

**Noise Diagnostics for Safety
Assessment Quarterly Progress Report
for July-September 1977**

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Prepared for the U.S. Nuclear Regulatory Commission,
Office of Nuclear Regulatory Research, under Interagency
Agreements ERDA 40-551-75 and 40-552-75.

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Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
Price: Printed Copy \$4.50 ; Microfiche \$3.00

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Contract No. W-7405-eng-26

INSTRUMENTATION AND CONTROLS DIVISION

NOISE DIAGNOSTICS FOR SAFETY ASSESSMENT
QUARTERLY PROGRESS REPORT FOR JULY-SEPTEMBER 1977

R. C. Kryter and K. R. Piety

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Prepared for the U.S. Nuclear Regulatory Commission,
Office of Nuclear Regulatory Research, under Interagency
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Manuscript Completed: November 9, 1977

Date Published - November 1977

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ABSTRACT

The development of advanced calculational methods for understanding and quantifying commonly encountered neutron noise sources (e.g., coolant boiling and vibrations of fuel, control rods, and other reactor internals) in LWRs is proceeding on schedule, with completion of the initial noise-equivalent source formulation and generation of a working cross section library being the major accomplishments this quarter. A work plan for investigating experimentally the propagation of acoustic disturbances in complex, massive, metallic structures is being formulated and a portion of the instrumentation and apparatus necessary to pursue this loose-parts-oriented work was obtained, but a full experimental program is not yet underway, owing to delays in obtaining certain key equipment. Development and application assessment activities for the year in the area of automated signature analysis methods for nuclear reactors and rotating machinery are summarized in a paper recently presented at an international specialists' conference.

1. RESEARCH ACTIVITIES

R. C. Kryter

1.1 In-Core Monitoring Methods

With assistance from the author of the MATTEO computer program and from numerical computations experts at ORNL, the inadequate convergence encountered earlier with the iterative solution of the two-phase flow continuity equations was overcome, and MATTEO now produces satisfactory results. An anticipated major modification to the TASK computer program, which solves the one-dimensional, multi-group kinetics equation to yield frequency-dependent fluxes, proved unnecessary. Close comparisons between steady-state fluxes produced by TASK and ANISN (a widely recognized neutron transport code) verified the problem formulation and data input.

Considerable effort was required to generate a suitable cross section set for input to the TASK and VART computer programs. We decided to use the NRC 218-group microscopic set instead of those compiled by Hansen and Roach because the NRC set will give a better treatment of neutron upscattering in the thermal energy region, which is important to the boiling sub-channel problem being studied. Subroutines for entering the collapsed (4-energy-group) cross sections produced by the AXMIX code into the variational method program VART were written and tested.

Significant progress was also made in the initial coding of VART, particularly the noise-equivalent source computation. This new code will be ready for initial testing early in the first quarter of FY 1978.

1.2 Loose-Parts Monitoring Systems

With the conclusion of interviews with LPMS suppliers and users having been completed late in July, confirmatory assessment needs in this area have come into sharper focus. Throughout this quarter we continued to refine the investigative program outlined in the third quarter progress report (ORNL/NUREG/TM-133), with particular attention given to task priorities, interfacing activities, and milestone accomplishments.

Procurement of instrumentation and apparatus necessary to pursue our identified development objectives proceeded, but somewhat more slowly than had been anticipated last quarter. Difficulties with two major items caused this delay: (1) no commercial source of supply could be found for a standardized impacting apparatus meeting our requirements, so this will have to be designed and assembled in-house, and (2) identification and purchase of a suitable commercial 4-channel transient signal capture instrument required more effort and time than originally estimated. However, our orders for high-temperature accelerometers, line-driving acoustic emission transducers, charge- and voltage-sensitive preamplifiers, cables, etc. were filled in early September, and magnetic mounting blocks were obtained (on loan) for preliminary measurements. Also, an *initial* impact test fixture (machinist's surface plate) was obtained and modifications to accept stud-type accelerometer mountings were initiated.

The frequency responses of the acoustic transducers and preamplifiers received were measured using sinusoidal excitation tests, and initial measurements of metallic impacts on the surface plate test fixture were made, using a conventional 2-channel analog oscilloscope in lieu of a more accurate digital transient signal capture instrument. These preliminary

tests, which will be confirmed later under more controlled conditions, verified the conformance of the equipment to the manufacturers' specifications but also revealed unit-to-unit response differences that will have to be accounted for in subsequent studies.

1.3 Surveillance and Monitoring by Noise Analysis

On the basis of programmatic guidance from NRC:RSR and the Noise Surveillance and Diagnostics Review Group, work objectives originally planned for this subtask (concerned with automated monitoring techniques for rotating machinery) were reconsidered. It was suggested that we examine the subjects of (1) pressure boundary crack and leak detection in LWRs, (2) stability monitoring of BWRs by means of nonperturbative or noise-related techniques, and (3) applicability of surveillance methods and failure-predictive statistical algorithms to reactor protection and engineered safeguard systems, all to be considered as candidates for possible expansion of our FY 1978 work scope. These areas were reviewed cursorily in August and September, and our findings will be presented at the next Review Group meeting.

Although no longer an on-going activity, as explained above, work directed towards automated signature analysis was carried out early in FY 1977 but not previously reported. We therefore consider it appropriate to devote this last quarter's focus report (Section 2) to this topic by reproducing a paper by K. R. Piety that was presented at the Second Specialists' Meeting on Reactor Noise (SMORN-II) in Gatlinburg, Tennessee, September 19-23, 1977.

2. FOCUS REPORT--TASK 3a, SURVEILLANCE AND DIAGNOSTICS BY NOISE ANALYSIS

STATISTICAL ALGORITHM FOR AUTOMATED SIGNATURE ANALYSIS OF POWER SPECTRAL DENSITY DATA^a

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INTRODUCTION

Scope of the Work

A statistical algorithm has been developed and implemented on a minicomputer system for on-line, surveillance applications. Power spectral density (PSD) measurements on process signals are the performance signatures that characterize the "health" of the monitored equipment. Statistical methods provide a quantitative basis for automating the detection of anomalous conditions. The surveillance algorithm has been tested on signals from neutron sensors, proximeter probes, and accelerometers to determine its potential for monitoring nuclear reactors and rotating machinery.

Background

Diagnostic information has been obtained from applications of signature analysis in both rotating equipment (1-5) and nuclear reactors (6-11). A drawback to utilizing the signature analysis method for surveillance activities is the demand it places on personnel. With an automated system, this burden would be relieved, and personnel could give their attention to equipment most in need of attention. Additionally, an automated signature analysis system would have sufficient sensitivity to provide an early warning of an incipient failure. This would allow better scheduling of maintenance and would enhance diagnosis of a problem condition by documenting its development.

An impediment to the automation of signature analysis for anomaly detection has been the lack of a quantitative basis for determining when significant changes have occurred. In earlier work at ORNL, we investigated the use of a hyperellipsoid enclosure algorithm (12-13) and a decoupled variables approach (14) to automate signature analysis for on-line monitoring applications. These algorithms incorporated general, multivariate statistical techniques to analyze a set of random and stationary variables whose composition was quite flexible. In contrast to the generality of these earlier algorithms, the statistical techniques utilized in this algorithm are derived strictly for the PSD descriptor, and will be referred to as the PSD statistical recognition (PSDREC) algorithm. The benefits gained that offset the loss in generality of this method are: (1) the learning time necessary to establish normal behavior is reduced; (2) all available PSD estimates (rather than a preselected few, limited by storage requirements) can be examined, thus minimizing preprocessing judgments; (3) the ability to differentiate the rate of change of monitored behavior is greater; and (4) identification of spectral changes is more direct.

^aPaper presented at the Second Specialists' Meeting on Reactor Noise, Gatlinburg, Tennessee, September 19-23, 1977.

METHODOLOGY OF SURVEILLANCE TECHNIQUE

During an initial learning period, PSDREC forms the initial baseline signatures and a statistical description of the normal variations that occur. The monitored system is assumed to be operating normally or at least acceptably during this period. The detection of anomalous conditions is determined by examining decision discriminants which quantitate certain types of spectral deviations. The limiting criteria that are applied to the discriminants are predicted from input parameters that define the monitoring procedure. The predicted criteria are corrected for the specific behavior observed during the initial learning period. Additional correction is allowed at specified intervals if normal conditions prevail.

Baseline Signatures

A baseline signature is the standard against which each set of PSD measurements is compared. Two separate baseline signatures (designated "Base PSD" and "Trend PSD") are maintained to describe the rate at which a change is occurring. The Base PSD characterizes operational behavior at the time monitoring is initiated; once the Base PSD includes a specified number of PSD estimates, it is never changed. Comparisons against the Base PSD allow maximum sensitivity to slowly developing phenomena. However, the value of Base PSD comparisons is reduced if normal cycles of change are present in the data.

The Trend PSD is the signature which characterizes the most recent period of normal operation. A candidate to replace the Trend PSD is calculated repetitively at specified intervals. During monitoring, this replacement is allowed only if comparisons of incoming PSD estimates against the Trend PSD during that interval did not detect significant variations. Although the Trend PSD adaptively follows slow changes, comparisons against the Trend PSD will detect abrupt or rapidly developing conditions. Comparisons of the candidate (Trend) PSD against the Trend PSD and the Base PSD allow the detection of the more gradual changes.

Decision Discriminants

PSDREC calculates eight statistical discriminants that are formed from ratios of a set of PSD measurements obtained on-line with a baseline set. The discriminants detect (1) fluctuations in the integral power of the spectrum, (2) spectral shape changes, (3) deviations in the magnitude of individual PSD estimates at a given frequency, and (4) shifts in the frequency of spectral peaks. The sensitivity of multiple discriminants to certain spectral changes is greater than that of a single global measure, such as the Mahalanobis distance (12-14). Use of several discriminants also offers a possibility of requiring certain combined or coincident deviations as means of preventing alarms during normal phenomena which produce changes that are statistically significant. The eight discriminants help to identify what type of spectral change is occurring, to quantify its magnitude, and to initiate plots that give a clear, visual indication of the detected variation. These discriminants can be formed on the complete set of PSD estimates or on any subset (or any collection of up to ten subsets).

Prediction and Correction

PSDREC initially calculates theoretical limiting criteria for the discriminants. These criteria can be strictly applied only if the data has a gaussian amplitude distribution and the individual PSD estimates are independent (additionally for one criterion, the frequency

spectrum is assumed to be white). During the learning period, the discriminants are tested against the predicted criteria in order to determine how closely the data follow the assumptions stated. Also, during learning, the means and variances calculated from the actual sampled population are used to correct, in a heuristic manner, the theoretically derived criteria values. In essence, the discriminant formed from the PSDs is transformed, using the sample mean and variance, to produce a distribution that has the mean and variance of the theoretical distribution that was assumed in deriving the limiting criterion. In practice, a transformation is applied in an equivalent fashion to alter the limiting criterion so as to maintain the chosen false alarm rate. This prediction-correction procedure maximizes the use of the information available from the original assumptions and reduces the amount of data required during learning to establish appropriate limiting values.

Two-Level Alarm Logic

The discriminants are checked against two sets of criteria values. The alert level criteria are less conservative, and violations at these limits must occur on two consecutive occasions before the significance of a change is acknowledged and an alarm is sounded. At the danger level, an exceedance of any criteria immediately generates an alarm. This type logic provides protection against alarming for an occasional statistical deviant without sacrificing sensitivity to excessive deviations even if they occur intermittently.

STATISTICAL CONSIDERATIONS

Discriminants Defined

The test discriminants are based on the ratio of a test PSD, $P(f_i)$, with an appropriate baseline PSD, $P^*(f_i)$. The independent variable, f_i , identifies the frequency of that estimate. The first discriminant, D_I , is an integral measure that is sensitive to differences in the integral power in a total of r estimates contained in the frequency intervals selected for analysis:

$$D_I = \log_{10} \frac{\sum_i P(f_i)}{\sum_i P^*(f_i)} .$$

This discriminant is dominated by the estimates with larger absolute magnitudes and is relatively unaffected by changes in estimates that are one or two orders of magnitude smaller.

The second and third discriminants examine the minimum and maximum ratios of individual estimates, respectively:

$$D_{II} = \text{Min} \left[\log_{10} \frac{P(f_i)}{P^*(f_i)} \right], \text{ for all } i$$

and

$$D_{III} = \text{Max} \left[\log_{10} \frac{P(f_i)}{P^*(f_i)} \right], \text{ for all } i .$$

Discriminants D_{II} and D_{III} are singular measures of deviations and are completely unaffected by the absolute magnitudes of the PSDs.

The fourth discriminant is the mean ratio constructed from the set of ratios at the r individual frequencies. This discriminant is a measure of the integral difference between spectra:

$$D_{IV} = \frac{1}{r} \sum_i \log_{10} \frac{P(f_i)}{P^*(f_i)} .$$

This discriminant gives equal weight to all components, regardless of their absolute magnitudes. It is sensitive to uniform spectral shifts; however, it is subject to cancellation effects from terms of opposite signs (i.e., ratios greater than or less than unity), and this limits its ability to detect spectral variations where offsetting deviations are present. Another composite ratio is constructed using the second moment of the ratios to eliminate this limitation:

$$D_V = \frac{1}{r} \sum_i \left[\log_{10} \frac{P(f_i)}{P^*(f_i)} \right]^2 .$$

This discriminant is a measure of the variance of the set of ratios or, alternatively, it is the average squared distance between the test and baseline spectra on a log scale.

The sixth discriminant is an application of the sign test (15,16) to the set of log ratios:

$$D_{VI} = \text{Larger}\{(\text{Number of Log Ratios} > \text{Median}) \text{ or } (\text{Number of Log Ratios} < \text{Median})\} .$$

The seventh and eighth discriminants are based on the number and length of sequences of consecutive log ratios (runs) above or below the median (median ≈ 0):

$$D_{VII} = (\text{Number of Runs} > \text{Median}) + (\text{Number of Runs} < \text{Median})$$

and

$$D_{VIII} = \text{Max Length}\{(\text{Runs} > \text{Median}) \text{ or } (\text{Runs} < \text{Median})\} .$$

These latter three discriminants are global measures of changes in spectral shape, and they are not influenced by the absolute magnitudes of the individual PSDs or the magnitude of a single ratio in any way.

Theoretical Limiting Criteria

The first five discriminants are derived as parametric tests based on an assumption that the time waveform, $x(t)$, of the monitored signal has a gaussian amplitude distribution. Since the Fourier transform is a linear process, the resulting real and imaginary Fourier components at each frequency, $X_R(f_i)$ and $X_I(f_i)$, are also independent gaussian variables. The PSD estimate at f_i is given by

$$\hat{P}(f_i) = X_R^2(f_i) + X_I^2(f_i) . \quad (1)$$

The sum of the squares of n independent gaussian variables results in a chi-square distribution, χ_n^2 , with n degrees of freedom; hence, it follows that

$$\frac{\hat{P}(f_i)}{P(f_i)} = \frac{\chi_n^2}{2} , \quad (2)$$

where $P(f_1)$ is the true (unknown) PSD at f_1 . To obtain a consistent estimate of the true PSD functions, a smoothed PSD estimate, \hat{P} , is constructed by averaging the estimates derived from different time records. Since chi-square variables have the property

$$\chi_{a+b}^2 = \chi_a^2 + \chi_b^2,$$

an ensemble averaged estimate over n records gives (17)

$$\frac{\hat{P}(f_1)}{P(f_1)} = \frac{\chi_{2n}^2}{2n}. \quad (3)$$

The ratio of two PSD estimates taken at different times, assuming that the true PSD of the signal has not been altered, yields a ratio of chi-square variables:

$$\frac{\hat{P}_1(f_1)/P(f_1)}{\hat{P}_2(f_1)/P(f_1)} = \frac{\hat{P}_1(f_1)}{\hat{P}_2(f_1)} = \frac{\chi_{2n_1}^2/2n_1}{\chi_{2n_2}^2/2n_2}. \quad (4)$$

The new random variable which results is an F-variable with $2n_1$ and $2n_2$ degrees of freedom. The F-variable can assume only nonnegative values, and has a nonsymmetric distribution for $n_2 > 2$. If a \log_{10} transformation is applied to an F-variable, the resulting distribution has improved symmetry characteristics, as shown for a particular case in Fig. 1. Additionally, the \log_{10} F-variable can assume negative and positive values with approximately equal likelihood. A functional approximation (18) is available to calculate the percentile points $L_p(v_1, v_2)$ for the $\log_{10} F_{2n_1, 2n_2}$ distribution:

$$L_p(v_1, v_2) = \log_{10} [F_p(v_1, v_2)] = \frac{2\omega}{2n(10)}, \quad (5)$$

where

$$\omega = \frac{\chi_p^2(h + \lambda)^{1/2}}{h} - \left(\frac{1}{v_1 - 1} - \frac{1}{v_2 - 1} \right) \left(\lambda + \frac{5}{6} - \frac{2}{3h} \right), \quad (6)$$

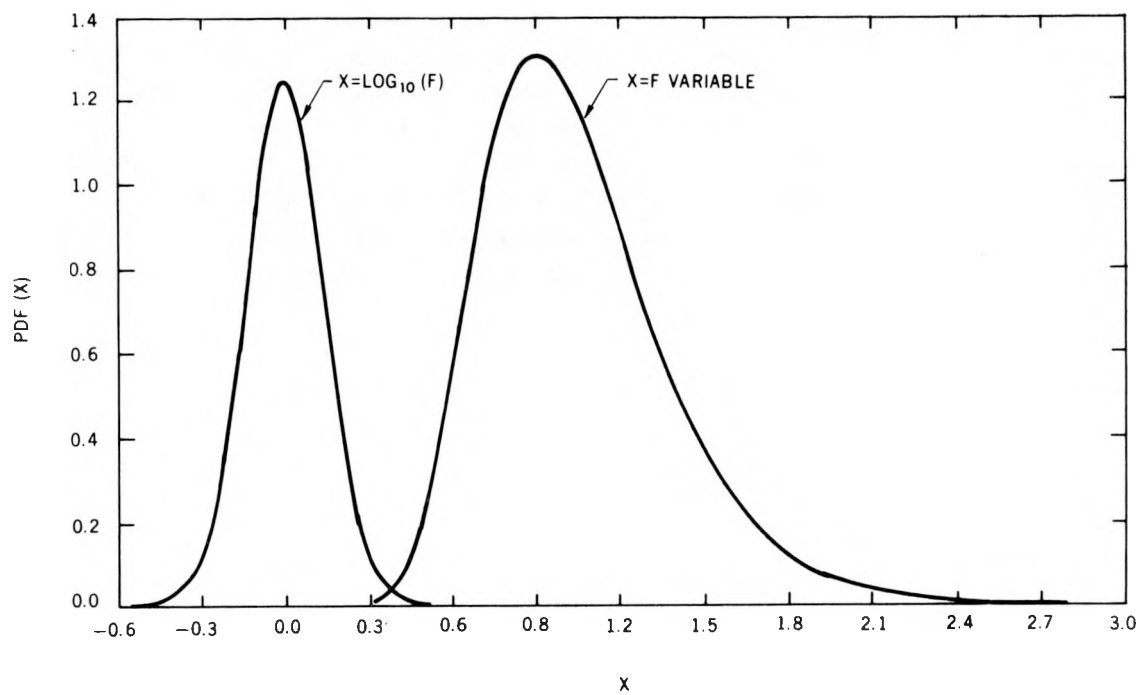
$$h = 2 \left(\frac{1}{v_2 - 1} + \frac{1}{v_1 - 1} \right)^{-1}, \quad (7)$$

and

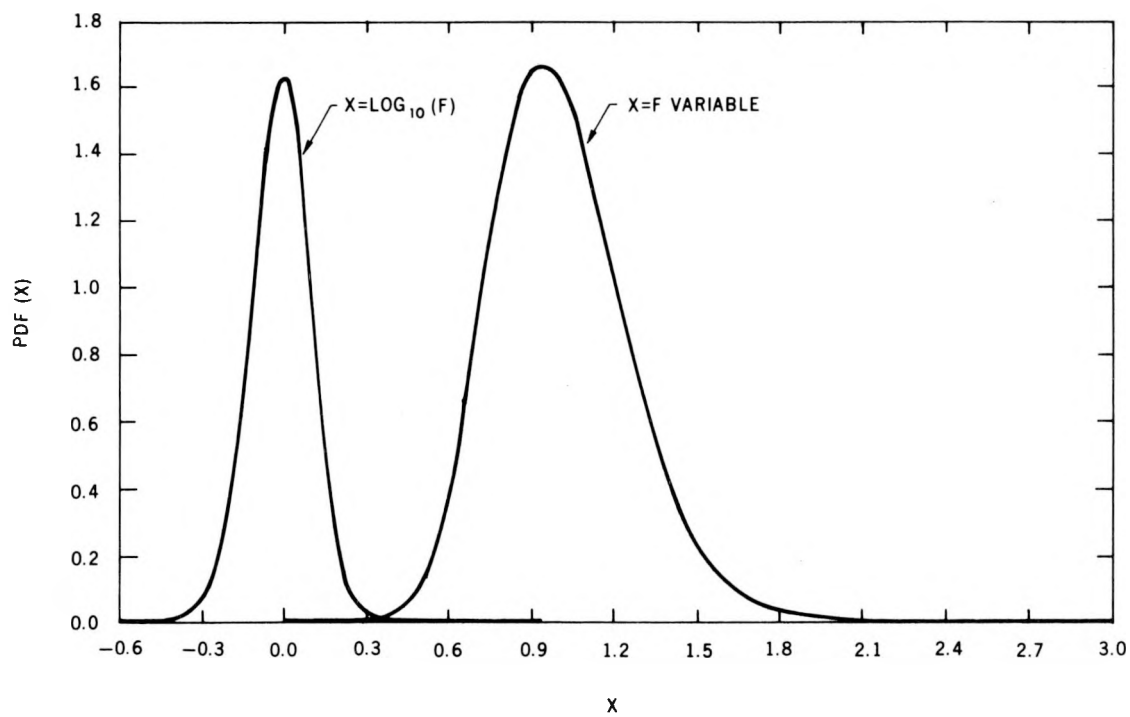
$$\lambda = \frac{\chi_p^2 - 3}{6}. \quad (8)$$

Here, χ_p is the corresponding percentile for the gaussian statistic.

This approximation can be used directly to predict the limiting criteria for the minimum and maximum log ratios for individual frequency estimates (discriminants D_{II} and D_{III}). If it is assumed that the time signal has a white frequency spectrum as well as a gaussian amplitude distribution, a predicted limit for the log of the ratio of integral power can be calculated from the same approximation. If the integral power is a sum over r independent estimates, then the degrees of freedom are increased to $v_1 = 2rn_1$ and $v_2 = 2rn_2$.



(a) $v_1 = 40; v_2 = 40$



(b) $v_1 = 40; v_2 = 200$

Fig. 1. Probability density function (PDF) of the F_{v_1, v_2} distribution vs the $\log_{10} F_{v_1, v_2}$.

The PSD estimates from an FFT calculation are assumed to be independent (if a Hanning window is applied to the time data, only every other estimate is independent), and, thus, the derived set of individual log ratios represents a set of independent variables having the same population distribution. These ratios may be viewed equivalently as a set of independently sampled values from the same probability distribution. The same is true for the set of squared log ratios. Thus, discriminants D_{IV} and D_V , which are formed from sums over the respective sets, meet the conditions for which the central limit theorem (CLT) applies. If we define

$$y = \log \frac{P_1(f)}{P_2(f)} \quad (9)$$

and

$$z = y^2, \quad (10)$$

then

$$D_{IV} = \frac{1}{r} \sum_{i=1}^r y_i \quad (11)$$

and

$$D_V = \frac{1}{r} \sum_{i=1}^r z_i. \quad (12)$$

By the CLT, D_{IV} and D_V are distributed approximately as gaussian variables (becoming more exact as $r \rightarrow \infty$) with

$$\mu(D_{IV}) = \mu_y, \quad (13)$$

$$\sigma(D_{IV}) = \frac{\sigma_y}{\sqrt{r}}, \quad (14)$$

$$\mu(D_V) = \mu_z, \quad (15)$$

and

$$\sigma(D_V) = \frac{\sigma_z}{\sqrt{r}}. \quad (16)$$

These theoretical means and variances can be expressed in terms of the polygamma functions for which functional approximations exist (18) (refer to Appendix A). The limiting criteria are then chosen, based on a gaussian distribution with the predicted means and variances. Discriminants D_{VI} through D_{VIII} , which are based on the signs, number of runs, and the longest run in the set of ratios, are nonparametric tests. The sign test (15,16) for the paired sample case hypothesizes the equality of the means for populations that are symmetrical and continuous. If the populations are not symmetric, the hypothesis applies to the medians. When two PSDs that have been averaged over a different number of blocks are compared, the paired samples are necessarily drawn from chi-squared distributions with different degrees of freedom, and this introduces skewness. The validity of the test can be

restored if the ratios are given "signs" based on their relationship to the median of the assumed F-distribution. Under this approach, each sign has an equal likelihood of occurrence. Therefore, the limiting criterion for this discriminant is based on the binomial distribution with a trial probability of 1/2. For large sample sets containing r ratios, the discriminant can be tested against the normal curve approximation to the binomial distribution:

$$Z_{VI} = \frac{D_{VI} - r\mu_{VI}}{\sigma_{VI}}, \quad (17)$$

where $\mu_{VI} = r/2$ and $\sigma_{VI} = \sqrt{r/4}$.

The two test runs have been proposed as techniques for testing the assumption of randomness in a set of sampled data. If the test PSD is statistically identical to the baseline PSD, the log ratios are a set of sampled values from the same population and should be randomly distributed about the median. On the other hand, if a small shift has occurred over some portion of the spectra, a long run may develop, or the number of runs may be altered significantly. When the sample size, r , is greater than 20, a standardized gaussian variable, Z_{VII} , can be constructed from D_{VII} and the expected number of positive, p , and negative, n , runs (15,16):

$$Z_{VII} = \frac{D_{VII} - \mu_{VII}}{\sigma_{VII}}, \quad (18)$$

where

$$\mu_{VII} = \frac{2pn}{n+p} + 1, \quad (19)$$

and

$$\sigma_{VII} = \left[\frac{2pn(2pn - p - n)}{(p+n)^2(p+n-1)} \right]^{1/2}. \quad (20)$$

If the predicted median is the true median of the population, p and n are equal to $r/2$.

The limiting criterion for the longest run is based on a formula that predicts $M(R_k)$, the mean number of runs of length k or greater than would be expected in r samples:

$$M(R_k) = \frac{r - k + 2}{2^k}. \quad (21)$$

If the expected number of runs of length k or greater is small, the probability of such an event occurring is unlikely. The value of k can be found in an iterative manner, choosing a value (~ 0.001) for $M(R_k)$ and using an initial guess for k given by

$$k = \frac{\ln \frac{M(R_k)}{r}}{\ln(0.5)}. \quad (22)$$

Correction with Learned Parameters

During the learning period, the means and variances are calculated from the sample data. These sample statistics are used to correct the criteria values, with an assumption that the shapes of the theoretical distributions are approximately correct and only the magnitudes of the first and second moments are in error. For the integral power discriminant, D_I , the interval of acceptability is altered by normalizing the theoretical criteria with the predicted mean and standard deviation and by normalizing the calculated discriminant values with the measured sample mean and standard deviation:

$$\frac{C_I^l - \mu_I}{\sigma_I} \leq \frac{D_I - M_I}{S_I} \leq \frac{C_I^u - \mu_I}{\sigma_I} . \quad (23)$$

The notation that will be adhered to in this paper designates theoretical means and standard deviations by μ and σ , and measured sample means and standard deviations by M and S . The criteria are denoted by C , and the superscripts u and l denote the upper and lower limits, respectively.

As above, the corrections for the minimum and maximum individual ratios (D_{II} and D_{III}) are normalizations of theoretical criteria and the calculated discriminant:

$$\frac{D_{II} - M_{II}}{S_{II}} > \frac{C_{II} - \mu_{II}}{\sigma_{II}} \quad (24)$$

and

$$\frac{D_{III} - M_{III}}{S_{III}} < \frac{C_{III} - \mu_{III}}{\sigma_{III}} . \quad (25)$$

Thus,

$$D_{II} > (C_{II} - \mu_{II}) \left(\frac{S_{LR}}{\sigma_{II}} \right) + M_{LR} \quad (26)$$

and

$$D_{III} < (C_{III} - \mu_{III}) \left(\frac{S_{LR}}{\sigma_{III}} \right) + M_{LR} , \quad (27)$$

where

$$M_{LR} = M_{II} = M_{III} = \text{sample mean of individual log ratios,}$$

$$S_{LR} = S_{II} = S_{III} = \text{sample standard deviation of the individual log ratios.}$$

The composite ratio discriminants are gaussian variables according to CLT, and the calculated parameters are substituted for the predicted values:

$$\frac{D_{IV} - M_{LR}}{S_{LR} \sqrt{r}} \leq C_{IV} , \quad (28)$$

and

$$\frac{D_V - M_{SLR}}{S_{SLR} \sqrt{r}} < C_V, \quad (29)$$

where M_{SLR} is the sample mean of the square of the individual log ratios, and S_{SLR} is the sample standard deviation of the square of the individual log ratios.

The nonparametric tests are strictly valid only if the true median of the population of log ratios is known. The predicted median for a \log_{10} F-distribution is obtained from Eqs. (5) through (8) by setting $\chi_p = 0.0$. Instead of calculating the sample median to correct the inadequacies of the predicted value, the relative proportion, f_p , of the population greater than the predicted median is tabulated. The correction for D_{VI} is altered by exchanging μ_{VI} in Eq. (17) for $M_{VI} \approx rf_p$. The theoretical standard deviation is retained because it is a conservative estimate. In a similar fashion, the D_{VII} correction procedure alters the predicted mean and variance by setting $p = rf_p$ and $n = r - p$.

The D_{VII} discriminant has a robust character and is relatively insensitive to small errors in the predicted median. The experience to date has indicated that a correction procedure is unnecessary; however, later applications may prove otherwise. This might be accomplished by replacing the factor $(1/2)^k$ in Eq. (21) by $(f_p)^{1/2}$, where f_p is the larger of f_p or $(1 - f_p)$.

RESULTS

Implementation and Program Evaluation

The surveillance algorithm has been implemented on a Digital Equipment Corporation PDP 11/20 minicomputer with 28K words of memory. The complementary hardware includes 2.5M words of disk storage, an analog-to-digital converter, a line printer, and a CRT terminal. The programming is largely in FORTRAN; assembly language routines are used only to accomplish functions not available through FORTRAN or, in the case of the FFT routine, to speed up execution.

Disk storage is required for storing programs and retaining selected data. Two data files are maintained; one stores the control and statistical parameters and the other stores the baseline signatures. These files require about $2N + 600$ words of disk storage for each signal analyzed (N = No. of PSD estimates calculated by the FFT routine). The program version implemented for evaluation monitors only one signal and will perform FFT analysis on data blocks of 2048 or less. After operation of the program is initiated, monitoring will continue in a repetitive fashion until the user interrupts operation from the CRT terminal. Operation of the program can be reinitiated as long as the data files are preserved.

The performance of the algorithm was evaluated using a test signal from a gaussian, white noise generator. The predicted criteria proved adequate in this situation, and negligible correction was required as a result of the statistics calculated during the learning period. The individual discriminants exhibited different detection sensitivities to a given spectral alteration, as expected from their formulations. Also, it was demonstrated that gradual

changes in the spectra could be tracked by the Trend PSD, but they were always detected, as planned, by the Base PSD comparisons.

Monitoring Signals from Neutron Sensors

Neutron data were recorded during an experiment performed at the High Flux Isotope Reactor (HFIR) that has been previously reported (12,13). This experiment included a 12-hr period during which the reactor operated normally, followed by several hours during which small, control-rod oscillations were induced artificially to create perturbations of less than 0.1% in the reactor power. The PSDREC algorithm established the baseline signatures and completed the learning phase within 1 hr. (In previous experiments with the more general algorithms (12-13), a 12-hr learning period was required.) During the 12 hr of normal operation, the algorithm verified that conditions were normal. Figure 2 shows the baseline spectrum and a spectrum measured during normal conditions. Rod oscillations were detected, and spectral changes were noted (Figs. 3-6). Discriminant values calculated during these anomalous conditions are listed in Table 1. A comparison of the table values with the spectra illustrates the ability of the discriminants to quantitate spectral variations.

For another test, data were recorded at the Edwin I. Hatch 1 nuclear power plant of the Georgia Power Company (13). Approximately 12 hr of data were recorded for four local power range monitors (LPRMs) in this boiling water reactor at a fixed operating condition. Additional data were also recorded on another occasion at different plant conditions (11). These data were used to test the hypothesis that the corrected criteria for any LPRM could be applied to another LPRM despite individual variations between their spectra. The hypothesis was proved to be true. The four LPRMs had been selected impartially. Their spectral signatures are displayed in Fig. 7a and b. Signals for the initial baseline signatures and the learning period were from LPRM 12-45C. When these signals were replaced with signals from LPRM 04-296, the algorithm immediately sounded an alarm. However, after the algorithm had been instructed to accept a new baseline, subsequent monitoring indicated that normal conditions prevailed. No additional learning was necessary. This same sequence of events was repeated for other LPRM signals recorded at different operating conditions. Monitoring under all tested conditions required forming a new baseline signature only; no additional learning was necessary to adjust the discriminant criteria. These results indicate that separate learning periods are not necessary for each LPRM. Thus, a large number of the LPRMs in a BWR might be monitored with minimal time required for learning. Further, an efficient way to handle different operating conditions might be to alter only the baseline signatures when process variables indicate that the reactor operator has modified the operational mode. If an anomalous condition were to be introduced by, or to occur at the same moment as, the operational change, the subsequent anomalous state would not be detected as long as it remained stable. However, the surveillance system would sense further deterioration if it occurs. For processes such as baseloaded power plants, where a given operational mode is maintained for a relatively long time period, such a procedure would be particularly attractive since a single learning period would be sufficient and the algorithm would not be required to retain statistical parameters and signatures for many different conditions.

Monitoring Signals from Rotating Machinery

Vibrations from rotating equipment are frequently measured with proximeter probes or accelerometers. Both types of sensors were installed on a small (1/20 hp), variable speed

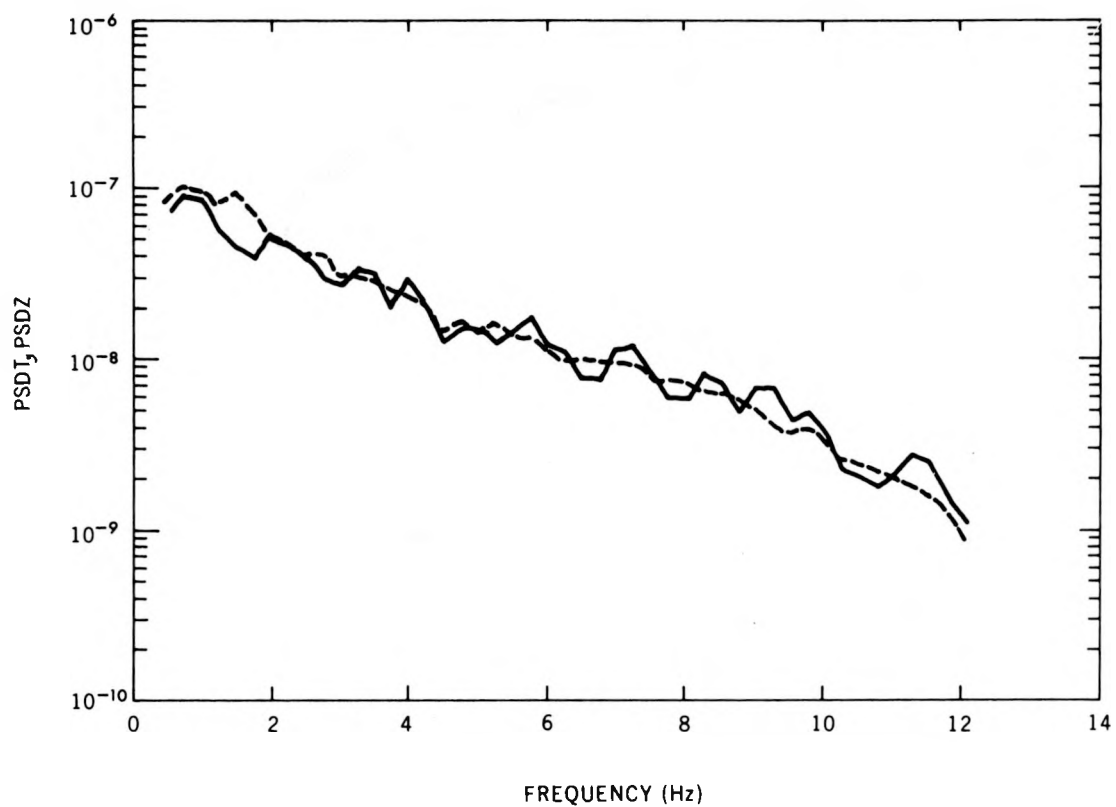


Fig. 2. Baseline signature (Trend PSD) and test PSD measured during normal conditions at the HFIR.

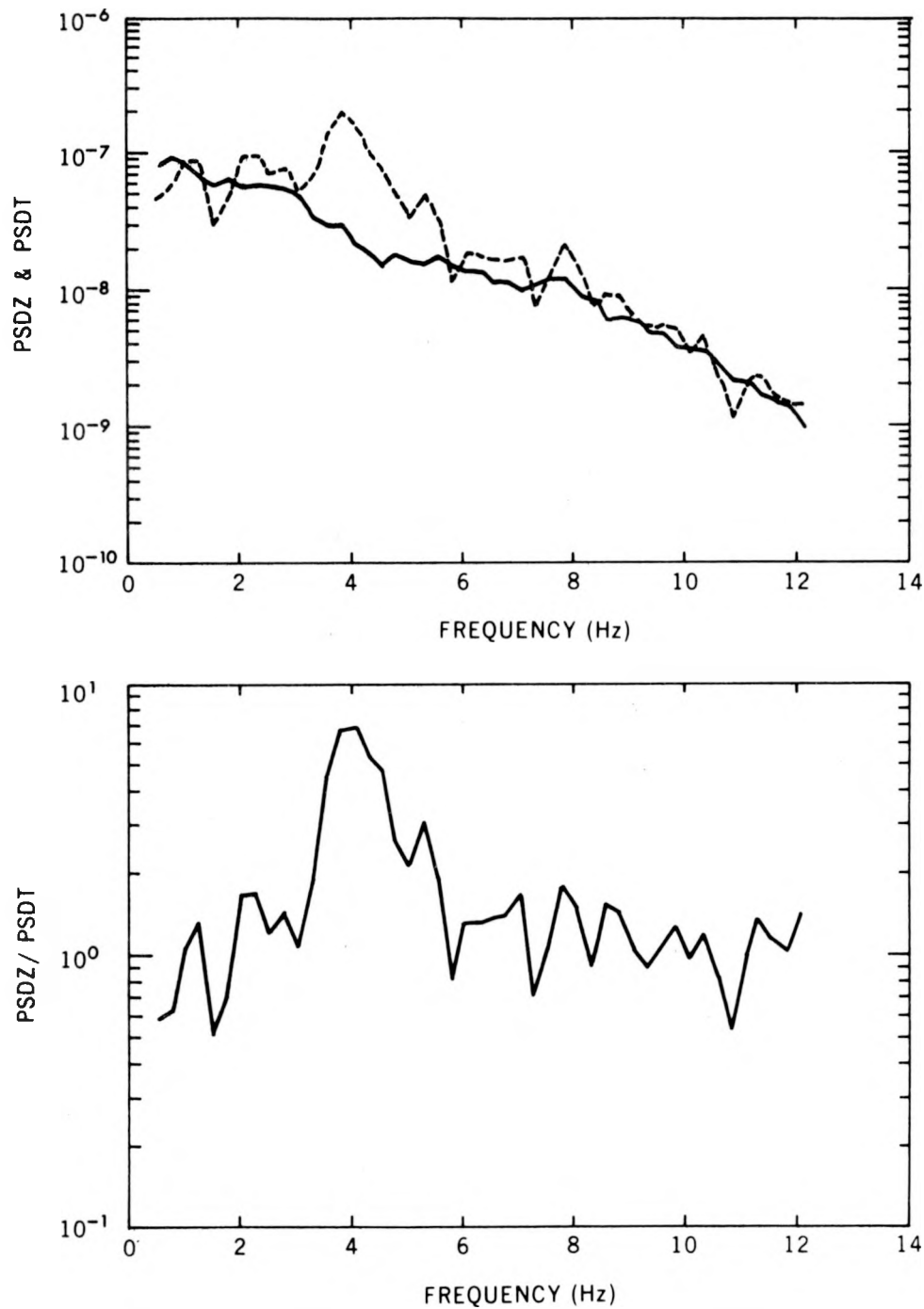


Fig. 3. Spectral density and spectral ratio plots generated as a result of the anomalous conditions created by test signal 1.

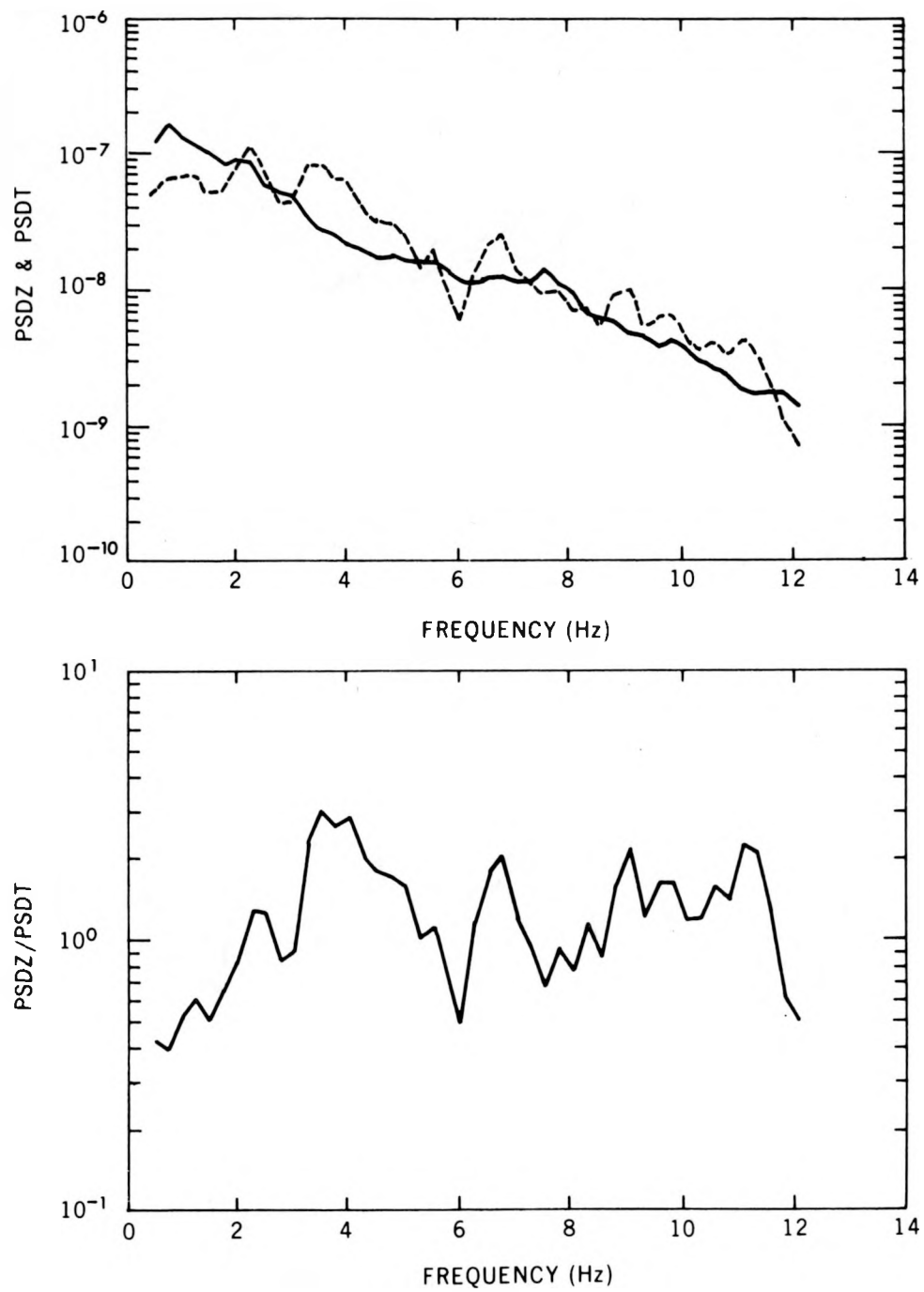


Fig. 4. Spectral density and spectral ratio plots generated as a result of the anomalous conditions created by test signal 2.

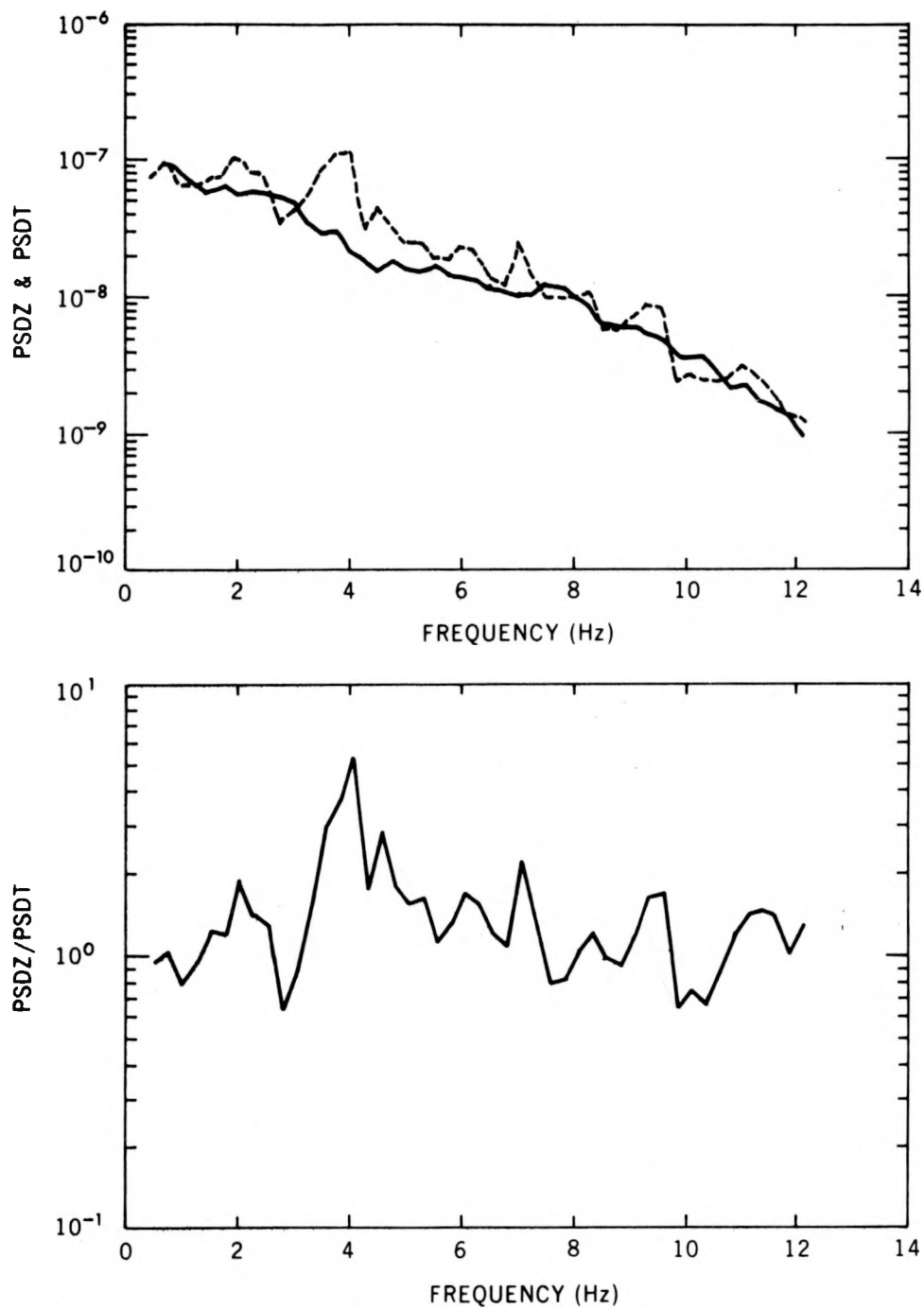


Fig. 5. Spectral density and spectral ratio plots generated as a result of the anomalous conditions created by test signal 3.

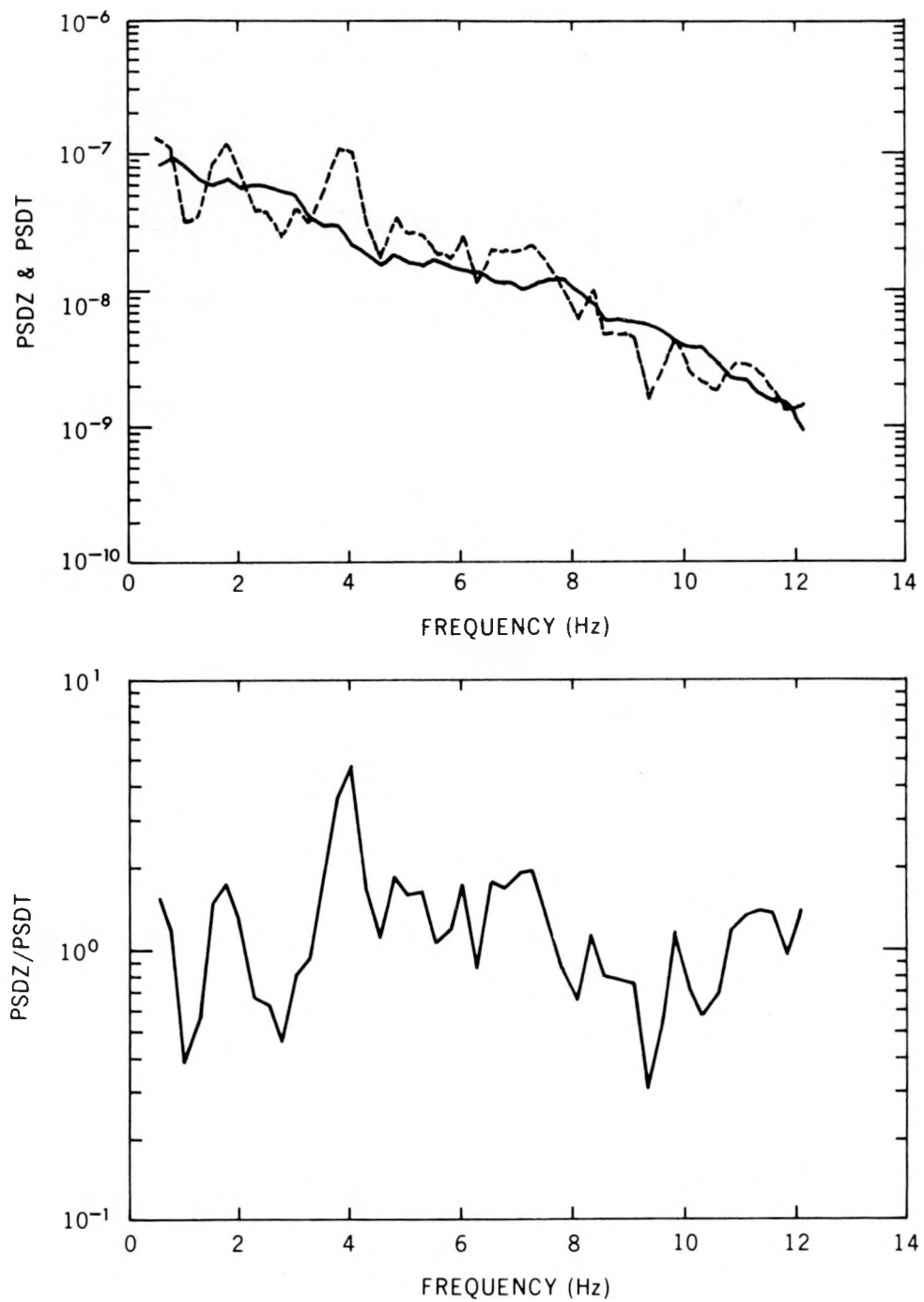


Fig. 6. Spectral density and spectral ratio plots generated as a result of the anomalous conditions created by test signal 4.

TABLE 1 Statistical discriminants resulting from rod oscillation anomalies at the HFIR^a

	D _I	D _{II}	D _{III}	D _{IV}	D _V	D _{VI}	D _{VII}	D _{III}
Normal								
Criteria: (0.603,1.610)	0.0561	5.1794	±4.13	±3.87	(5,21)	(5,21)	14	
TS1	1.619 ^b	0.516	6.96 ^b	5.82 ^b	8.98 ^b	19	6	16 ^b
TS3	1.479	0.637	5.34 ^b	5.21 ^b	4.74 ^b	17	10	8
TS4	1.262	0.305	4.77	2.34	4.09 ^b	15	7	9
TS2	0.923	0.3947	3.021	2.24	5.19 ^b	14	9	6

^aListed in order of decreasing severity.

^bIndicates value out of normal bounds.

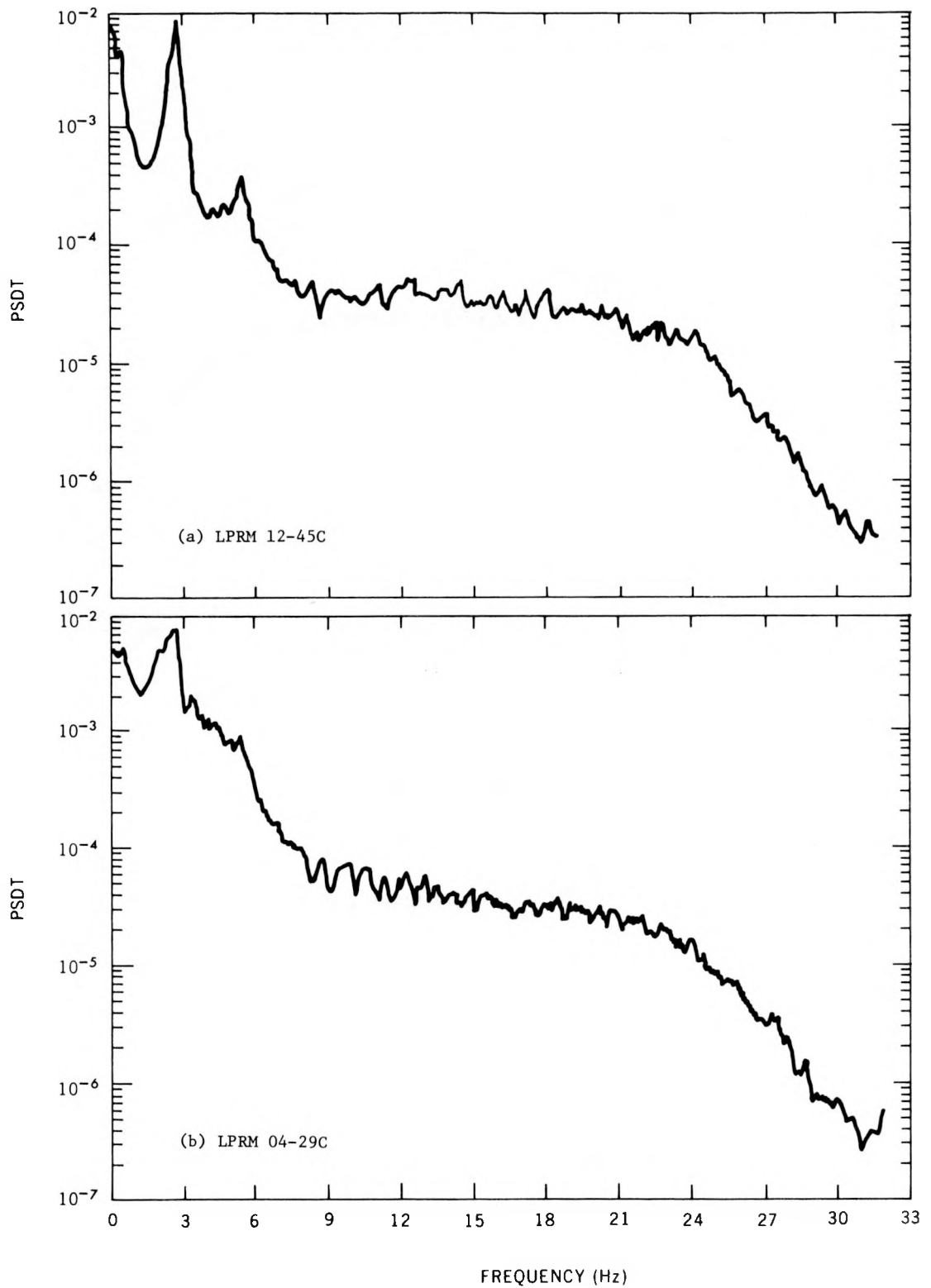


Fig. 7. Baseline signature from LPRMs in the Hatch Unit I Reactor.

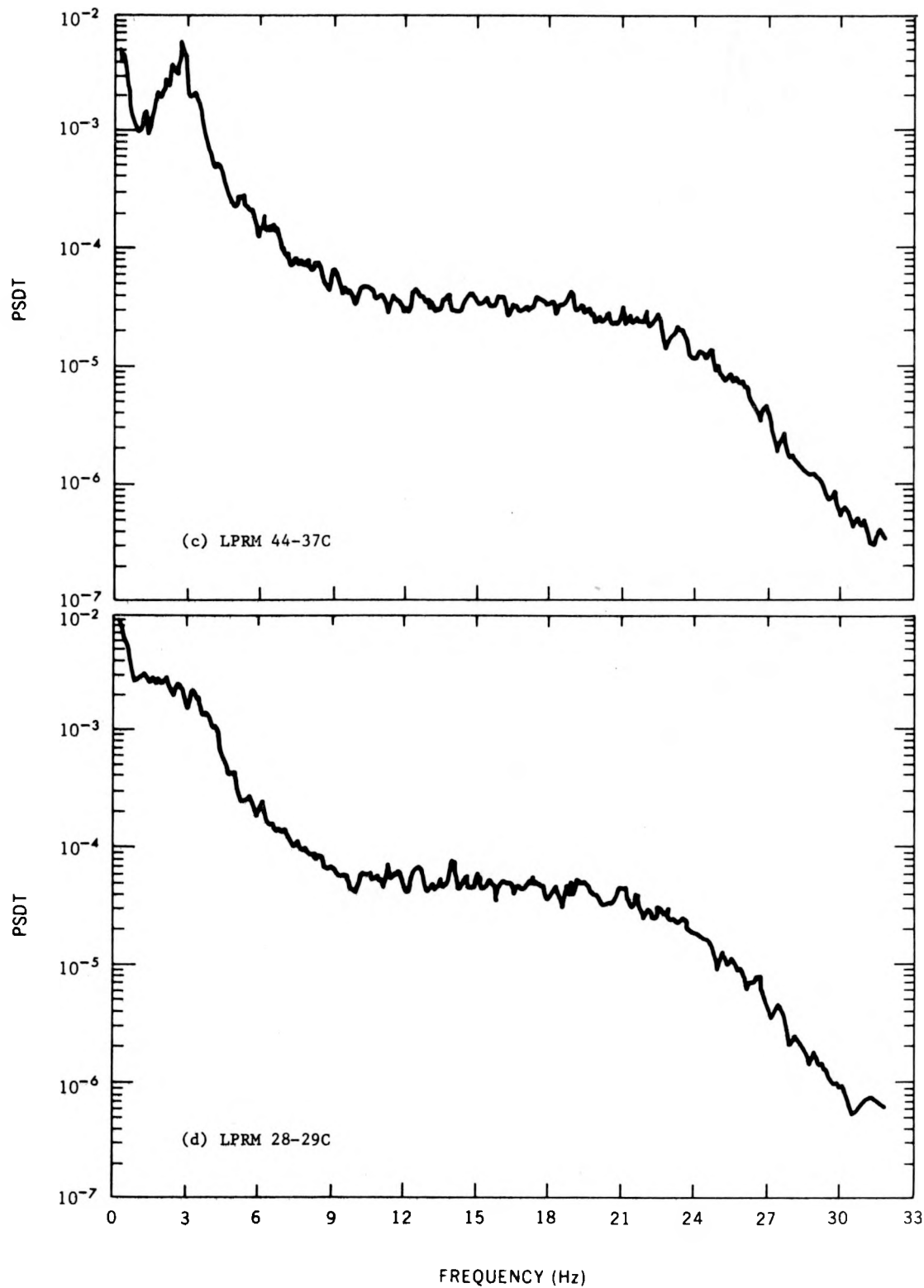


Fig. 7. Baseline signature from LPRMs in the Hatch Unit I Reactor (continued).

electric motor in a laboratory. A slotted wheel was attached to the drive shaft to provide, with the aid of a light source and photocell, 24 pulses per revolution. These pulses established the rate at which the computer sampled the vibrational signals. This synchronization of the sampling with motor shaft rotation allowed the FFT analysis to be performed as a function of harmonic orders instead of frequency.

Vibrational signals from rotating equipment have a different character from neutron noise. Neutron noise spectra generally resemble filtered white noise, rolling off at varying slopes over the frequency interval. If resonant structures are present, they appear as broad peaks in the spectra. In contrast to this, vibrational signals appear to be a composite of sinusoidal signals, many having a nearly constant amplitude. The spectra thus comprise a series of sharp spikes. Two phenomena complicate surveillance of these relatively periodic signals. First, some of the sinusoids are rotationally related, and others are structurally related. Thus, some peaks are fixed in frequency, and others are fixed in order. When varying speed operation is analyzed, the locations of some peaks shift, regardless of whether the analysis is based on order or frequency. However, such shifting of can be categorized only as normal. The effect of this shifting is important because perfectly constant speed operation is not achievable. Small variations ($\sim 1\%$) will result in a detectable shift, even though the variations of the rotational frequency are less than the frequency resolution of the analysis employed.

The second phenomenon is the appearance of small step-changes in peak amplitudes. The amplitude of an estimate has a small variance for a period and then changes abruptly to a new level (again with a small variance), even though the operating conditions of the motor remain unchanged. The amplitude may later return to its original value, or change to some other value. This type of variation has been observed on a time scale from hours to days. Although these variations are small, their nonrandom manner of appearance causes problems for statistical methods. More work is necessary to find the best way to deal with these effects.

Two anomalous conditions, unbalance and shaft rubs, were purposely introduced into this setup. A 4-g weight 4 cm from the center of the shaft was detected. A plot of the ratio of the PSD measured with the motor out of balance (using a proximeter probe) to the Trend PSD is shown in Fig. 8. A flexible wire sprung against the shaft introduced a much smaller deviation from normal operation. A ratio plot for this anomaly is shown in Fig. 9. The differences between discriminant values for these anomalies and all normal variations (including two phenomena discussed above) are apparent and could be established by human evaluation of the results. However, the criteria values selected by the program, even after learning, were not adequate (there were too many false alarms). It may be that preprocessing the signal or additional detection logic based on empirical results would provide sufficient discrimination and therefore acceptable false alarm rates, but this has not been attempted to date.

Planned Surveillance Demonstration in Commercial Nuclear Power Plant

A 6-month demonstration of these surveillance techniques is planned for the winter of 1977. An expanded version of the PSDREC algorithm will be used to monitor four to eight neutron sensors in an operating power reactor (the site is uncertain). The demonstration will be completely passive and will require no reactor operational changes of any type. The purpose of the demonstration will be to verify that the algorithm can automatically accommodate

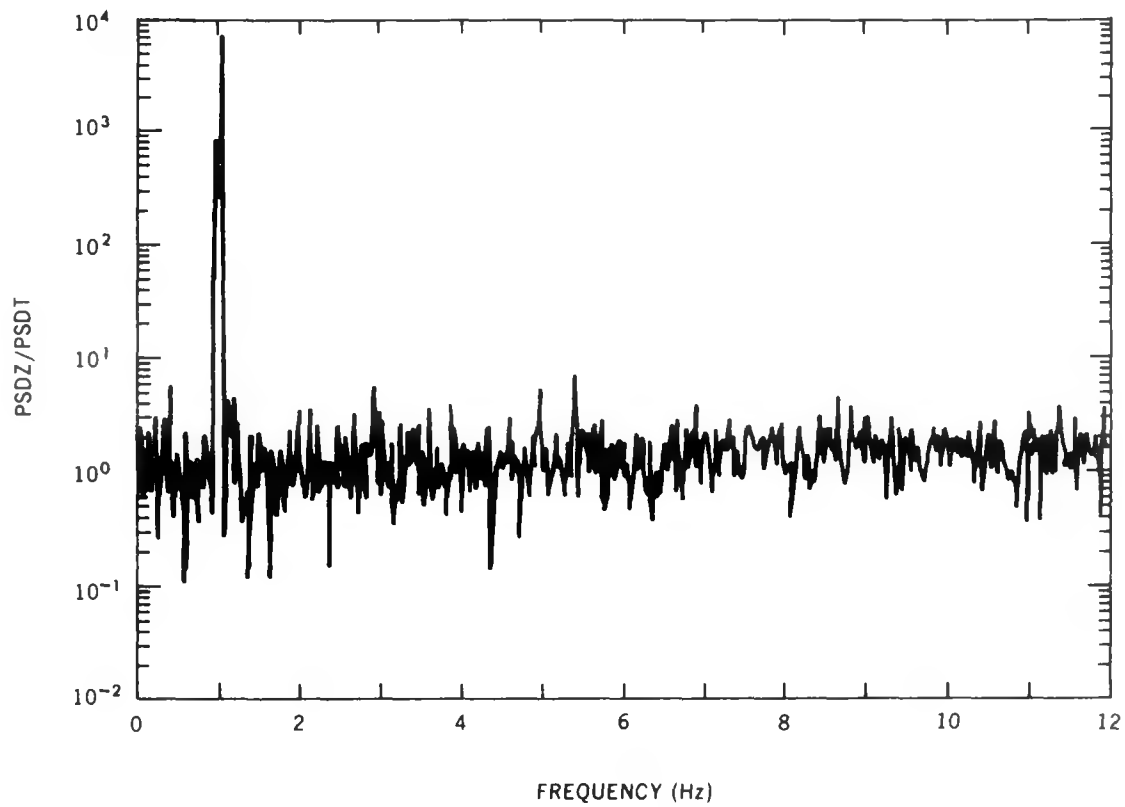


Fig. 8. Spectral ratio plot generated as a result of an unbalance anomaly introduced into an electric motor.

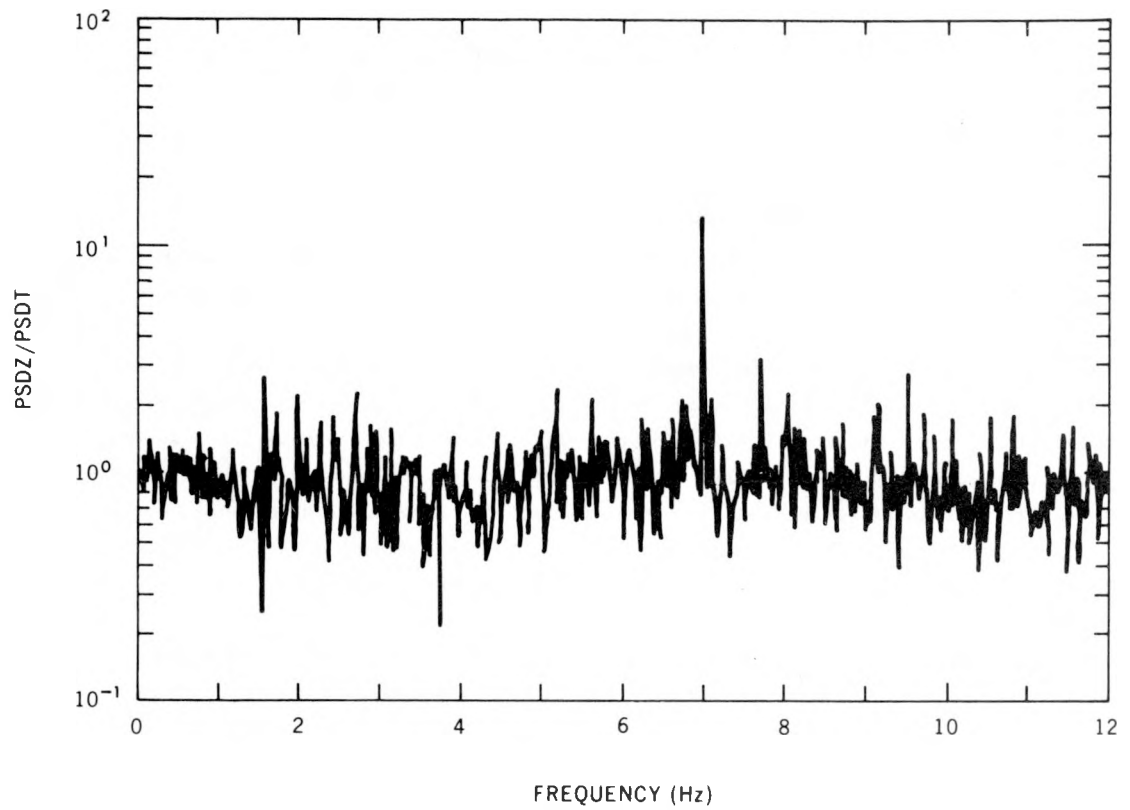


Fig. 9. Spectral ratio plot generated as a result of a rub anomaly introduced into an electric motor.

normal operational changes or to identify conditions for which additional algorithm adjustment or development is necessary. Although no anomalous conditions will be introduced or even simulated in the planned demonstration, we anticipate that we will be able to use operational tests to some extent to verify the ability of the algorithm to detect unusual conditions. This was the case in a test at the Hatch 1 nuclear plant where a turbine bypass valve test and a rod cycling test were detected (13).

SUMMARY AND RECOMMENDATIONS

The PSDREC algorithm was demonstrated to be sensitive for spectral effects which could indicate incipient malfunction. The algorithm is limited to surveillance applications for which the PSD is an adequate descriptor. The ability of the algorithm to automatically establish criteria for the statistical discriminants was adequate for neutron surveillance applications where the signals have a random, broad-band character. However, in rotating machinery applications, the criteria determined by the algorithm produce too many false alarms. Nevertheless, at least for some anomalies (such as unbalance), the changes in the discriminants are clearly distinguishable from the normal variations. The addition of some heuristic detection logic, such as requiring certain patterns of discriminant deviations, may alleviate the problem with false alarms. More work in this area is in progress.

The logic structure and the statistical tests implemented in PSDREC quantify spectral changes and serve as a statistical model against which the variations of a given monitored system can be compared. This comparison determines if the behavior of the monitored system is consistent with the random, stationary model that was assumed; if not, the analyst may be required to incorporate additional logic to handle unanticipated variations. The performance of the algorithm can serve as a guide for the decisions made in this tailoring process.

The effectiveness of PSDREC is untested for situations where many normal operational modes are possible. When the operational states are maintained for relatively long intervals, the algorithm can exchange baseline signatures if there is access to the process control parameters. The demonstration planned for a commercial nuclear power plant is expected to validate this approach.

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APPENDIX A

The expressions necessary to calculate the mean and the variance of the gaussian distribution which describes the D_{IV} discriminant are given first:

$$D_{IV} = \frac{1}{r} \sum_i y_i, \quad (A-1)$$

where

$$y_i = \log_{10} \left[\frac{P_1(f_i)}{P_2(f_i)} \right] = \log_{10} \left[\frac{x_{2n_1}^2 / 2n_1}{x_{2n_2}^2 / 2n_2} \right]. \quad (A-2)$$

Then

$$M(D_{IV}) = Y_y, \quad (A-3)$$

and

$$\sigma(D_{IV}) = \sigma_y / \sqrt{r}, \quad (A-4)$$

since

$$y = \log \left(x_{2n_1}^2 / 2n_1 \right) - \log \left(x_{2n_2}^2 / 2n_2 \right). \quad (A-5)$$

Let

$$y_1 = \ln \left(\chi_{2n_1}^2 / 2n_1 \right) \quad (\text{A-6})$$

and

$$y_2 = \ln \left(\chi_{2n_2}^2 / 2n_2 \right) . \quad (\text{A-7})$$

Then

$$\mu_y = \frac{1}{\ln(10)} \left[\mu_{y1} + \mu_{y2} \right] \quad (\text{A-8})$$

and

$$\sigma_y = \frac{1}{\ln(10)} \left[\sigma_{y1}^2 + \sigma_{y2}^2 \right]^{1/2} . \quad (\text{A-9})$$

According to Ref. 16, the first and second moments of Eqs. (A-6) and (A-7) are given by the digamma and trigamma functions,

$$\mu_{y1} = \psi(n_1) - \ln(n_1) \quad \text{and} \quad \sigma_{y1} = \psi^1(n_1) ; \quad (\text{A-10})$$

$$\mu_{y2} = \psi(n_2) - \ln(n_2) \quad \text{and} \quad \sigma_{y2} = \psi^1(n_2) . \quad (\text{A-11})$$

The asymptotic expansions for ψ and ψ^1 are

$$\psi(n) - \ln(n) = -\frac{1}{2n} - \frac{1}{12n^2} + \frac{1}{120n^4} - \frac{1}{252n^6} + \dots , \quad (\text{A-12})$$

$$\psi^1(n) = \frac{1}{n} + \frac{1}{2n^2} + \frac{1}{6n^3} - \frac{1}{30n^5} + \frac{1}{42n^7} - \frac{1}{30n^9} + \dots . \quad (\text{A-13})$$

The expressions necessary to calculate the mean and variance of D_V is an extension of those for D_{IV} :

$$D_V = \frac{1}{r} \sum_{i=1}^r z_i , \quad (\text{A-14})$$

where

$$z = \left\{ \log \left[\frac{P_1(f)}{P_2(f)} \right] \right\}^2 = Y^2 . \quad (\text{A-15})$$

Thus, CLT yields

$$\mu(D_V) = \mu_z \quad (\text{A-16})$$

and

$$\sigma(D_V) = \frac{\sigma_z}{\sqrt{r}} . \quad (\text{A-17})$$

Since

$$\mu_z = E(z) = E(y^2), \quad (\text{A-18})$$

where $E(\cdot)$ is the expectation operator,

$$\mu_z = E \left[(y - \mu_y)^2 + 2Y \mu_y - \mu_y^2 \right], \quad (\text{A-19})$$

$$\mu_z = E \left[(y - \mu_y)^2 \right] + 2\mu_y E(y) - \mu_y^2, \quad (\text{A-20})$$

$$\mu_z = \sigma_y^2 + \mu_y^2, \quad (\text{A-21})$$

$$\sigma_z^2 = E \left[(z - \mu_z)^2 \right] = E \left\{ \left[y^2 - (\sigma_y^2 + \mu_y^2) \right]^2 \right\}, \quad (\text{A-22})$$

$$= E \left\{ (y - \mu_y)^2 + \left[2\mu_y(y - \mu_y) - \sigma_y^2 \right]^2 \right\}, \quad (\text{A-23})$$

$$= E \left\{ (y - \mu_y)^4 + 4\mu_y(y - \mu_y)^3 - 2\sigma_y^2(y - \mu_y)^2 + \left[2\mu_y(y - \mu_y) - \sigma_y^2 \right]^2 \right\}, \quad (\text{A-24})$$

$$= \overline{Y_c^4} + 4\mu_y \overline{Y_c^3} - 2\sigma_y^4 + E \left[4\mu_y^2 (y - \mu_y)^2 - 4\mu_y \sigma_y^2 (y - \mu_y) + \sigma_y^4 \right], \quad (\text{A-25})$$

$$\sigma_z^2 = \overline{Y_c^4} + 4\mu_y \overline{Y_c^3} - \sigma_y^4 + 4\mu_y^2 \sigma_y^2. \quad (\text{A-26})$$

The mean and variance of Y are available from the expressions for D_{IV} ; however, expressions for the third and fourth central moments of Y must be derived. By definition,

$$\overline{Y_c^4} = E(y - \mu_y)^4 = \left[\frac{1}{\ln(10)} \right]^4 E \left[(y_1 - \mu_{y1}) - (y_2 - \mu_{y2}) \right]^4. \quad (\text{A-27})$$

Let

$$\delta_{y1} = y_1 - \mu_{y1} \quad (\text{A-28})$$

and

$$\delta_{y2} = y_2 - \mu_{y2}. \quad (\text{A-29})$$

Then

$$\overline{Y_c^4} = E \left[(\delta_{y1} - \delta_{y2})^4 \right] \left[\frac{1}{\ln(10)} \right]^4. \quad (\text{A-30})$$

Assuming y_1 and y_2 are independent,

$$= E \left(\delta_{y1}^4 - 4\delta_{y1}^3 \delta_{y2} + 6\delta_{y1}^2 \delta_{y2}^2 - 4\delta_{y1} \delta_{y2}^3 + \delta_{y2}^4 \right) \left[\frac{1}{\ln(10)} \right]^4 . \quad (A-31)$$

$$\overline{Y_c^4} = \left(\overline{Y_{1c}^4} + 6\sigma_{y1}^2 \delta_{y2}^2 + \overline{Y_{2c}^4} \right) \left[\frac{1}{\ln(10)} \right]^4 . \quad (A-32)$$

Similarly,

$$\overline{Y_c^3} = E \left[(\delta_{y1} - \delta_{y2})^3 \right] \left[\frac{1}{\ln(10)} \right]^3 , \quad (A-33)$$

$$\overline{Y_c^3} = E \left(\delta_{y1}^3 - 3\delta_{y1}^2 \delta_{y2} + 3\delta_{y1} \delta_{y2}^2 - \delta_{y2}^3 \right) \left[\frac{1}{\ln(10)} \right]^3 , \quad (A-34)$$

$$\overline{Y_c^3} = \left(\overline{Y_{1c}^3} - \overline{Y_{2c}^3} \right) \left[\frac{1}{\ln(10)} \right]^3 . \quad (A-35)$$

Reference (16) defines $\overline{Y_{1c}^3}$, $\overline{Y_{2c}^3}$, $\overline{Y_{1c}^4}$, and $\overline{Y_{2c}^4}$ in terms of the polygamma functions:

$$\overline{Y_{1c}^3} = \psi''(n_1) , \quad \overline{Y_{2c}^3} = \psi''(n_2) , \quad (A-36)$$

$$\overline{Y_{1c}^4} = \psi'''(n_1) + 3\sigma_{y1}^4 , \quad \overline{Y_{2c}^4} = \psi'''(n_2) + 3\sigma_{y2}^4 , \quad (A-37)$$

and gives asymptotic expansions as follows:

$$\psi''(n) = -\frac{1}{n^2} - \frac{1}{n^3} - \frac{1}{2n^4} + \frac{1}{6n^6} - \frac{1}{6n^8} + \frac{3}{10n^{10}} - \dots , \quad (A-38)$$

$$\psi'''(n) = \frac{2}{n^3} + \frac{3}{n^4} + \frac{2}{n^5} - \frac{1}{n^7} + \frac{4}{3n^9} - \frac{3}{n^{11}} + \dots . \quad (A-39)$$

To summarize the preceding discussion, the calculational procedure is:

1. Calculate numerical values for μ_{y1} , μ_{y2} , σ_{y2} , $\overline{Y_{1c}^3}$, $\overline{Y_{2c}^3}$, $\overline{Y_{1c}^4}$, and $\overline{Y_{2c}^4}$, using the asymptotic expansions.
2. Calculate by combining the above parameters: μ_y , σ_y , $\overline{Y_c^3}$, and $\overline{Y_c^4}$, using Eqs. (A-8), (A-9), (A-35), and (A-32).
3. Calculate μ_y , using Eq. (A-21); and σ_y using Eq. (A-26).
4. Calculate $\mu(D_{IV})$, using Eq. (A-3); $\sigma(D_{IV})$ using Eq. (A-4); $\mu(D_V)$, using Eq. (A-16); and $\sigma(D_V)$, using Eq. (A-17).

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