity except through the identity of supply and demand $PC \cdot C = \sum_{1}^{n} PI \cdot CI$, where the left-hand term represents the value of consumption and the right-hand term represents the sum of the value of consumer goods in each industry. Similar equations are necessary for

[5/] A major criticism of the Hudson-Jorgenson analysis has to do with forecasting the impact of future energy price changes. Since the translog function represents an approximation around a point of expansion, the use of the translog function is only justified for estimating smoothly changing series and obtaining local estimates of the specified function. Thus, its ability to predict is greatly hampered as data move further away from the point of expansion. This is a serious flaw in making projections in the energy sector, particularly from estimates over the sample period used by Hudson-Jorgenson (1947-71). Since future price changes in petroleum may be expected to be more dramatic than those during the sample period or occur in discrete jumps rather than conform to historical patterns, the translog approximation may be quite poor in the range of projected price changes, especially if techniques cannot be devised to maintain the same degree of substitutability between factors as in the sample period. The prescription may be to analyze actual technical possibilities rather than relying on past market performance. These shortcomings are not damaging when applied to transportation. In contrast to the very large movements of energy prices, which differ greatly from those in the estimated period, alterations in transportation prices can be expected to be small and not far removed from experienced values. Thus, present technological relations can adequately adjust to the scenarios suggested by regulatory changes.

government, investment and imports. As Phoebus Drymes (1976) notes, the macroeconomic and interindustry relations can be solved quite independently save for aggregate summation constraints.

This suspicion of more fundamental linkages between the interindustry model, which determines input-output coefficients, and the macroeconometric model, which determines primary factor prices and final demands, leads to a further major criticism from a theoretical level. Because of the existence of more than one primary factor of production, the nonsubstitution theorem of Samuelson has been misapplied. Hudson and Jorgenson contend that "This theorem states that for given prices of the factors of production and competitive imports, the prices of domestic availability of the output of each sector are independent of the composition of final demand." (1974, p. 468). While true, this result is inappropriate for their or the present analysis. With more than one primary factor, this statement can have one of two meanings. First, all primary factors except one can be traded for that one at constant prices. This, however, is equivalent to the statement that all factors except one can be transformed into that one by a simple production scheme: Factor $j = a_j \times$ Factor 1 for all j primary factors. As originally pointed out by Samuelson (1966, p. 522), this strains the concept of a "primary" factor and actually returns us to the one true primary factor case. If this meaning is incorporated into the estimation of the price possibility frontier, we would have to expect perfect multicollinearity in the specified price possibility frontier since $PK = a_K PL$ and $P_T = a_T PL$ for all other "primary factors" except labor, which we designate as the true primary factor.

Alternatively, we can treat capital, labor and transportation as true primary factors and permit their relative prices to vary over the sample period. In this case, the usual nonsubstitution theorems do not apply, and there is not a unique technologically-determined price vector that can allocate resources independently of the composition of final demand. As demand for goods whose production uses one of the factors intensively rises, the return to that factor will generally rise as well causing changes in relative producer prices, which should lead to changes in the composition of final demand. In the terminology of the Leontief system, the price vector of the economy is the "normal" vector to the production possibility surface. With more than one primary factor, the production possibility surface is piecewise linear and consequently has more than one normal vector. Thus there is no unique producer price vector that is independent of final demand. The theorem cited by Hudson-Jorgenson can only be meaningful as a tautology such as "given the prices of all goods which determine factor prices, the price of domestic availability is independent of final demand." The fact that the factor prices are determined in a different model does not circumvent the need to determine the price vector jointly with final demand.

This independence causes problems with the econometric specification of the price frontier. If factor prices are dependent on final demand, the price possibility frontier should be written as:

$$PI = PI(PK(Y),\ PL(Y),\ PT(Y),\ p1(Y).....PN(Y)) \qquad (5.12)$$

where Y represents the vector of final demands. In terms of input-output analysis this can be written as:

$$P = w(Y)[I-A(w(Y))]^{-1} \tag{5.12a}$$

But the final demand vector itself depends upon producer prices:

$$Y = Y(P1...PN) \tag{5.13}$$

As these equations suggest, the problem is that the proposed metholodogy attempts to estimate prices separately when in fact they are part of a simultaneously determined system with a high proportion of jointly determined variables. This would lead to biased estimates, and make the economic implications of the estimated price functions suspect.

A further difficulty with the Hudson-Jorgenson approach involves its level of aggregation. While highly aggregated studies are frequently criticized for the loss of usefulness in policy making, some theoretical problems are also worth noting. As mentioned before, the price possibility frontier is defined only when production is separable. This implies that aggregation has been carried out and that shares within the aggregate are independent of prices outside the aggregate. With industry groups as broad as "durable manufactures" or "agriculture, construction and nonfuel mining" it is not likely that this holds. Indeed, all the duality theorems such as Shephard's Lemma relating factor demands to cost fucntions rely on the single firm as the unit of analysis. It is not clear that these theorems are at all applicable to broad commodity classes.[6/] This level of aggregation may well introduce severe specification error in the estimation.

[6/] For a good discussion of this point, see Sewall (1976).

B. Econometric Results

In spite of these theoretical problems, the present study attempted to fit the price possibility frontier directly to the translog specification using only price data. The data used were indices reported quarterly (1953-1973) on the prices of the nine commodity groups and five aggregates: capital, labor, energy, materials, and transportation services. All data came from the Department of Commerce Survey of Current Business except for the price of capital services and the price of transportation services.[7/]

The price possibility frontiers were estimated with a nonlinear technique which allowed the appropriate homogeneity and symmetry constraints to be imposed[8/] along with the exogenously determined rate of technical progress in each sector. In addition, corrections were made for autocorrelation (which was very high), and instrumental variables were used to correct for problems of simultaneity in the materials and energy sectors.

By using the factor share equations given above (eq. (5.11)), it is possible to obtain an independent check of the plausibility of the results obtained from estimating the production possibility frontier directly using price data. Unfortunately, although the estimated

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7/ The price of capital services was provided by the MFP (MIT, Federal Federal Reserve, University of Pennsylvania) macroeconometric model; the price of transportation services was constructed from prices of railroad services (Association of American Railroads "Yearbook of Rail way Facts"); inland waterways (American Waterway Operators, Inc., "Inland Waterborne Commerce Statistics"); and regulated motor carriers (ICC, "Transport Statistics in the U.S.").

8/ See Chapter 3, above, for a discussion of these constraints.

price equations were reasonable in terms of standard statistical tests, the implied factor shares were inadmissible since they often did not lie between zero and one. Table 5.1 gives the estimated equation for durable manufactures, which was typical of all of those estimated. While the R^2 is high and the coefficients are generally statistically significant, the economic implications of the equations are unacceptable. Since the α_i's represent the factor share of the respective aggregate input evaluated at the mean of the prices, the results are clearly incorrect. In addition, the series of implied factor shares varied widely over the sample period, contrary to the historical record.

That the results obtained using pure price data were incorrect is not surprising. Usually, quantity-related data, such as measures of capital utilizaiton in cost function or joint estimation of factor demands and cost functions, are needed to supplement the estimation of duality relationships. Consequently, while formally equivalent to the factor share equations estimated by Hudson-Jorgenson, the estimation of the price possibility frontier by price data alone is not likely to yield admissible results[9/] and data on actual factor shares or interindustry coefficients are needed. Unfortunately, however, such data do not exist to permit a time series analysis of interindustry relationships. Consequently, we are forced to return to more conventional methods.

[9/] For a good discussion of this point, see Burgess (1975). Dhrymes (1976) has made a similar point.

Table 5.1

Estimated Equations for Durable Manufactures

Linear Terms			Quadratic Terms				
			K	L	E	M	T
α_K	-.045 (.0005)	K	-.225 (.003)	.111 (.008)	.854 (.001)	-2.510 (.019)	1.800 (.013)
α_L	-.119 (.002)	L		-14.6 (.054)	-6.87 (.031)	21.8 (.063)	-.466 (.040)
α_E	-.071 (.001)	E			-3.67 (.032)	9.29 (.073)	.403 (.053)
α_M	1.042 (.003)	M				-25.2 (.165)	-3.38 (.094)
α_T	.193 (.003)	T					1.64 (.076)

R^2 = .997

Autocorrelation Coefficient = .934

(Standard Error = .001)

Note: Figures in parentheses represent standard errors.

IV. A Conventional Input-Output Analysis

Since the incorporation of flexible coefficients does not appear to be feasible, we now turn to a traditional input-output analysis with exogenously altered technical coefficients. The limitations of the assumption of fixed coefficients do not impinge severely on an analysis of transportation industries for two main reasons. First, as mentioned earlier, changes in transportation prices or technical relations are generally small, and since expenditures on the transportation sector are usually a small proportion of industrial inputs, it is not unreasonable to expect very small adjustments to the change in price of such services. Second, the peculiar nature of transportation services and its competing commodities in the factor demand markets make the assumption of no substitution less damaging. The main "substitute" for transportation services is generally not the usual alternative mix of inputs, but rather a change of location of a producing firm. A firm may well respond to changes in transportation costs (especially of one mode vis a vis another) by relocating at a point where total transport costs for its inputs and marketable outputs are at a minimum. In the short run, however, the ability to change location is strictly limited since there is often a large fixed capital investment tying the firm to a specific point in space. Because the savings in transportation are probably small, relocation is not warranted. However, second-order effects of changes in freight rates may influence substitution; as rates rise, industries may substitute away from factors intensive in transportation services.

Nevertheless, we will have to tolerate the problems of this effect.

The method of analysis takes the form of simulating technical change (literally interpreted as a change of techniques) in the relevant transportation industries. This posits a change in the structure of production due to regulatory intervention in the transportation industries.

The method is best illustrated by example. It is well known that the railroad industry is characterized by chronic excess capacity and therefore, that firms operate on a short run, rather than long run cost curve. Changes in regulatory practices, which induce the firm to operate efficiently, act as technical progress. The coefficients corresponding to the "new" technique may then be incorporated into the input-output matrix by reducing the elements of the column of the railroad industry by the appropriate amount. In particular, this requires reducing the amount of the fixed factor used per unit output and changing the coefficient for certain capital equipment in the railroad column. By assuming that the vector of final demand is unchanged, it is possible to calculate the following by the formulae given in eqs. (5.1) and (5.2): fixed new total production requirements; intermediate demands; and price vectors.

Thus, the impact of federal transportation policy upon interindustry relationships could be determined in a multi-step procedure. First, the change in policy must be translated into a change in the relevant cost functions, demand functions, or market structure. Second, a new equilibrium with respect to rates, outputs and factor usage (both primary and produced) must be calculated from the regional transportation models.[10] Third, these changes in factor utilizations must be translated into the appropriate changes in the technical coefficients of the columns of the input-output matrix. Fourth, the change in outputs among modes could be translated into changes in the row values of the input-output matrix for the transportation industries. Finally, given the changed column and row vectors of the transportation industries, new solutions can be obtained for outputs and producer prices. Thus, while less elegant than the Hudson-Jorgenson formulation, this approach should permit some interindustry response to changes in federal transportation policy.

To estimate the impact of changes in federal transportation policy upon interindustry relationships, the 1967 BEA input/output table was consolidated into a forty industry table with seven disaggregate transportation sectors: rail, urban transportation services, trucking, inland water transportation, air, pipeline and other transportation services. This table, reported in dollar transactions rather than in technical coefficients, was computed in producer's prices. Thus, transportation services used are credited to the transportation sector and the interindustry effects of transportation can be obtained.

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[10] For a full discussion of these models see Chapter 2, below.

For illustrative purposes we postulate a 5 percent increase in the efficiency of the rail and trucking industries, that is, 5 percent less of all inputs are needed for the same output. This implies, of course, that the columns for the rail and trucking industries are reduced by 5 percent in the input-output matrix. Whi unrealistic for evaluating the impact of changes in regulatory policy, this example gives some feel for the sensitivity of the system to changes in the interindustry coefficients of the transportation industries.

Table 5.2 presents the percentage change in the prices of each industry, respectively engendered by a 5 percent increase in the efficiency of the rail or trucking industry, while Table 5.3 present similar percentage changes in total output. In both cases, the char in prices and outputs yield reasonable and expected results. The greatest savings from a more efficient railroad sector (besides transportation services) occur in heavy, transportation intensive industries such as lumber, paper products, stone, clay and glass products, primary metals and metal products. Greater reduction on average occurred with changes in the trucking sector, the largest ones being in industries similar to those most affected by rail witl the addition of livestock and processed foods. The larger average reductions over a broader sample of industries in response to technical changes in trucking seem intuitively plausible, since smaller users would tend to use the more flexible and smaller-load trucking services over rail. Least affected, understandably, were industries where material inputs are a small fraction of total inputs. These include wholesale and retail trade, insurance, banking, and services. Thus, we can see that the freight transportation industries have greatest affect on the industrial production sectors of the national economy and will most likely affect the income distribution between regions through increasing the competitive advantage of industrial areas.

Table 5.2

Percentage Price Reduction with Uniform 5 Percent Increase in Efficiency in Rail and Trucking Industries

	Rail	Trucking
Livestock	.041	.090
Grain	.036	.049
Other Agriculture, Forestry, Fishing	.025	.048
Mining (exc. Coal, Petroleum)	.049	.041
Coal Mining	.046	.040
Crude Petroleum	.017	.019
Construction	.031	.048
Ordinance	.046	.051
Food Products	.036	.073
Textile Products	.024	.053
Lumber/Furniture	.086	.055
Paper Products	.059	.050
Chemicals, Plastics	.050	.045
Petroleum refining	.023	.043
Rubber, leather	.047	.059
Stone, Clay, Glass	.105	.160
Primary Metals	.086	.061
Metal Products	.056	.054
Machinery	.029	.034
Electrical equipment	.027	.034
Motor vehicles	.058	.055
Aircraft	.028	.036
Other transportation equipment	.069	.067
Other equipment	.037	.046
Railroad	3.36	.052
Passenger trains	.048	.087
Motor freight	.044	4.15
Water transport	.041	.162

Table 5.2 (cont'd.)

Air transport	.021	.064
Pipeline	.030	.070
Transportation Services	.013	.016
Communications/Utilities	.031	.023
Wholesale and retail trade	.004	.011
Finance, insurance, real estate	.007	.007
Services	.008	.012
State and Federal Government	.108	.125
Imports	0	0
Dummy industries	.044	.082
Rest of world	0	0
Government, household industry	0	0

Table 5.3

Percent Savings in Total Output with 5 Percent Increase in Efficiency in Rail and Trucking Industries

	Rail	Trucking
Livestock	.005	.008
Grain	.006	.023
Other Agriculture, Forestry, Fishing	.009	.010
Mining (exc. Coal, Petroleum)	.047	.019
Coal Mining	.031	.034
Petroleum (crude)	.077	.160
Construction	.061	.012
Ordinance	.001	.001
Food Products	.003	.005
Textile Products	.011	.012
Lumber, Furniture	.027	.010
Paper Products	.023	.040
Chemicals, Plastics	.023	.026
Petroleum Refining	.089	.192
Rubber, Leather	.016	.083
Stone, Clay, Glass	.044	.018
Primary Metals	.059	.015
Metal Products	.039	.015
Machinery	.024	.011
Electrical Equipment	.014	.011
Motor Vehicles	.003	.015
Aircraft	.002	.002
Other Transportation Equipment	.106	.007
Other Equipment	.011	.015
Railroads	.308	.094
Passenger Trains (Urban)	.105	.024
Motor Freight	.043	.731
Water Transport	.028	.044

Table 5.3 (cont'd.)

Air Transport	.044	.038
Pipeline	.085	.202
Transportation Services	.615	1.30
Communications/Utilities	.035	.049
Wholesale and Retail Trade	.013	.042
Finance, Insurance, Real Estate	.036	.039
Services	.019	.051
State and Federal Government	.040	.018
Imports	0	0
Dummy Industries	.044	.083
Rest of World	0	0
Government, Household Industry	0	0

Since the calculations were made with the fixed coefficient assumption intact, these results serve as a lower bound on the potential savings on intermediate goods. Any substitution will offer opportunities to save additional amounts as inputs are chosen with an eye to economizing in the face of price changes. Therefore, these results give conservative estimates of interindustry effects. Furthermore, since many of the same industries appear to be tied to both rail and truck, it is likely that further intermodal competition and substitution of producing firms will yield substantial savings. The logical next step of this research is to incorporate the specific knowledge of the freight transportation sectors obtained in the regional transportation models in order to specify correctly the change in the direct interindustry relations engendered by regulatory changes.

Chapter Six

Models of Income Determination

Having outlined the regional transportation model and the interindustry model, let us now consider the linkages among regional transportation outputs and prices, interindustry relationships, and the level of regional and national incomes. These interrelationships can be made explicit by considering a model of regional income determination, discussed in Part I of this chapter, and a small-scale national macroeconometric model, discussed in Part II.

I. Regional Income Model

Let us now consider the interrelationships between the regional transportation model and the regional income model. Briefly stated, the equilibrium in the regional transportation market affects the levels of regional economic activity in two important ways. First, the demand for labor in the transportation industries has a direct impact upon regional employment and income. Second, the transportation rate structure in any region relative to that of the nation as a whole can influence the location and investment decision of firms and thus affect regional income and employment. Similarly, regional income levels can have a direct impact upon the demand for transportation services, while regional wage structures can affect the demand for labor within the transportation industries. Thus, if we view the transportation industries as only one sector within a regional economy, it is clear that there are bound to be many linkages between the equilibrium in the transportation industries and that of the entire regional economy.

This analysis attempts to capture the major linkages and concentrates upon the interrelationships among regional income, employment, and transportation. To this end, we will develop employment, wage, and personal income relationships and show how they interact with the regional transportation model. In doing this, our goal is not to develop a fully specified model of regional income determination, but rather to utilize a somewhat aggregative model that will capture the main elements of the problem.

A. The Structure of Regional Models

Macroeconometric modeling has a well established tradition on the national level. The pioneering work of Klein (1955) in the early 1950's has been expanded and developed until there are now a number of large scale econometric models of the national economy that can be used for forecasting levels of economic activity and for evaluating alternative monetary and fiscal policies.[1/] While these models do not have a perfect track record with respect to all sectors of the economy, on the whole their performance indicates that it is possible to forecast the vehavior of the national economy with a reasonable degree of accuracy.

It is only in the past decade that serious econometric modeling on the state or regional level has been developed. On the state level, models of Massachusetts (1975), Michigan (1965), California (1972) and Mississippi (1975) have been developed, while on the local level a model of Philadelphia (1975) has been developed and one of Boston is currently under construction (1976). Although none of these models has been as fully developed as the more established national models (e.g., the Wharton or the FMP models), the forecasting record of these models indicates that it is possible to explain and forecast the economic behavior of a state or a region reasonably well.

The theory underlying the national macroeconometric models is generally well defined. All of these models rely on a Keynsian aggregate demand framework; thus, much of these models is built around the national-income equilibrium condition that income equals the sum of final demand (consumption, investment, government expenditures and the net trade balance). The analysis then proceeds to explain the components of final demand on a disaggregate level, the determinants of aggregate supply, and the relationships between the "real" and the monetary sectors.

[1/] See, for example, the FMP Model (1968), the Wharton Model (1967), the U.S. Department of Commerce, B.E.A. Model (1966), as well as a number of models developed by private consulting firms, e.g., Chase Econometrics and Data Resources, Inc.

One of the under pinnings of all national macroeconometric models is the national income accounts, which not only provide the data but an analytical, Keynsian framework on which to develop a model. Unfortunately, there are no data equivalent to the national income accounts on the state or regional level, making the analytical framework upon which to build a state or regional macroeconometric model less obvious. While the Michigan Model (1965) attempted to analyze regional income determination in a Keynsian framework, recent regional models (e.g. the Massachusetts Model (1975), the Mississippi Model (1975), and the California Model (1972)) have been built around the excellent regional data bases in employment and wages. In this case, the models tend to take on more of a supply-oriented, microeconomic general equilibrium character than the demand-oriented, macroeconometric national models. Indeed, their analytical under pinnings lie in the neoclassical theory of the firm and the simultaneous determination of employment, output, wages, and producers' prices.

Since the level of transportation rates relative to other prices plays a key role in an integrated transportation policy model, it seems logical to adopt a neoclassical approach, which incorporates relative price differentials, in modeling regional income levels. As such, we draw upon the analytical framework developed in the Massachuse Model (1975) and its predecessors. Because, however, the focus of this analysis is the interrelationships among the transportation industries and the rest of the regional economy, transportation rates and employment will play a central role in this modeling effort that they have not had in previous regional models.

The structure of the regional income model is illustrated in Figure 6.1. Regional employment is assumed to depend upon regional factor costs (transportation, labor, capital, and energy) relative to those of the nation and regional income. Regional wages are related to national wages and regional employment growth. Given wages and employment, we can then determine labor income, and from that, we can derive measures of gross state product. Personal income is given by the sum of labor and nonlabor income. Finally, the regional consumer price index is determined by the regional transportation rate structure and the national CPI.

Since employment plays a central role in this analysis, we first discuss its determination. We then consider the other components of the regional income model: wages, income, output, and consumer prices; and show how the transportation industries and the rest of the regional economy are linked.

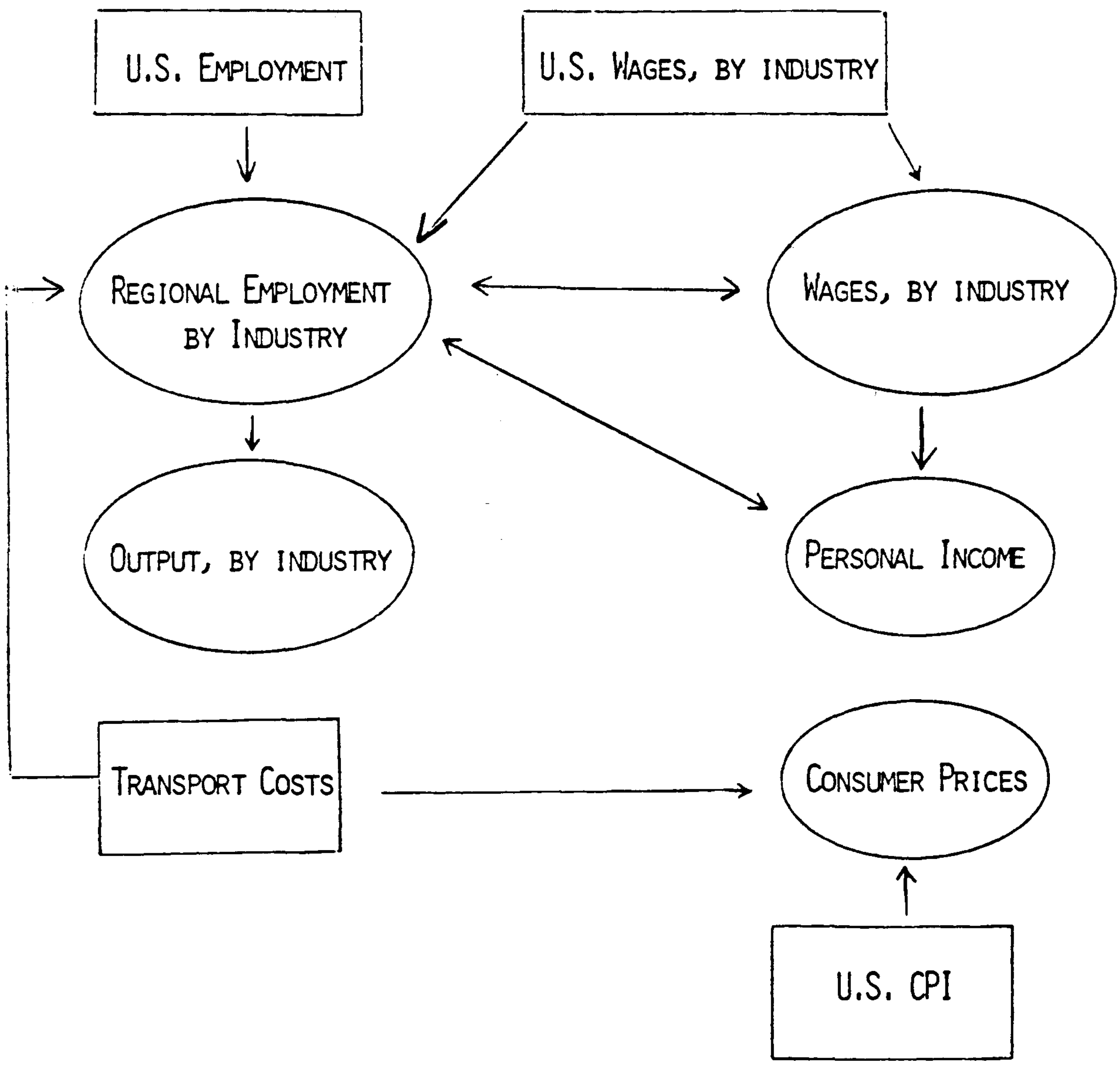
B. Employment

In analyzing employment, it is useful to distinguish between the so-called export industries, whose output levels are highly sensitive to regional cost differentials, and the so-called home industries, whose output is tied to the local, regional economy. Manufacturing industries are generally thought to fall into the former group, as are agricultural and extractive activites with a natural resource base. Activities such as services, construction, retail and whoesale trade are generally thought to fall into the latter category.

In so far as the manufacturing industries are not tied to any specific resource base, they are highly mobile and will tend to be

FIGURE 6.1

REGIONAL INCOME MODEL



responsive to regional differentials in factor costs, including transportation. Since the extractive and agricultural industries are clearly tied to a resource base, they are not particularly mobile. Nevertheless, because transportation costs play a large role in their delivered price, it is likely that transportation rates can have a significant impact upon their levels of output and employment. Hence, it seems reasonable to model the behavior or each of the export industries in a similar fashion.

In contrast, the home industries are largely tied to the local economy and should not be particularly sensitive to regional differentials in factor prices. Thus, we will analyze this group of industries in a manner somewhat different from that followed with respect to the export industries.

1. The Export Industries

For the purpose of the regional income analysis, it is convenient to combine the agriculture and livestock categories into a single agricultural sector and to combine the coal and petroleum categories into a single extractive sector. The regional model will thus have the following four industries: durable manufacturers, nondurable manufacturers, agriculture, and energy raw materials.

In order to assess the impact of regional differentials in transportation rates upon regional income levels, we should know how these differentials affect flows of capital and labor in and out of the region. This, in turn, requires a full general equilibrium analysis of the simultaneous determination of output, employment, capital utilization, and factor migration. Since, however, data are not available to support the analysis on this level of detail, we must be content with a reduced-form determination of employment instead of using a fully-specified structural model.

The extent to which reduced-form equations are used in the analysis in large part depends upon the adequacy of the data. While data on employment and wages are generally available and reliable in a fairly disaggregate level, data on investment, capital, and real output are quite poor. Moreover, regional data on producer's prices are non-existent. Consequently, it seems reasonable to adopt a reduced form approach that substitutes for output, capital, and producer's prices in the employment equations. While this approach has the admitted defect that it does not make explicit the structure through which changes in relative factor costs affect the level of economic activity, it has the distinct advantage of limited data requirements.

Because the export sectors primarily produce for the national market, we analyze the share of national employment in industry i that takes place in region d. In structuring these employment equations, our general behavioral postulate is that firms locate to maximize profits and will thus be sensitive to production and distribution costs. Production costs are reflected in relative labor costs, relative capital costs, relative fuel costs, and relative transportation costs; distribution costs are reflected in relative levels of disposable income.

Our formal point of departure is the neoclassical theory of the firm. We assume that production is characterized by a constant-returns-to-scale, Cobb-Douglas production function utilizing capital (K), labor (L), energy (E) and transportation (T), with neutral technological change.[2/] Thus:

$$Q_{id} = A_{id} e^{\lambda_{id} t} L_{id}^{\beta_1^{id}} E_{id}^{\beta_2^{id}} T_{id}^{\beta_3^{id}} K_{id}^{1-\beta_1^{id}-\beta_2^{id}-\beta_3^{id}} \qquad (6.1)$$

where Q_{id} represents the output of industry i in region d; L_{id}, E_{id}, T_{id} and K_{id} respectively represent the physical amounts of labor, energy, transportation, and capital used by industry i in region d; λ_{id} represents a technological change factor of industry i in region d; A_{id} represents a scale factor associated with industry i in region d; and β_1^{id}, β_2^{id}, β_3^{id} represent the parameters of the production function.

If the firm maximizes profits, it sets the marginal product of each factor equal to the real factor price. Using these marginal productivity conditions, we can eliminate the producer's price variable and express capital, energy, and transportation in terms of labor. By substituting the resulting expression into the production function and solving for employment we obtain the following expression (note that we omit the industry and regional subscripts for notational simplicity):

$$L = (A^{-1})(e^{-\lambda t})\left(\frac{1-\beta_1-\beta_2-\beta_3}{\beta_1}\right)^{\beta_1+\beta_2+\beta_3-1} \left(\frac{\beta_2}{\beta_1}\right)^{-\beta_2} \left(\frac{\beta_3}{\beta_1}\right)^{-\beta_3}$$

$$\left(\frac{w}{r}\right)^{\beta_1+\beta_2+\beta_3-1} \left(\frac{w}{f}\right)^{-\beta_2} \left(\frac{w}{t}\right)^{-\beta_3} \cdot Q \qquad (6.2)$$

where w,r,f, and t respectively represent the factor prices of labor, capital, energy, and transportation.

[2/] The basic analytical framework is quite similar to the one utilized in the national interindustry model. In that case, however, we dealt with prices and translog approximations of the underlying technology, while in this case we deal with physical quantities and a specific functional characterization of technology.

If national employment in that industry is also determined by similar considerations, we obtain a similar expression for national employment. If we, furthermore, assume that technology does not differ among regions, we can express regional employment as a proportion of national employment by the following expression:

$$\frac{L}{L_u} = \frac{A^{-1}}{A_u^{-1}} \cdot \frac{e^{-\lambda t}}{e^{-\lambda_u t}} \cdot \left(\frac{w}{w_u}\right)^{\beta_1 - 1} \left(\frac{f}{f_u}\right)^{\beta_2} \left(\frac{t}{t_u}\right)^{\beta_3} \left(\frac{r}{r_u}\right)^{1-\beta_1-\beta_2-\beta_3} \frac{Q}{Q_u} \qquad (6.3)$$

where the subscript "u" refers to the national variable.

Equation (6.3) thus relates the regional share of employment in a given industry to regional differences in factor prices, output levels, and technological change.

While data on regional output as measured by gross regional product exist, they are seriously flawed; not only because they are derived from data series in labor income, but also because they are measured in terms of current dollars.[3/] Consequently, we have elected to eliminate the output variable by substitution. We thus postulate that the proportion of total output produced in any given region depends upon relative factor costs and the levels of personal income and write:

3/ The fact that they are derived from series as labor income creats substantial problems of simultaneous equation bias; the fact that they are expressed in money terms requires that they be deflated by an appropriate price index. Unfortunately, however, data on producers' prices do not exist, and use of a proxy could create substantial measurement errors.

$$\frac{Q}{Q_u} = \alpha_o\left(\frac{w}{w_u}\right)^{\alpha_1}\left(\frac{f}{f_u}\right)^{\alpha_2}\left(\frac{t}{t_u}\right)^{\alpha_3}\left(\frac{r}{r_u}\right)^{\alpha_4}\left(\frac{Y}{Y_u}\right)^{\alpha_5} \qquad (6.4)$$

While the specific form of this equation is not uniquely related to cost-minimization under specific assumptions, the choice of the explanatory variables is intended to reflect the factors that are relevant in the locational decision. In eq. (6.4), w,f,t and r respectively represent the costs of labor, energy, transportation, and capital, with the subscripts again reflecting the national variables. Thus, α_1, α_2, α_3, and α_4 are expected to be negative since an increase in the cost of any factor relative to its national average should cause a movement away from that region. Similarly, α_5 should be positive since an increase in regional income relative to that of the nation should increase employment in that region.

Substituting eq. (6.4) into eq. (6.3) collecting terms, and taking logs yields the following estimating equation:

$$\ell n(L/L_u) = \mu_o + \mu_1\tau + \mu_2\,\ell n(w/w_u) + \mu_3\,\ell n(f/f_u) \qquad (6.5)$$

$$+ \mu_4\,\ell n(t/t_u) + \mu_5\,\ell n(r/r_u) + \mu_6\,\ell n(Y/Y_u)$$

where τ represents a time trend and the coefficients are defined as follows:

$$\mu_o = \ell n\,\alpha_o + \ell nA_u - \ell nA$$

$$\mu_1 = \lambda_u - \lambda$$

$$\mu_2 = \beta_1 + \alpha_1 - 1$$

$$\mu_3 = \beta_2 + \alpha_2$$

$$\mu_3 = \beta_3 + \alpha_3$$

$$\mu_4 = 1-\beta_1-\beta_2-\beta_3+\alpha_4$$

$$\mu_5 = \alpha_5$$

The expected sign of μ_0 is not known a priori. The sign of μ_1 will be positive or negative, depending upon whether neutral technological change in the region is less than or greater than that of the nation as a whole. The sign of μ_2 should be negative since both the factor substitution effect (β_1-1) and the location effect (α_1) should be negative. The expected signs of μ_3, μ_4, and μ_5 are indeterminate and depend upon whether the negative location effects (α_2, α_3, and α_4) dominate the positive factor-substitution effects (β_2, β_3, and 1-β_1-β_2-β_3). The sign of μ_6 is expected to be positive since it reflects the positive location effect.

Equation (6.5) thus relates the share of employment of a given industry to the level of regional factor costs relative to that of the nation and the level of regional income relative to that of the nation. Consequently, by estimating these equtions, it should be possible to quantify the impact of changes in regional transportation rates upon regional levels of employment in the export industries.

2. Transportation

As explained in Chapter Two, above, it is possible to derive factor demand equations from the transportation estimated cost functions Consequently, by using the regional cost functions derived for each mode, we can derive regional employment functions for each mode. By summing the implied labor demands we can thus obtain estimates of the employment in the regulated transportation industries in each region.

3. Home Industries

In our analysis, we have combined all of the remaining industries into a category which we have called "other". This includes the service trade, construction and related industries, as well as the non-regulated transportation industries. Since these industries are oriented to the local market, it is unlikely that regional differentials in factor prices will have much bearing on their output or employment levels.

Again, we assume that we can characterize production of the home industries by the following constant-returns-to-scale, Cobb-Douglas production function:

$$Q_{hd} = A_{hd}\, e^{\lambda_{hd} t}\, L_{hd}^{\beta_1^{hd}}\, E_{hd}^{\beta_2^{hd}}\, T_{hd}^{\beta_3^{hd}}\, K_{hd}^{1-\beta_1^{hd}-\beta_2^{hd}-\beta_3^{hd}} \qquad (6.6)$$

where the subscripts h and d respectively refer to the home industry and the relevant region. If we assume cost minimization, we can utilize the marginal productivity conditions and solve for L_{hd} to derive an expression that is identical in form to eq. (6.2). Unfortunately, however, this contains the variable Q_{hd}, which represents the real value of output in the home industries and for which we have no data series. Nevertheless, by using personal income as a proxy for this variable and taking logs, we can utilize the following equation to estimate the employment in the home industries, which we represent by the "other" category:

$$\begin{aligned} \ln L_{hd} = \gamma_0 + \gamma_1 \ln\tau + \gamma_2 \ln(w_d/r_d) + \gamma_3 \ln(w_d/f_d) \\ + \gamma_4 \ln(w_d/t_d) + \gamma_5 \ln Y_d \end{aligned} \qquad (6.7)$$

where τ represents a time trend; γ_0 represents a linear combination of the production coefficients, γ_1 represents a measure of neutral technical change, $\gamma_2 = \beta_1+\beta_2+\beta_3-1$, $\gamma_3 = -\beta_2$, $\gamma_4 = -\beta_3$ and $\gamma_5 = 1$.

Thus, using eqs. (6.5) and (6.7) we can estimate regional employment in the export and home industries (exclusive of the regulated transportation industries). By utilizing the factor demand equations derived in conjunction with the modal cost functions, we can also estimate regional employment in the regulated transportation industries and thus obtain measures of total employment by broad industry aggregate and by region.

B. Wage and Income Relationships

Although specific data on hourly wages exist for the manufacturing industries, they do not exist for the other sectors. Hence, it is more convenient to utilize the BEA's data on personal income, which gives wage and salary income by sector. We can thus define aggregate wage in sector i in region d by:

$$ws_i^d \equiv Yws_i^d/E_i^d$$

where Yws_i^d represents earned labor income in sector i in region d; E_i^d represents employment in industry i in region d; and ws_i^d represents the average labor payments in sector i in region d. We thus postulate that average wage and salary payments are related to their national counterparts, the change in regional employment, and the ratio of the regional CPI to the national CPI and thus specify:

$$ws_i^d = ws_i^d[ws_i^u,\ \Delta E_i^d,\ CPI^d/CPI^u] \qquad (6.8)$$

where ws_i^d and ws_i^u respectively represent average wages and salaries in industry i in region d and in the nation as a whole; ΔE_i^d represents the change of employment in region d and industry i; and CPI^d and CPI^u

respectively represent the regional and national CPI.

Since total labor income represents the product of employment and average wage and salary payments, we can simply derive total regional labor income as:

$$YL^d = \sum_i (ws_i^d)(E_i^d) \quad (6.9)$$

where i ranges over the export, transportation, and home industries.

Total personal income is the sum of labor income, proprietor's income, property income, and transfers less contributions for social insurance. Equation (6.9) gives the labor component of this total, but neglects the other categories. At this point, it is not clear whether it is necessary to estimate a separate equation for each category or whether it is acceptable to estimate an equation for non-labor personal income. Since transportation does not affect non-labor personal income, little seems to be gained by disaggregating it into its components. Hence, it seems reasonable to estimate a simple relationship for non-labor personal income and relate this variable to its national counterpart.

As indicated above, gross regional product by sector is derived from wage and salary income by sector according to a standard formula.[4/] Consequently, there is no real linkage flowing from gross regional product to employment by definition. Therefore, although it might prove desirable to include estimates of gross regional product for the sake of completeness, such estimates add little information that is not already contained in the estimates of total labor income.

[4/] This is described in detail in Friedlaender, Treyz, and Tresch (1975).

We now turn to questions of price determination. In so far as transportation rates may affect consumer prices, it is useful to introduce prices explicitly into the analysis. We thus postulate that the regional CPI depends upon regional transportation rates, the level of personal income relative in the region relative to that of the nation, and the regional unemployment rate and thus write:

$$CPI^d = CPI^d(CPI^u,\ t^d,\ Y^d/Y^u,\ u^d) \qquad (6.10)$$

where CPI^d and CPI^u respectively represent the regional and national CPI, t^d represents the regional structure of transportation rates, Y^d and Y^u respectively represent regional and national personal income, and u^d represents the regional unemployment rate.

To close the model we need to estimate relationships for the regional unemployment rate. The most theoretically sound approach to this problem involves specifying a labor supply equation and then estimating unemployment as a residual between labor demanded and supplied. However, this would require the specification of a large number of equations having to do with population growth, migration, etc., and would extend the analysis in a direction that is probably unnecessary for the problem at hand. Hence, it is probably a reasonable alternative to explain the regional unemployment rate directly, relating it to the national rate and the rate of growth of regional personal income and write:

$$u^d = u^d(U^u,\ \Delta Y^d) \qquad (6.11)$$

where u^d and U^u represent regional and national unemployment rates and ΔY^d represents the growth in regional income.

We have now specified the foundations of the following three models: a regional transportation model; a national interindustry model; and a regional income model. In each case, we have specified the relationships in the sub-model in terms of its own relevant variables and in terms of those that feed in from the other sub-models, with the exception of a number of national macroeconomic variables having to do with income, factor prices, consumer price and so forth. Hence, to close the system we must develop a national macroeconomic model and specify the missing variables.

B. Macroeconometric Model

A number of variables are required to close the various sub-models. The national interindustry model needs data on final demand by sector and the price of capital and labor, while the regional model needs data on national personal income, consumer prices, and the unemployment rate. Since these variables are all interrelated, we must develop a small-scale macroeconometric model to specify these interrelationships and to estimate equations for these variables.

As indicated above, the art of macroeconometric model building is well advanced, and there are a large number of existing models that range in size from the small-scale Fair model (1971) to the enormous FMP model (1968). Since questions associated with fiscal and monetary policy are not particularly relevant to the problem at hand, it probably makes sense to deal with fairly aggregative models that do not consider in great detail the channels through which monetary or fiscal policy work. Thus it may be reasonable to adapt the Fair model (1971) to our analysis. As an alternative, we could also adapt the model developed by Hudson and Jorgenson (1974) in their analysis of energy policy.

Since we have not fully explored the structure of the existing small-scale macroeconometric models, little would be gained from making a specification of such a model de novo. Clearly such a model would require the determination of gross national product by broad sector and its components: consumption, investment, government, and net exports. It would similarly require the determination of sectoral wages, consumer prices, the interest rate and the unemployment rate. These are the traditional elements of a full Keynsian model, and their analysis and estimation is well grounded in macroeconomic theory and its applications in the existing macroeconometric models. Thus, although we have not yet developed the specification of the macroeconometric model needed to close the system, this is a straightforward task that we will undertake at the appropriate time.

Chapter Seven

Summary and Extensions

This volume has described our first year's work in developing a number of integrated models that can be used to evaluate a wide range of federal transportation policies affecting the surface freight industries.

By simulating the response of the system to changes in federal transportation policies, these models are specifically aimed at evaluating the impact of a wide range of transportation policies upon the following kinds of variables: traffic allocations, rates, profitability, costs, employment by transportation industries; outputs, employment, prices, and factor prices by industries for the nation as a whole; employment, income, and wage by industry and by region. This analysis will provide a vehicle for quantifying the impact of transportation policy upon a wide range of fairly aggregative economic variables that not only provide measures of economic efficiency, but also provide measures of the gainers and losers of a given change in transportation policy by industry (both within transportation and elsewhere), by region, and by factor.

To this end, this volume contained the general specifications of the following four models:

- a model of the regional transportation markets;
- an interindustry model;
- a model of regional incomes and employment;
- a national small-scale macroeconometric model.

Before we can fully evaluate the impact of policy, we must calibrate these models. Thus, the bulk of our effort in the second year of this project will go toward data gathering and calibration of these models. At the moment, we have concentrated upon the estimation of demand and cost functions for the trucking and rail industries in the Official Territory, and Chapter Four contained a report on our preliminary analysis of trucking costs. In terms of methodology, probably the most striking finding is the need to take quality of output into account in estimating cost functions as well as demand functions. Unless the relevant quality variables are taken into account in the estimated cost functions, serious misspecifications and biases may result, leading to incorrect policy conclusions. During the coming year, we plan to extend this analysis to all the regions in the country and, if resources permit, to the water and pipeline modes.

We have also made a preliminary analysis of interindustry relationships, which was discussed in Chapter Five. This indicates that instead of estimating the interindustry coefficients endogenously, they will have to be determined exogenously. Thus, considerable effort must be spent determining how changes in the relevant modal cost functions, demand functions, and market equilibria can be translated into changes in the interindustry coefficients.

We have just begun the implementation of the regional and national macroeconometric models. Reports on our progress in these areas will be made during the coming year.

Since these models are being developed for policy analysis, we hope to be able to perform some policy simulations during the coming year. Thus a major effort will be devoted to the area of scenario development and the determination of a number of alternative policies that can be evaluated in the context of our models. A large number of these will be based on the legislation currently pending and recently passed concerning deregulation of the air, trucking, and rail industries. We will not limit ourselves to regulatory proposals, however, but will also consider the implications of removing some of the current inconsistencies of federal transportation policy with respect to the various modes. For example, we can analyze the implications of treating all modes equally with respect to user charges or the provision of infrastructure. In addition, we plan to assess the impact of the establishment of rate bands in the rail and trucking industries, subsidies for light density service, rail consolidation, trucking merger policies, etc.

Since this phase of the research is aimed at specific policy analysis, we hope to work closely with members of government during this time to ensure that we are analyzing the most relevant transportation policies.

Obviously, the development of these models for all modes and regions is an enormous task, and their integration into an integrated simulation model is an even bigger one. Nevertheless, preliminary analysis already indicates that new and important insights into the impacts of policy can be obtained from these models. Thus, it is hoped that these models can be used for actual policy evaluation in the coming year.

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