BINDING ENERGIES OF HYPERNUCLEI AND Λ-NUCLEAR INTERACTIONS

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Variational Monte Carlo calculations have been made for the s-shell hypernuclei and also of $^{9}_{\Lambda}$Be with a 2α + Λ model. The well depth is calculated variationally with the Fermi hypernetted chain method. A satisfactory description of all the relevant experimental Λ separation energies and also of the Λp scattering can be obtained with reasonable TPE ΛN and ΛNN forces and strongly repulsive dispersive ΛNN forces which are preferred to be spin dependent. We discuss variational calculations for $^{6}_{\Lambda\Lambda}$He and $^{10}_{\Lambda\Lambda}$Be with α + 2Λ and 2α + 2Λ models, and the results obtained for the ΛΛ interaction and for $^{6}_{\Lambda\Lambda}$He from analysis of $^{10}_{\Lambda\Lambda}$Be. Coulomb effects and charge symmetry breaking in the A=4 hypernuclei are discussed.

1. INTRODUCTION

We report here mostly on recent work on the long-standing problem of the binding energies of Λ hypernuclei and their interpretation in terms of two- and three-body ΛN and ΛNN potentials, and more briefly on studies of ΛΛ hypernuclei and of charge symmetry breaking (CSB) in the A=4 hypernuclei. (Much of the earlier work on these topics is reviewed in ref. 1.) Such efforts are parallel to work for nonstrange nuclei, and in fact lean strongly on the techniques and physics learned from these. Of course much less is known about the hyperon-nucleon interactions than about the NN interaction, with a consequent difference in emphasis. Our approach, which is a hadrodynamic one in Walecka's terminology, attempts to obtain a consistent phenomenological description of hypernuclear binding energies and low-energy Λp scattering in terms of reasonable ΛN and ΛNN forces, where reasonable means consistent with meson-exchange models. Effects of baryon quark structure are assumed to be of short-range and capable of parameterization in the conventional way through repulsive cores and cutoffs. Of course, our potentials are to be considered as effective

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interactions. In particular, our ΛNN forces are considered to be the result of eliminating Σ, Δ, ... degrees of freedom (from a coupled channel approach which includes these and which represents a more sophisticated level of hadrodynamic phenomenology) to obtain a reduced description in terms of only Λ and N degrees of freedom.

We have studied the following problems:

I. Binding energies of Λ hypernuclei with ΛN and ΛNN forces.

1. Λp scattering and the s-shell hypernuclei: $A=3,4,4^*,5$ where 4^* denotes the excited state of the A=4 hypernuclei.

2. Λ binding in nuclear matter ($A=\infty$), i.e. the Λ well depth.

3. Selected intermediate mass hypernuclei with zero-spin core nuclei: ${}^{9}_{\Lambda}Be$, ${}^{13}_{\Lambda}C$.

II. The ΛΛ hypernuclei ${}^{6}_{\Lambda\Lambda}He$ and ${}^{10}_{\Lambda\Lambda}Be$ and the ΛΛ interaction.

III. Coulomb effects and charge symmetry breaking for the mirror pair ${}^{4}_{\Lambda}H$, ${}^{4}_{\Lambda}He$.

We shall concentrate mostly on I, especially on I.1 and I.2, for which preliminary reports have been published[2,3] but where much of the work we discuss is new. We only briefly summarize the work on II and III since complete accounts of these have been published[4,5].

2. BINDING ENERGIES OF THE S-SHELL HYPERNUCLEI

2.1 ΛN potential and Λp scattering

We use a (charge symmetric) central ΛN potential with a theoretically reasonable attractive tail due to two-pion exchange (TPE) in accord with the Urbana-type potentials[6]:

$$V_{\Lambda N} = V_{2\pi} = V_C - (\bar{V} - \frac{1}{4} V_\sigma \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N)\, T_\pi^2 \, .$$

T_π is the OPE tensor shape with cutoff ($c = 2\ fm^{-2}$) and T_π^2 corresponds to a TPE mechanism (fig. 1). V_C is a Woods-Saxon repulsive core[2] taken from the NN potential[6]. Such a V_C is needed with an attractive TPE tail in order to fit the Λp scattering (giving an intrinsic range $b \simeq 2$ fm). It is convenient to use the spin-average and spin-dependent strengths $\bar{V}$, V_σ to parameterize $V_{\Lambda N}$:

$$\bar{V} = \frac{1}{4} V_s + \frac{3}{4} V_t \, , \quad V_\sigma = V_s - V_t$$

(s, t denotes singlet and triplet). For hypernuclei with zero-spin core nuclei, e.g. ${}^{5}_{\Lambda}He$, effectively only the spin-average $\bar{V}$ enters. The spin dependence V_σ is assumed positive consistent with hypernuclear spins.

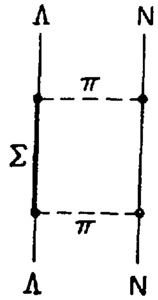

Figure 1
Representative diagram for TPE ΛN potential.

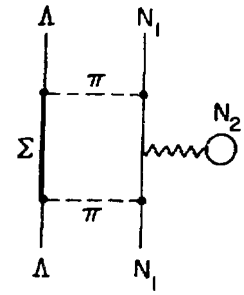

Figure 2
Representative diagram for dispersive ΛNN potential.

Low energy Λp scattering determines the spin-average s-wave elastic cross section $\bar{\sigma}^{\Lambda p} = (\sigma_s + 3\sigma_t)/4$. (This effectively determines a spin-average scattering length and effective range: $\bar{a}^{\Lambda p} \approx -1.9$ fm, $\bar{r}_0^{\Lambda p} \approx 3.4$ fm). Then $\bar{\sigma}^{\Lambda p}$ determines $\bar{V}^{\Lambda p} = 6.2 \pm 0.05$ MeV reasonably well, whereas the spin dependence is effectively undetermined ($0 \lesssim V_\sigma \lesssim 0.5$ MeV). With charge symmetry breaking determined from the A=4 hypernuclei (section 6 and ref. 5) one obtains for the (charge symmetric) strength $\bar{V}_{scatt} = 6.15 \pm .05$ MeV.

2.2 ΛNN potentials

These arise from projecting out Σ, Δ, ... degrees of freedom from a coupled channel formalism. This gives two types of ΛNN forces (see e.g. ref. 1).

1. Dispersive ΛNN forces $V^D_{\Lambda NN}$. These are associated with suppression of the TPE ΛN potential arising from modifications ("dispersion") of the intermediate Σ, N, ... by the medium (a "2nd" nucleon N_2) as in fig. 2. Consistent with suppression, $V^D_{\Lambda NN}$ is expected to be repulsive. We consider two phenomenological forms:

Spin independent[2,7]: $V^D_{\Lambda NN} = WT^2_\pi\ (r_{1\Lambda})T^2_\pi\ (r_{2\Lambda})$

Spin dependent[2]: $V^{DS}_{\Lambda NN} = V^D_{\Lambda NN}\ [1 + \frac{1}{6}\ \vec{\sigma}_\Lambda \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)]$

$V^D_{\Lambda NN}$ and $V^{DS}_{\Lambda NN}$ are equivalent for spin zero core nuclei (e.g. ${}^5_\Lambda$He, ${}^9_\Lambda$Be, D). $V^{DS}_{\Lambda NN}$ is obtained by assuming the dispersive (suppressive) modifications act only for triplet ΛN_1 states (fig. 2), and then symmetrizing between N_1 and N_2. This spin dependence is a simple phenomenological representation of effects discussed previously, in particular by Gibson and Lehman, which arise from suppression of $V_{\Lambda N}$ predominantly in the triplet state as a result of assuming that $V_{\Lambda N}$ is dominated by the OPE ΛN-ΣN transition potential with its characteristic strong tensor component[8,9].

2. TPE ΛNN forces $V^{2\pi}_{\Lambda NN}$ (fig. 3)[1,10,11]

We use the form appropriate for p-wave pion interactions and to assuming only relative s states in our s-shell wave functions.

$$V^{2\pi}_{\Lambda NN} = C_p \, [1 + (3\cos^2\theta - 1) \, T_\pi(r_{1\Lambda}) \, T_\pi(r_{2\Lambda})] \, Y_\pi(r_{1\Lambda}) \, Y(r_{2\Lambda}) \; ,$$

where Y(r) is the OPE Yukawa function (with cutoff) and $\cos\theta = \hat{r}_{1\Lambda} \cdot \hat{r}_{2\Lambda}$. Theoretical estimates give $C_p \approx$ 1-2 MeV. Note that $V^{2\pi}_{\Lambda NN}$ has no spin dependence.

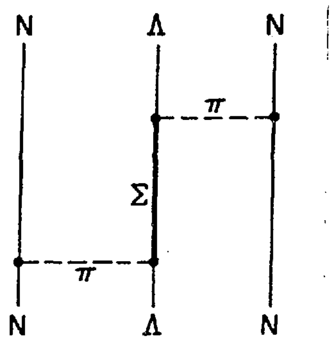

Figure 3
Diagram for TPE ΛNN potential.

Our Hamiltonian is then

$$H^{(A)} = H_N^{(A-1)} + T_\Lambda + \sum_{i=1}^{A-1} V_{\Lambda N}(i\Lambda)$$
$$+ \sum_{i<j}^{A-1} [V^{D(\text{or } DS)}_{\Lambda NN}(ij\Lambda) + V^{2\pi}_{\Lambda NN}(ij\Lambda)]$$

where

$$H_N^{(A-1)} = \sum_{i=1}^{A-1} T_N(i) + \sum_{i<j}^{A-1} V_{NN}(ij)$$

is the Hamiltonian of the A-1 core nucleus. For V_{NN} we use (central) Mafliet-Tjon potentials[12], suitably adjusted where necessary. These give a good description both for the energies and radii of the core nuclei (^{2}H, ^{3}H, ^{4}He). The four strengths $\bar{V}$, V_σ, C_p, W are considered as adjustable parameters; also two values were considered (c = 2 and 3 fm^{-2}) of the cutoff for $V^{2\pi}_{\Lambda NN}$.

2.3 Variational calculations

The Λ separation energies are given by

$$- B_\Lambda = \frac{(\psi^{(A)} | H^{(A)} | \psi^{(A)})}{(\psi^{(A)}, \psi^{(A)})} - \frac{(\phi^{(A-1)} | H_N^{(A-1)} | \phi^{(A-1)})}{(\phi^{(A-1)}, \phi^{(A-1)})} \; .$$

We use standard-type correlated trial wave functions based on procedures developed by the Urbana group[5,7,13],

$$\psi^{(A)} = \prod_{i=1}^{A-1} f_{\Lambda N}(r_{i\Lambda}) \prod_{i<j}^{A-1} f_{NN}(r_{ij}) \prod_{i<j}^{A-1} f_{\Lambda NN}(ij\Lambda)$$

$$\phi^{(A-1)} = \prod_{i<j}^{A-1} f_{NN}(r_{ij}) \ .$$

The two-body correlation functions f_{NN}, $f_{\Lambda N}$ (taken to be spin-independent) are obtained from Schrödinger equations which contain effective potentials through which the variational parameters enter; f_{NN} is allowed to be different for $\phi^{(A-1)}$ and $\psi^{(A)}$. A new feature in $\psi^{(A)}$ is the three-body ΛNN correlations. For these we use

$$f_{\Lambda NN} = f^{D}_{\Lambda NN}\ f^{2\pi}_{\Lambda NN}$$

$$f^{D}_{\Lambda NN} = 1 - \alpha\, Y(r_{1\Lambda})\ Y(r_{2\Lambda}),$$

$$f^{2\pi}_{\Lambda NN} = 1 - \beta(3\cos^2\theta - 1)\ Y(r_{1\Lambda})\ Y(r_{2\Lambda}) \ ,$$

where $f^{D}_{\Lambda NN}$ is appropriate for $V^{D}_{\Lambda NN}$ and $f^{2\pi}_{\Lambda NN}$ for $V^{2\pi}_{\Lambda NN}$. $Y(r)$ are Yukawa functions with cutoff; this, the range of Y, and the correlation strengths α and β are variational parameters. The introduction of $f^{2\pi}_{\Lambda NN}$ turns out to be quite essential and qualitatively changes the contribution of $V^{2\pi}_{\Lambda NN}$.

The integrations necessary for B_Λ are made by standard Monte Carlo (MC) procedures with typical statistical errors of $\approx$ 0.02 MeV for ^{3}H and $\approx$ .05-.15 MeV for A=4,4*,5.

The Λ separation energies B_Λ for A<5 are given below. The values for A=4,4* are averages for $^4_\Lambda$H, $^4_\Lambda$He.

A	$3(^3_\Lambda H)$	$4(^4_\Lambda H,\ ^4_\Lambda He)$	$4^*(^4_\Lambda H^*,\ ^4_\Lambda He^*)$	$5(^5_\Lambda He)$
J^P	$\frac{1}{2}^+$	0^+	1^+	$\frac{1}{2}^+$
B_Λ(MeV)	0.13±.05	2.22±.04	1.12±.06	3.12±.02
V_A	$\bar{V} + \frac{1}{2} V_\sigma$	$\bar{V} + \frac{1}{4} V_\sigma$	$\bar{V} - \frac{1}{12} V_\sigma$	$\bar{V}$
W_A	$\frac{1}{3}W$	$\frac{2}{3}W$	$\frac{10}{9}W$	W

V_A is the effective ΛN attractive strength (i.e. $V_{\Lambda N} = V_C - V_A\, T^2_\pi$) after the spin expectation values have been taken. (This assumes spin dependent correlations in $\Psi^{(A)}$ are negligible.) Similarly, W_A is the effective strength of $V^{DS}_{\Lambda NN}$ (= $W_A\, T^2_\pi\, T^2_\pi$) after the spin expectation values have been taken. Thus, the results for the spin dependent forces $V^{DS}_{\Lambda NN}$ are reduced to those for the spin independent forces $V^{D}_{\Lambda NN}$ but with the modified strengths W_A. Note especially the factors for A=4,4* which are in the direction expected for suppression of $V_{\Lambda N}$ only in the triplet state which is more important for A=4* than for A=4. This difference between W_4 and W^*_4, coming from the spin dependence of $V^{DS}_{\Lambda NN}$, can contribute significantly to the $0^+ - 1^+$ spin flip splitting. For the spin independent forces $V^{D}_{\Lambda NN}$ this splitting is mostly due to the ΛN spin dependence V_σ apart from minor wave function effects.

The results of our variational calculations show very similar trends with V_A, C_p, W for all the s-shell hypernuclei. (B_Λ increases with V_A, decreases with W and may either slightly decease or significantly increase with C_p.) The dependence on these strengths becomes progressively larger from A=3 to 5, resulting from the progressively larger B_Λ, and hence less extended Λ wave function, as well as from the decrease in radii of the core nuclei.

Particularly significant is the importance of three-body correlations $f^{2\pi}_{\Lambda NN}$ on the effect of $V^{2\pi}_{\Lambda NN}$. These correlations reduce the moderately repulsive contribution of $V^{2\pi}_{\Lambda NN}$ for $f^{2\pi}_{\Lambda NN} = 1$ to a small repulsive or even an attractive contribution and give a strong nonlinear dependence on the strength C_p. For $V^{D}_{\Lambda NN}$ the three-body correlations $f^{D}_{\Lambda NN}$ have only a small effect; however reoptimization of the two-body correlations $f_{\Lambda N}$ is quite important, more so than for $V^{2\pi}_{\Lambda NN}$.

2.4 Discussion of the variational results

With only ΛN forces ($C_p = W = 0$) a (barely) consistent description is obtained for A=3,4,4* and $\bar{V}_{scatt}$ (namely $\bar{V} \simeq 6.10$ MeV, $V_\sigma \simeq 0.34$ MeV). This result agrees with previous analyses in particular that of ref. 10.

${}^5_\Lambda$He with only $V_{\Lambda N}$ gives $\bar{V} = 6.015$ MeV; conversely using $\bar{V}_{scatt}$ gives $B_\Lambda = 6.1\pm1$ MeV, i.e. almost twice the experimental value. This is the well-known large overbinding of ${}^5_\Lambda$He (e.g. refs. 1, 10). It is important to observe that this result depends only on $\bar{V}_{scatt}$ which is fairly well determined by Λp scattering (together with CSB corrections). These results clearly show the need for strongly repulsive ΛNN forces in our approach.

With both ΛNN + ΛNN forces we first consider ${}^5_\Lambda$He. $B_\Lambda({}^5_\Lambda\text{He})$ depends only on $\bar{V}$, C_p, W and not on V_σ. The experimental B_Λ then determines a relation between these: $\bar{V} = \bar{V}(C_p, W)$. For a given value of $\bar{V}$ this in turn determines a relation between C_p and W. Thus, B_Λ together with the limits on $\bar{V}$ from scattering impose restrictions on the values of C_p, W - depicted in fig. 4 by the region between the full lines, the upper line corresponding to $\bar{V} = 6.2$ MeV and the lower one to $\bar{V} = 6.1$ MeV, appropriate to the scattering limits. We conclude the following from these results:

1. TPE ΛNN forces $V^{2\pi}_{\Lambda NN}$ alone cannot account for the overbinding of ${}^{5}_{\Lambda}$He. Thus, for no reasonable values of C_p (for W=0) is it possible to obtain agreement with $B_{\Lambda}({}^{5}_{\Lambda}He)$. This conclusion is consistent with that of ref. 10, but is much stronger because of the inclusion of three-body correlations, since these result in a net contribution from $V^{2\pi}_{\Lambda NN}$ which is at best only slightly repulsive and which becomes increasingly attractive for larger C_p. (Furthermore, results obtained for ordinary nuclei[7] strongly suggest that the contribution from the neglected parts of $V^{2\pi}_{\Lambda NN}$ which explicitly involve tensor operators (and which require the inclusion of a ΛN tensor force and associated tensor correlation) will also be attractive, thus making the overbinding situation for ${}^{5}_{\Lambda}$He even worse).

2. ${}^{5}_{\Lambda}$He requires strongly repulsive ΛNN dispersive forces whose strength does not depend strongly on C_p.

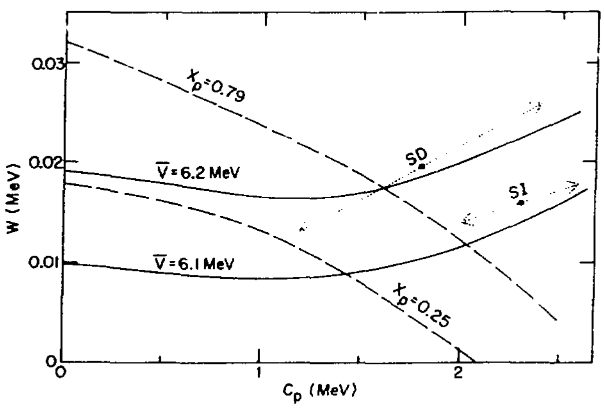

Figure 4
Acceptable values of C_p, W for a cutoff c = 3 fm^{-2} for $V^{2\pi}_{\Lambda NN}$. The full lines are for the limits on $\bar{V}$ from scattering and the dashed lines for those on x_p. The dotted lines represent the s-shell acceptable solutions, with the best solutions shown circled.

The requirement that all the s-shell B_Λ be adequately fitted (i.e. within the errors due to those in B_Λ and in the MC calculations) strongly restricts the allowable interactions. We denote the solutions by SI(c) and SD(c) where SI and SD refer to the spin independent and spin dependent ΛNN forces $V^{D}_{\Lambda NN}$ and $V_{\Lambda NN}$, respectively, and where c = 2 or 3 fm^{-2} is the cutoff for $V^{2\pi}_{\Lambda NN}$. There are then four solutions: SI(2), SI(3) and SD(2), SD(3). The s-shell acceptable solutions are shown in the table and also depicted (for c=3 fm^{-2}) in fig. 4 by the diagonal dotted lines and where the "best" solution, corresponding to the central values of B_Λ, is shown circled. If the restrictions on $\bar{V}$ from scattering are now imposed on these solutions one obtains those solutions which are jointly allowed by both the s-shell B_Λ and by scattering. These s-shell + scattering acceptable solutions are shown in the table; in fig. 4 they correspond to those segments of the dotted lines which are between the full lines corresponding to $\bar{V}$ = 6.1 and 6.2 MeV.

TABLE. Acceptable Interactions (MeV)

	s-shell acceptable solutions						s-shell + scattering acceptable solutions			
	C_p	W	$\bar{V}$	V_σ	D	x_p	C_p	W	$\bar{V}$	x_p
SI(2)	3.5±.9	.02±.01	6.33±.25	.35±.01	25∓10	1.23±.45	2.6-2.7	.0115-.012	6.19-6.20	.77-.8
SI(3)	2.[illegible]±.3	.016±.02	6.14±.01	.30∓.015	29∓5	1.05±.22	2.0-2.6	.014-.018	6.13-6.15	.83-1.3
SD(2)	2±.7	.010±.004	6.16±.06	.23±.003	41∓8.5	.40±.43	1.3-2.4	.0065-.012	6.11-6.20	.25-.7
SD(3)	1.8±.6	.0195±.006	6.20±.05	.185∓.02	31∓10	.89±.50	1.2-1.6	.0135-.0175	6.16-6.20	.40-.75

We make the following observations:

1. All the s-shell acceptable solutions are also consistent with scattering. However, the solution SI(2) is barely acceptable (the minimum value of $\bar{V}$ is close to 6.2 MeV), whereas the others are well consistent with scattering. Representative Λp scattering lengths (in fm) for the s-shell + scattering acceptable solutions are for SI(3): $\bar{a} = -1.93$, $a_s = -3.02$, $a_t = 1.56$; for SD(2): $\bar{a} = -1.89$, $a_s = -2.66$, $a_t = -1.63$; for SD(3): $\bar{a} = -2.05$, $a_s = -2.75$, $a_t = -1.81$. To be noted is that even for the SD solutions, for which V_σ is ≈ 1/3 less than for the SI solutions, there is still quite a sizable difference between a_s and a_t.

A major uncertainty in the s-shell acceptable solutions comes from the experimental error in $B_\Lambda(^3_\Lambda H)$ and a somewhat lesser but still appreciable one from the B_Λ for A=4,4*. Thus improved accuracy for these B_Λ would be quite significant. Clearly, a better determination of just the low energy total elastic cross section $\bar{\sigma}^{\Lambda p}$ would be very helpful in determining $\bar{V}_{scatt}$ more precisely.

2. The outstanding difference between the spin dependent ΛNN dispersive force $V^{DS}_{\Lambda NN}$ and the spin independent force $V^{D}_{\Lambda NN}$ is that the ΛN spin dependence V_σ obtained for $V^{DS}_{\Lambda NN}$ is reduced by ≈ 1/3 and that consequently also ≈ 1/3 of the 0^+ to 1^+ splitting of 1.1 MeV for A=4,4* comes from the spin dependence of $V^{DS}_{\Lambda NN}$. To resolve this important issue of the spin dependence of the ΛNN force, a determination of the spin dependence of the low energy Λp scattering would be required. Also, the improvements mentioned above would again be very desirable. Direct knowledge of V_σ from scattering together with the spin flip excitation in A=4 would then shed important light on the suppression mechanism associated with ΛN-ΣN coupling.

The spin dependence obtained from the s-shell may be reconciled with the quite small spin dependence obtained from the p-shell hypernuclei because of the different combination of Talmi integrals which enter[14].

3. Particularly significant for the "reasonableness" of our interactions is

that the allowable strengths $C_p \simeq 1.2\text{-}2.7$ MeV of the TPE ΛNN potential are very well consistent with theoretical expectations; this is particularly so for those obtained with $V_{\Lambda NN}^{DS}$, i.e. the SD solutions, for which $C_p \approx 1.2\text{-}2.4$ MeV. Further support for such reasonableness is that our values of W are comparable with those ($\simeq 0.03$ MeV) obtained for ordinary nuclei[7]), after allowing for a factor of 3 from symmetrization of $V_{\Lambda NN}^{D}$.

3. THE Λ WELL DEPTH

The empirical value is $D \simeq 30 \pm 3$ MeV. The Hamiltonian we use is effectively that described previously for the s-shell hypernuclei as is the variational wave function, except that Φ_N is now multiplied by an uncorrelated Fermi gas wave function (with $\rho = 0.177\ \text{fm}^{-3}$) and that the usual nuclear matter limit is taken. The expression for D is then calculated with the Fermi hypernetted-chain (FHNC) method. This is described in ref. 15. for two-body forces, and has been extended to include ΛNN forces and ΛNN correlations. The so-called elementary diagrams are neglected, which is justified because nuclear matter is dilute. The rearrangement energy was shown in ref. 15 to be negligible. (This also has the consequence that the details of V_{NN} are unimportant and in fact the V_{NN} of ref. 15 was used.)

In contrast to the s-shell hypernuclei, the three-body correlations for both $V_{\Lambda NN}^{D}$ and $V_{\Lambda NN}^{2\pi}$ are found to have only a small effect ($\lesssim 2\%$ of D), presumably because the wave function in nuclear matter is much more constrained by the boundary conditions than that of the loosely bound s-shell hypernuclei. As a consequence, the contribution of $V_{\Lambda NN}^{2\pi}$ is now moderately repulsive and approximately linear with C_p ($\simeq 7$ MeV for $C_p = 1$ MeV).

Figure 5 shows the results for D for those values sets of of $\bar{V}$, C_p, W which reproduce $B_\Lambda({}^5_\Lambda\text{He})$, i.e. which satisfy the relation $\bar{V} = \bar{V}(C_p, W)$ determined by ${}^5_\Lambda$He. If the relative effectiveness of $V_{\Lambda N}$ and $V_{\Lambda NN}$ were the same for ${}^5_\Lambda$He and D then all the values of D so obtained would be identical. In fact, D decreases with $\bar{V}$, C_p, W. This shows that the ΛNN forces are relatively more effective for D than for ${}^5_\Lambda$He. ($\langle V_{\Lambda N}\rangle$ involves a factor ρ, whereas $\langle V_{\Lambda NN}\rangle$ involves a factor of ρ^2 in addition to the implicit dependence on ρ arising from the correlations.)

A feature of the interaction which becomes important for D (and generally for A>5) is the ΛN interaction for $\ell>0$, in particular the p-state potential. For our $V_{\Lambda N} = V_{2\pi}$, this contributes $\langle V_{\Lambda N}\rangle_{\ell=1} \approx -20$ MEV almost independently of the ΛNN forces (to within $\simeq 0.5$ MeV). The d-state contribution is $\langle V_{\Lambda N}\rangle_{\ell=2} \approx -2$ MeV. We denote by $x_p = V_{\Lambda N}^{(\ell=1)}/V_{\Lambda N}^{(\ell=0)}$ the relative strength of the ΛN p- and s-state potentials. Λp scattering at moderate energies gives $x_p \simeq 0.5$ with large uncertainties; we use the limits $0.25 < x_p < 0.75$ as consistent with scattering. With the empirical value D = 30 MeV and for $\langle V_{\Lambda N}\rangle_{\ell=1} = -20$ MeV, these limits then impose restrictions on C_p, W corresponding to the region between the dashed lines in fig. 4. It is quite significant that these limits on x_p (and even the more generous ones $0 < x_p < 1$) imply strongly repulsive ΛNN forces quite independently of the values of $\bar{V}$ determined by scattering. This seems strong confirmation of the need for repulsive ΛNN forces. The restrictions on the interaction strengths imposed jointly by $B_\Lambda(^5_\Lambda He)$ and D (for $0.25 < x_p < 0.75$) and by $\bar{V}$ from scattering ($6.1 \lesssim \bar{V} \lesssim 6.2$ MeV) result in the region common to both sets of allowed regions in fig. 4. This is seen to imply a quite restrictive set of parameters. (Thus, e.g. for $C_p = 0$ the acceptable values of W are limited to 0.018-0.019 MeV). In particular, upper limits are placed on the values of C_p: $\lesssim 2.7$ and 2 MeV for c = 2 and 3 fm^{-2}, respectively.

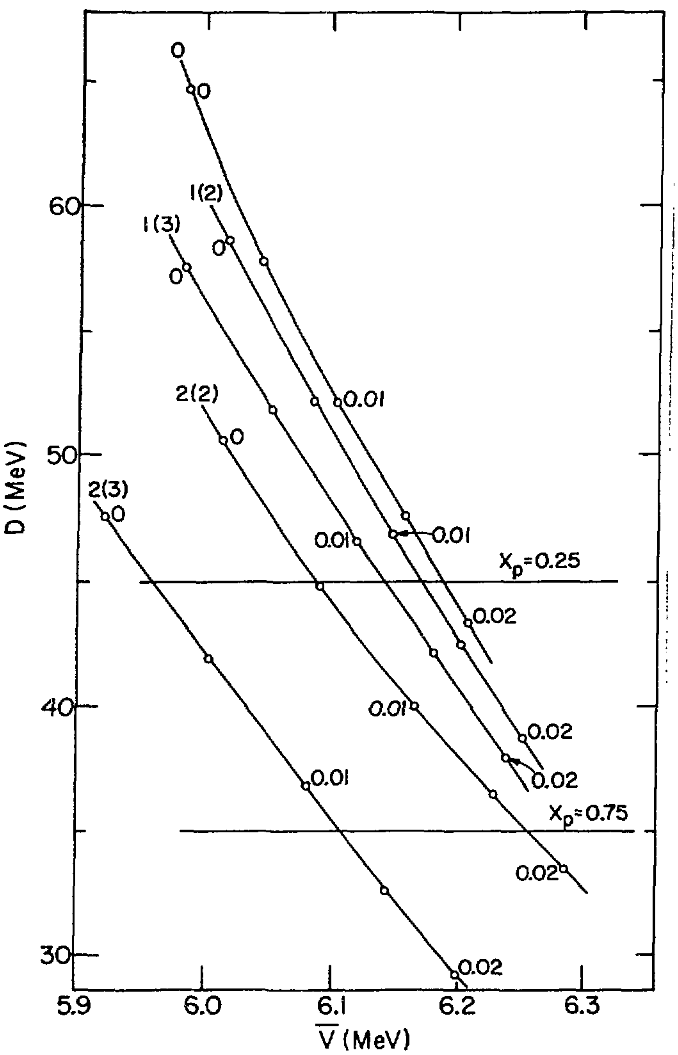

Figure 5
The well depth D for values of C_p, W, $\bar{V}$ which reproduce $B_\Lambda(^5_\Lambda He) = 3.12$ MeV. The curves for D vs. $\bar{V}$ are for different C_p(c) with the appropriate values of W indicated. The horizontal lines are for the limits $x_p = 0.25$, 0.75.

Finally, we consider the predictions for D of the solutions obtained in section 2 together with the implications of the empirical value of D. The values of D for the s-shell acceptable solutions are shown in the table. We also give, equivalently, the values of x_p which give the empirical value D = 30 MeV using $\langle V_{\Lambda N}\rangle_{\ell=1}$ = -20 MeV. These values of x_p are within the limits 0.25-0.75 for the SD solutions, i.e. those with $V^{DS}_{\Lambda NN}$. However, for the SI solutions, i.e. those for the spin independent force $V^{D}_{\Lambda NN}$, the values of x_p are outside the limits. Thus the SI solutions are excluded by these limits, although only barely by the upper limit. Thus, the well depth + limits on x_p from scattering favor $V^{DS}_{\Lambda NN}$. These solutions are also the ones which correspond to the theoretically most reasonable values of C_p (≃ 1.2-2.4 MeV).

It is satisfactory and is of course a necessary condition for the adequacy of our model that it gives a satisfactory description for scattering, the s-shell hypernuclei, and for D. Clearly, better scattering data at moderate energies relevant to the p-state interaction would be very valuable. As illustrated by fig. 4 such data could provide significant further restrictions, in particular on the mix of dispersive and TPE ΛNN forces, especially if better values of $\bar{V}$ were also available from scattering.

Some further support for the reasonableness of our interactions is provided by coupled channel (ΛN-ΣN) G-matrix calculations of D which give a suppression[8] of ≈ 20 MeV. This is consistent with our FHNC result for $\langle V^{D}_{\Lambda NN}\rangle$ of ≈ 25 MeV. However, there is some doubt about the adequacy of the lowest order G-matrix calculations, which could substantially overestimate the suppression.

4. INTERMEDIATE MASS HYPERNUCLEI: ${}^{9}_{\Lambda}$Be

Calculations have been made for ${}^{9}_{\Lambda}$Be (B_Λ = 6.71 ± .04 MeV) with a 2α + Λ model, implemented by variational calculations of the same type as described for the s-shell hypernuclei[2]. So far only dispersive ΛNN forces have been considered. The αα potential used[16] gives a good fit to αα scattering. The αΛ potential $V_{\alpha\Lambda} = V^{(2)}_{\alpha\Lambda} + V^{(3)}_{\alpha\Lambda}$ has contributions both from the ΛN and ΛNN potentials and is obtained by a folding procedure which uses effective ΛN and ΛNN potentials. This procedure makes use of the nuclear matter ΛN correlations, and is consistent with the MC calculations of ${}^{5}_{\Lambda}$He. The three-body potential $V^{D}_{\Lambda NN}$ gives not only a contribution $V^{(3)}_{\alpha\Lambda}$ to $V_{\alpha\Lambda}$ but also gives rise to an effective three-body ααΛ potential $V_{\alpha\alpha\Lambda}$ due to the interaction of the Λ (via $V^{D}_{\Lambda NN}$) with pairs of nucleons each in a different α. $V_{\alpha\alpha\Lambda}$, which is also obtained by folding, is proportional to W and is completely determined for a given $V_{\alpha\Lambda}$. For potentials $V_{\Lambda N} + V^{D}_{\Lambda NN}$, which via the corresponding $V_{\alpha\Lambda}$ fit ${}^{5}_{\Lambda}$He and are also consistent with $\bar{V}_{scatt}$, one obtains B_Λ ≈ 7.8 MeV if $V_{\alpha\alpha\Lambda}$ is neglected. $V_{\alpha\alpha\Lambda}$ (which is repulsive because $V^{D}_{\Lambda NN}$ is) contributes ≈ 0.9 MeV and results in B_Λ ≈ 6.9 MeV. An improved calculation of the ΛN p-state contribution gives, with $x_p = 0.5$, a reduction of 0.3 MeV (close to the 0.4 MeV estimated in ref. 2) thus giving $B_\Lambda \approx 6.6$ MeV, in excellent agreement with the experimental value. It is tempting to consider this situation as a strong indication of the presence of repulsive $\alpha\alpha\Lambda$ and hence of repulsive ΛNN forces, since a repulsive contribution of ≈ 1 MeV is needed to avoid ${}^9_\Lambda$Be being overbound by this amount when only $\alpha\Lambda$ potentials which fit ${}^5_\Lambda$He are used. It seems unlikely that the inclusion of $V^{2\pi}_{\Lambda NN}$ will significantly change these results for ${}^9_\Lambda$Be which would then be consistent with those for the s-shell hypernuclei as well as for D.

Calculations for the excited states based on the 2^+ state of ^{8}Be are in progress. Preliminary estimates give ≈ 3 MeV for the excitation energy in agreement with experiment.

Our version of α-cluster calculations of ${}^9_\Lambda$Be, and also of ${}^6_{\Lambda\Lambda}$He and ${}^{10}_{\Lambda\Lambda}$Be discussed below, uses realistic potentials which accurately reproduce the pertinent data, together with accurate two-body correlations. Such an approach is particularly suitable for accurate calculations of the ground and certain low-lying states. Other versions, in particular that of Bando and collaborators[17], are more suitable for an overall description of spectra, reduced widths and transition probabilities, especially of those states which involve excitations of the α-clusters and which cannot be described in our approach.

In summary, we have found that Λp scattering, the s-shell binding energies, the well depth, and probably also ${}^9_\Lambda$Be representative of intermediate mass hypernuclei, can all be fitted with ΛN plus ΛNN forces consistent with meson-exchange models. Both TPE ΛNN forces consistent with theoretical expectations and strongly repulsive dispersive ΛNN forces are required. The spin dependent form of these is strongly preferred by the well depth results when use is made of the limits on the p-state ΛN potential obtained from scattering. The ΛNN spin dependence reduces the spin dependence of the ΛN force by $\approx 1/3$, and correspondingly contributes $\approx 1/3$ to the 0^+-1^+ splitting of A=4. Clearly, better Λp scattering data (as well as Λn data) are most desirable; specifically in the context of our model in order to determine the ΛN spin and p-wave dependence, as well as a more precise value of the spin average s-state interaction. More accurate binding energies for A=3,4,4* would also be very valuable in restricting the interactions.

So far we have considered mostly only central forces (ΛN and NN as well as ΛNN) on which our analysis is based. Tensor ΛN forces, e.g. due to kaon exchange, give only a small reduction ($\lesssim 4$ MeV) in D and (≈ 0.5 MeV) in $B_\Lambda({}^5_\Lambda\mathrm{He})$[18], although especially for A=5 better estimates are required and calculations are needed for A=3, 4, 4*. However, it should be noted that the relatively small suppression effects expected from ΛN tensor forces are already implicitly included in our dispersive ΛNN forces. NN tensor forces were included in some of our calculations of ${}^{3}_{\Lambda}H$ and have a small effect ($\lesssim$ .02 MeV for B_{Λ}). Preliminary calculations for A=4 indicate a contribution from NN tensor forces of $\approx 0.2 \pm .02$ to B_{Λ}; for ${}^{5}_{\Lambda}He$ a similar contribution may be expected. They are expected to give only a small contribution to D, occuring only through higher-order terms in the FHNC calculation related to the small rearrangement energy contribution. Perhaps a more significant effect of ΛN and NN tensor forces could be through the effect of the associated tensor correlations on the contribution of the neglected tensor components of $V^{2\pi}_{\Lambda NN}$. This needs study, and as already mentioned is expected to make an attractive contribution, but probably appreciably less so than for the nuclear case[7] in view of the relatively weak ΛN tensor force.

5. THE $\Lambda\Lambda$ HYPERNUCLEI ${}^{6}_{\Lambda\Lambda}He$ AND ${}^{10}_{\Lambda\Lambda}Be$ AND THE $\Lambda\Lambda$ INTERACTION

Our calculations for ${}^{6}_{\Lambda\Lambda}He$ and ${}^{10}_{\Lambda\Lambda}Be$ are described in ref. 4, and will be only briefly discussed. We use an $\alpha + 2\Lambda$ model for ${}^{6}_{\Lambda\Lambda}He$ and a $2\alpha + 2\Lambda$ model for ${}^{9}_{\Lambda}Be$. For $V_{\Lambda\Lambda}$ we use a variety of shapes and ranges both for the repulsive core V_C and for the attractive part V_A.

${}^{10}_{\Lambda\Lambda}Be$ ($B_{\Lambda\Lambda} = 17.71 \pm .08$) is the best established and most critically examined $\Lambda\Lambda$ hypernucleus event[19]. Variational four-body calculations for ${}^{10}_{\Lambda\Lambda}Be$ determine one parameter of $V_{\Lambda\Lambda}$ (e.g. the strength V_A). "Reasonable" $V_{\Lambda\Lambda}$ (repulsive core V_C comparable to that for $V_{\Lambda N}$, reasonable ranges for V_A) give $a^{\Lambda\Lambda} \approx -(2.5\text{-}3.5)$ fm, $r_0^{\Lambda\Lambda} \approx 2.6\text{-}3.1$ fm. Thus, the $\Lambda\Lambda$ interaction is strongly attractive, comparable to or even more attractive than the ΛN force, and is not far from giving a bound $\Lambda\Lambda$ state (H dibaryon!?). Meson-exchange models obtained by the Nijmegen group[20] predict $a^{\Lambda\Lambda} \approx -0.26$ fm, i.e. a very weakly attractive $V_{\Lambda\Lambda}$. This discrepancy could be tentative evidence for a 6q state with the quantum numbers of a ${}^{1}S_{0}$ $\Lambda\Lambda$ state and not too far above the $\Lambda\Lambda$ threshold.

${}^{6}_{\Lambda\Lambda}He$ and ${}^{10}_{\Lambda\Lambda}Be$. Calculations of $B_{\Lambda\Lambda}$ for both these with a large number of different $V_{\Lambda\Lambda}$, having different shapes, ranges and strengths, give an approximately linear relation between the calculated values of $B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}He)$ and $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}Be)$. For the experimental value of $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}Be)$ this relation predicts significantly too small values of $B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}He) \approx 9.7$ MeV as compared with the quoted experimental value[21] of $10.9 \pm .6$ MeV.

6. COULOMB EFFECTS AND CHARGE SYMMETRY BREAKING (CSB) FOR THE A=4 HYPERNUCLEI

This work is described in detail in ref. 5.

6.1 Coulomb corrections

The Coulomb repulsion between the protons in ${}^4_\Lambda$He contributes $\Delta B_c < 0$ to $\Delta B_\Lambda = B_\Lambda({}^4_\Lambda\text{He}) - B_\Lambda({}^4_\Lambda\text{H})$. To 1st order in the coulomb interaction V_c: $\Delta B_c = -\Delta E_c = -[E_c({}^4_\Lambda\text{He}) - E_c({}^3\text{He})]$, where ΔE_c is the increase in the Coulomb energy of the ^{3}He core due to its compression by the Λ. ΔB_c has been obtained from MC variational calculations of ${}^4_\Lambda$He and ^{3}He for several values of q^2 in the range $0 < q^2 < 9$, where $V_c = q^2e^2/r$; i.e. the Coulomb repulsion was artificially boosted. The charge symmetric ΛN potential used was $V_{2\pi}$. For $q^2 \lesssim 3$, the dependence on q^2 is linear and interpolation to $q^2 = 1$ gives the He values with improved accuracy, needed because of the statistical MC errors. We obtain the rather small values: $\Delta B_c = -0.05 \pm .02$ MeV for the ground state, and $\Delta B_c^* = -0.025 \pm .015$ MeV for the excited state; these are also consistent with the calculated values of ΔE_c. Our values are consistent with those of ref. 22, but significantly smaller in magnitude than those of ref. 10. Subtracting our values from the appropriate experimental values of ΔB_Λ then gives the following values to be attributed to CSB effects:

$$\Delta B_\Lambda = 0.40 \pm .06 \text{ MeV}, \quad \Delta B_\Lambda^* = 0.27 \pm .06 \text{ MeV} .$$

6.2 Phenomenological charge symmetry breaking potential

For this we consider a $T_\pi^2(r)$ shape. This is to be used together with our CS potential $V_{2\pi}$. Fitting to the above values of ΔB_Λ, ΔB_Λ^* gives (in MeV)

$$V^{CSB} = -0.054\ \tau_3\ T_\pi^2[(1\pm 0.11) + (0.054 \pm 0.14)\ \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N] .$$

Thus the CSB potential is effectively spin independent. For $a_s = a_t \approx -1.9$ fm this potential gives $\Delta a_s \approx 0.39$ fm, $\Delta a_t \approx 0.36$ fm, where $\Delta a = -(a^{\Lambda p} - a^{\Lambda n})$ and is positive if the Λp interaction is more attractive than the Λn interaction. (We have checked that for our shape of V^{CSB} the connection between ΔB_Λ, ΔB_Λ^* and Δa_s, Δa_t is in good agreement with that obtained in ref. 23.)

6.3 Comparison with meson-exchange models

An instructive model is one for which the charge symmetric potential V_K^{CS} has kaon exchange (with $g^2_{\Lambda NK} = 16$), which gives a ΛN tensor force, and is otherwise adjusted to give a ≈ -2 fm, and where the CSB potential V_π^{CSB} is the OPE potential due to Λ-Σ^0 mixing[1,24]. This model gives $\Delta a_s \approx -0.1$ fm, $\Delta a_t \approx 0.15$ fm, qualitatively similar to the results of the more complete models of Nagels et al.[20] which also include ρ and δ exchange and the effect of the Σ^+, Σ^- mass difference, and which give $\Delta a_s \approx -0.3$ fm, $\Delta a_t \approx 0.1$-0.2 fm (for their models B, D, F). It is important to note that the major contribution to Δa_t in our model comes from the CSB tensor part acting together with the CS tensor part. This contribution is proportional to $V_T^{CSB}\ V_T^{CS}$, i.e. to $V_{K,T}^{CS}\ V_{\pi,T}^{CSB}$ for our simple model, giving the major contribution of 0.12 fm to $\Delta a_t \approx 0.15$ fm. Thus uncertainties in the CS tensor part (e.g. in $g^2_{\Lambda NK}$) will give corresponding uncertainties in Δa_t. Furthermore (probably moderate) corrections can arise from many-body and nuclear structure effects. Since in any case there is no major discrepancy between the meson-exchange and phenomenological values of Δa_t, we conclude that the triplet CSB interaction obtained from the A=4 hypernuclei is consistent with meson-exchange models.

For the singlet value Δa_s there is no uncertainty corresponding to that arising from V_T^{CS} for Δa_t. Furthermore, many-body and nuclear structure effects are expected to be less than for Δa_t. The large differences (even the opposite sign) between the meson-exchange and phenomenological results for Δa_s then strongly suggest that meson-exchange models of the singlet CSB interaction are inconsistent with the A=4 data, indicating that there may be important quark structure contributions.

Complete calculations with ΛN and NN tensor forces would be desirable for the A=4 hypernuclei in order to definitely establish that nuclear structure effects do not change the above conclusions.

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